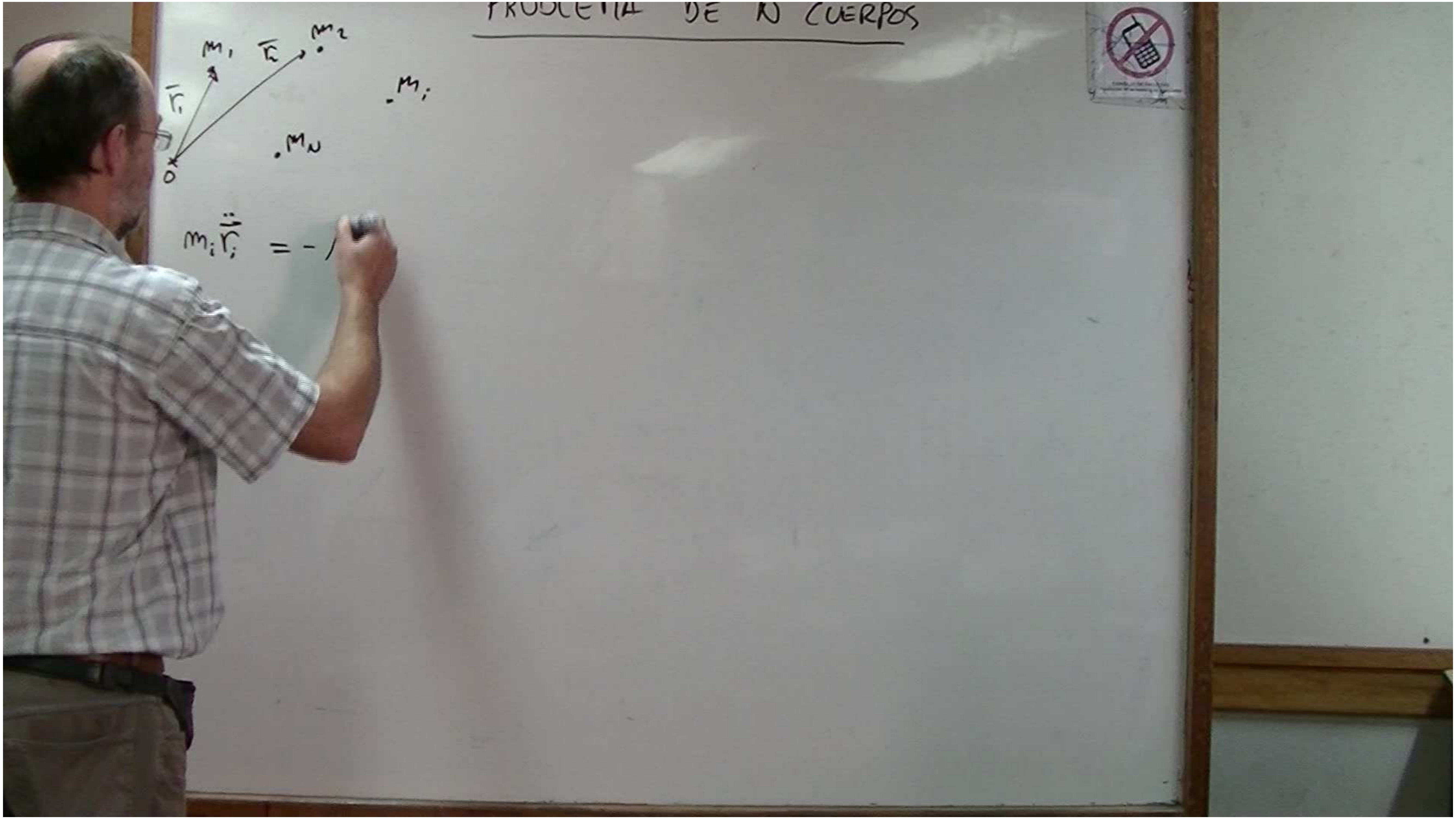
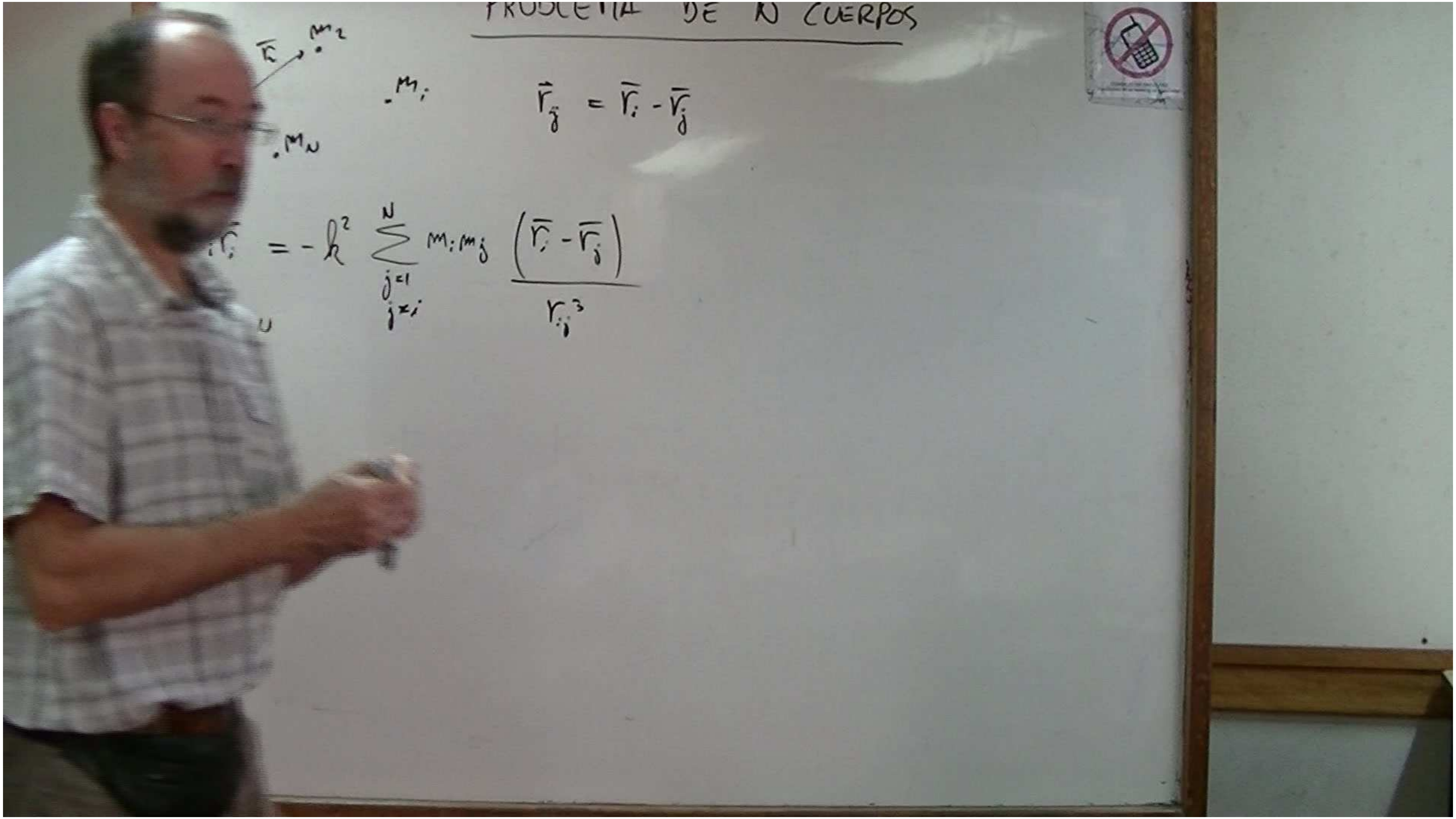
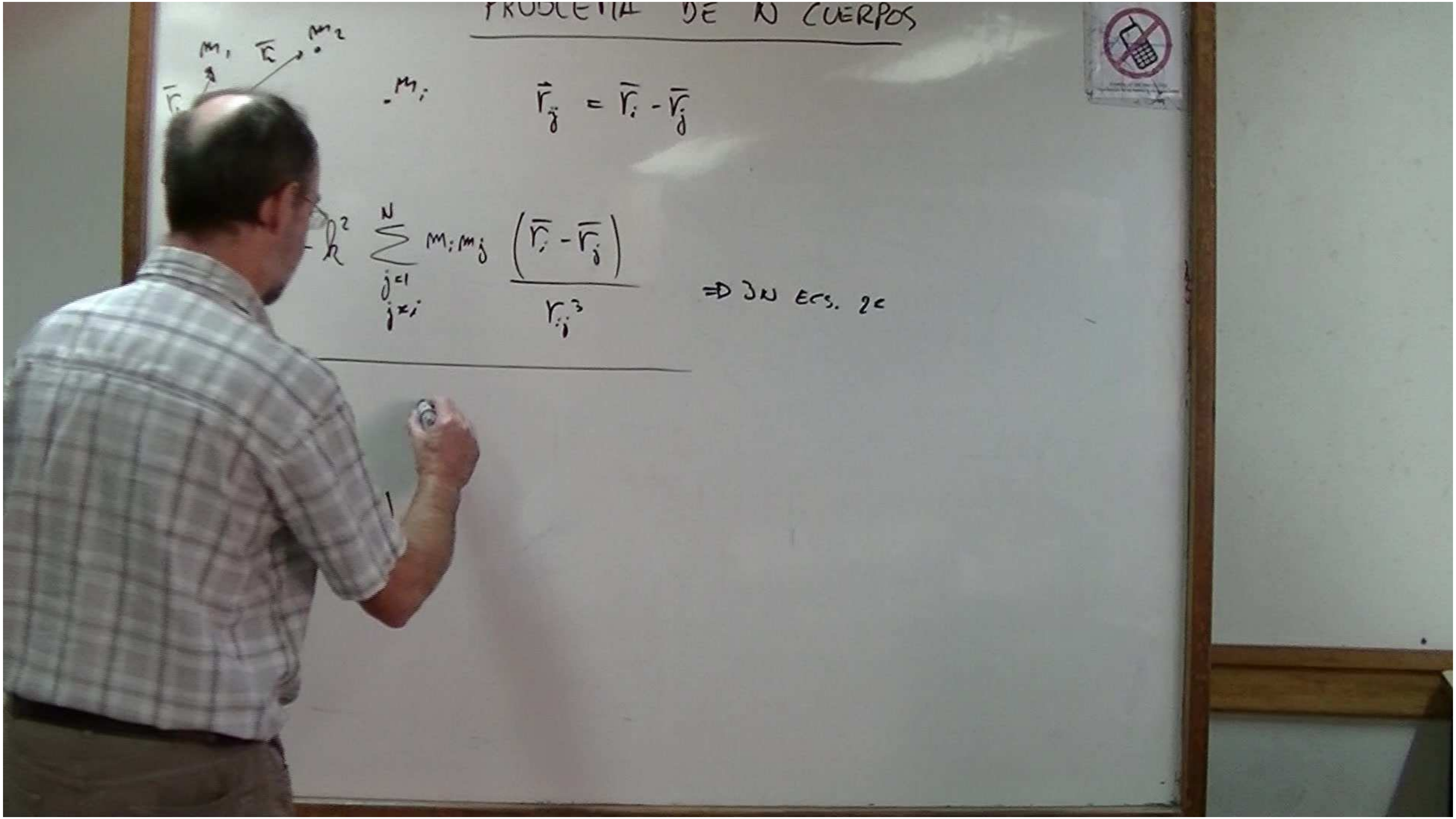
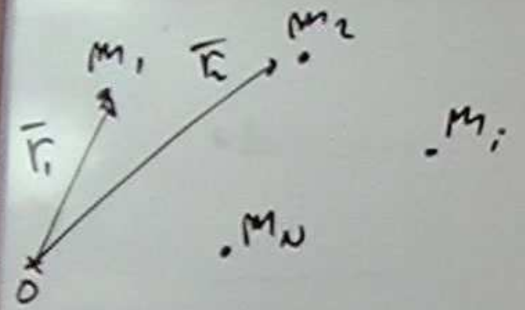

PROBLEMA DE N CUERPOS


DE N CUERPOS
$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$
$= -k^2 \sum_{j=1, j \neq i}^{N} m_i m_j \frac{(\vec{r}_i - \vec{r}_j)}{r_{ij}^3}$


PROBLEMA DE N CUERPOS
⇒ 3N Ecs. 2º

PROBLEMA DE N CUERPOS



$$\vec r_{ij} = \bar r_i - \bar r_j$$

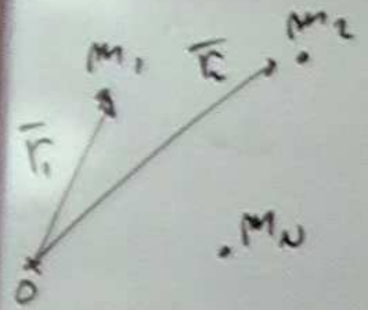
$$m_i \ddot{\vec r}_i = -k^2 \sum_{\substack{j=1 \\ j\neq i}}^{N} m_i m_j \frac{(\bar r_i - \bar r_j)}{r_{ij}^3} \quad \Rightarrow 3N \text{ Ecs. } 2^{\circ}$$

$$\forall_{i=1,N}$$

$$\left.\begin{aligned} \dot{\vec r}_i &= \vec V_i \\ \dot{\vec V}_i &= f_i(\bar r_1, \dots \bar r_N) \end{aligned}\right\} 6N \text{ ec.}$$

$$\dot{\vec X} = F(\bar X)$$

PROBLEMA DE N CUERPOS



$$\vec r_{ij} = \bar r_i - \bar r_j$$

$$m_i \ddot{\bar r}_i = -k^2 \sum_{\substack{j=1 \\ j \neq i}}^{N} m_i m_j \frac{(\bar r_i - \bar r_j)}{r_{ij}^3} \qquad \forall_{i=1,N}$$

$\Rightarrow$ 3N ecs. 2º

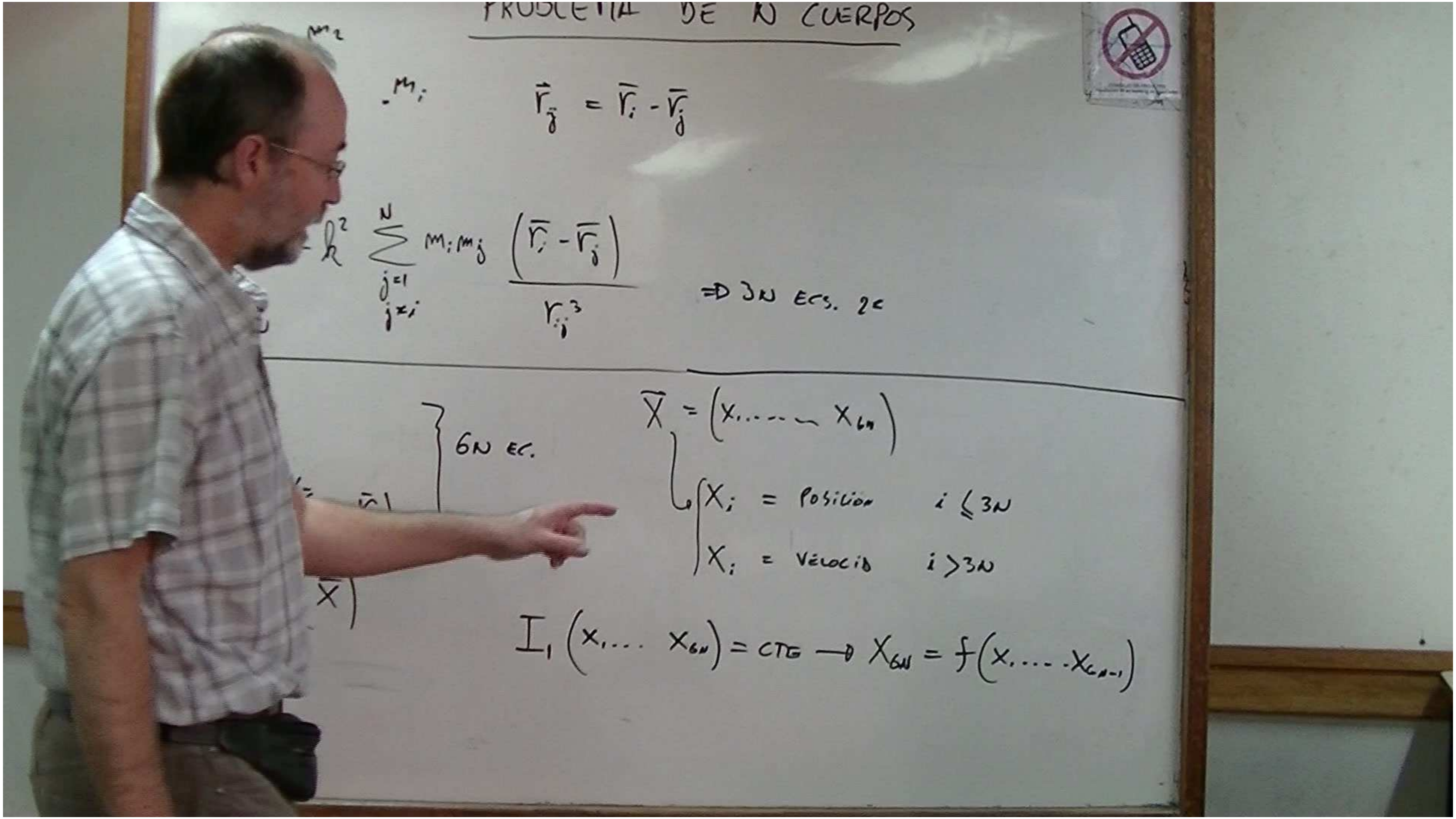
$$\left.\begin{aligned} \dot{\bar r}_i &= \vec V_i \\ \dot{\bar V}_i &= f_i(\bar r_1, \dots \bar r_N) \end{aligned}\right\} \text{6N ec.}$$

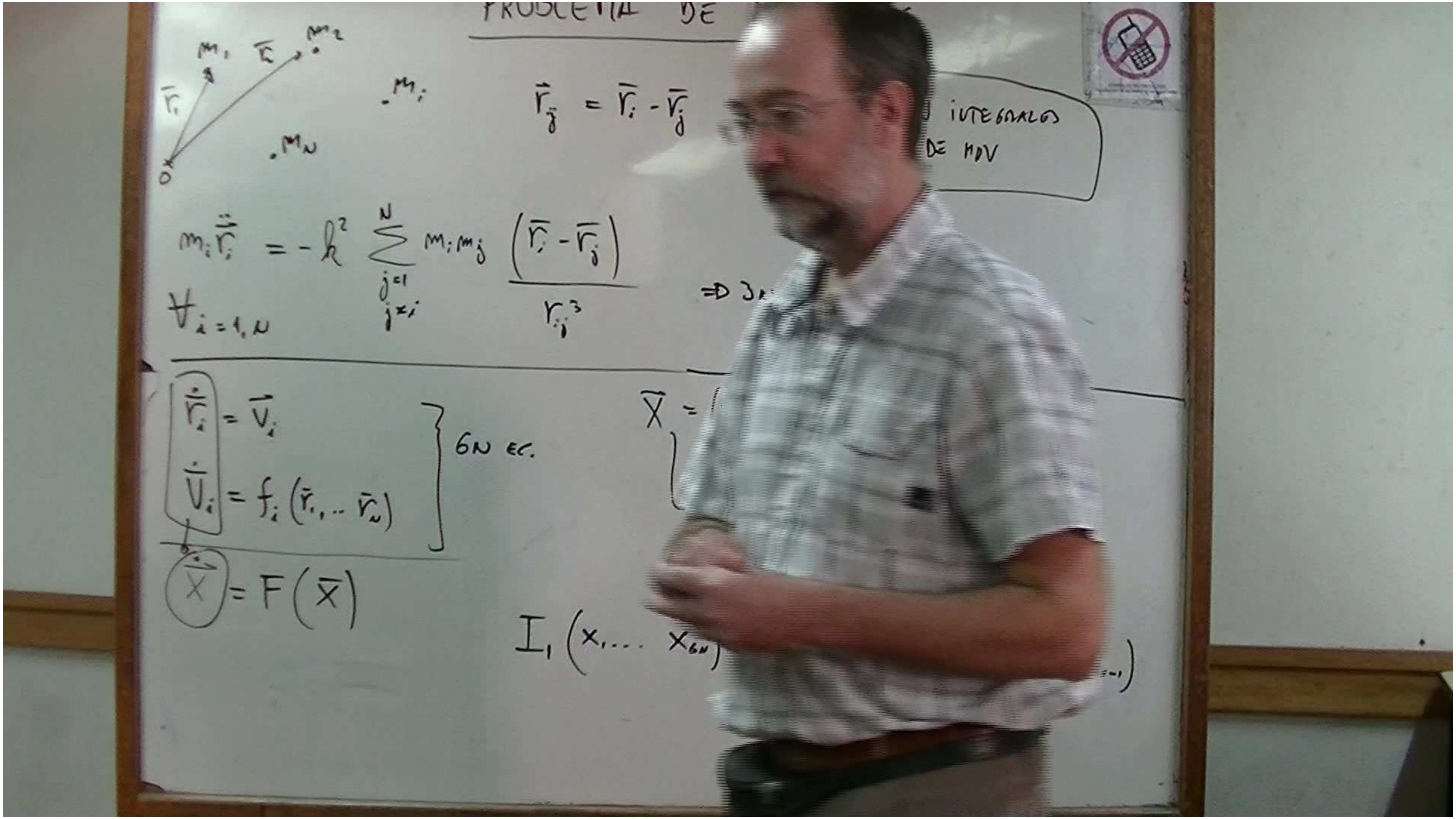
$$\dot{\bar X} = F(\bar X)$$

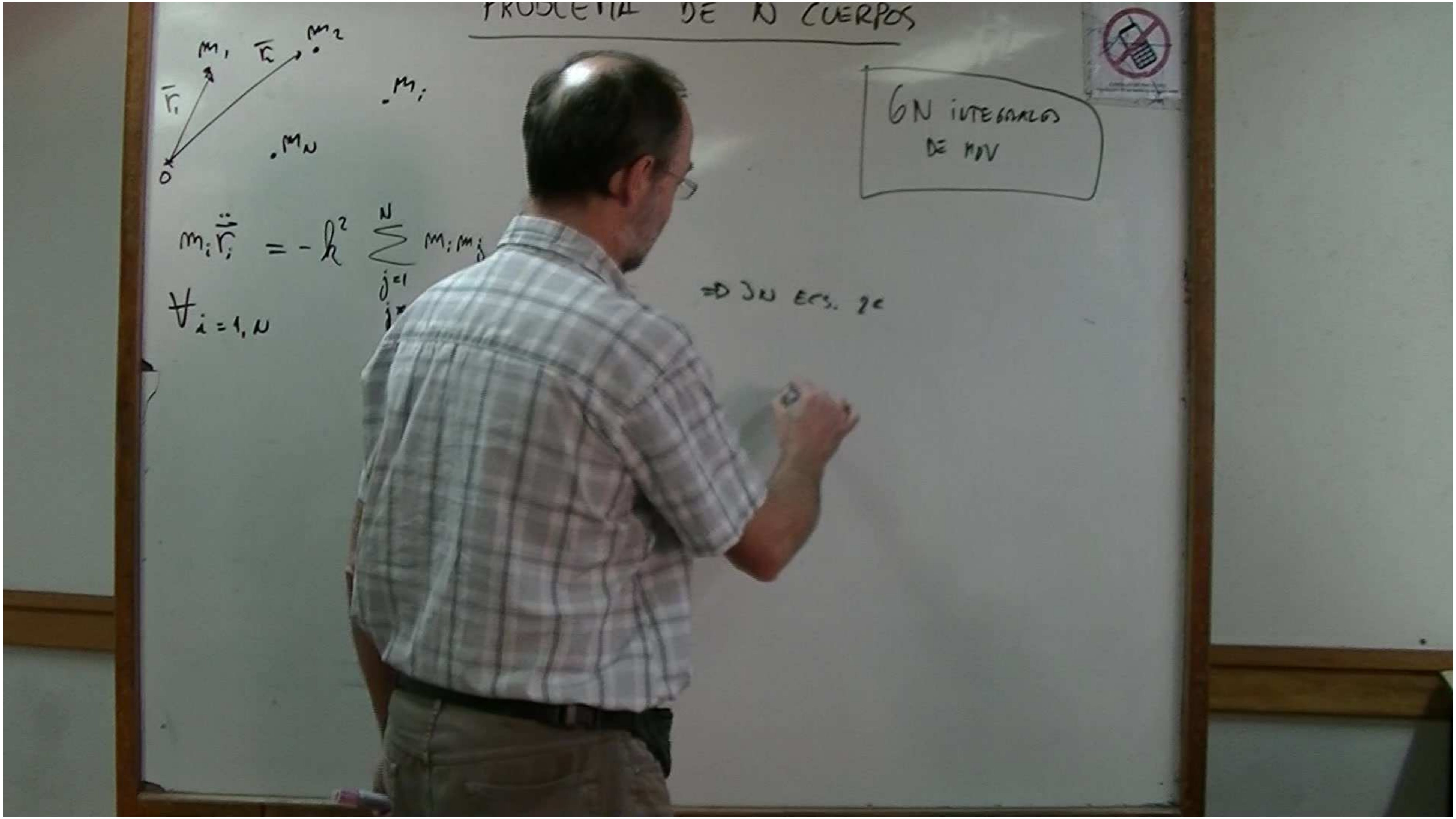
$$\bar X = (X_1 \dots\dots X_{6N})$$

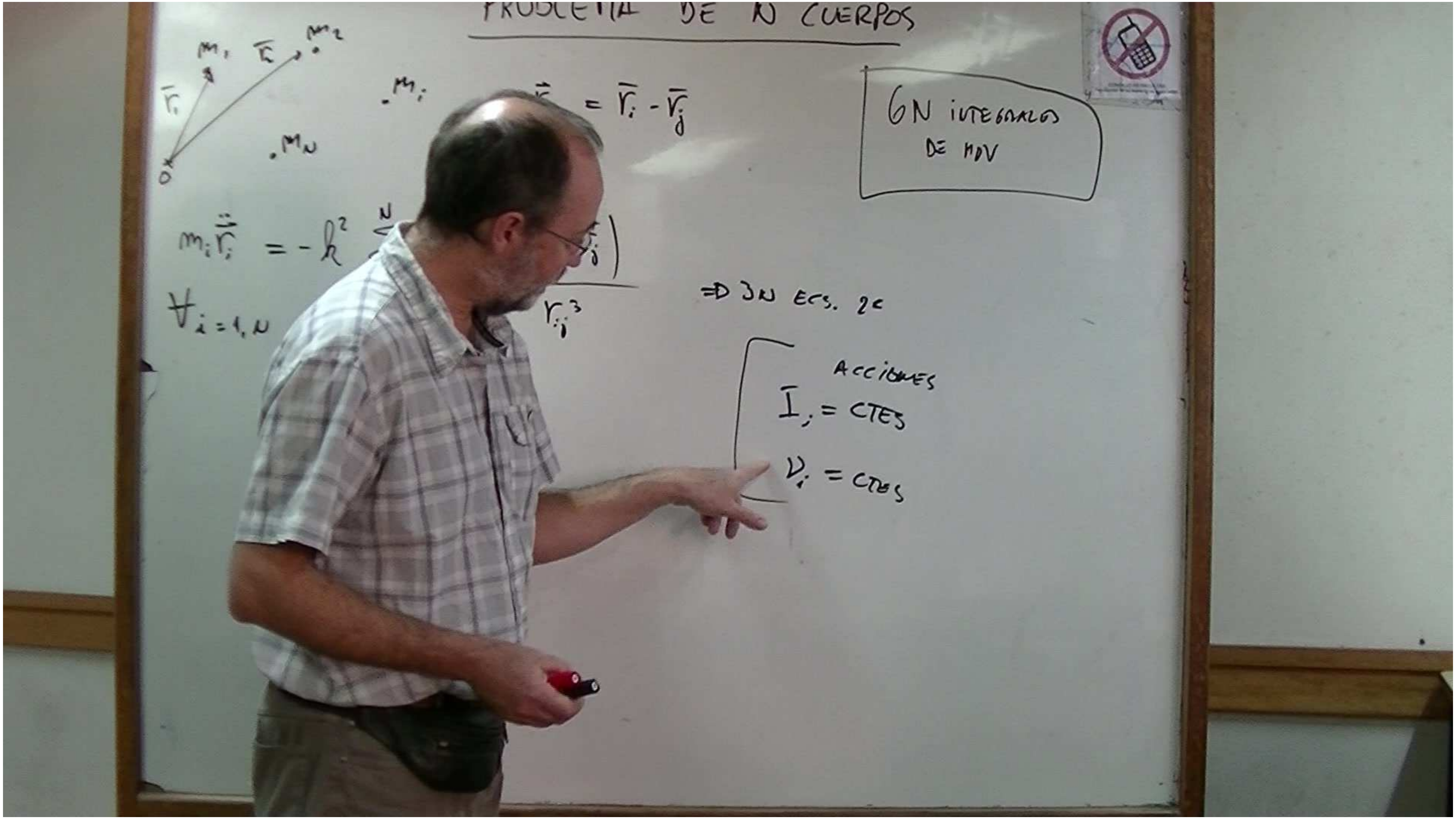
X_i = posición $\quad i \leq 3N$

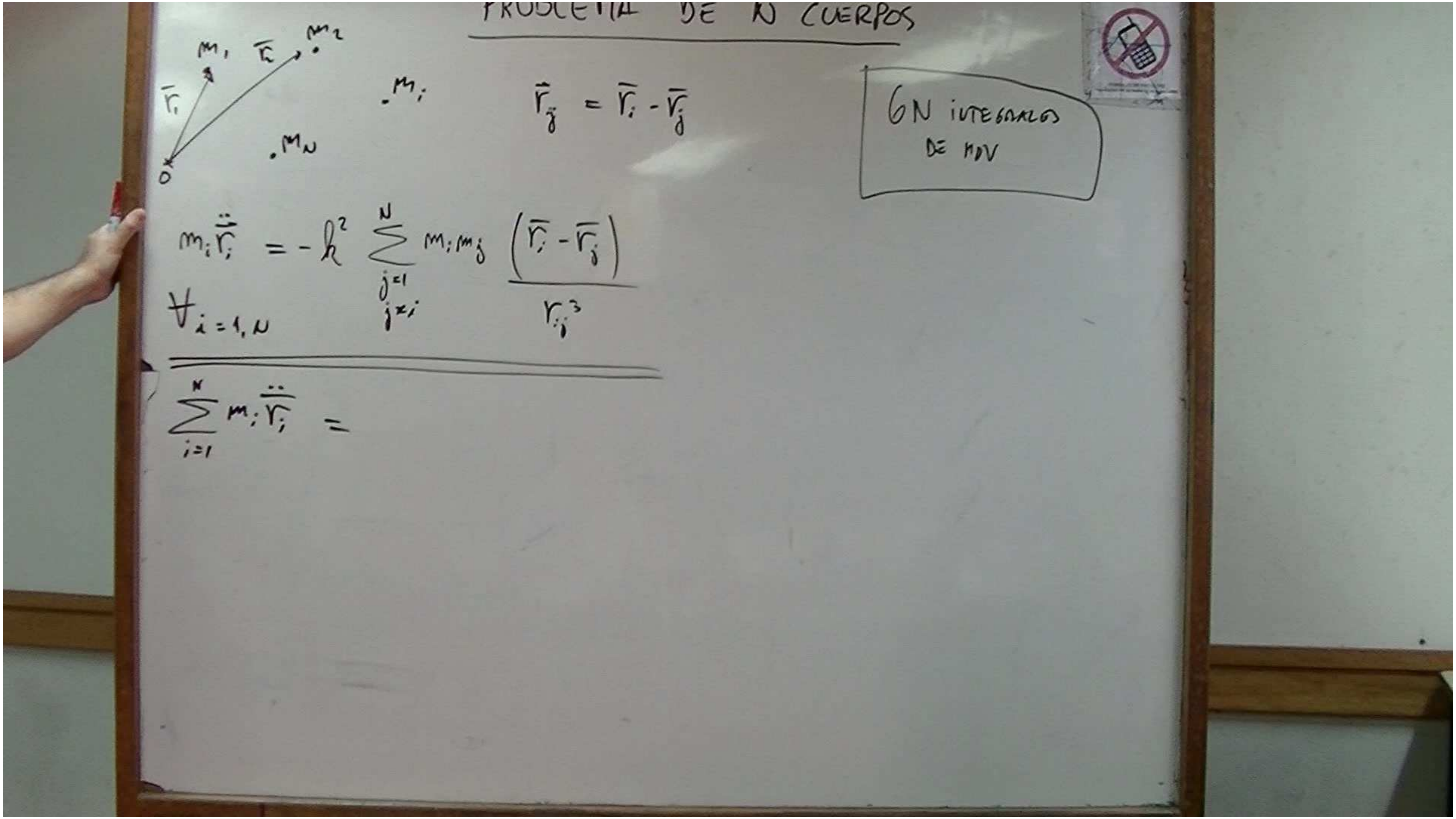
X_i = velocid $\quad i > 3N$

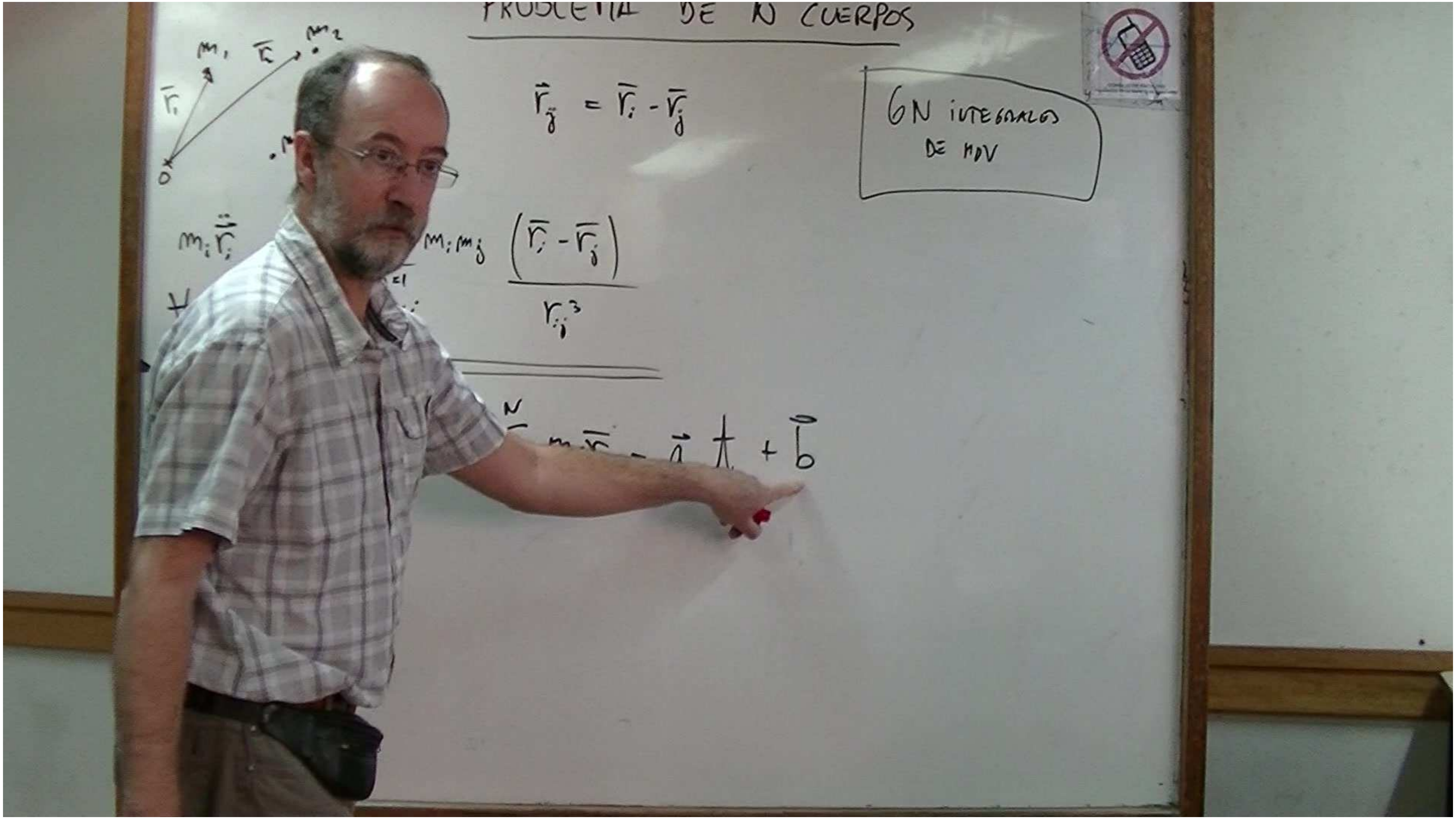

PROBLEMA DE N CUERPOS
m_2
m_i
$\vec{r}_{ij} = \bar{r}_i - \bar{r}_j$
$k^2 \sum_{j=1, j\neq i}^{N} m_i m_j \frac{(\bar{r}_i - \bar{r}_j)}{r_{ij}^3}$
$\Rightarrow$ 3N ECS. 2°
6N EC.
$\bar{X} = (x_1 \ldots x_{6N})$
X_i = Posicion $i \leq 3N$
X_i = Velocid $i > 3N$
$I_1(x_1 \ldots x_{6N}) = CTE \rightarrow X_{6N} = f(x_1 \ldots x_{6N-1})$

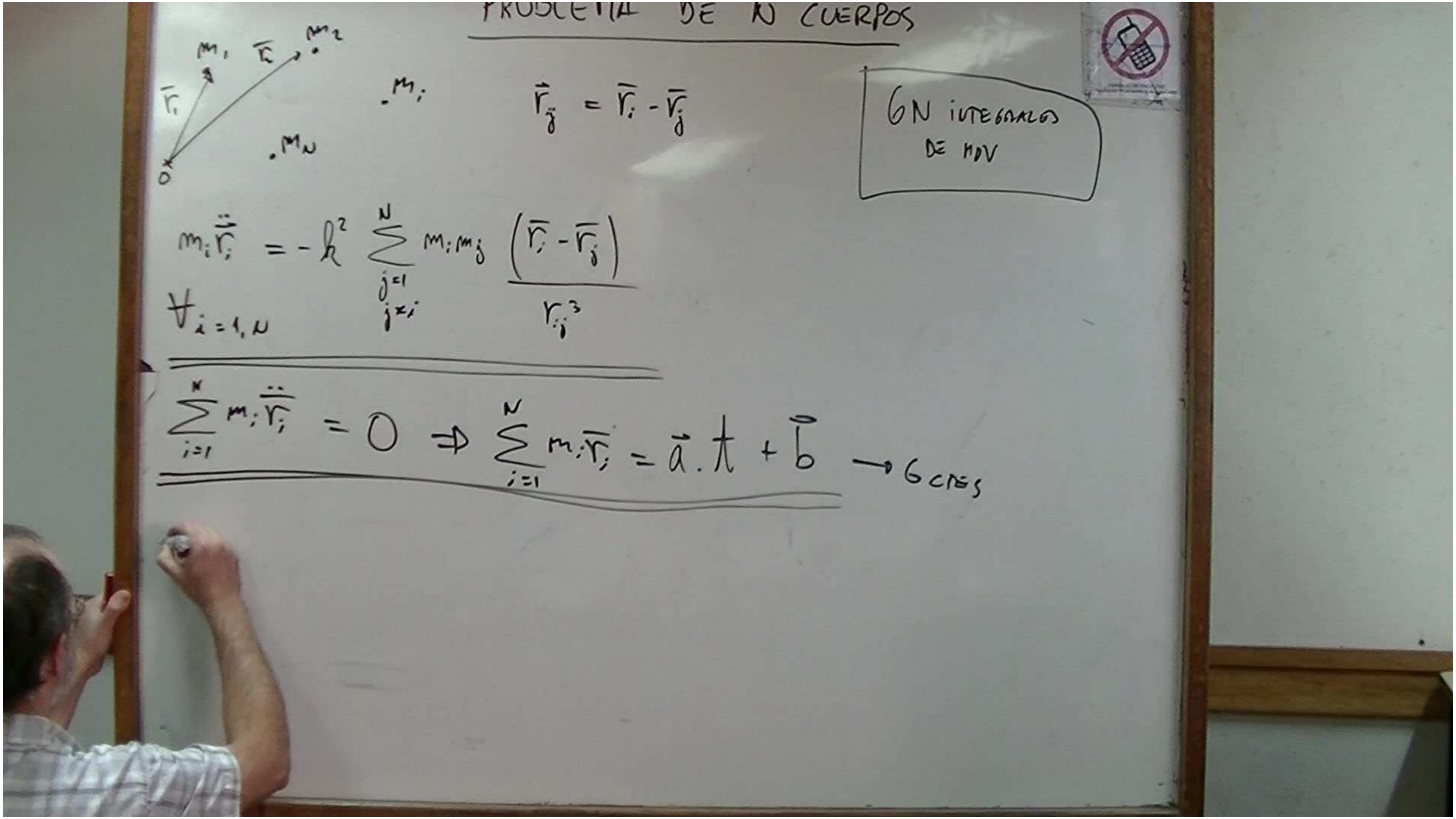

PROBLEMA DE
m1
m2
mi
mN
O
INTEGRALES
DE MOV
6N ec.


PROBLEMA DE N CUERPOS
6N INTEGRALES
DE MOV

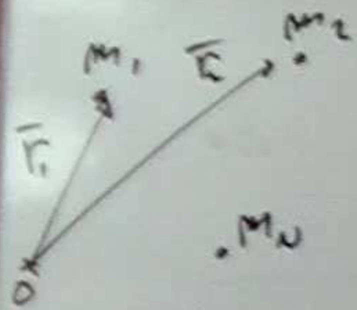

PROBLEMA DE N CUERPOS
6N INTEGRALES DE MOV
⇒ 3N ECS. 2º
ACCIONES
I_i = CTES
ν_i = CTES
$\forall\, i = 1, N$


PROBLEMA DE N CUERPOS
m_1 $\bar{r}_2$ m_2 $\bar{r}_1$ m_i m_N O
$\bar{r}_{ij} = \bar{r}_i - \bar{r}_j$
6N INTEGRALES DE MOV
$m_i \ddot{\bar{r}}_i = -k^2 \sum_{j=1, j \neq i}^{N} m_i m_j \frac{(\bar{r}_i - \bar{r}_j)}{r_{ij}^3}$
$\forall_{i=1,N}$
$\sum_{i=1}^{N} m_i \ddot{\bar{r}}_i =$


PROBLEMA DE N CUERPOS
6N INTEGRALES
DE MOV


PROBLEMA DE N CUERPOS
m_1
$\bar r_{12}$
m_2
$\bar r_1$
O
m_i
m_N
$\vec r_{ij} = \bar r_i - \bar r_j$
6N INTEGRALES DE MOV
$m_i \ddot{\bar r}_i = -k^2 \sum_{j=1, j\neq i}^{N} m_i m_j \frac{(\bar r_i - \bar r_j)}{r_{ij}^3}$
$\forall_{i=1,N}$
$\sum_{i=1}^{N} m_i \ddot{\bar r}_i = 0 \Rightarrow \sum_{i=1}^{N} m_i \bar r_i = \vec a . t + \bar b \rightarrow$ 6 ctes

PROBLEMA DE N CUERPOS



$$\vec r_{ij} = \bar r_i - \bar r_j$$

6N INTEGRALES DE MOV

$$m_i \ddot{\bar r}_i = -k^2 \sum_{\substack{j=1\\ j\neq i}}^{N} m_i m_j \frac{(\bar r_i - \bar r_j)}{r_{ij}^3}$$

$\forall_{i=1,N}$

$$\sum_{i=1}^{N} m_i \ddot{\bar r}_i = 0 \Rightarrow \sum_{i=1}^{N} m_i \bar r_i = \vec a . t + \vec b \longrightarrow 6 \text{ ctes}$$

$$\boxed{m_i \bar r_i \wedge \ddot{\bar r}_i} = k^2 m_i \sum_{\substack{j=1\\ j\neq i}}^{N} m_j \frac{\bar r_i \wedge \vec r_j}{r_{ij}^3}$$

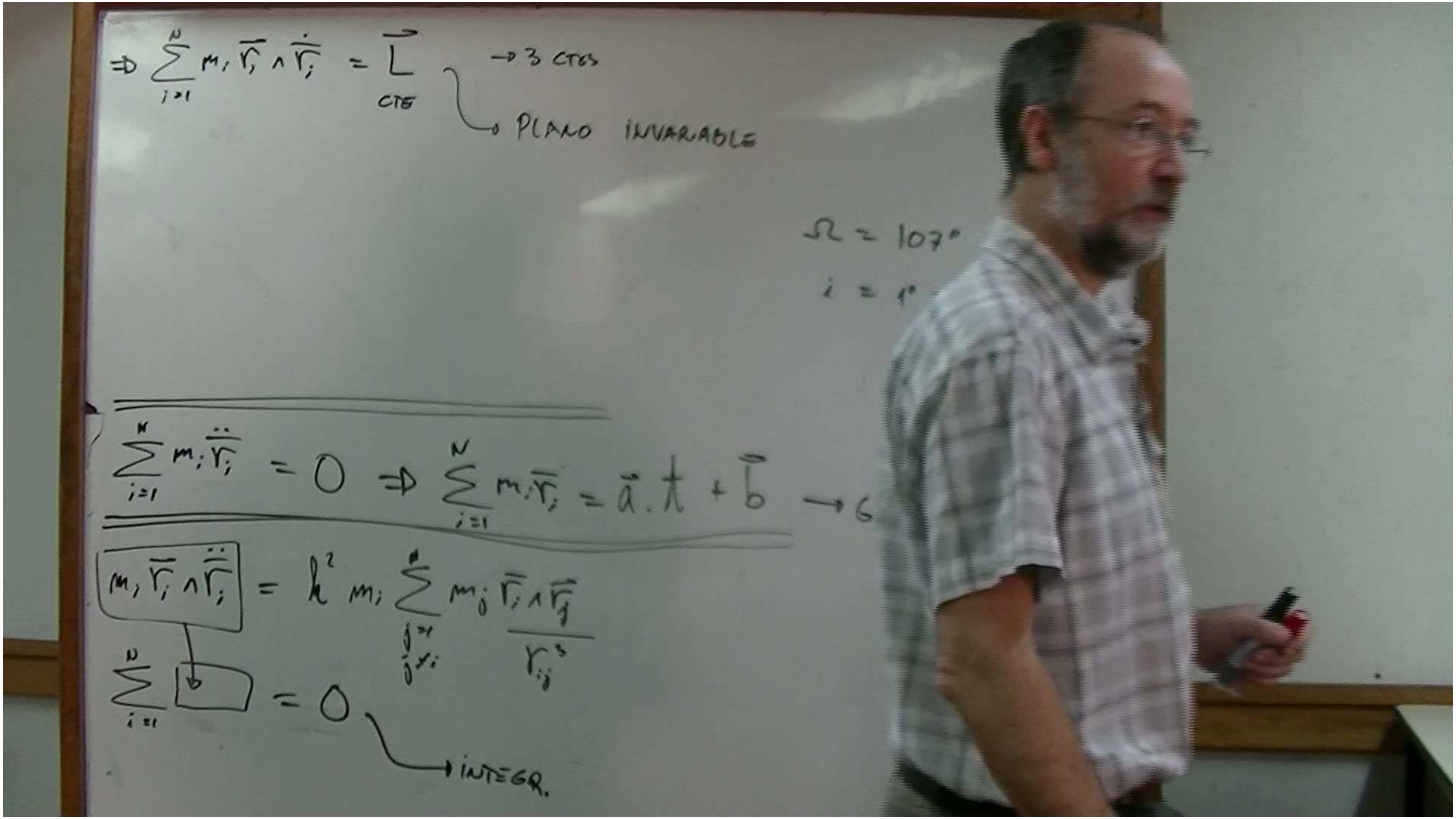
$$\sum_{i=1}^{N} \boxed{\ } =$$

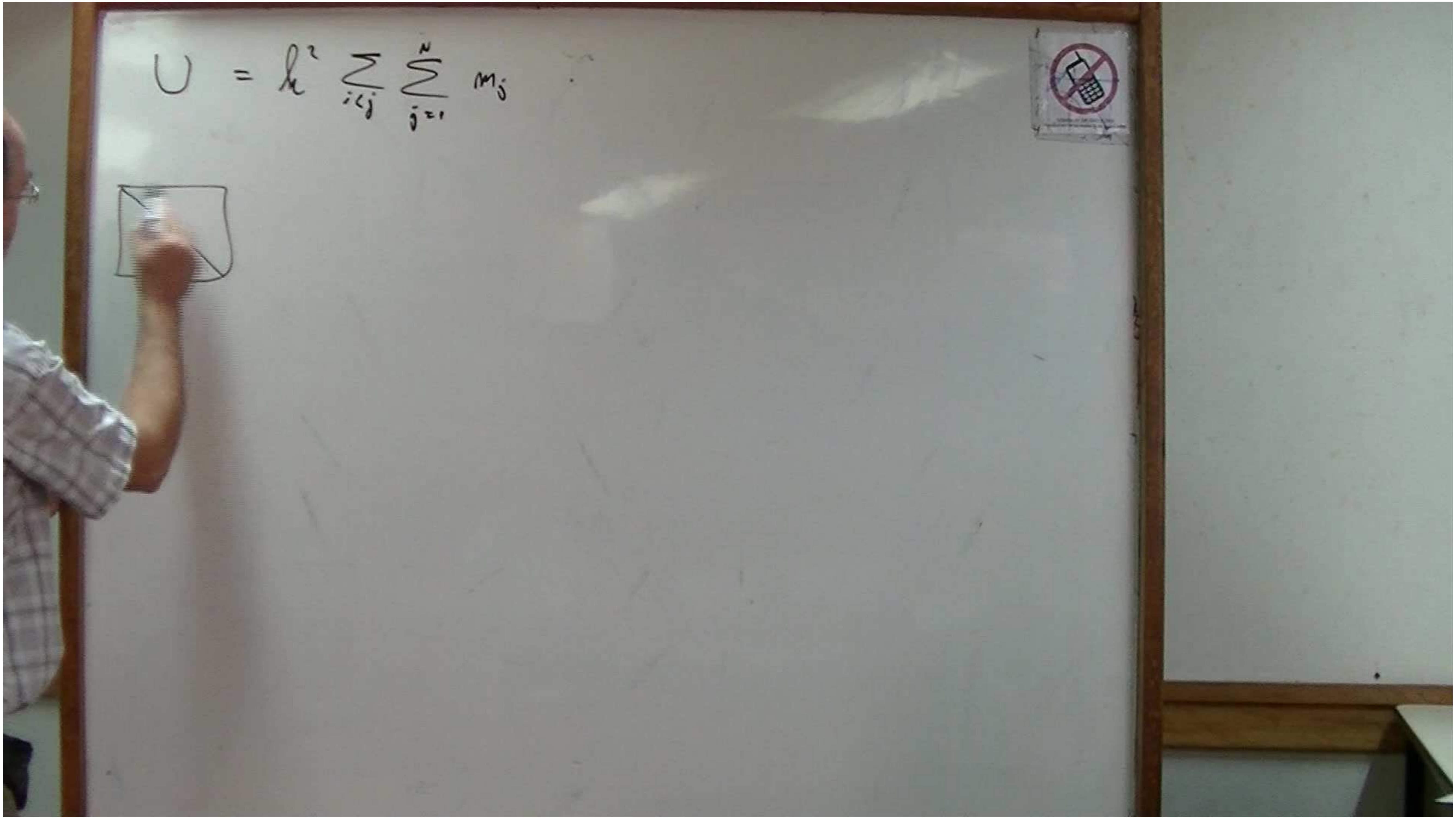
$$\Rightarrow \sum_{i=1}^{N} m_i \vec{r}_i \wedge \dot{\vec{r}}_i = \underbrace{\vec{L}}_{\text{CTE}} \longrightarrow 3 \text{ CTES}$$

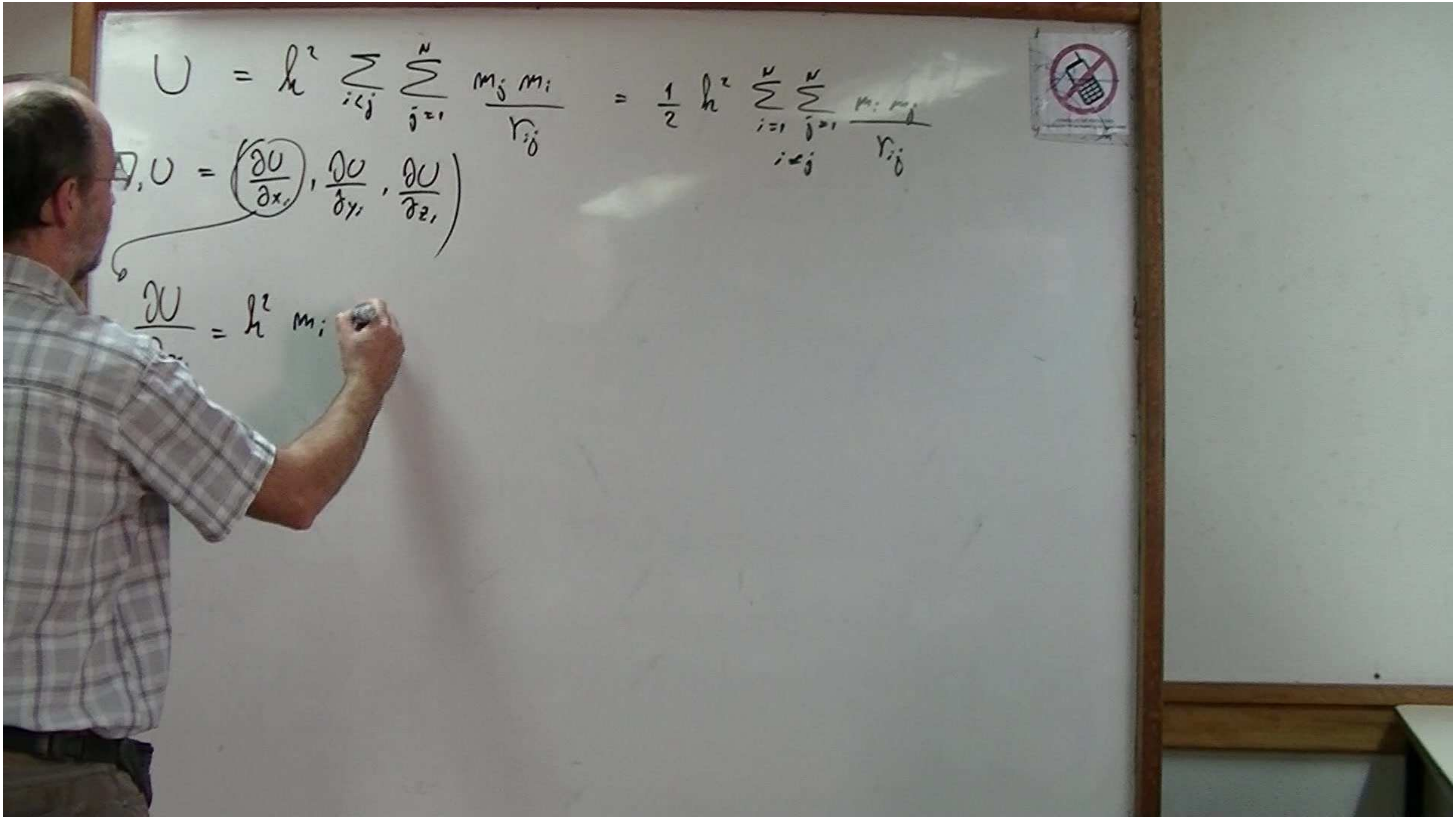
$$\sum_{i=1}^{N} m_i \ddot{\vec{r}}_i = 0 \Rightarrow \sum_{i=1}^{N} m_i \vec{r}_i = \vec{a}\,.\,t + \vec{b} \longrightarrow 6 \text{ CTES}$$

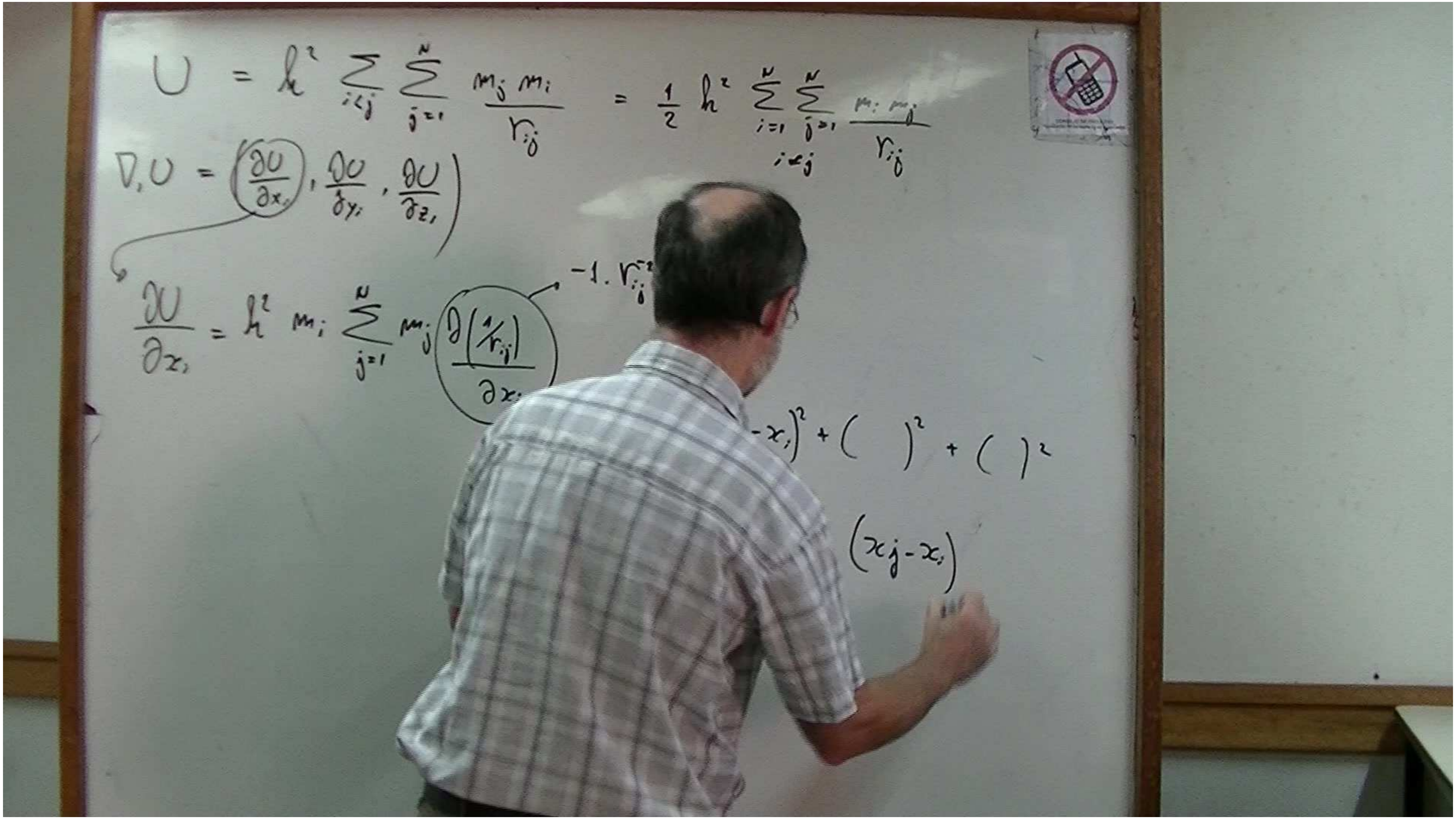
$$\boxed{m_i \vec{r}_i \wedge \ddot{\vec{r}}_i} = k^2 m_i \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\vec{r}_i \wedge \vec{r}_j}{r_{ij}^3}$$

$$\sum_{i=1}^{N} \boxed{} = 0 \longrightarrow \text{INTEGR.}$$


$\Rightarrow \sum_{i=1}^{N} m_i \bar{r}_i \wedge \dot{\bar{r}}_i = \underset{CTE}{\vec{L}}$
$\rightarrow$ 3 CTES
$\rightarrow$ PLANO INVARIABLE
$\Omega = 107°$
$i = 1°$
$\sum_{i=1}^{N} m_i \ddot{\bar{r}}_i = 0 \Rightarrow \sum_{i=1}^{N} m_i \bar{r}_i = \vec{a} . t + \bar{b} \rightarrow 6$
$\boxed{m_i \bar{r}_i \wedge \ddot{\bar{r}}_i} = k^2 m_i \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\bar{r}_i \wedge \vec{r}_j}{r_{ij}^3}$
$\sum_{i=1}^{N} \boxed{} = 0 \rightarrow$ INTEGR.


$U = k^2 \sum_{i<j} \sum_{j=1}^{N} m_j$


$U = k^2 \sum_{i<j} \sum_{j=1}^{N} \frac{m_j m_i}{r_{ij}} = \frac{1}{2} k^2 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{m_i m_j}{r_{ij}}$
$\nabla_1 U = \left(\frac{\partial U}{\partial x_1}, \frac{\partial U}{\partial y_1}, \frac{\partial U}{\partial z_1} \right)$
$\frac{\partial U}{\partial x_1} = k^2 m_1$


$U = k^2 \sum_{i<j} \sum_{j=1}^{N} \frac{m_j m_i}{r_{ij}} = \frac{1}{2} k^2 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{m_i m_j}{r_{ij}}$
$\nabla_i U = \left(\frac{\partial U}{\partial x_i}, \frac{\partial U}{\partial y_i}, \frac{\partial U}{\partial z_i} \right)$
$\frac{\partial U}{\partial x_i} = k^2 m_i \sum_{j=1}^{N} m_j \frac{\partial (1/r_{ij})}{\partial x_i}$
$-1 . r_{ij}^{-2}$
$-x_i)^2 + (\quad)^2 + (\quad)^2$
$(x_j - x_i)$

$$U = k^2 \sum_{i<j} \sum_{j=1}^{N} \frac{m_j m_i}{r_{ij}} = \frac{1}{2} k^2 \sum_{i=1}^{N} \sum_{\substack{j=1\\ i\neq j}}^{N} \frac{m_i m_j}{r_{ij}}$$

$$\nabla_i U = \left(\frac{\partial U}{\partial x_i}, \frac{\partial U}{\partial y_i}, \frac{\partial U}{\partial z_i}\right)$$

$$\frac{\partial U}{\partial x_i} = k^2 m_i \sum_{j=1}^{N} m_j \frac{\partial\left(1/r_{ij}\right)}{\partial x_i}$$

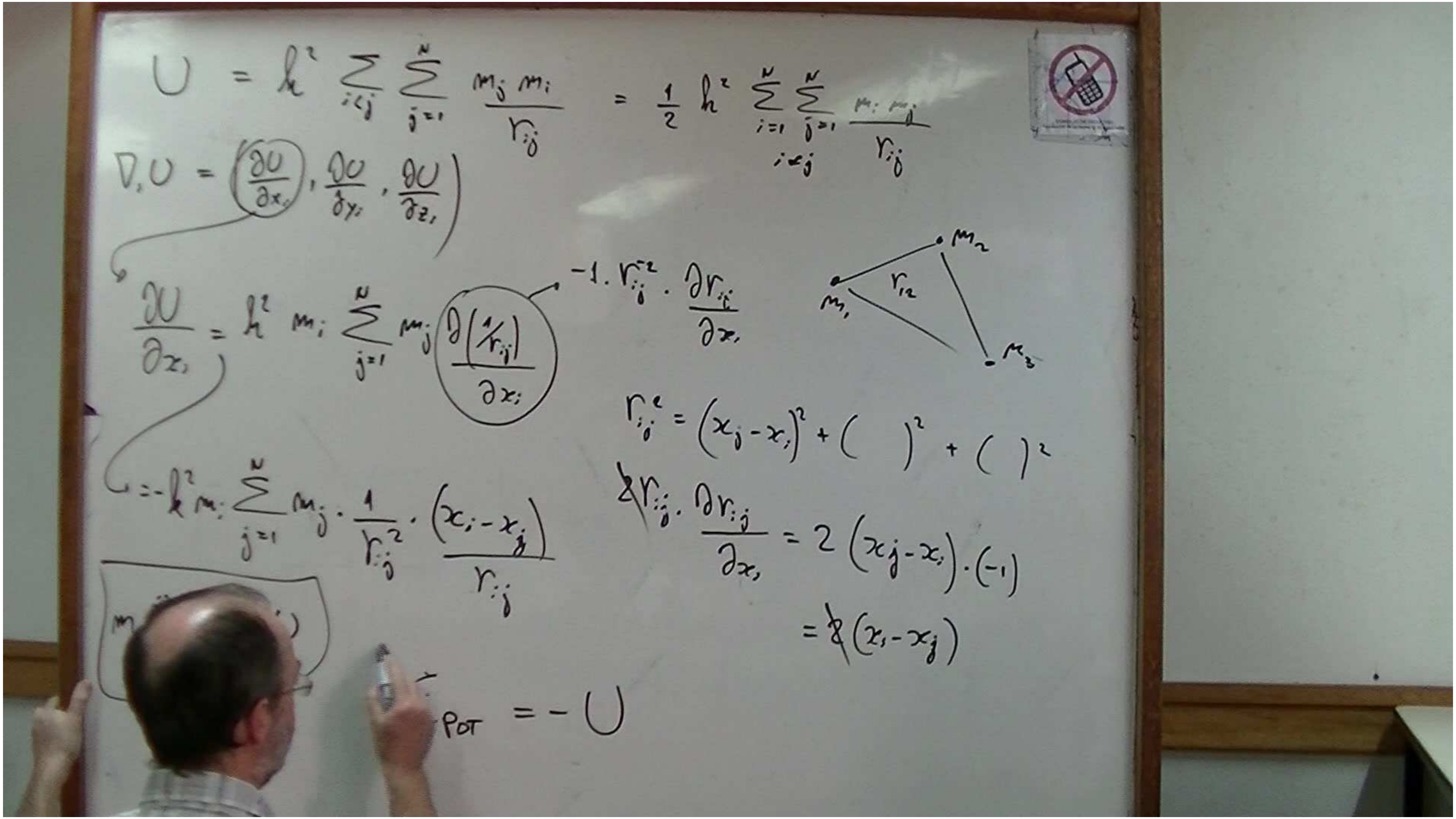
$$-1 \cdot r_{ij}^{-2} \cdot \frac{\partial r_{ij}}{\partial x_i}$$

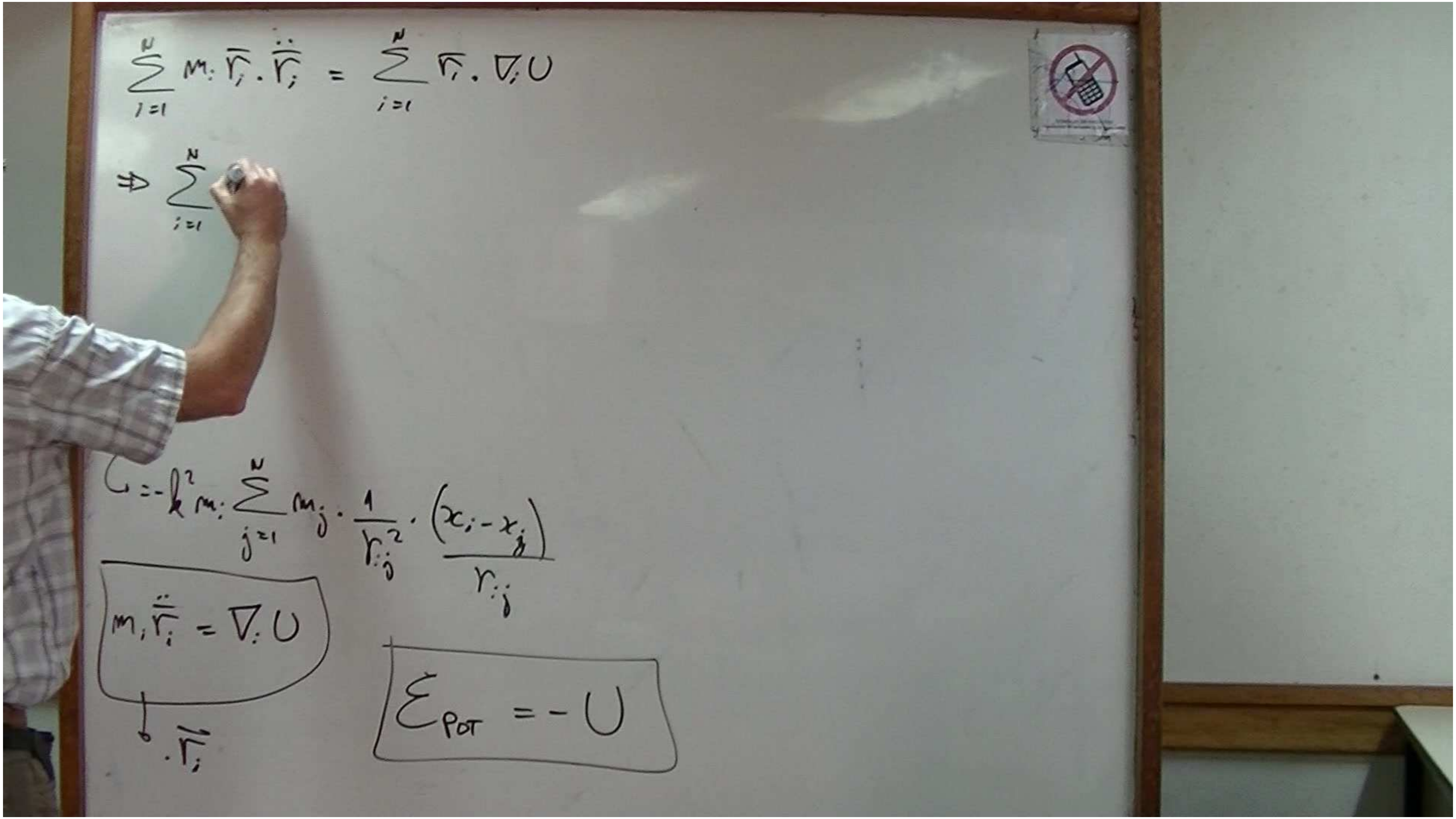
$$= -k^2 m_i \sum_{j=1}^{N} m_j \cdot \frac{1}{r_{ij}^2} \cdot \frac{(x_i - x_j)}{r_{ij}}$$

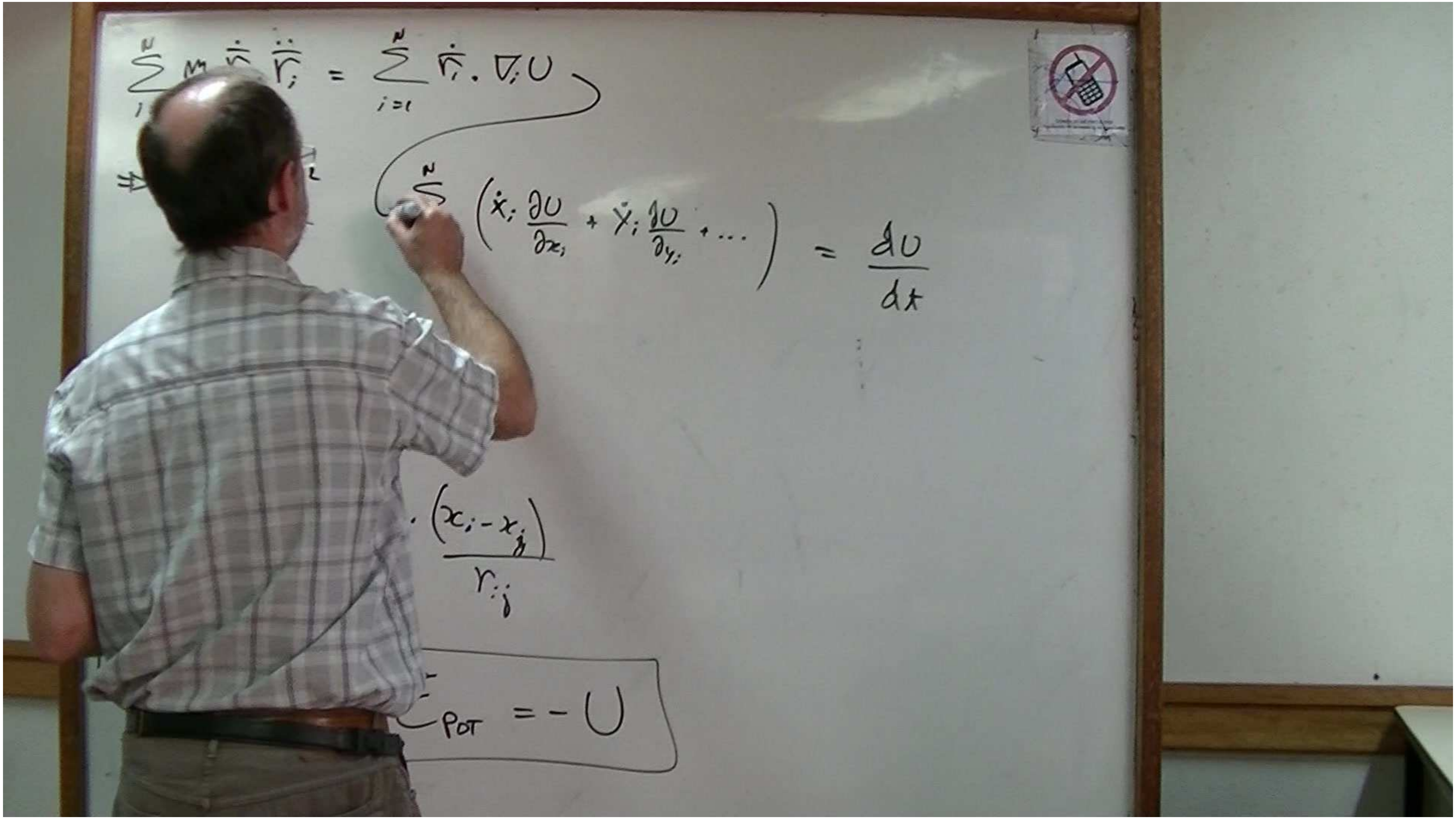
$$r_{ij}^2 = (x_j - x_i)^2 + (\quad)^2 + (\quad)^2$$

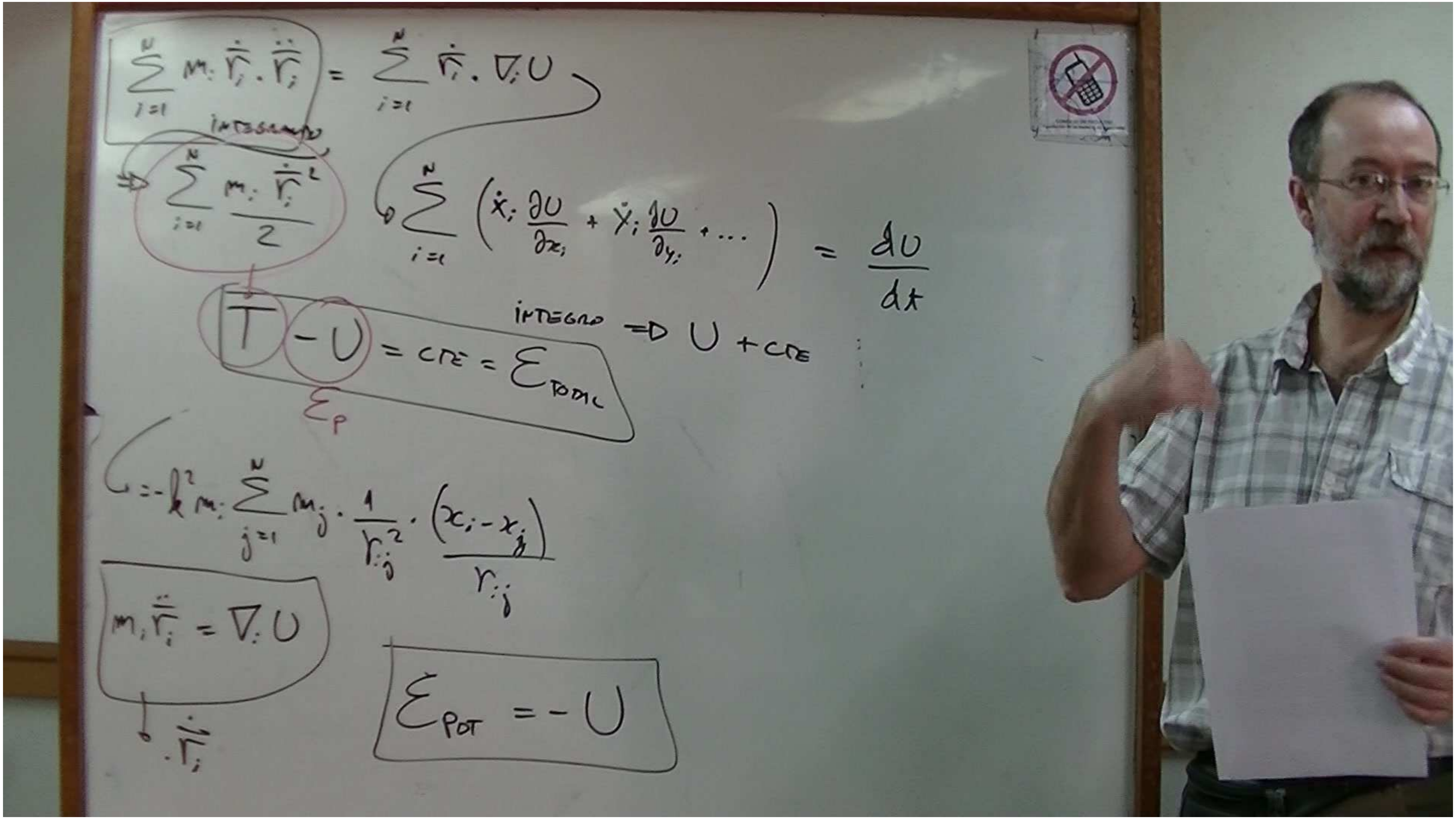
$$\cancel{2} r_{ij} \cdot \frac{\partial r_{ij}}{\partial x_i} = 2(x_j - x_i) \cdot (-1)$$

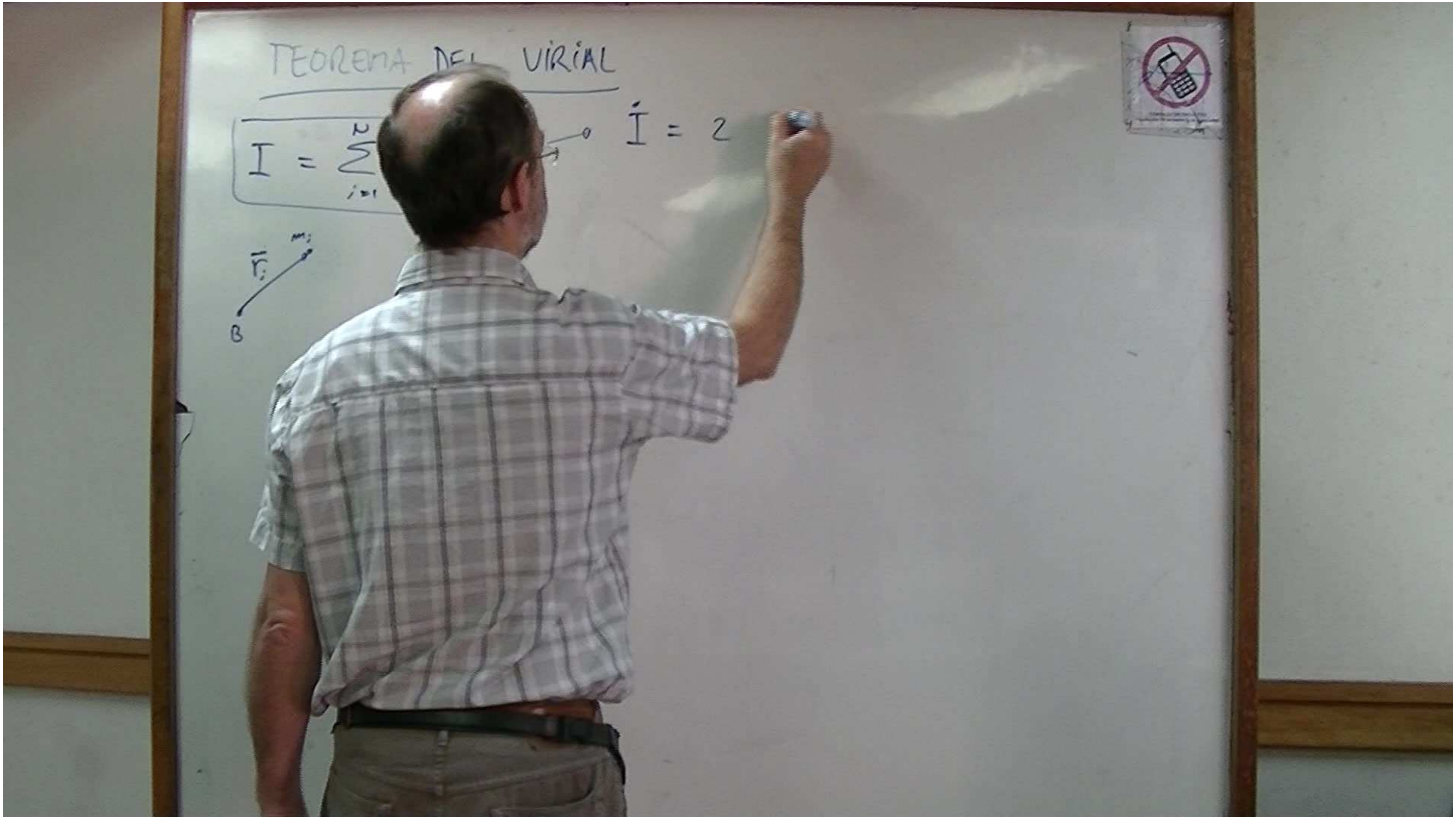
$$= \cancel{2}(x_i - x_j)$$

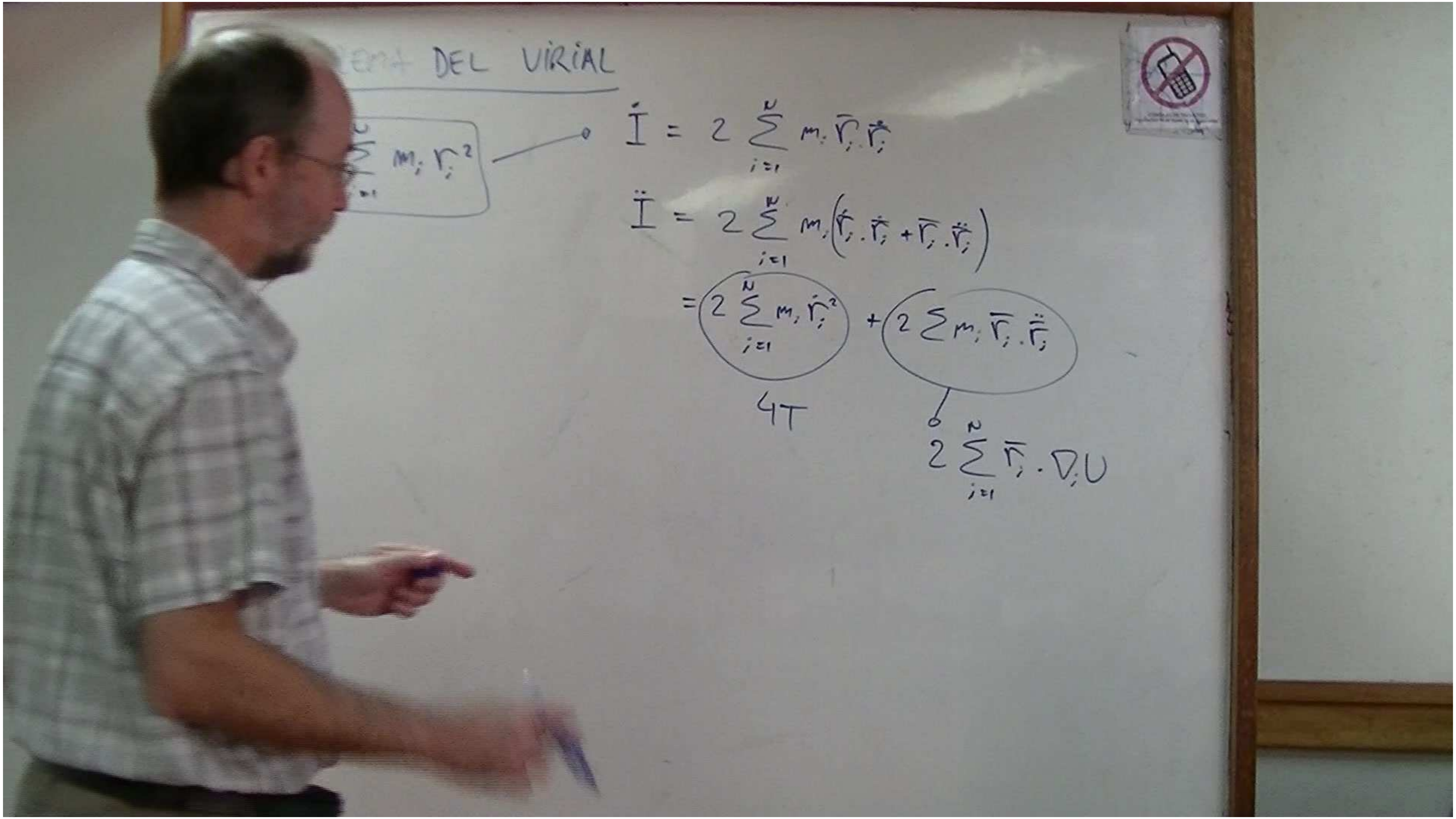

$U = k^2 \sum_{i<j} \sum_{j=1}^{N} \frac{m_j m_i}{r_{ij}} = \frac{1}{2} k^2 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{m_i m_j}{r_{ij}}$
$\nabla_i U = \left(\frac{\partial U}{\partial x_i}, \frac{\partial U}{\partial y_i}, \frac{\partial U}{\partial z_i} \right)$
$\frac{\partial U}{\partial x_i} = k^2 m_i \sum_{j=1}^{N} m_j \frac{\partial (1/r_{ij})}{\partial x_i}$
$-1 \cdot r_{ij}^{-2} \cdot \frac{\partial r_{ij}}{\partial x_i}$
$= -k^2 m_i \sum_{j=1}^{N} m_j \cdot \frac{1}{r_{ij}^2} \cdot \frac{(x_i - x_j)}{r_{ij}}$
m_1
m_2
m_3
r_{12}
$r_{ij}^2 = (x_j - x_i)^2 + (\quad)^2 + (\quad)^2$
$\cancel{2} r_{ij} \cdot \frac{\partial r_{ij}}{\partial x_i} = 2 (x_j - x_i) \cdot (-1)$
$= \cancel{2} (x_i - x_j)$
$_{POT} = -U$

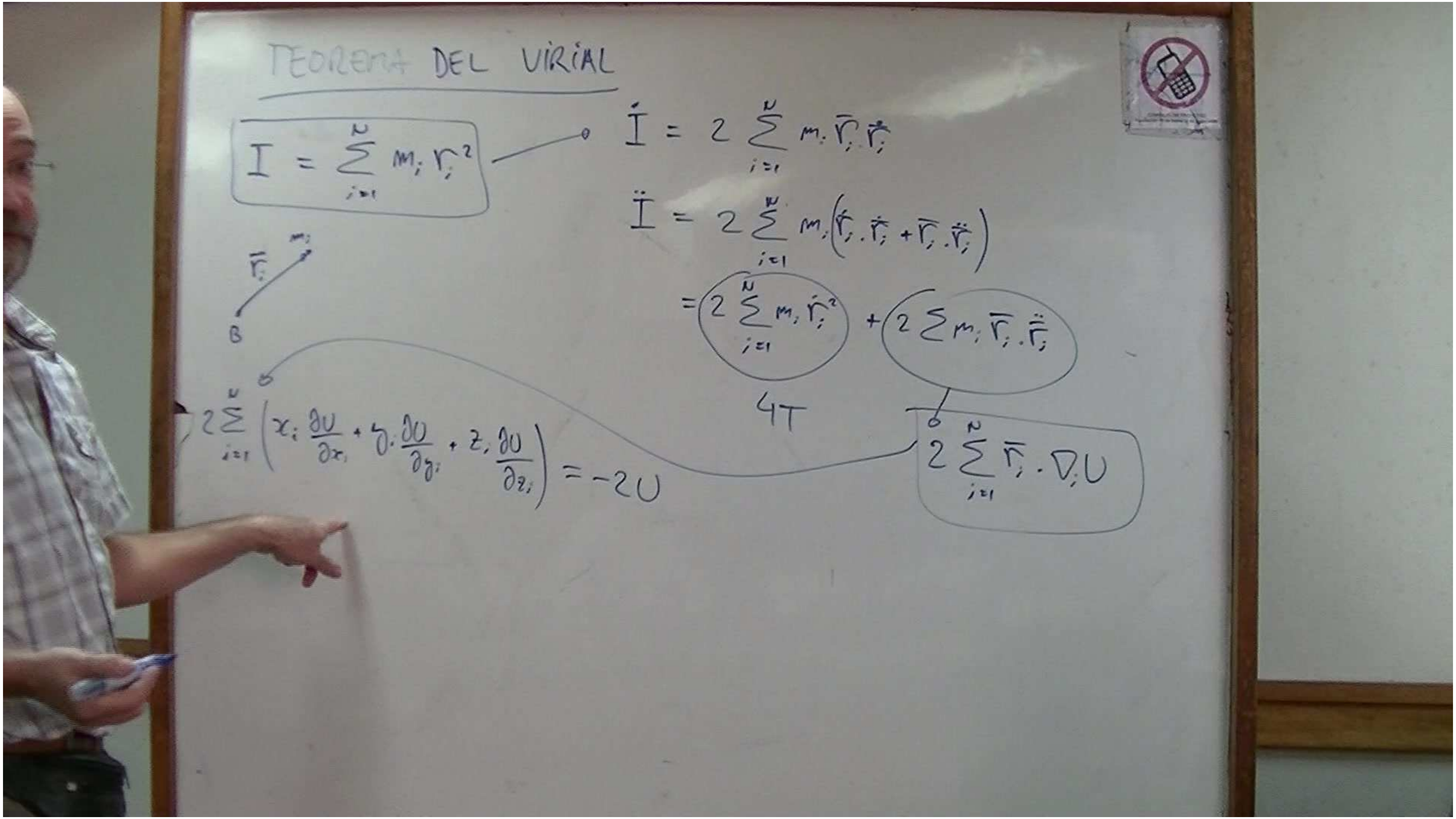

$\sum_{i=1}^{N} m_i \bar{r}_i \cdot \ddot{\bar{r}}_i = \sum_{i=1}^{N} \bar{r}_i \cdot \nabla_i U$
$\Rightarrow \sum_{i=1}^{N}$
$= -k^2 m_i \sum_{j=1}^{N} m_j \cdot \frac{1}{r_{ij}^2} \cdot \frac{(x_i - x_j)}{r_{ij}}$
$\boxed{m_i \ddot{\bar{r}}_i = \nabla_i U}$
$\cdot \bar{r}_i$
$\boxed{\mathcal{E}_{pot} = -U}$

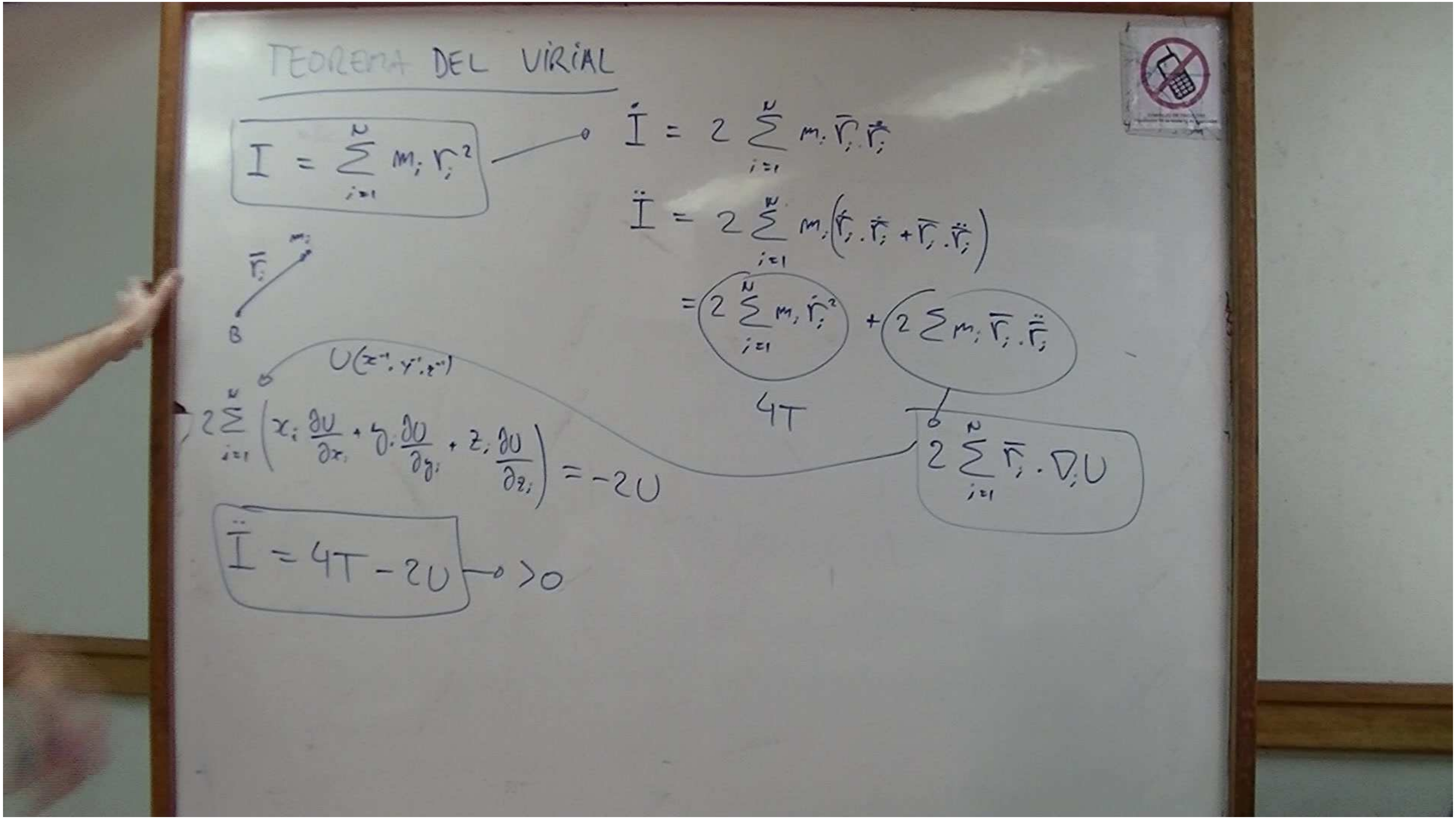

$= \sum_{i=1}^{N} \dot{r}_i \cdot \nabla_i U$
$\sum^{N} \left(\dot{x}_i \frac{\partial U}{\partial x_i} + \dot{y}_i \frac{\partial U}{\partial y_i} + \dots \right) = \frac{dU}{dt}$
$\frac{(x_i - x_j)}{r_{ij}}$
$E_{POT} = -U$

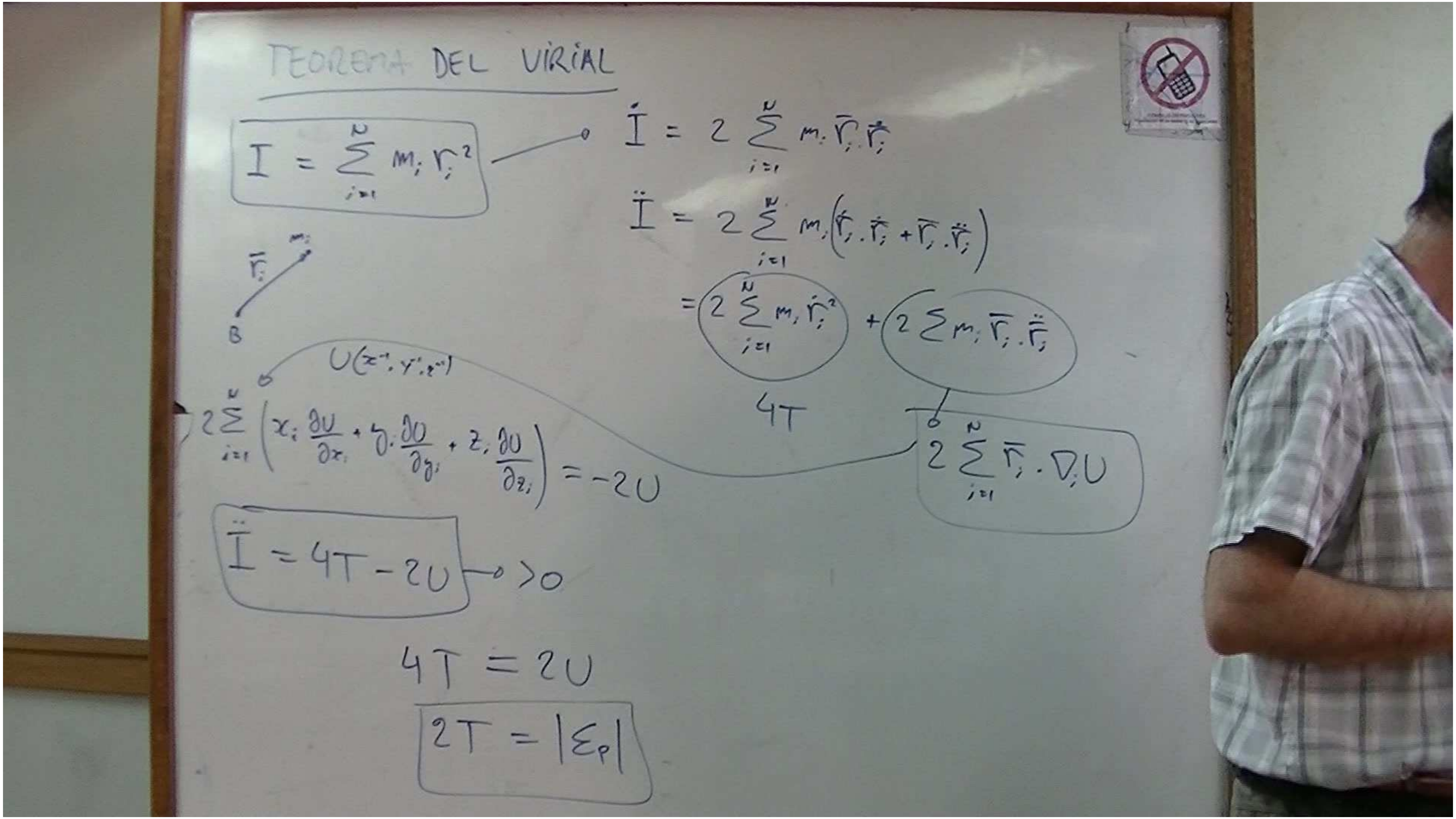

$\sum_{i=1}^{N} m_i \dot{\vec{r}}_i \cdot \ddot{\vec{r}}_i = \sum_{i=1}^{N} \dot{\vec{r}}_i \cdot \nabla_i U$
INTEGRANDO
$\Rightarrow \sum_{i=1}^{N} m_i \frac{\dot{\vec{r}}_i^{\,2}}{2}$
$\sum_{i=1}^{N} \left(\dot{x}_i \frac{\partial U}{\partial x_i} + \dot{y}_i \frac{\partial U}{\partial y_i} + \dots \right) = \frac{dU}{dt}$
INTEGRO $\Rightarrow U + \text{CTE}$
$T - U = \text{CTE} = \mathcal{E}_{\text{TOTAL}}$
$\mathcal{E}_p$
$= -k^2 m_i \sum_{j=1}^{N} m_j \cdot \frac{1}{r_{ij}^2} \cdot \frac{(x_i - x_j)}{r_{ij}}$
$m_i \ddot{\vec{r}}_i = \nabla_i U$
$\cdot \dot{\vec{r}}_i$
$\mathcal{E}_{\text{POT}} = -U$

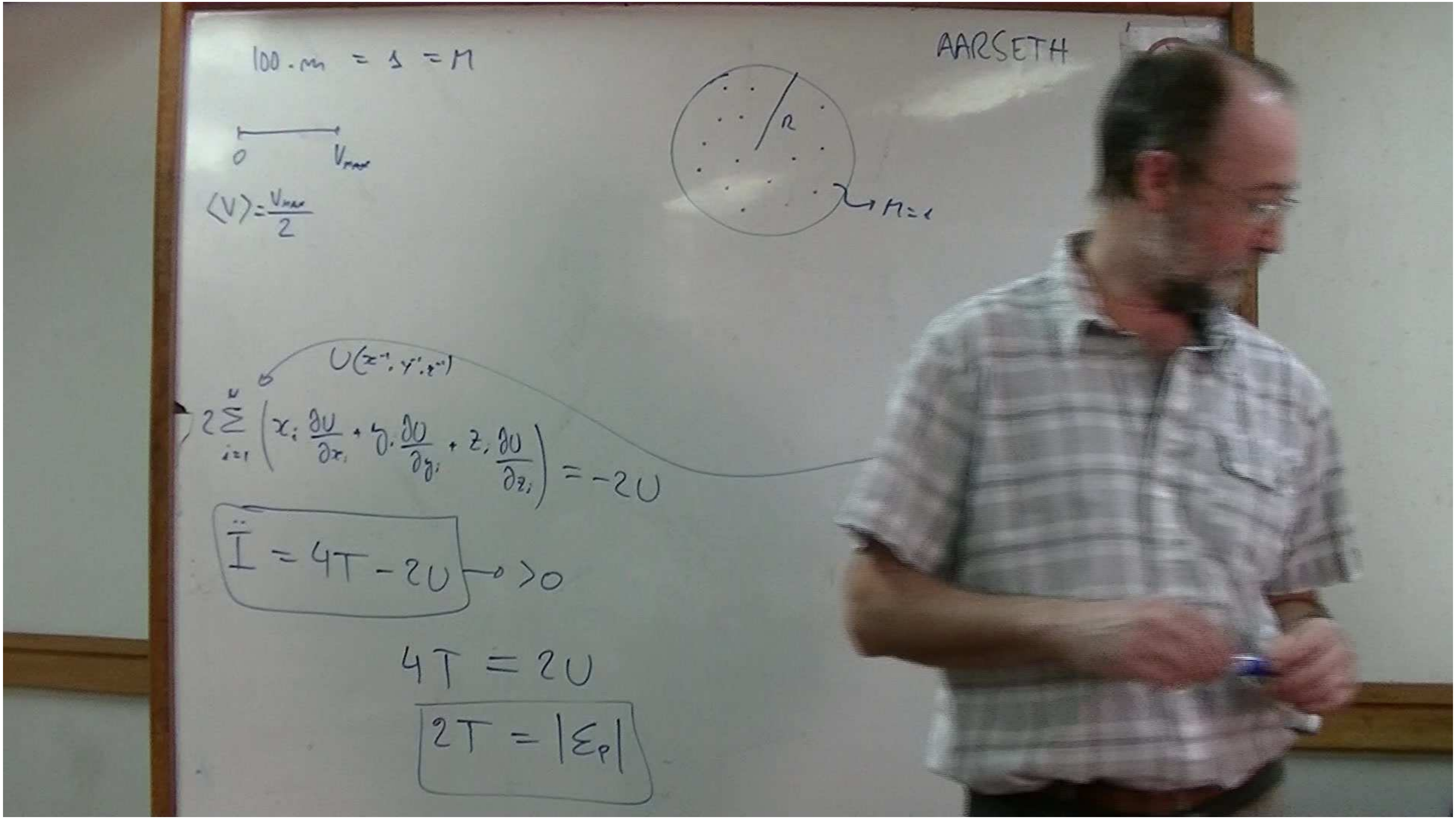

TEOREMA DEL VIRIAL
I = Σ
İ = 2
B


DEL VIRIAL
$\sum m_i r_i^2$
$\dot{I} = 2 \sum_{i=1}^{N} m_i \bar{r}_i \cdot \dot{\bar{r}}_i$
$\ddot{I} = 2 \sum_{i=1}^{N} m_i \left(\dot{\bar{r}}_i \cdot \dot{\bar{r}}_i + \bar{r}_i \cdot \ddot{\bar{r}}_i \right)$
$= 2 \sum_{i=1}^{N} m_i \dot{r}_i^2 + 2 \sum m_i \bar{r}_i \cdot \ddot{\bar{r}}_i$
$4T$
$2 \sum_{i=1}^{N} \bar{r}_i \cdot \nabla_i U$

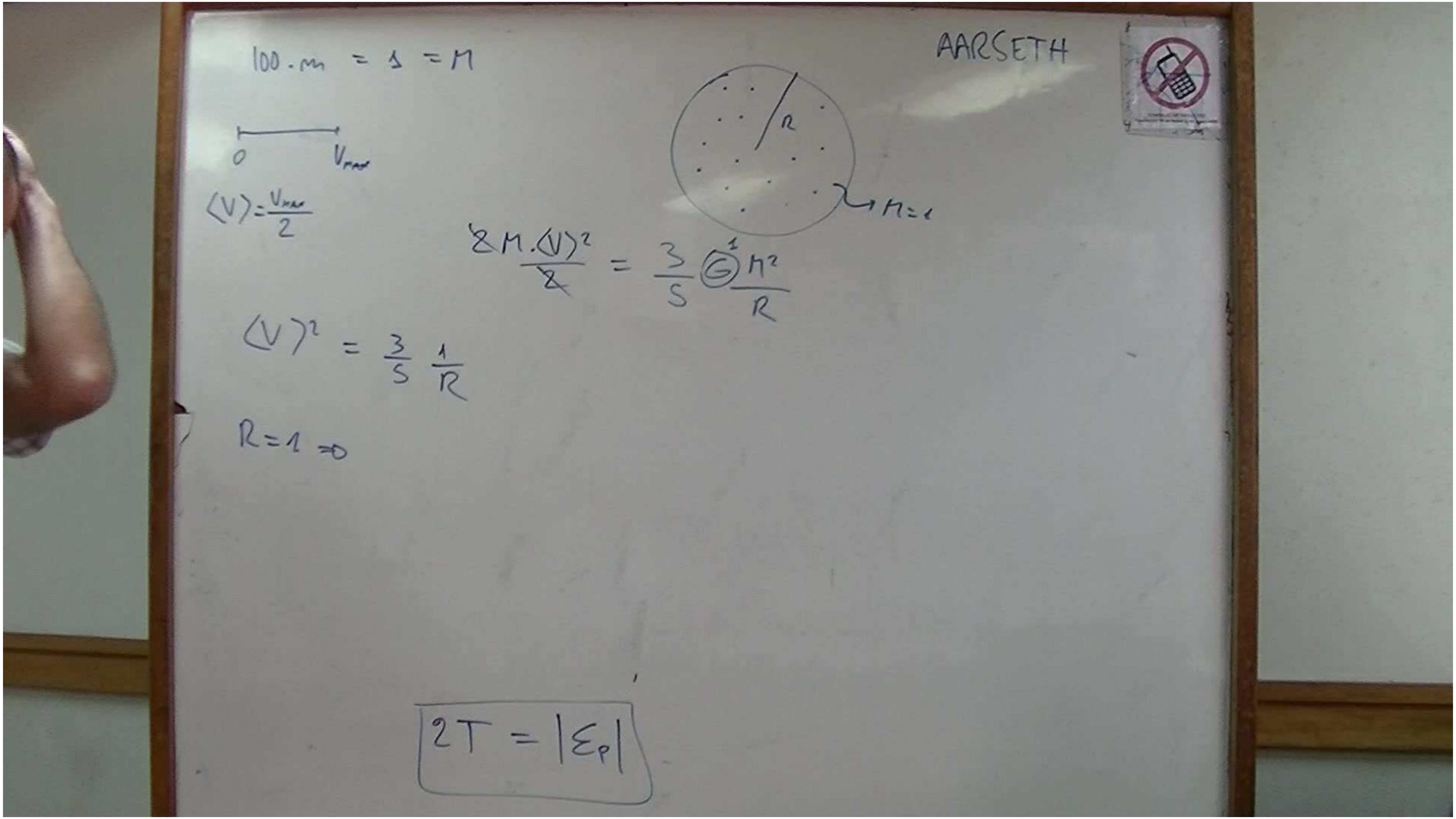

TEOREMA DEL VIRIAL
$I = \sum_{i=1}^{N} m_i r_i^2$
$\dot{I} = 2 \sum_{i=1}^{N} m_i \bar{r}_i \dot{\bar{r}}_i$
$\ddot{I} = 2 \sum_{i=1}^{N} m_i (\dot{\bar{r}}_i \cdot \dot{\bar{r}}_i + \bar{r}_i \cdot \ddot{\bar{r}}_i)$
$= 2 \sum_{i=1}^{N} m_i \dot{r}_i^2 + 2 \sum m_i \bar{r}_i \cdot \ddot{\bar{r}}_i$
$4T$
$2 \sum_{i=1}^{N} \bar{r}_i \cdot \nabla_i U$
$\bar{r}_i$
m_i
B
$2 \sum_{i=1}^{N} \left(x_i \frac{\partial U}{\partial x_i} + y_i \frac{\partial U}{\partial y_i} + z_i \frac{\partial U}{\partial z_i} \right) = -2U$

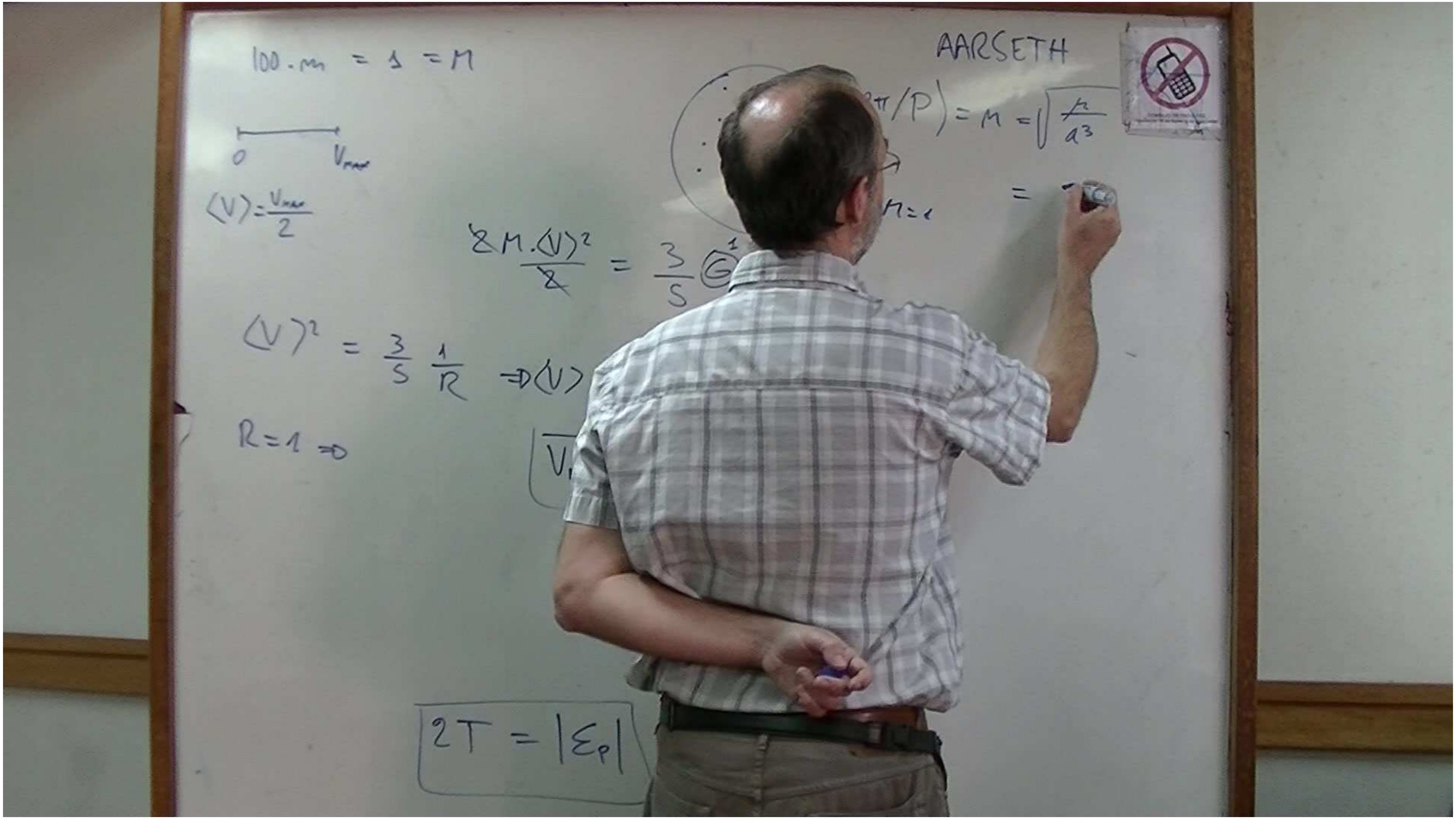

TEOREMA DEL VIRIAL
$I = \sum_{i=1}^{N} m_i r_i^2$
$\dot{I} = 2 \sum_{i=1}^{N} m_i \bar{r}_i \cdot \dot{\bar{r}}_i$
$\ddot{I} = 2 \sum_{i=1}^{N} m_i \left(\dot{\bar{r}}_i \cdot \dot{\bar{r}}_i + \bar{r}_i \cdot \ddot{\bar{r}}_i \right)$
$= 2 \sum_{i=1}^{N} m_i \dot{r}_i^2 + 2 \sum m_i \bar{r}_i \cdot \ddot{\bar{r}}_i$
$4T$
$2 \sum_{i=1}^{N} \bar{r}_i \cdot \nabla_i U$
$\bar{r}_i$
m_i
B
$U(x^{-1}, y^{-1}, z^{-1})$
$2 \sum_{i=1}^{N} \left(x_i \frac{\partial U}{\partial x_i} + y_i \frac{\partial U}{\partial y_i} + z_i \frac{\partial U}{\partial z_i} \right) = -2U$
$\ddot{I} = 4T - 2U$
> 0

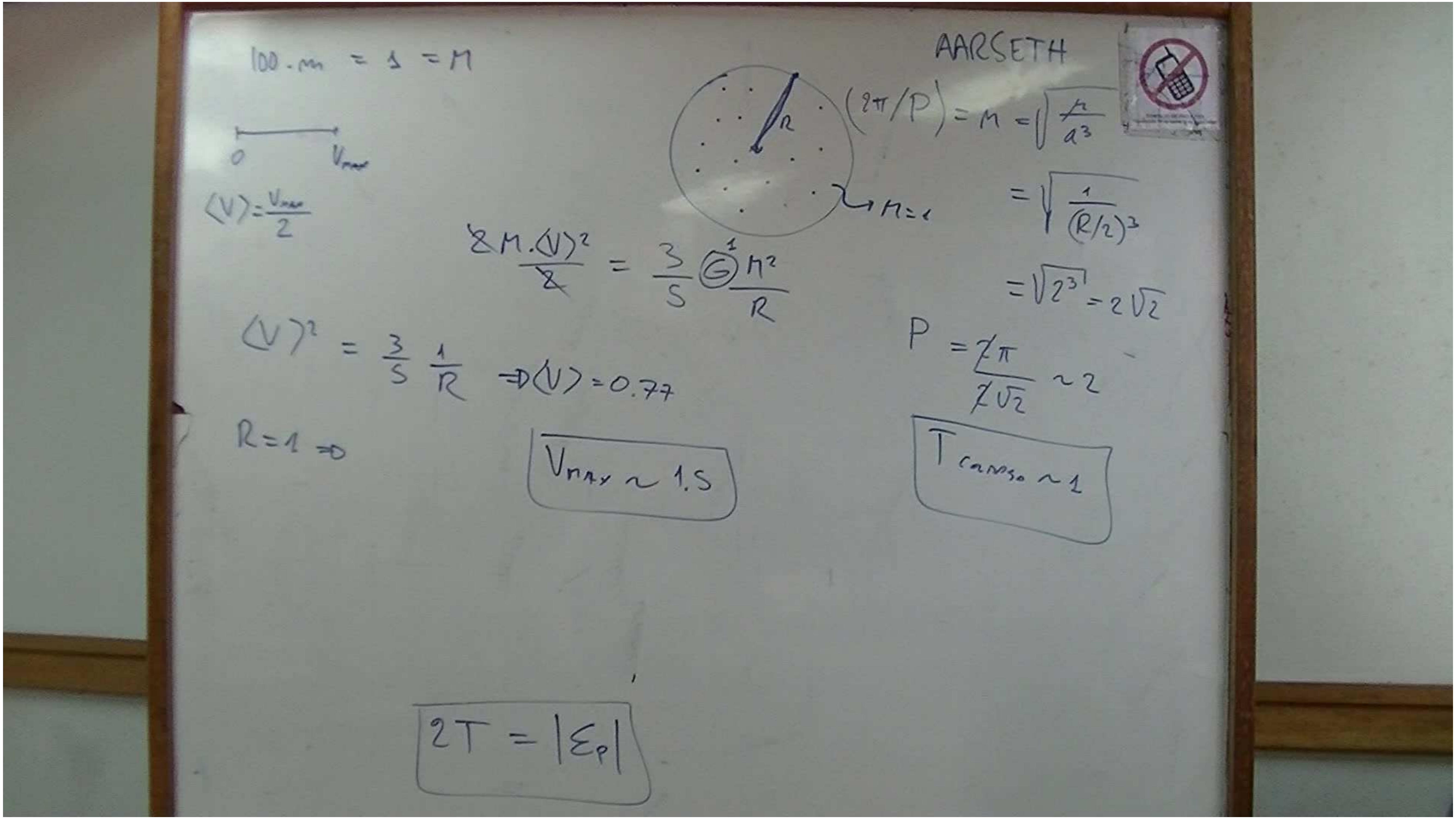

TEOREMA DEL VIRIAL
$I = \sum_{i=1}^{N} m_i r_i^2$
$\dot{I} = 2 \sum_{i=1}^{N} m_i \bar{r}_i \cdot \dot{\bar{r}}_i$
$\ddot{I} = 2 \sum_{i=1}^{N} m_i (\dot{\bar{r}}_i \cdot \dot{\bar{r}}_i + \bar{r}_i \cdot \ddot{\bar{r}}_i)$
$= 2 \sum_{i=1}^{N} m_i \dot{r}_i^2 + 2 \sum m_i \bar{r}_i \cdot \ddot{\bar{r}}_i$
$4T$
$2 \sum_{i=1}^{N} \bar{r}_i \cdot \nabla_i U$
m_i
$\bar{r}_i$
B
$U(x^{-1}, y^{-1}, z^{-1})$
$2 \sum_{i=1}^{N} \left(x_i \frac{\partial U}{\partial x_i} + y_i \frac{\partial U}{\partial y_i} + z_i \frac{\partial U}{\partial z_i} \right) = -2U$
$\ddot{I} = 4T - 2U \rightarrow > 0$
$4T = 2U$
$2T = |\varepsilon_p|$

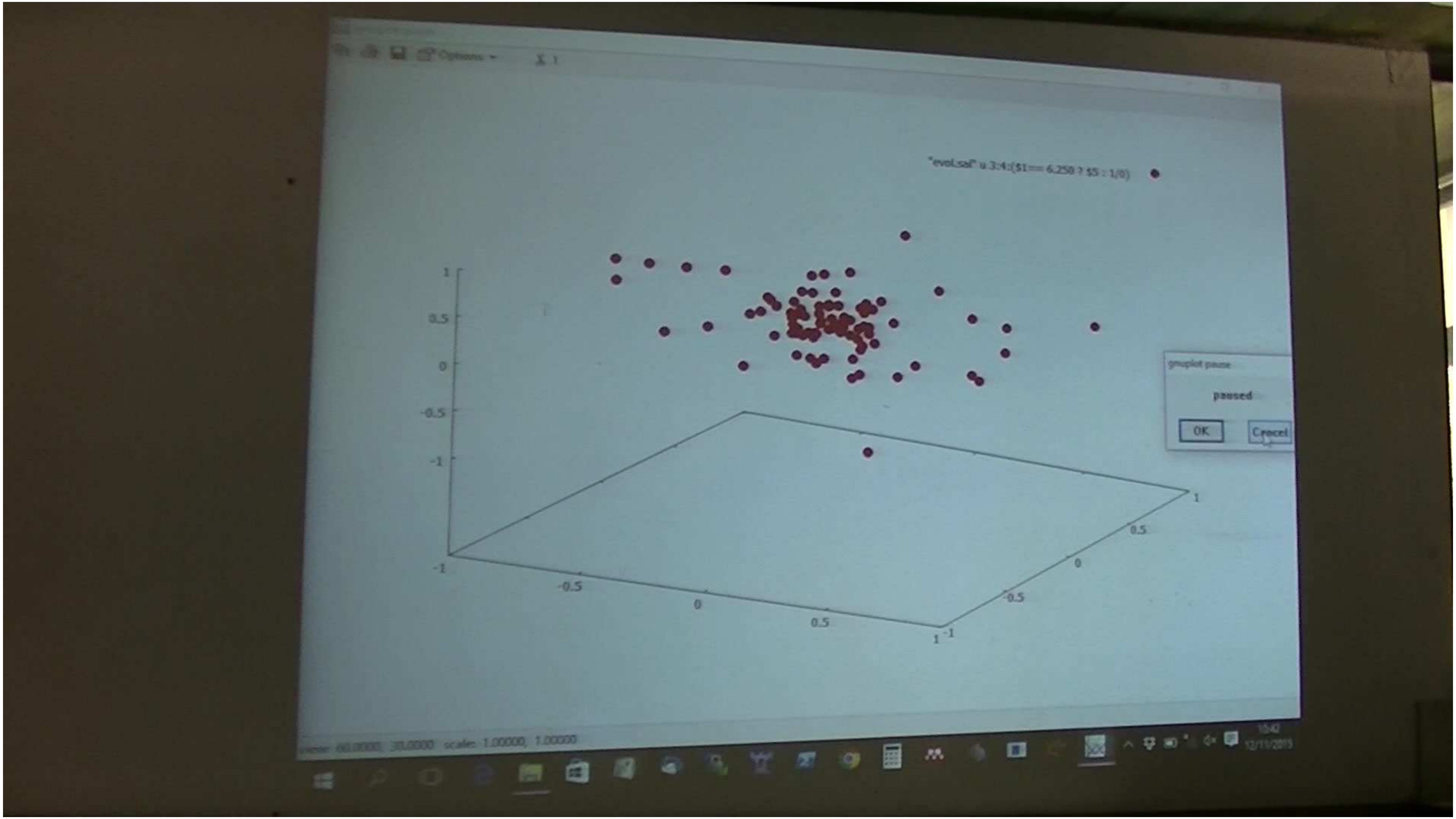


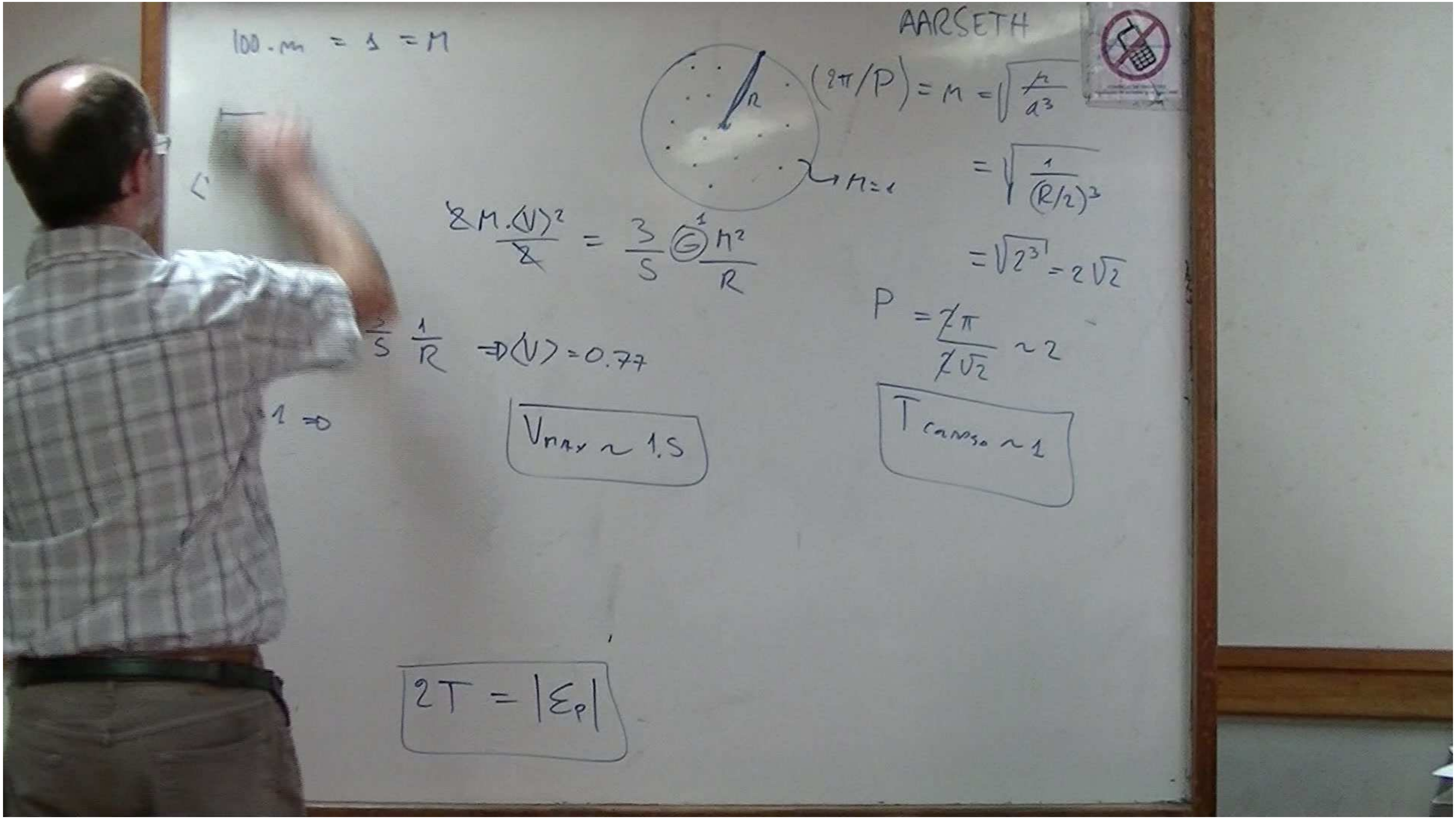
AARSETH
$100 \cdot m = 1 = M$
0
V_{max}
$\langle V \rangle = \frac{V_{max}}{2}$
R
$M = 1$
$2\sum_{i=1}^{N}\left(x_i \frac{\partial U}{\partial x_i} + y_i \frac{\partial U}{\partial y_i} + z_i \frac{\partial U}{\partial z_i}\right) = -2U$
$\ddot{I} = 4T - 2U$
> 0
$4T = 2U$
$2T = |E_p|$

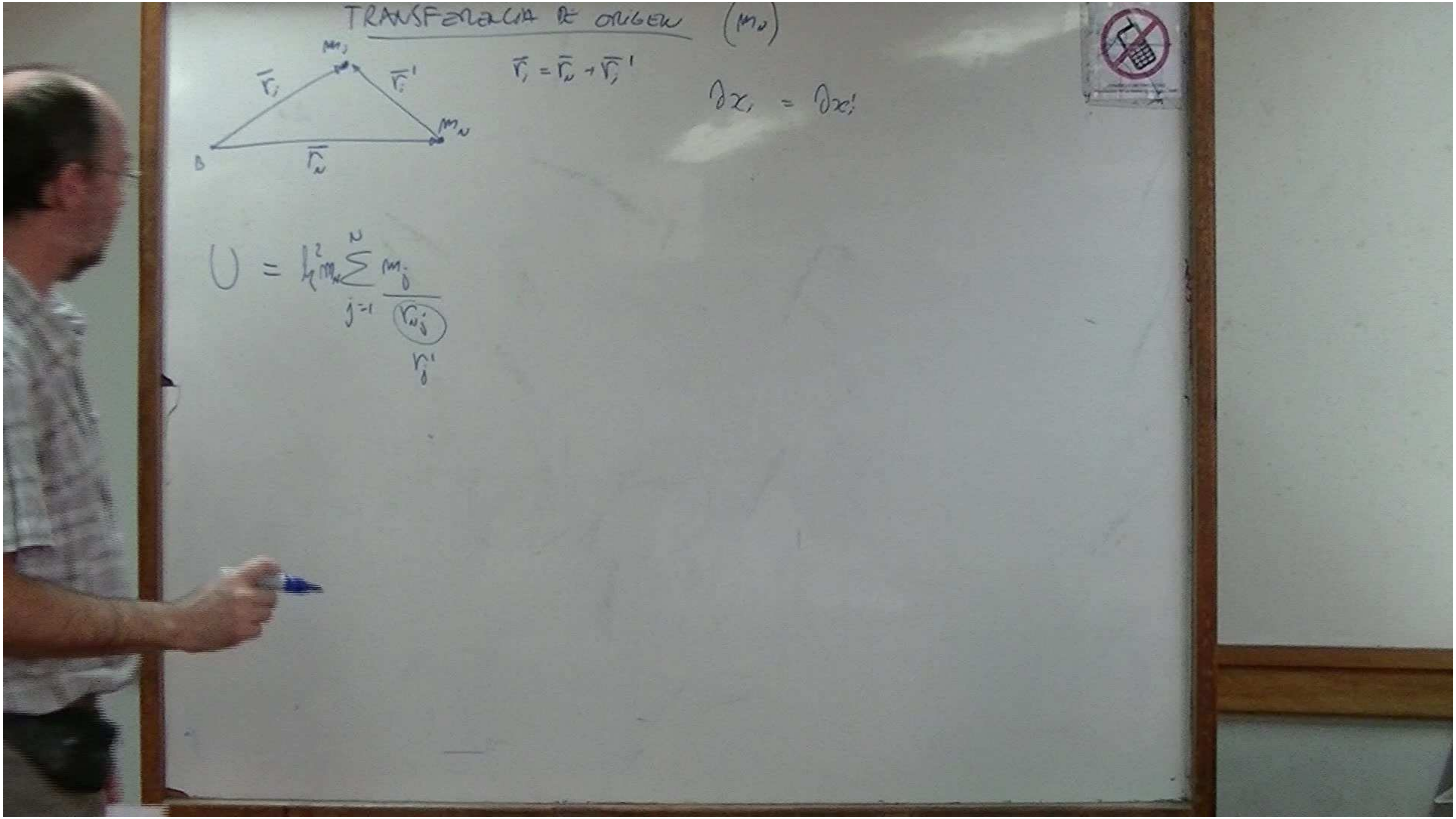

AARSETH
100 · m = 1 = M
0
Vmax
R
M = 1
⟨V⟩ = Vmax / 2
2 M ⟨V⟩² / 2 = 3/5 G M² / R
⟨V⟩² = 3/5 1/R
R = 1 ⇒
2T = |Ep|

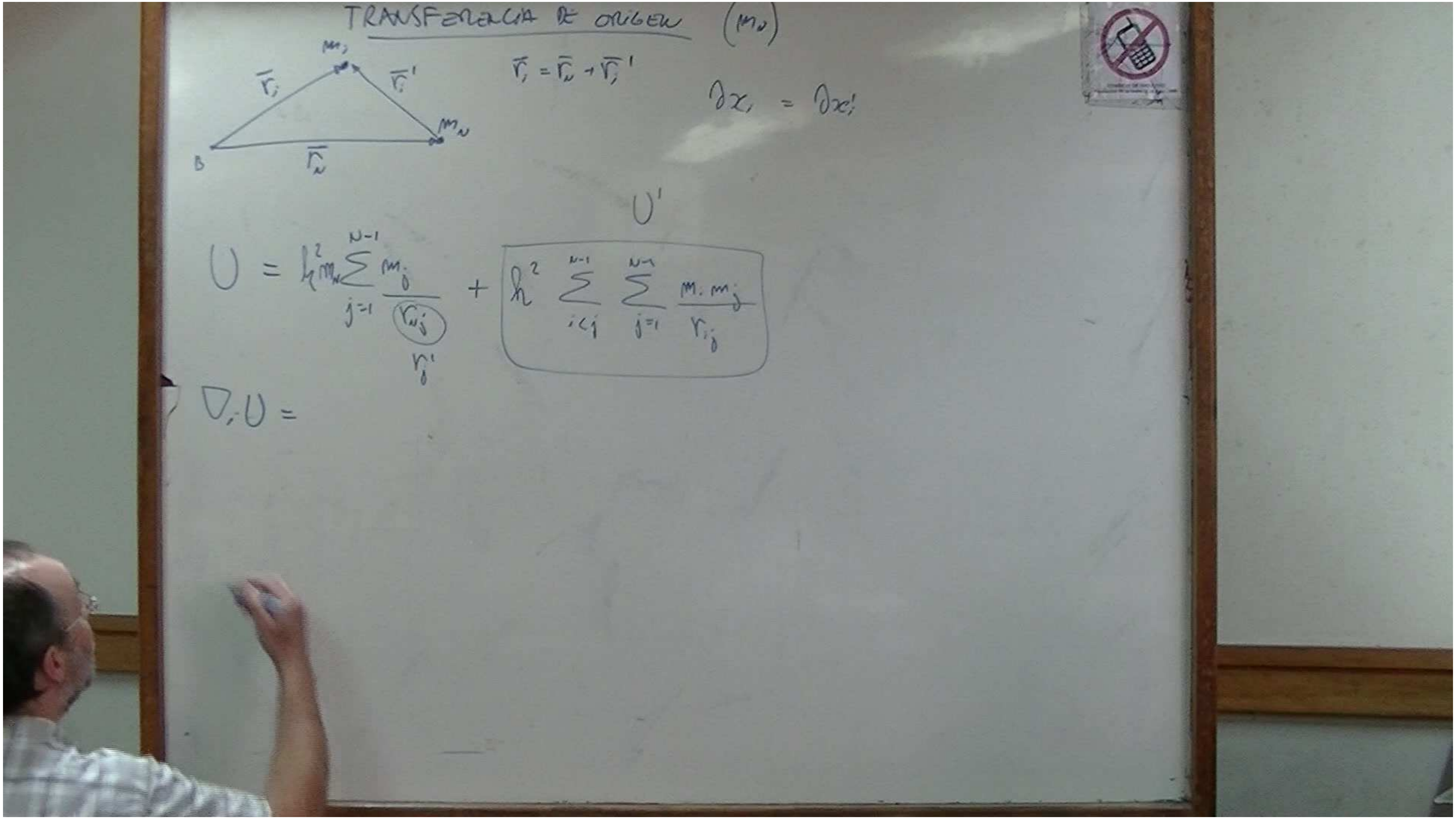

AARSETH
100 . m = 1 = M
0
V max
⟨V⟩ = V max / 2
R = 1 ⇒
2T = |E p|

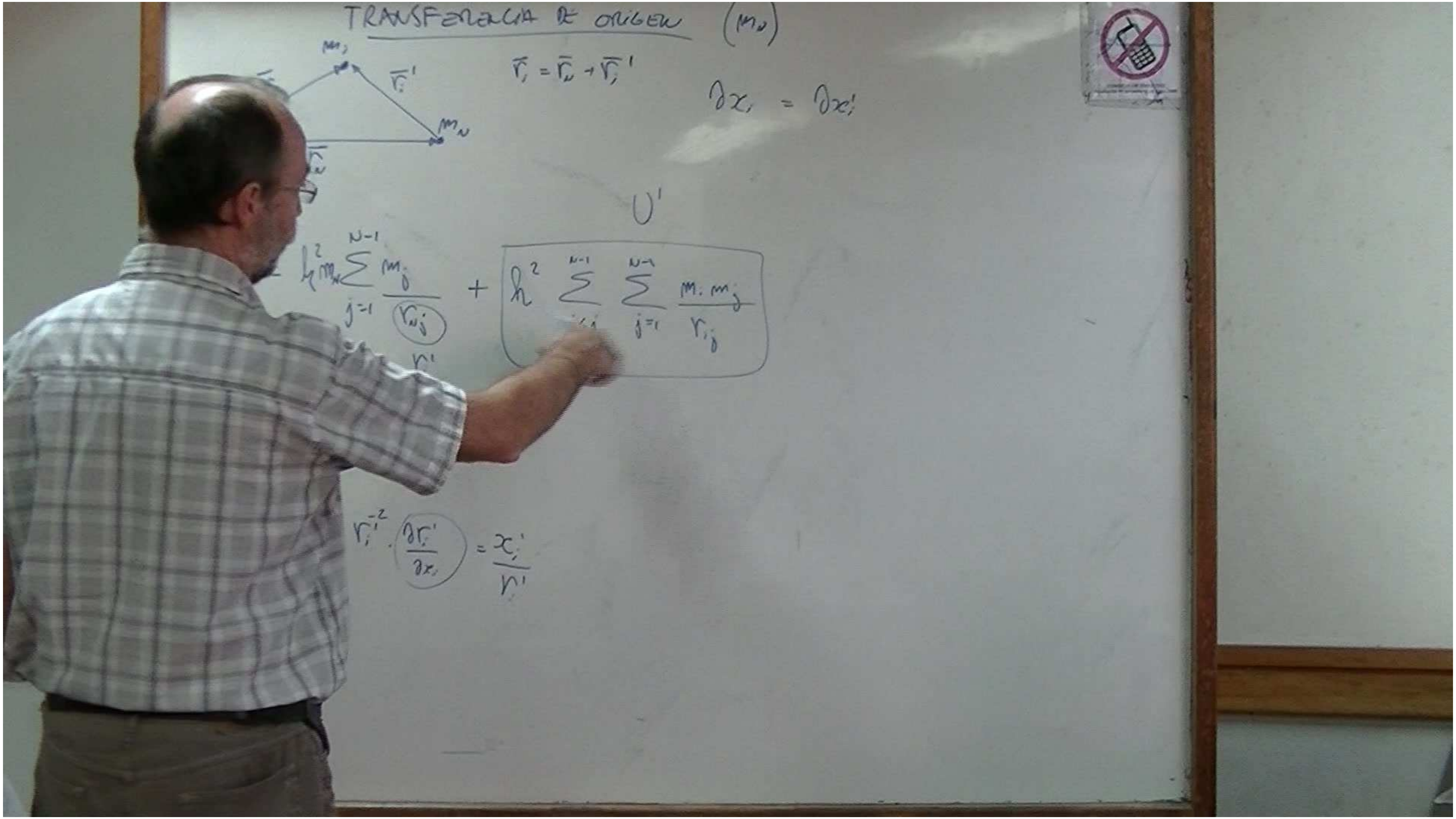

100 . m = 1 = M
0
V_{max}
$\langle V \rangle = \frac{V_{max}}{2}$
$\frac{2M \cdot \langle V \rangle^2}{2} = \frac{3}{5} G \frac{M^2}{R}$
$\langle V \rangle^2 = \frac{3}{5} \frac{1}{R} \Rightarrow \langle V \rangle = 0.77$
$R = 1 \Rightarrow$
$V_{max} \sim 1.5$
$2T = |E_p|$
R
M = 1
AARSETH
$(2\pi/P) = n = \sqrt{\frac{M}{a^3}}$
$= \sqrt{\frac{1}{(R/2)^3}}$
$= \sqrt{2^3} = 2\sqrt{2}$
$P = \frac{2\pi}{2\sqrt{2}} \sim 2$
$T_{cruce} \sim 1$

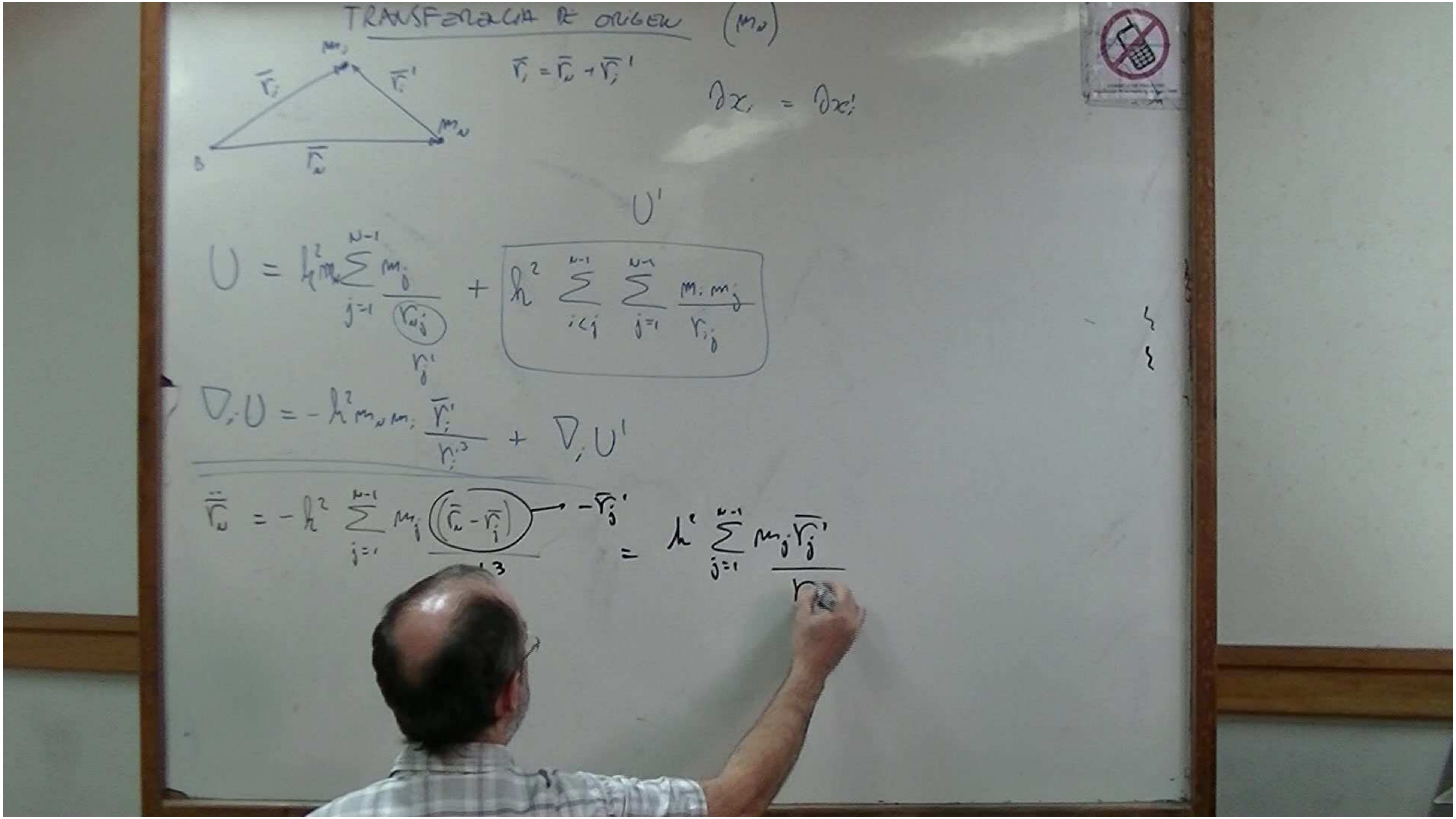

"evol.sal" u 3:4:($1== 6.250 ? $5 : 1/0)
gnuplot pause
paused
OK
Cancel

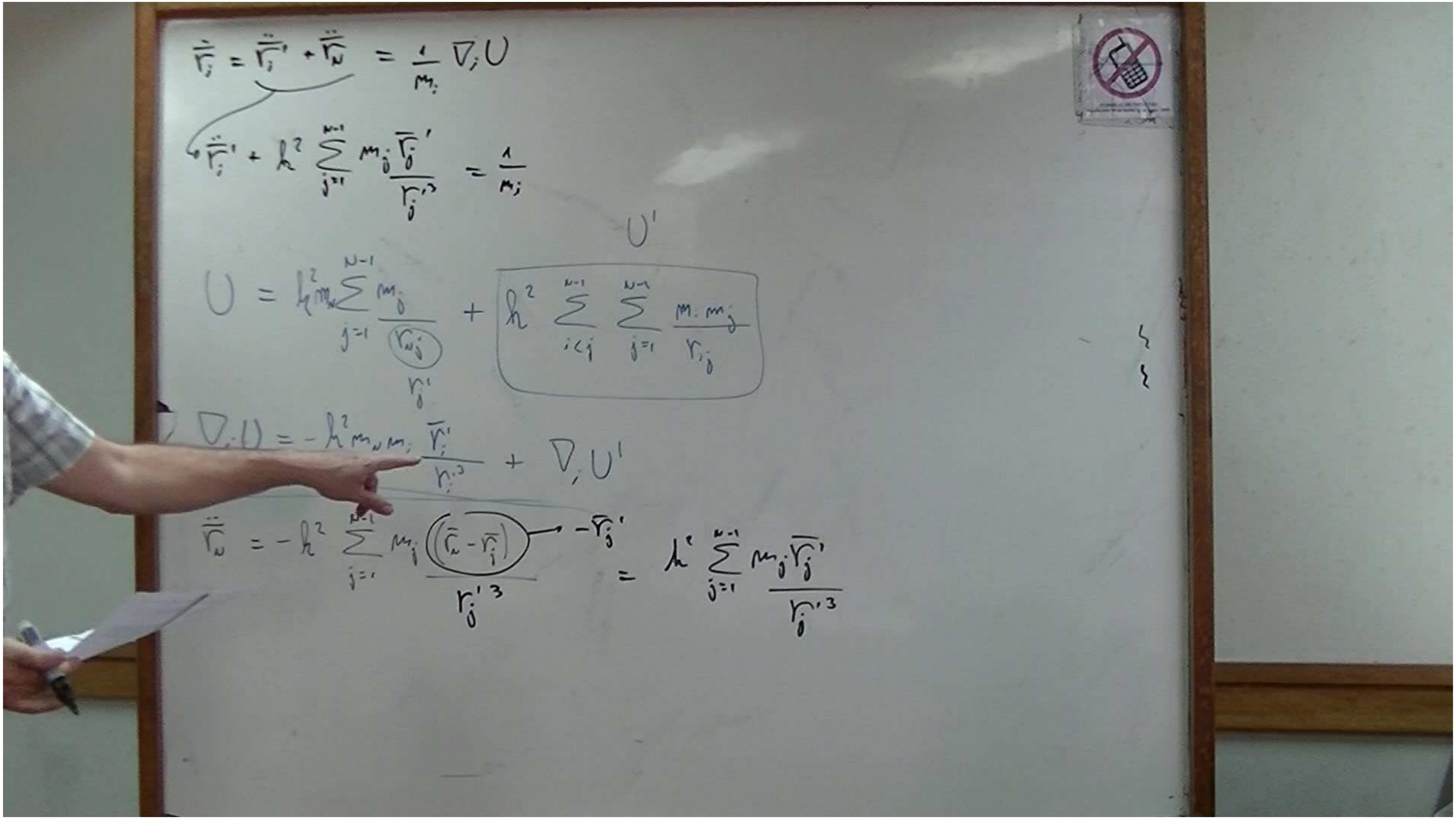

100 . m = 1 = M
AARSETH
R
M=1
(2π/P) = n = √(μ/a³)
= √(1/(R/2)³)
= √2³ = 2√2
P = 2π / 2√2 ~ 2
2M.⟨V⟩² / 2 = 3/5 G M² / R
3/5 1/R ⇒ ⟨V⟩ = 0.77
V max ~ 1.5
T cruce ~ 1
2T = |Ep|

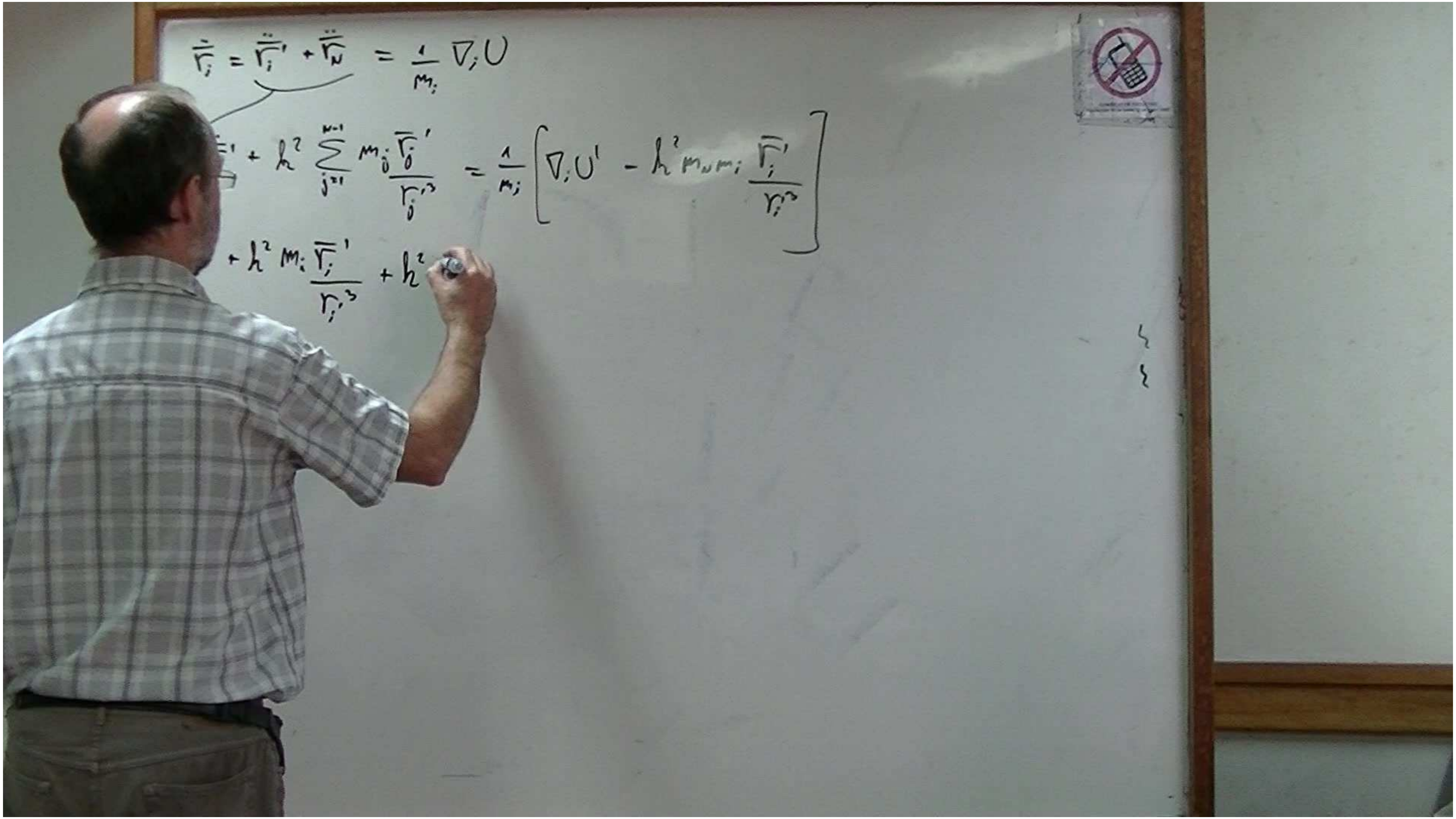

TRANSFERENCIA DE ORIGEN (mN)

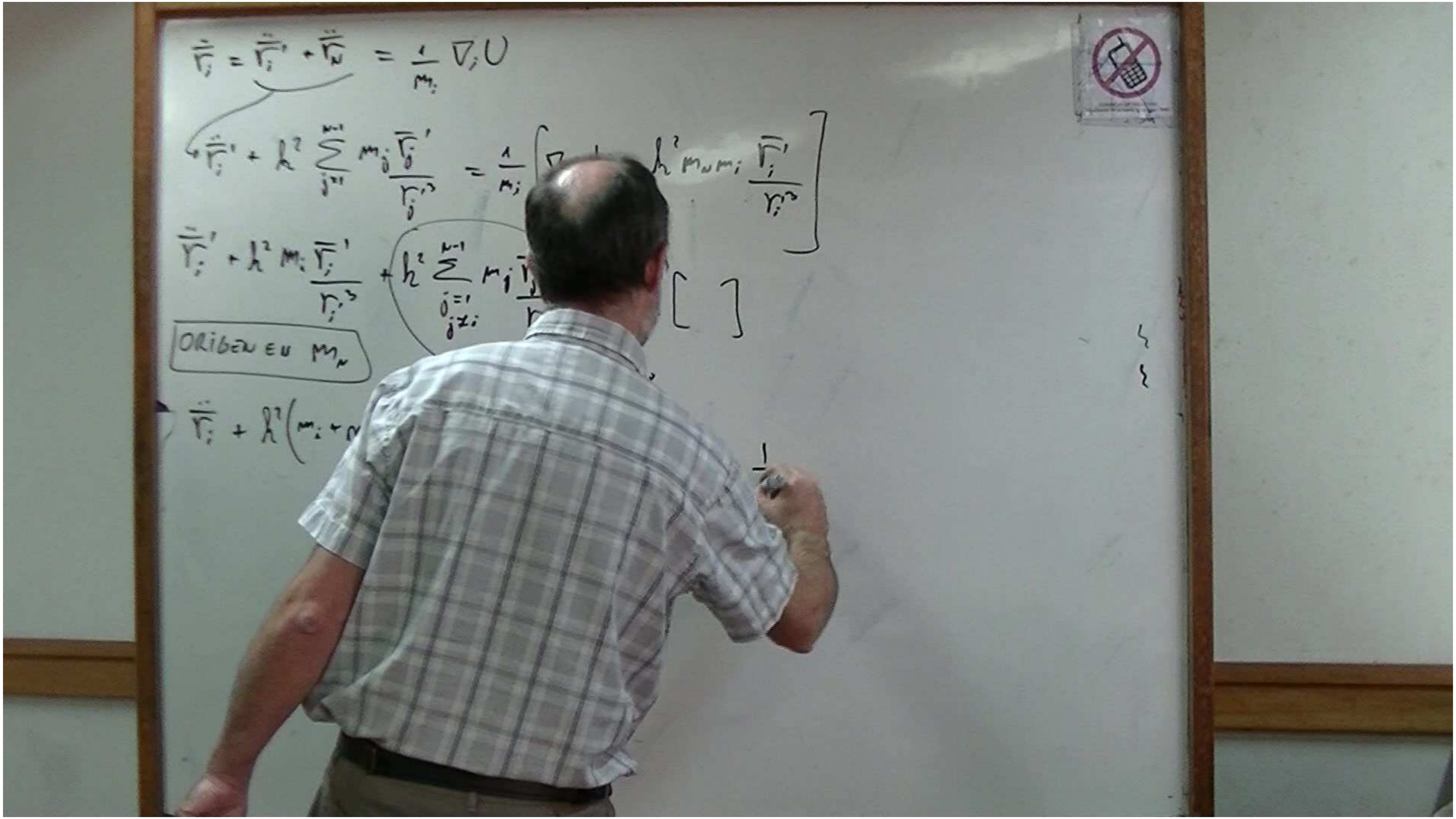

TRANSFERENCIA DE ORIGEN (m_N)
m_j
$\bar{r}_j$
$\bar{r}_j'$
m_N
B
$\bar{r}_N$
$\bar{r}_j = \bar{r}_N + \bar{r}_j'$
$\partial x_i = \partial x_i'$
U'
$U = k^2 m_N \sum_{j=1}^{N-1} \frac{m_j}{r_{Nj}} + \boxed{k^2 \sum_{i<j}^{N-1} \sum_{j=1}^{N-1} \frac{m_i m_j}{r_{ij}}}$
r_j'
$\nabla_i U =$

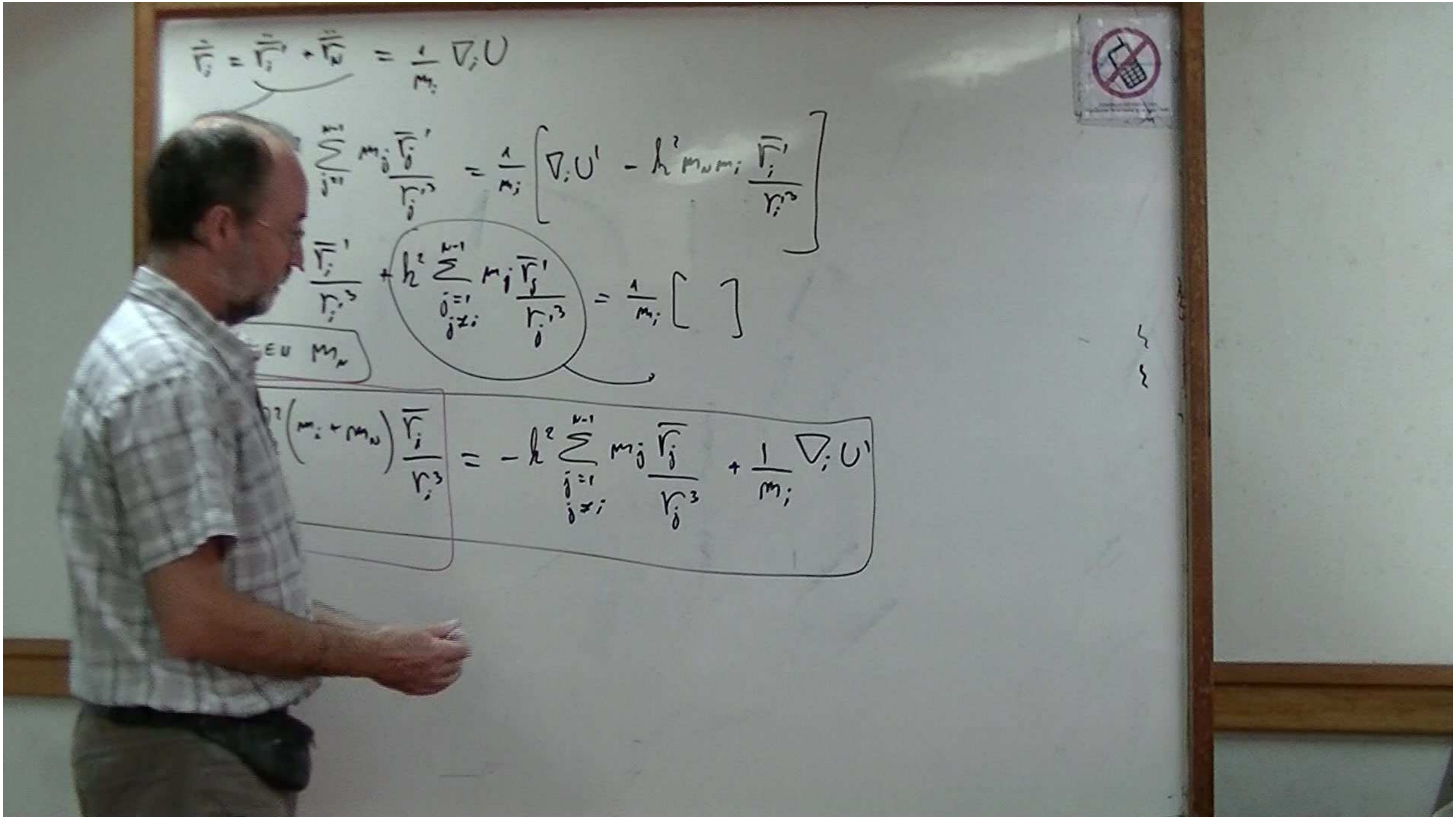

TRANSFERENCIA DE ORIGEN
(m_N)

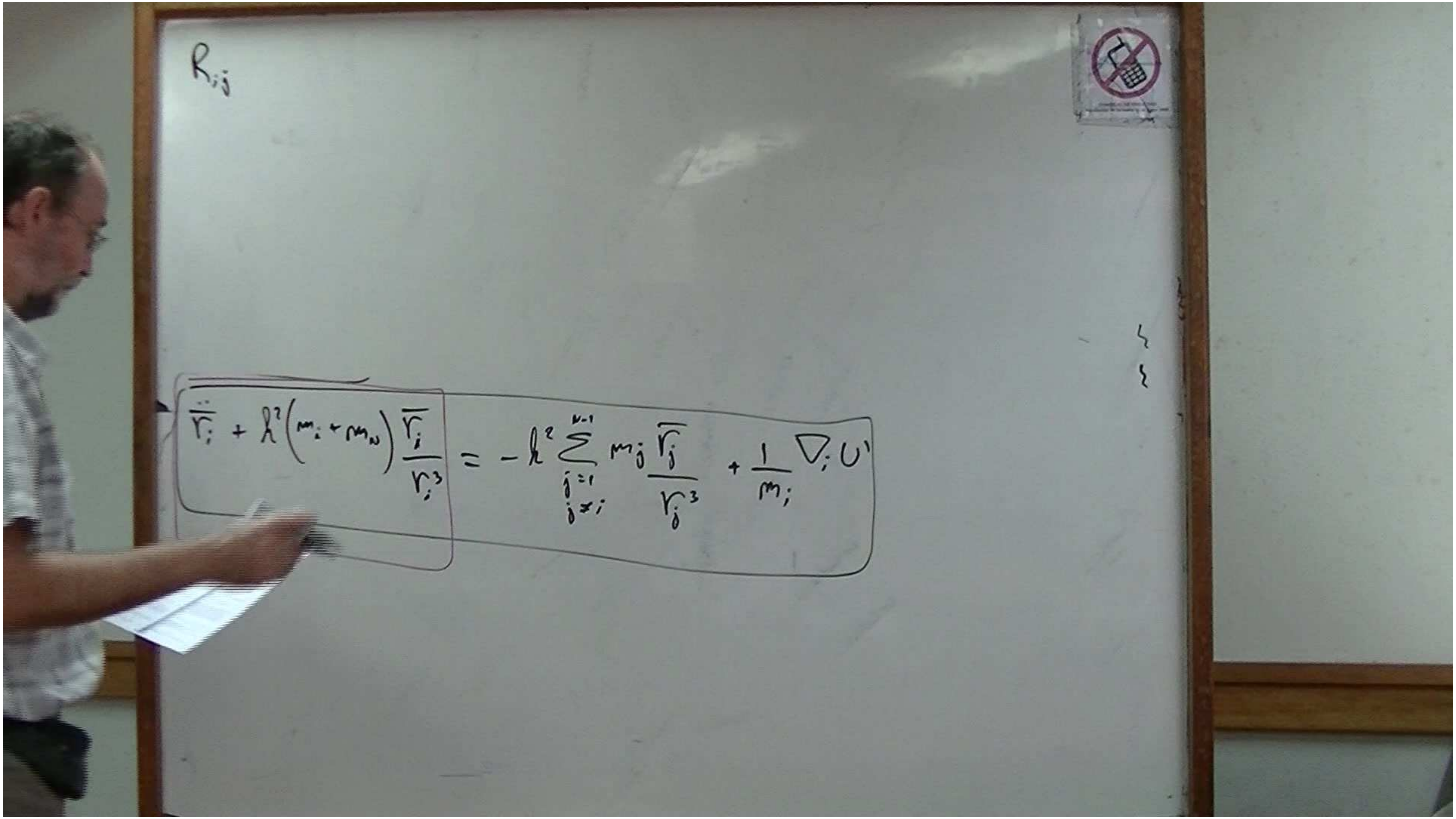

TRANSFERENCIA DE ORIGEN (m_N)
r_i = r_N + r_i'
∂x_i = ∂x_i'
U = k² m_N Σ_{j=1}^{N-1} m_j / r_Nj + k² Σ_{i<j}^{N-1} Σ_{j=1}^{N-1} m_i m_j / r_ij
U'
∇_i U = - k² m_N m_i r_i' / r_i'³ + ∇_i U'
r̈_N = - k² Σ_{j=1}^{N-1} m_j (r_N - r_j) / r³ = k² Σ_{j=1}^{N-1} m_j r_j' / r_j'³

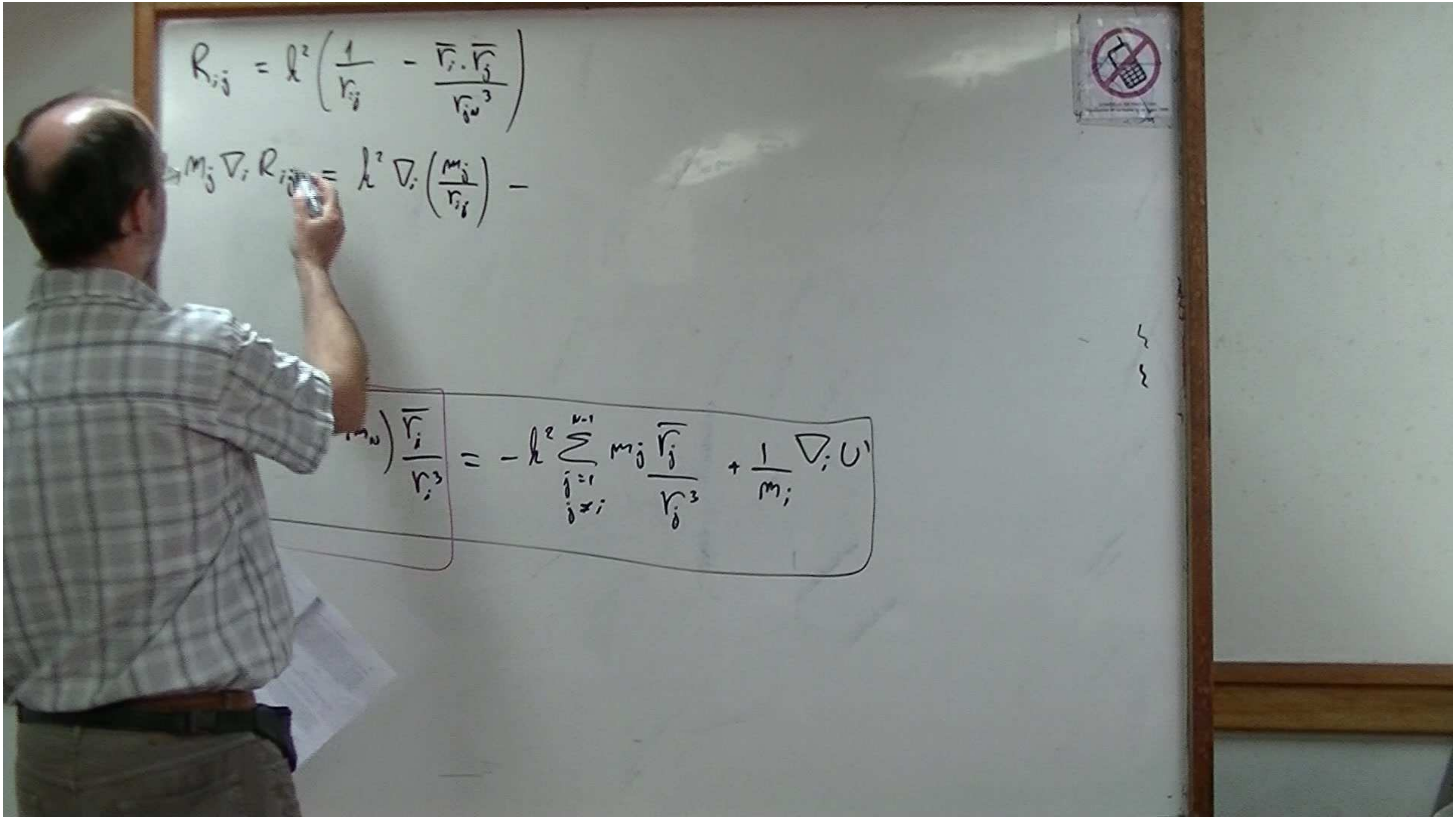

$\ddot{\bar{r}}_i = \ddot{\bar{r}}_i' + \ddot{\bar{r}}_N = \frac{1}{m_i} \nabla_i U$
$\ddot{\bar{r}}_i' + k^2 \sum_{j=1}^{N-1} m_j \frac{\bar{r}_j'}{r_j'^3} = \frac{1}{m_i}$
U'
$U = k^2 m_N \sum_{j=1}^{N-1} \frac{m_j}{r_{Nj}} + \boxed{k^2 \sum_{i<j}^{N-1} \sum_{j=1}^{N-1} \frac{m_i m_j}{r_{ij}}}$
r_j'
$\nabla_i U = -k^2 m_N m_i \frac{\bar{r}_i'}{r_i'^3} + \nabla_i U'$
$\ddot{\bar{r}}_N = -k^2 \sum_{j=1}^{N-1} m_j \frac{(\bar{r}_N - \bar{r}_j)}{r_j'^3}$
$-\bar{r}_j'$
$= k^2 \sum_{j=1}^{N-1} m_j \frac{\bar{r}_j'}{r_j'^3}$

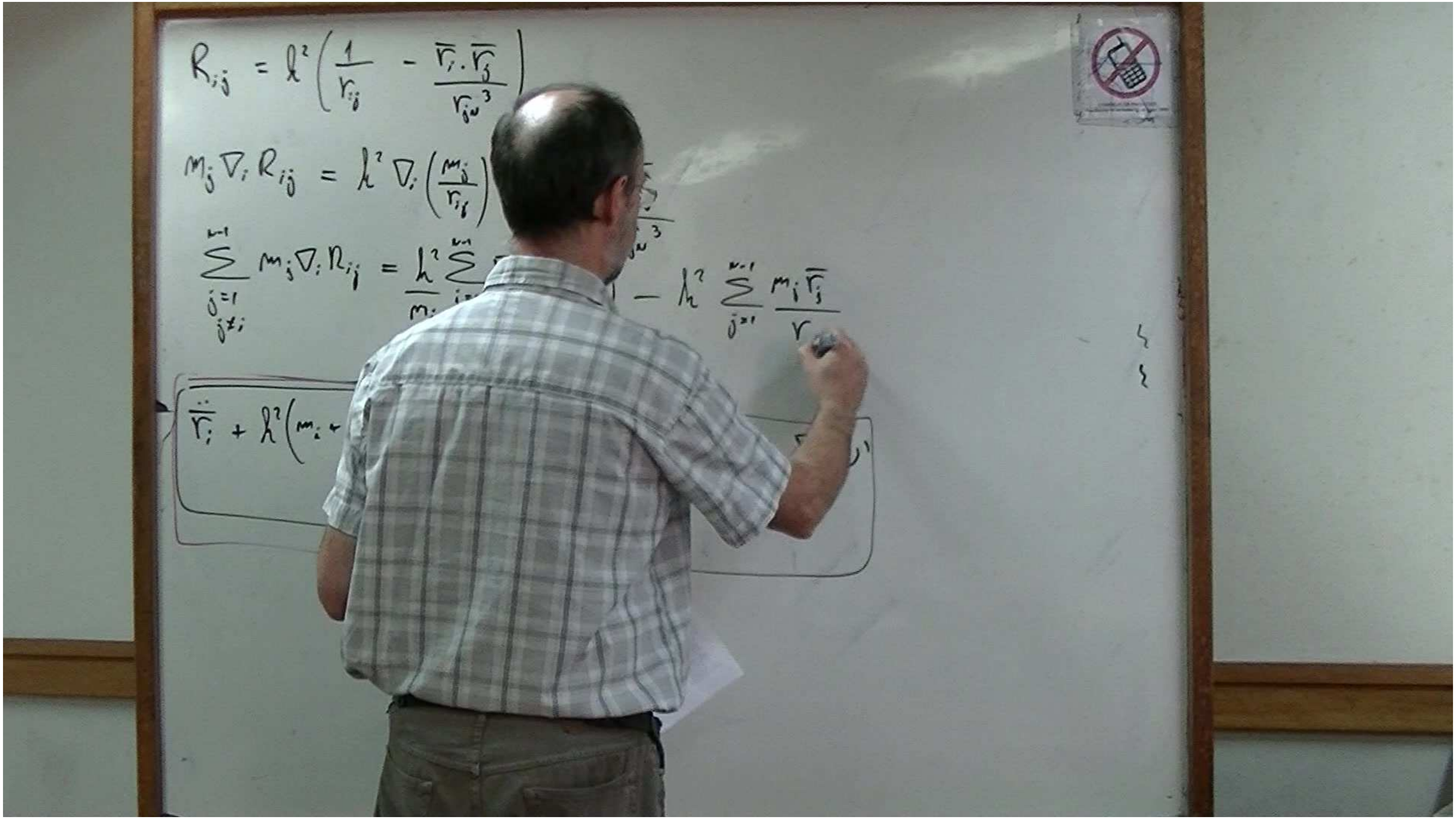

$\ddot{\bar{r}}_i = \ddot{\bar{r}}_i' + \ddot{\bar{r}}_N = \frac{1}{m_i} \nabla_i U$
$+ k^2 \sum_{j=1}^{N-1} m_j \frac{\bar{r}_j'}{r_j'^3} = \frac{1}{m_i} \left[\nabla_i U' - k^2 m_N m_i \frac{\bar{r}_i'}{r_i'^3} \right]$
$+ k^2 m_i \frac{\bar{r}_i'}{r_i'^3} + k^2$

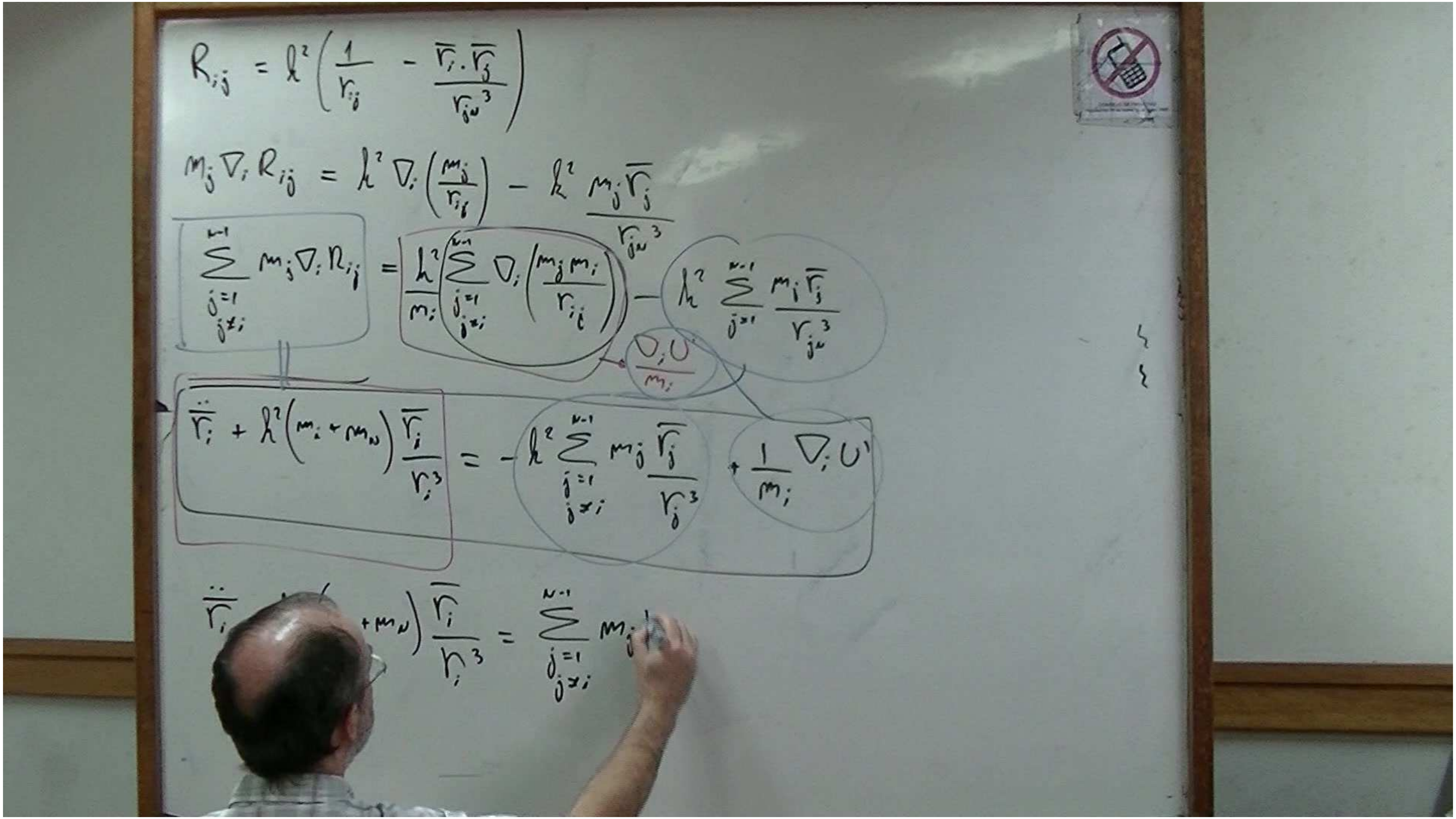

ORIGEN EN m_N


$\ddot{\bar r}_i = \ddot{\bar r}_i' + \ddot{\bar r}_N = \frac{1}{m_i}\nabla_i U$
$\sum_{j=1}^{N-1} m_j \frac{\bar r_j'}{r_j'^3} = \frac{1}{m_i}\left[\nabla_i U' - k^2 m_N m_i \frac{\bar r_i'}{r_i'^3}\right]$
$\frac{\bar r_i'}{r_i'^3} + k^2 \sum_{\substack{j=1\\ j\neq i}}^{N-1} m_j \frac{\bar r_j'}{r_j'^3} = \frac{1}{m_i}[\quad]$
eu m_N
$(m_i + m_N)\frac{\bar r_i}{r_i^3} = -k^2 \sum_{\substack{j=1\\ j\neq i}}^{N-1} m_j \frac{\bar r_j}{r_j^3} + \frac{1}{m_i}\nabla_i U'$


R_{ij}
$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = -k^2 \sum_{\substack{j=1 \\ j \neq i}}^{N-1} m_j \frac{\bar{r}_j}{r_j^3} + \frac{1}{m_i} \nabla_i U'$


$R_{ij} = k^2 \left(\frac{1}{r_{ij}} - \frac{\bar{r}_i \cdot \bar{r}_j}{r_{jN}^3} \right)$
$m_j \nabla_i R_{ij} = k^2 \nabla_i \left(\frac{m_j}{r_{ij}} \right) -$
$m_N) \frac{\bar{r}_i}{r_i^3} = -k^2 \sum_{\substack{j=1 \\ j \neq i}}^{N-1} m_j \frac{\bar{r}_j}{r_j^3} + \frac{1}{m_i} \nabla_i U'$





$R_{ij} = k^2\left(\frac{1}{r_{ij}} - \frac{\bar{r}_i \cdot \bar{r}_j}{r_{jN}^3}\right)$
$m_j \nabla_i R_{ij} = k^2 \nabla_i \left(\frac{m_j}{r_{ij}}\right) - k^2 \frac{m_j \bar{r}_j}{r_{jN}^3}$
$\sum_{\substack{j=1 \\ j\neq i}}^{N-1} m_j \nabla_i R_{ij} = \frac{k^2}{m_i} \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \nabla_i \left(\frac{m_j m_i}{r_{ij}}\right) - k^2 \sum_{j=1}^{N-1} \frac{m_j \bar{r}_j}{r_{jN}^3}$
$\frac{\nabla_i U}{m_i}$
$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = -k^2 \sum_{\substack{j=1 \\ j\neq i}}^{N-1} m_j \frac{\bar{r}_j}{r_j^3} + \frac{1}{m_i} \nabla_i U'$
$\ddot{\bar{r}}_i \quad + m_N) \frac{\bar{r}_i}{r_i^3} = \sum_{\substack{j=1 \\ j\neq i}}^{N-1} m_j$

$$R_{ij} = k^2\left(\frac{1}{r_{ij}} - \frac{\bar{r}_i \cdot \bar{r}_j}{r_{jN}^3}\right)$$

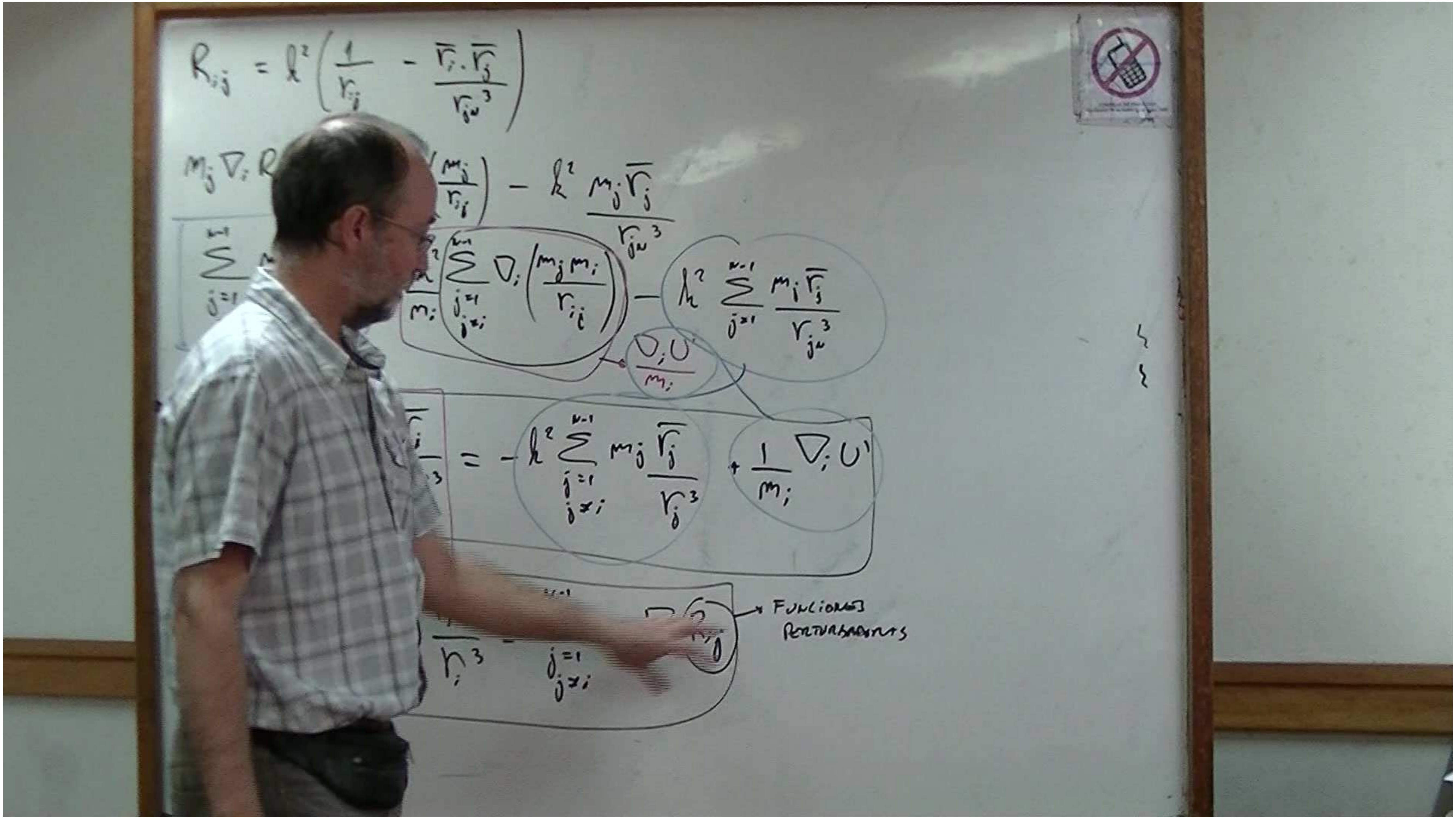
$$m_j \nabla_i R_{ij} = k^2 \nabla_i\left(\frac{m_j}{r_{ij}}\right) - k^2 \frac{m_j \bar{r}_j}{r_{jN}^3}$$

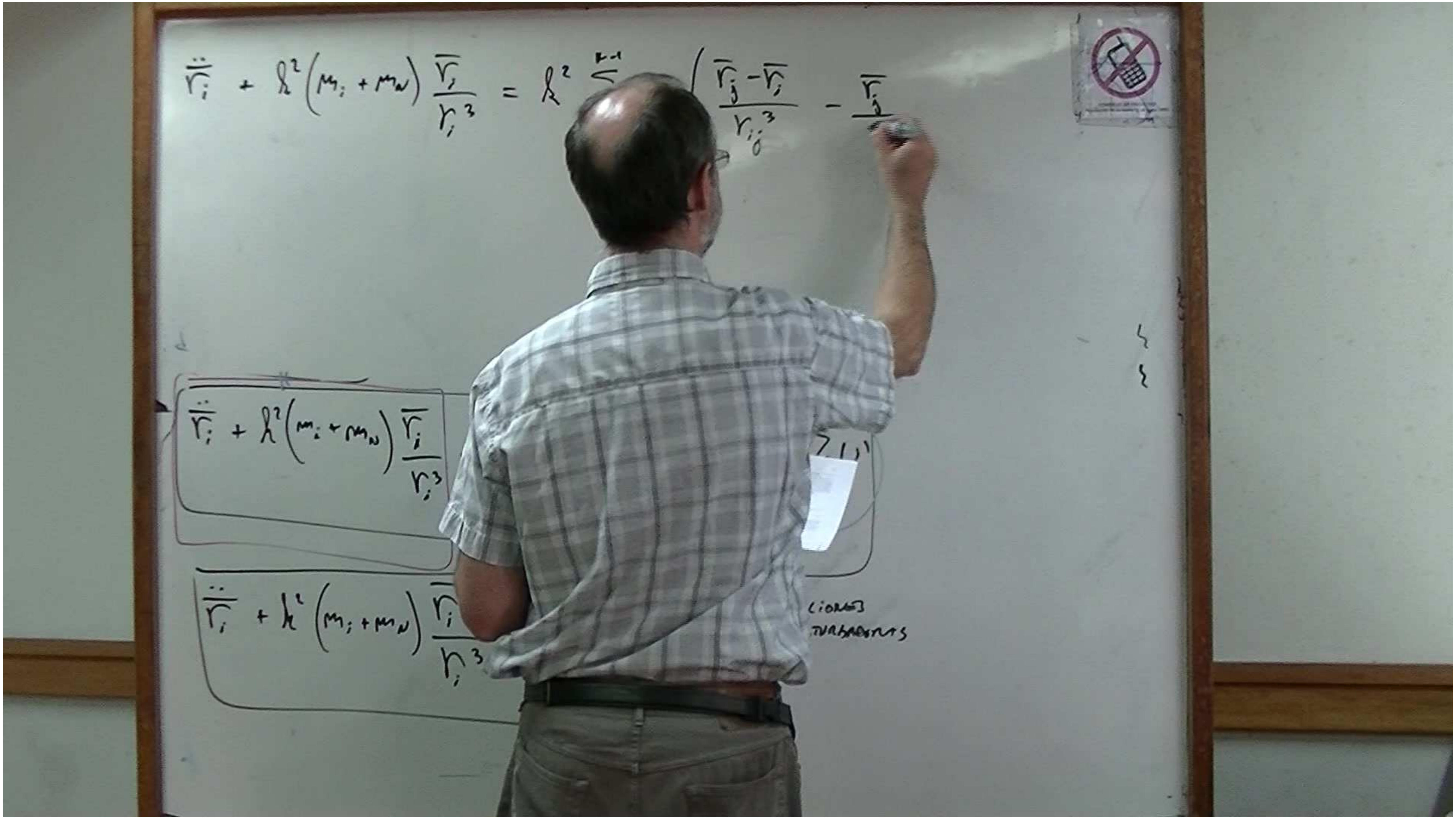
$$\sum_{\substack{j=1\\ j\neq i}}^{N-1} m_j \nabla_i R_{ij} = \frac{k^2}{m_i} \underbrace{\sum_{\substack{j=1\\ j\neq i}}^{N-1} \nabla_i\left(\frac{m_j m_i}{r_{ij}}\right)}_{\frac{\nabla_i U'}{m_i}} - k^2 \sum_{j=1}^{N-1} \frac{m_j \bar{r}_j}{r_{jN}^3}$$

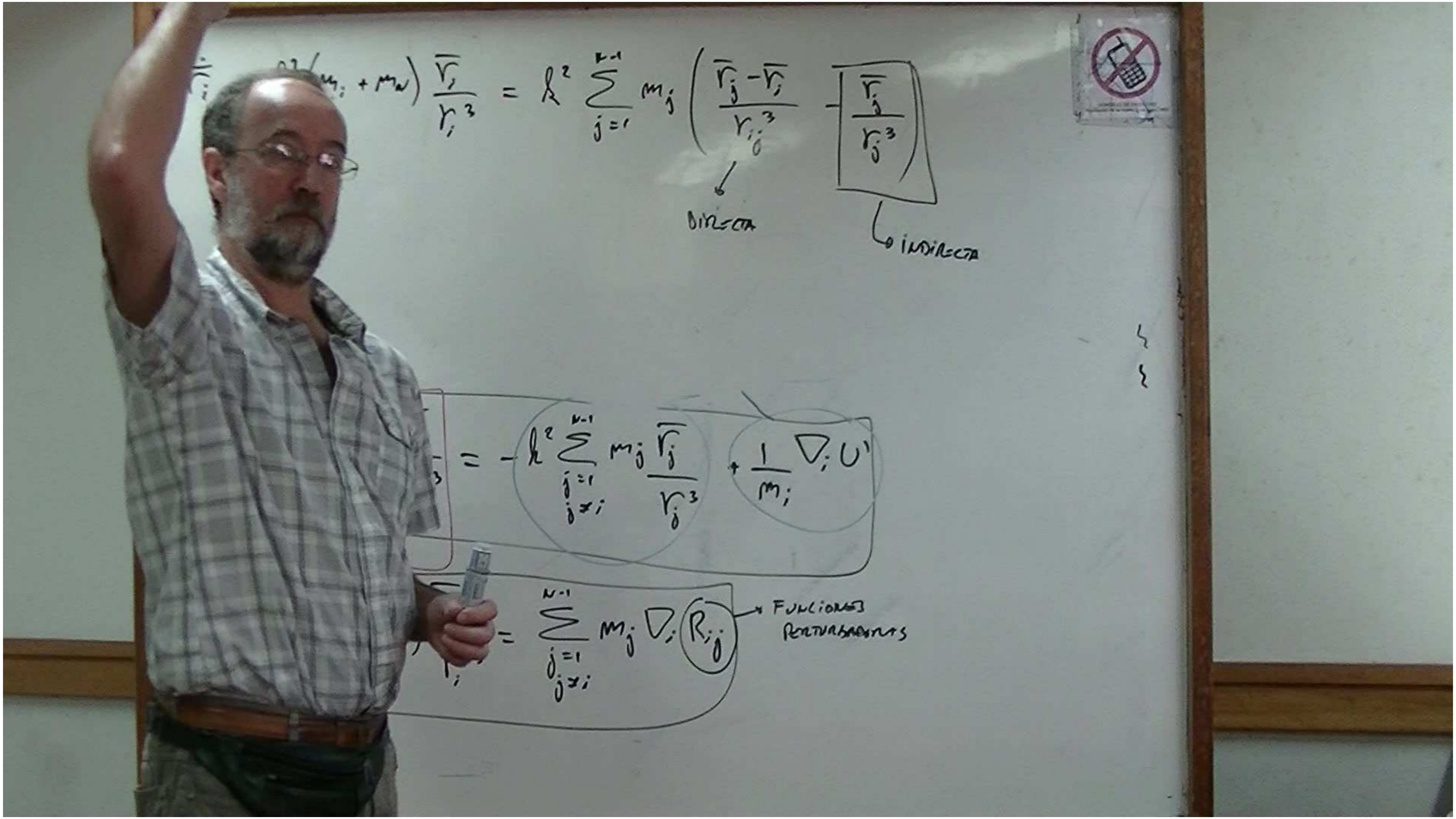
$$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = -k^2 \sum_{\substack{j=1\\ j\neq i}}^{N-1} m_j \frac{\bar{r}_j}{r_j^3} + \frac{1}{m_i} \nabla_i U'$$

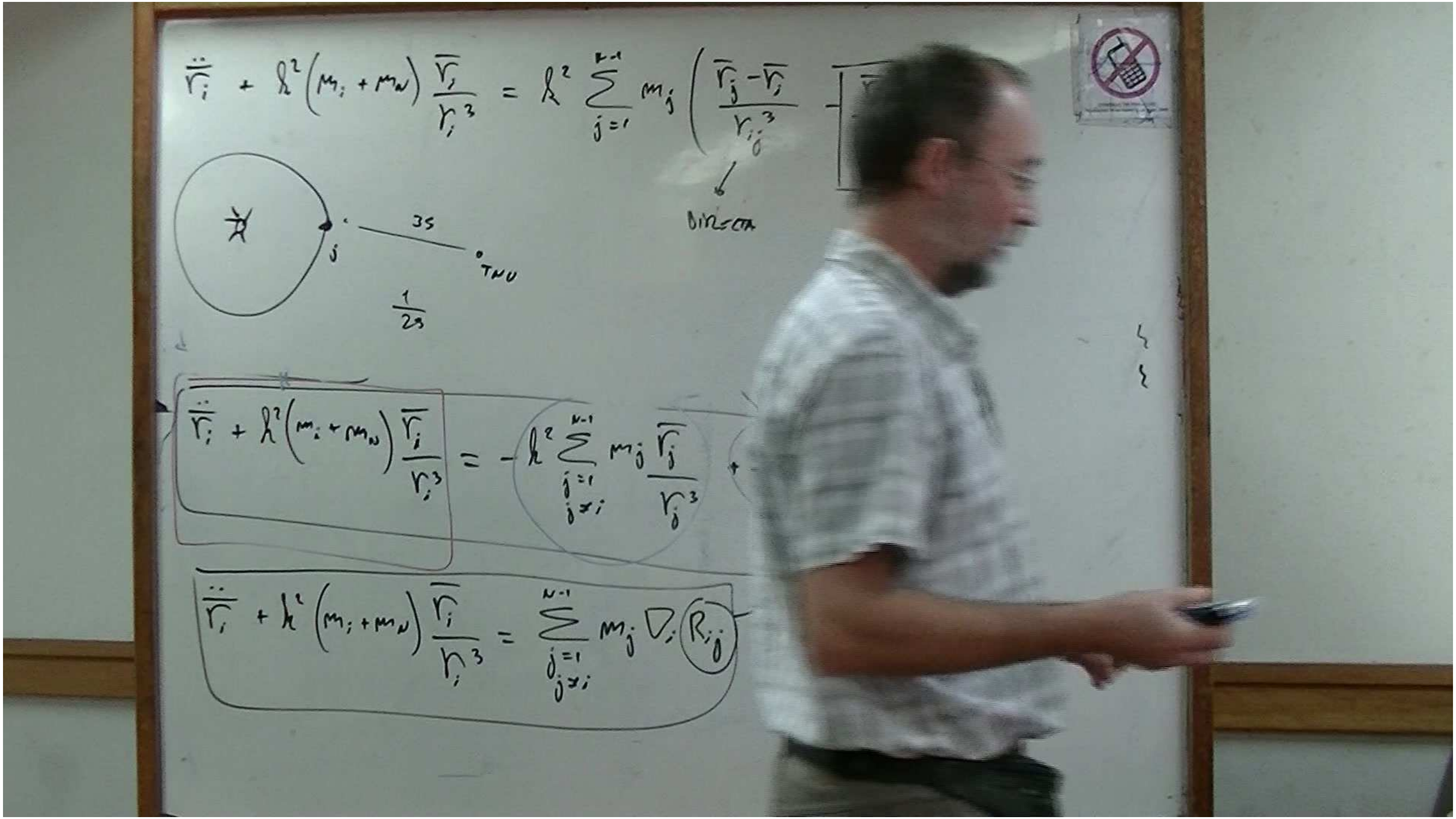
$$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = \sum_{\substack{j=1\\ j\neq i}}^{N-1} m_j \nabla_i R_{ij}$$

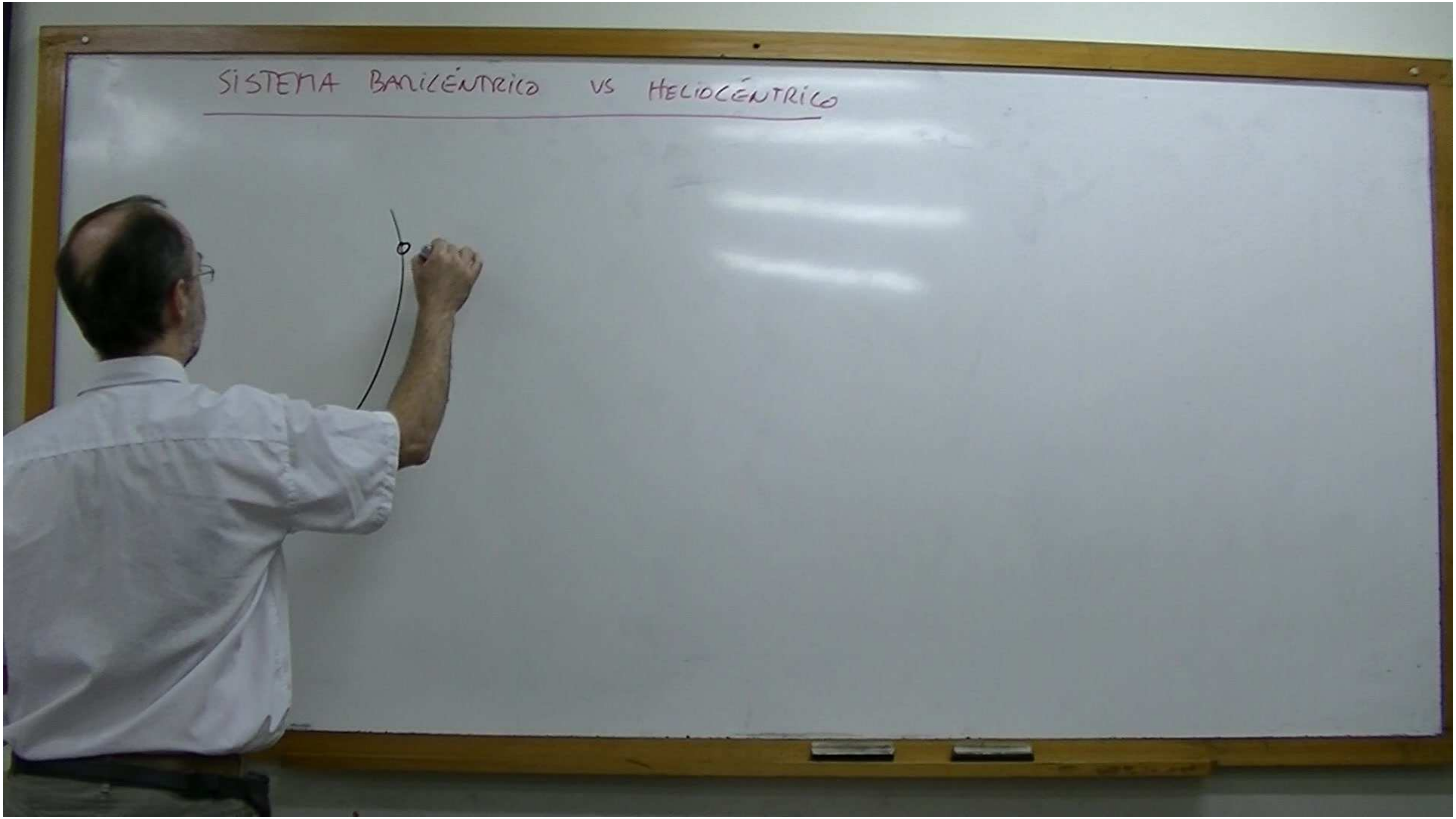
$R_{ij} \rightarrow$ FUNCIONES PERTURBADORAS

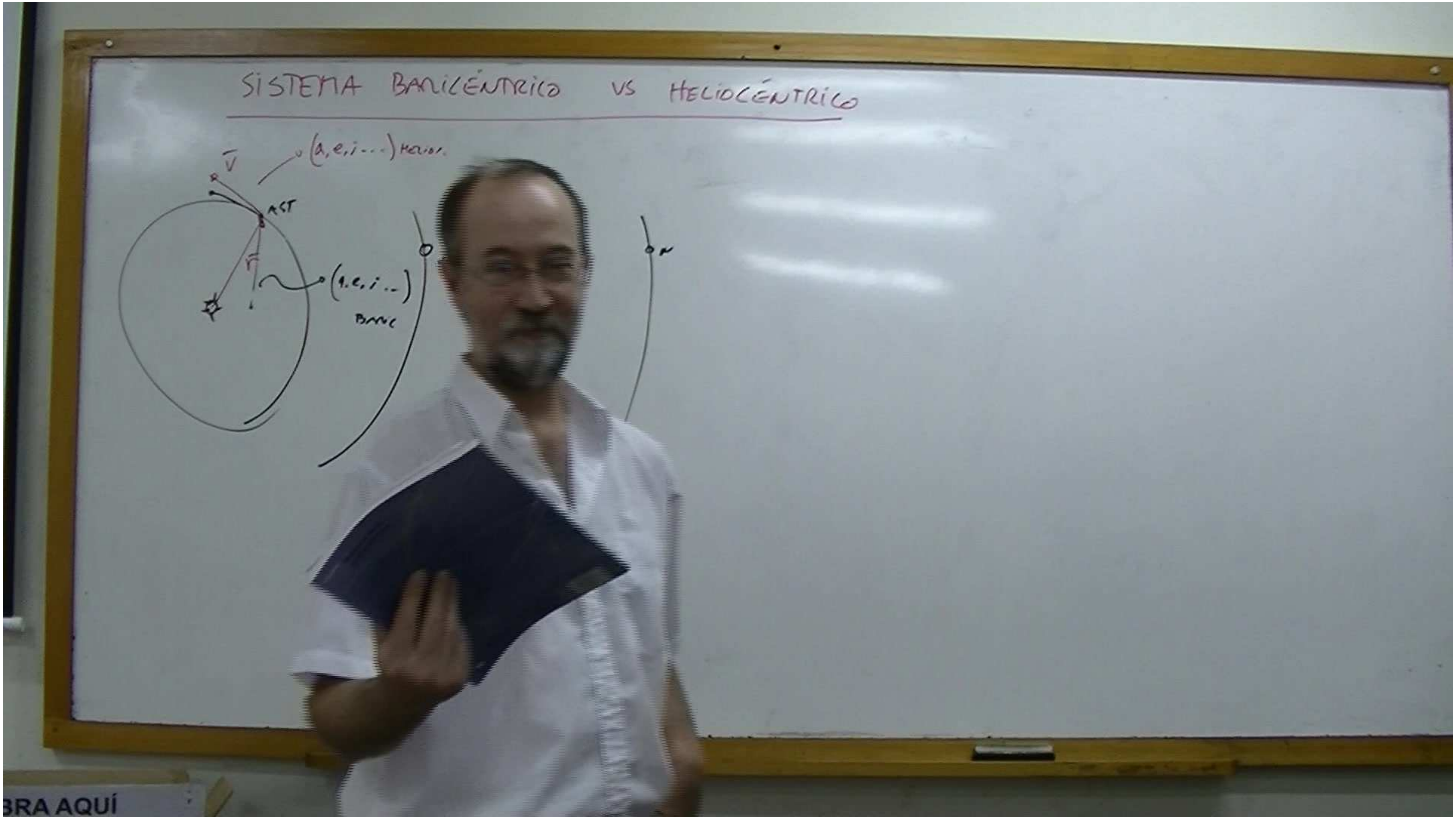

FUNCIONES
PERTURBADORAS

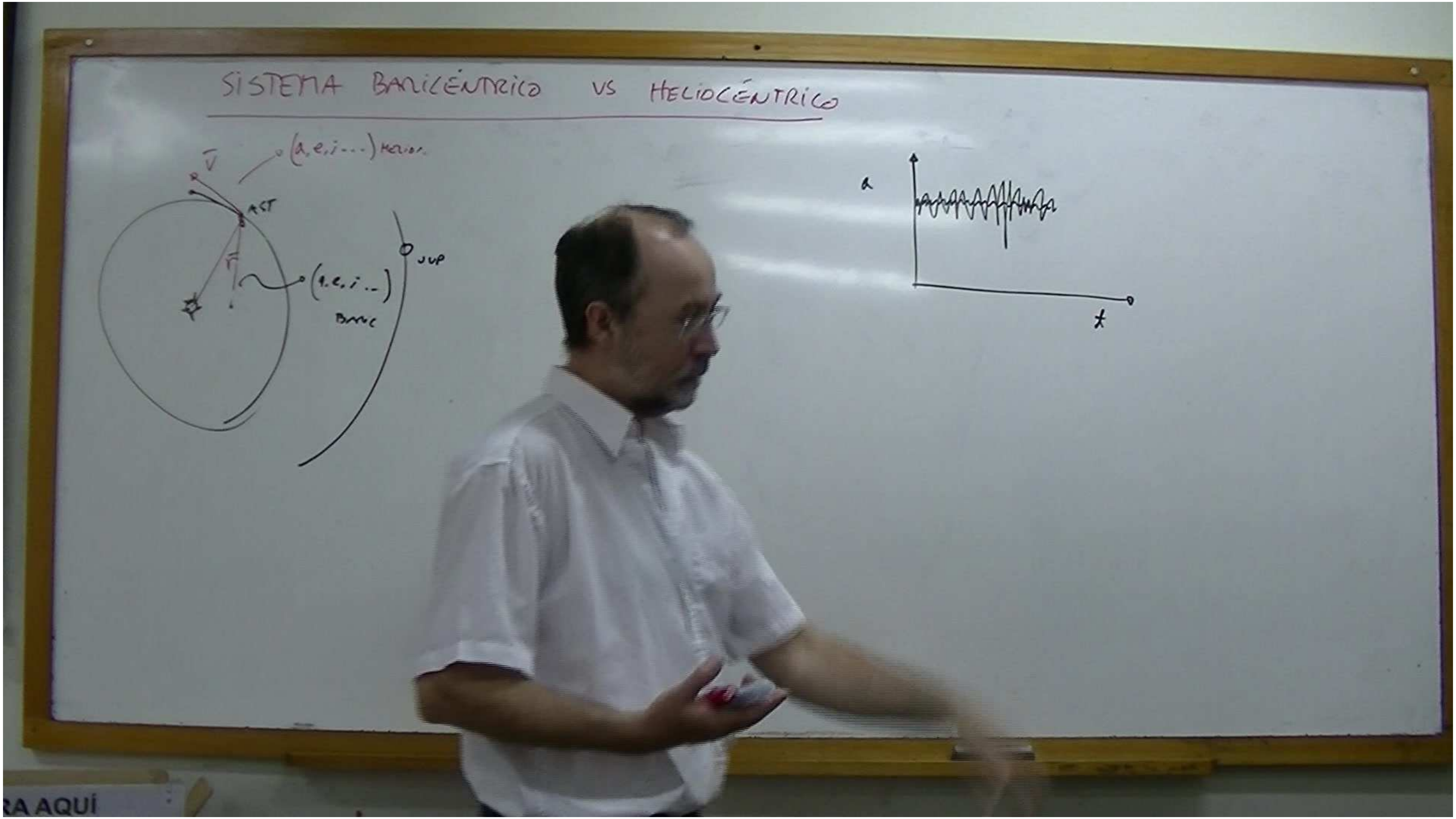


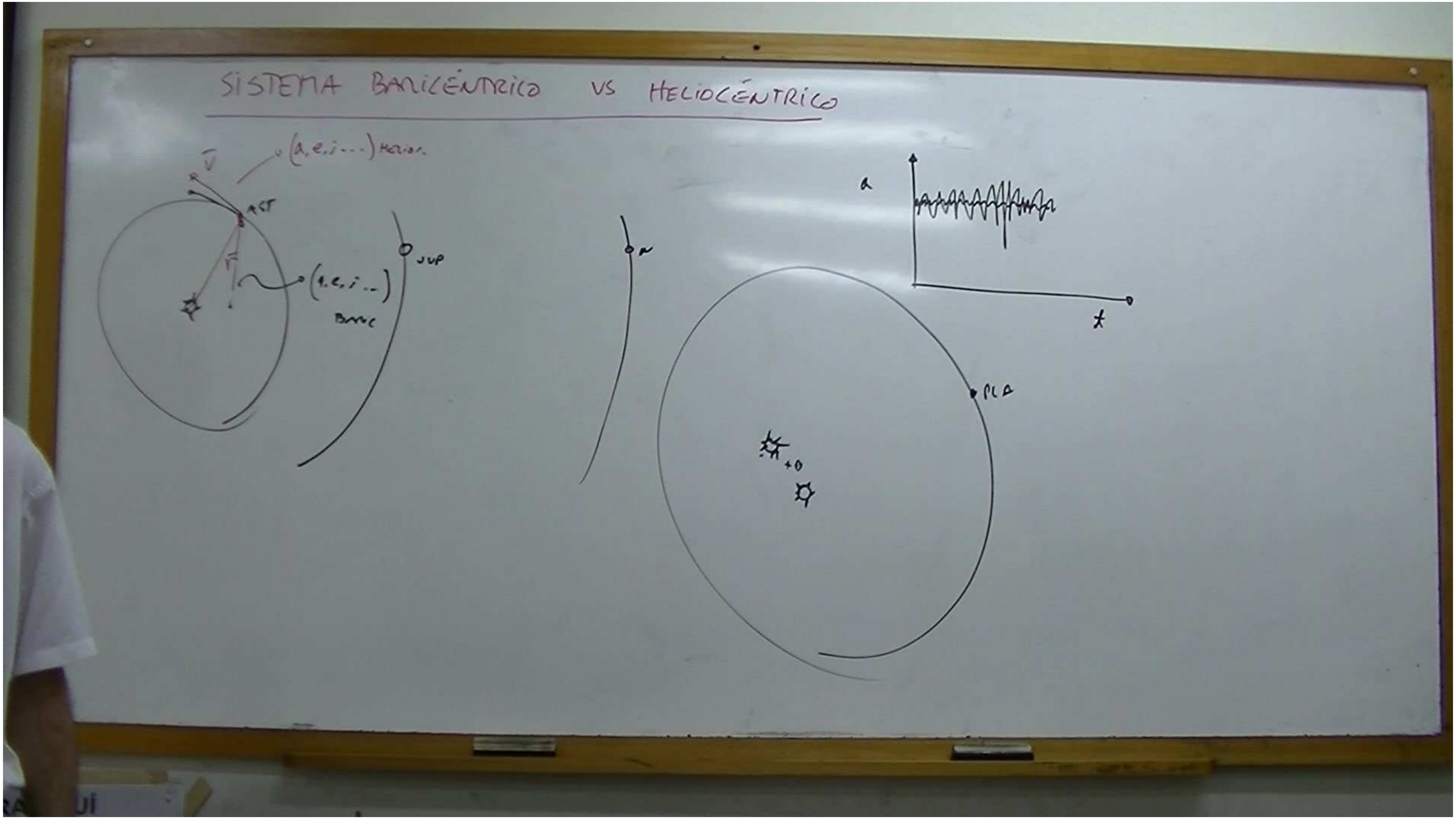

DIRECTA
INDIRECTA
FUNCIONES
PERTURBADORAS

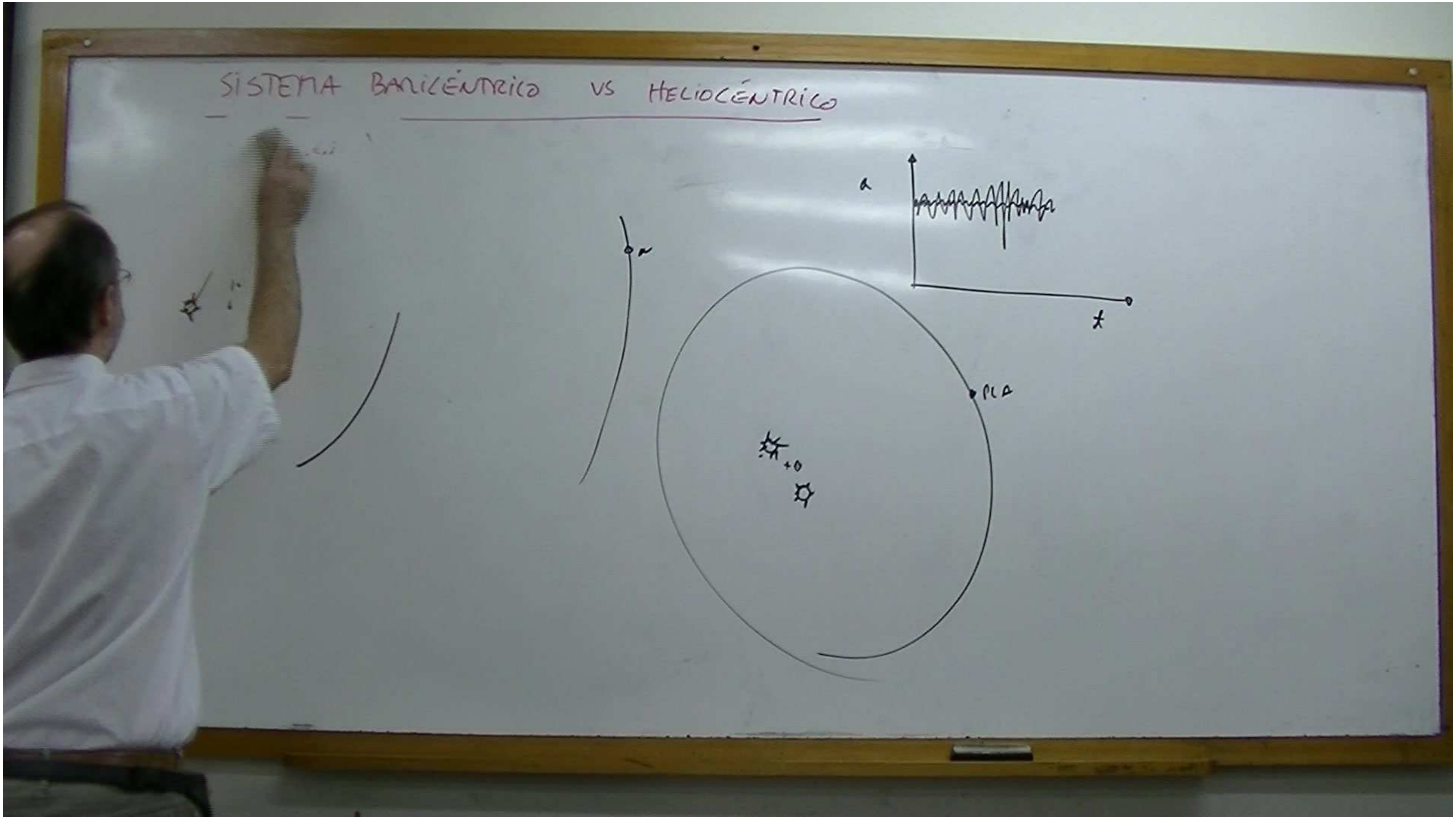

$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = k^2 \sum_{j=1}^{N-1} m_j \left(\frac{\bar{r}_j - \bar{r}_i}{r_{ij}^3} - \right.$
DIRECTA
35
TNU
j
$\frac{1}{23}$
$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = -k^2 \sum_{\substack{j=1 \\ j \neq i}}^{N-1} m_j \frac{\bar{r}_j}{r_j^3} +$
$\ddot{\bar{r}}_i + k^2 (m_i + m_N) \frac{\bar{r}_i}{r_i^3} = \sum_{\substack{j=1 \\ j \neq i}}^{N-1} m_j \nabla_i R_{ij}$

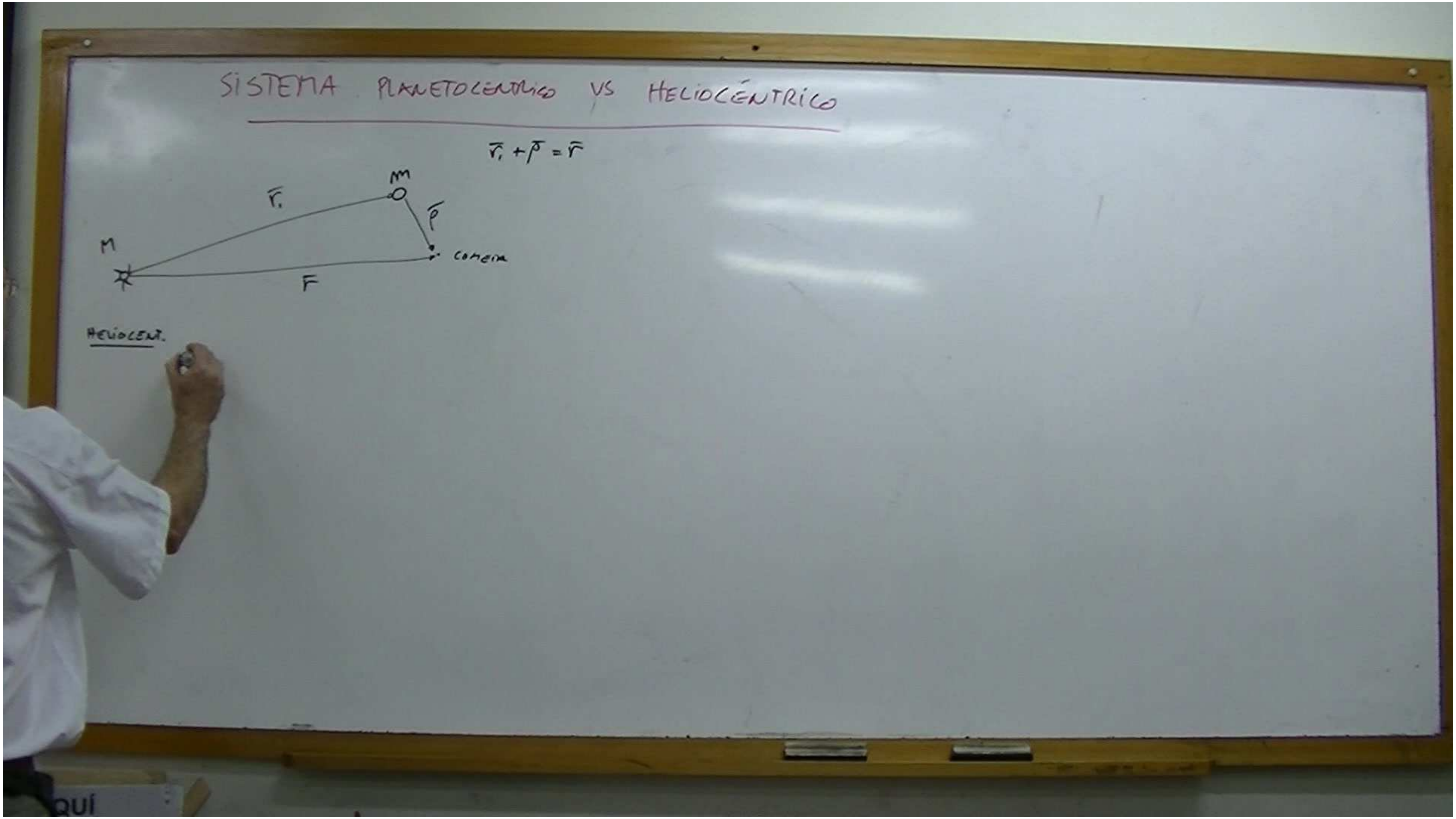

SISTEMA BARICÉNTRICO VS HELIOCÉNTRICO

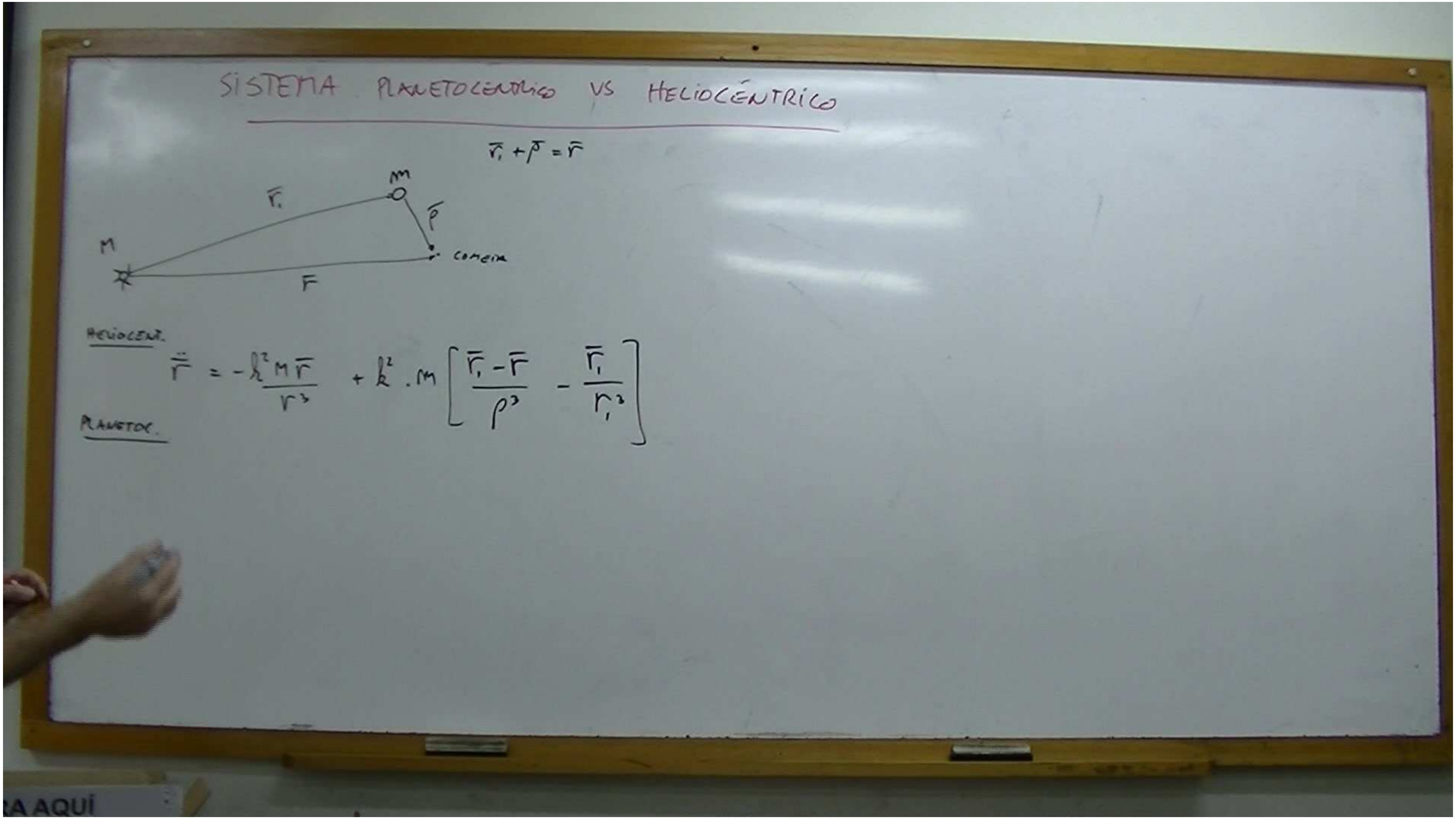

SISTEMA BARICÉNTRICO VS HELIOCÉNTRICO
(a, e, i ...) Helios.
AST
(a, e, i ..)
BARIC

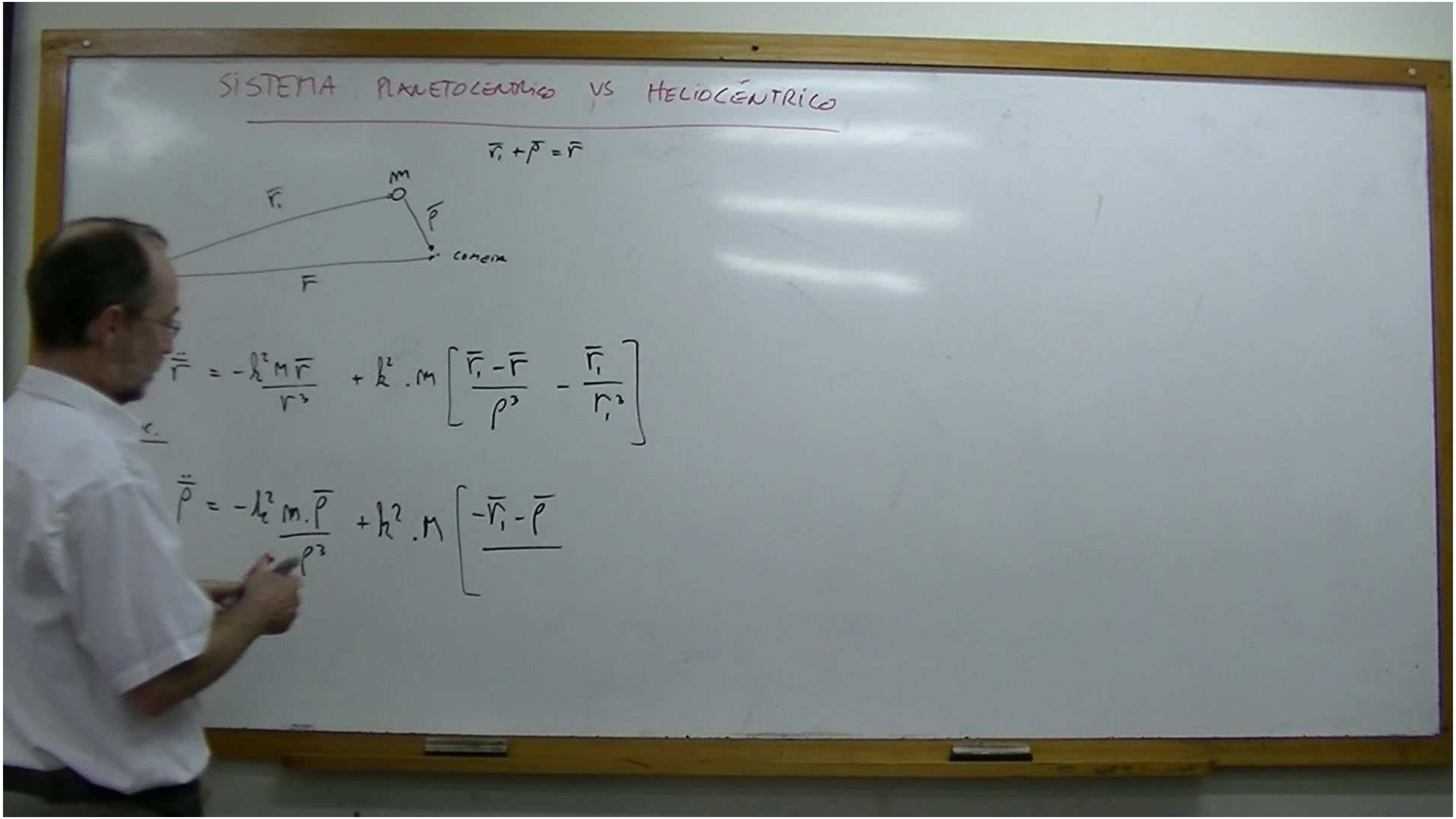

SISTEMA BARICÉNTRICO VS HELIOCÉNTRICO
(a, e, i . . .)
AST
JUP
(a, e, i . .)
a
t
RA AQUÍ

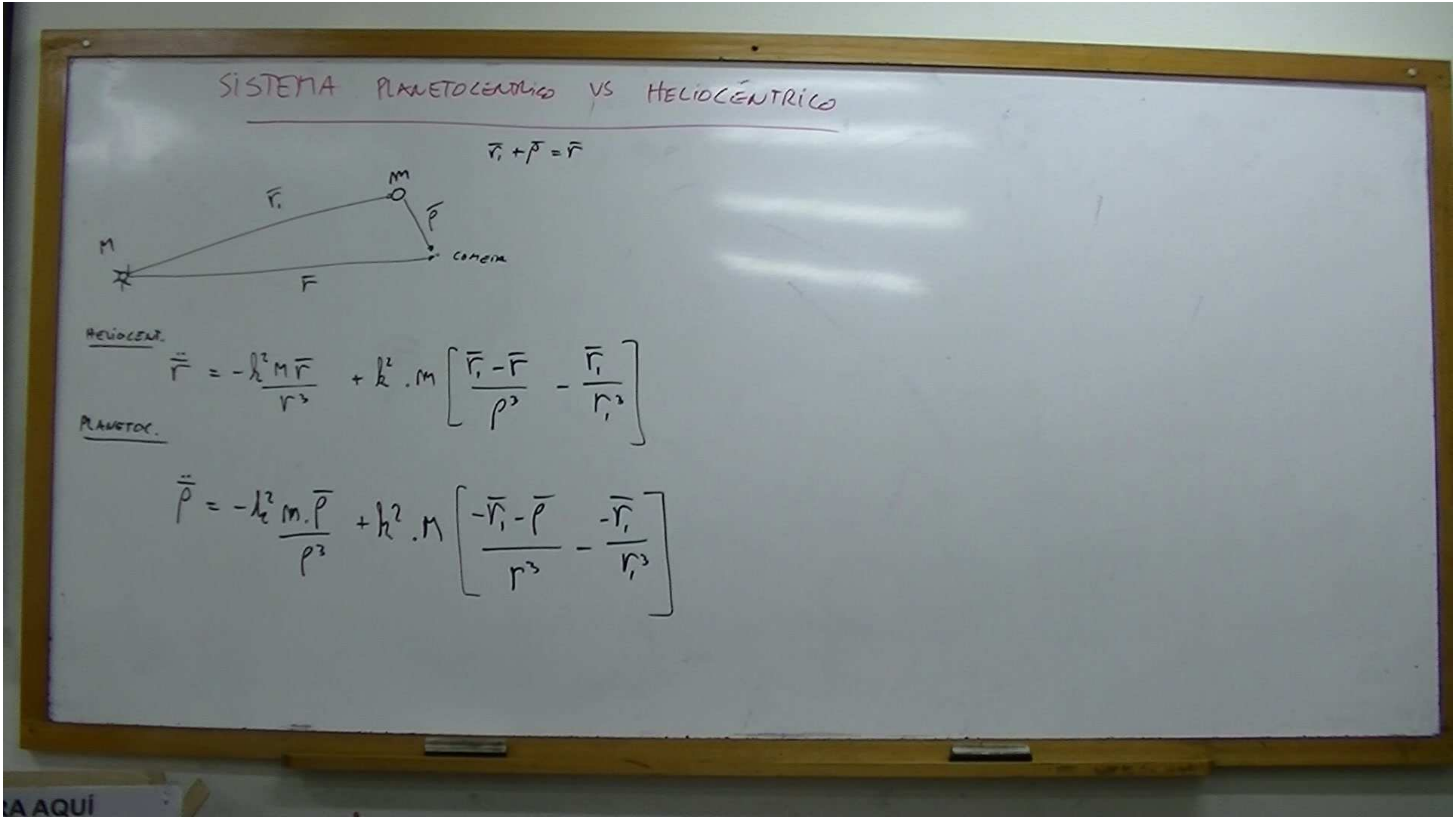

SISTEMA BARICÉNTRICO VS HELIOCÉNTRICO
(a, e, i . . .) Helios.
AST
JUP
(a, e, i ..)
Baric
PLA
a
t

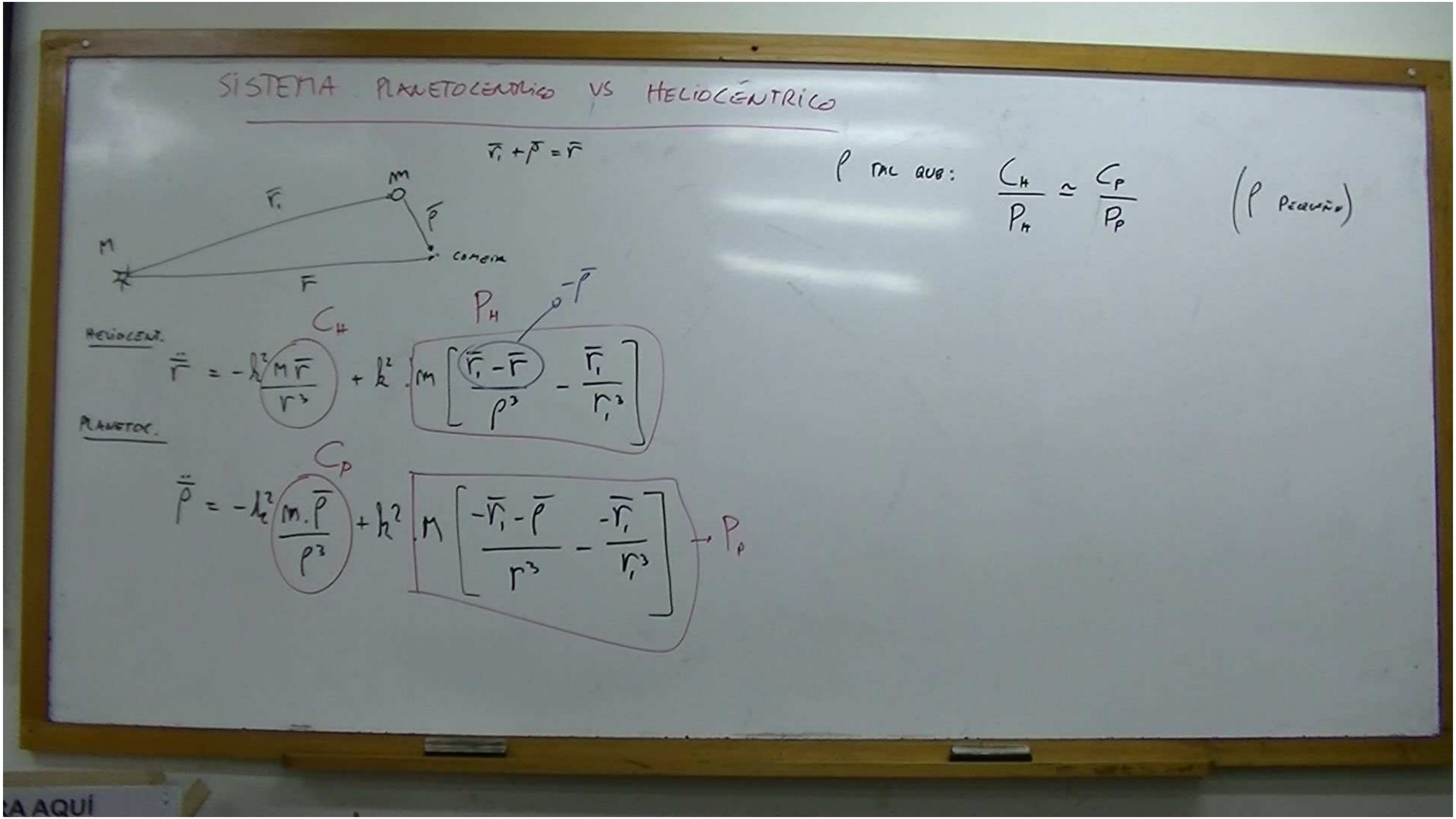

SISTEMA BARICÉNTRICO VS HELIOCÉNTRICO
a
t
PLA

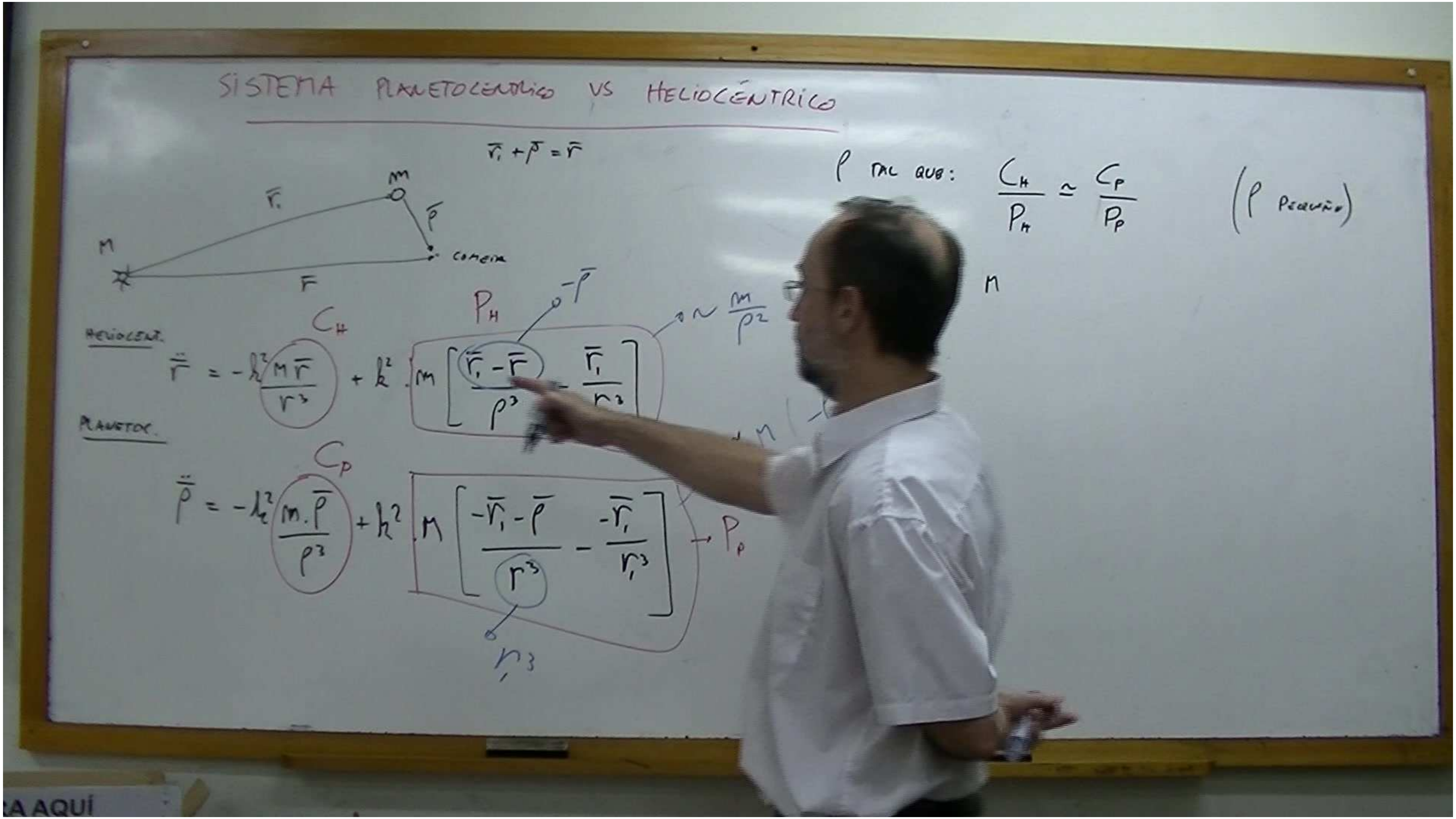

SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
r̄1 + ρ̄ = r̄
m
r̄1
ρ̄
M
cometa
r̄
HELIOCÉNT.

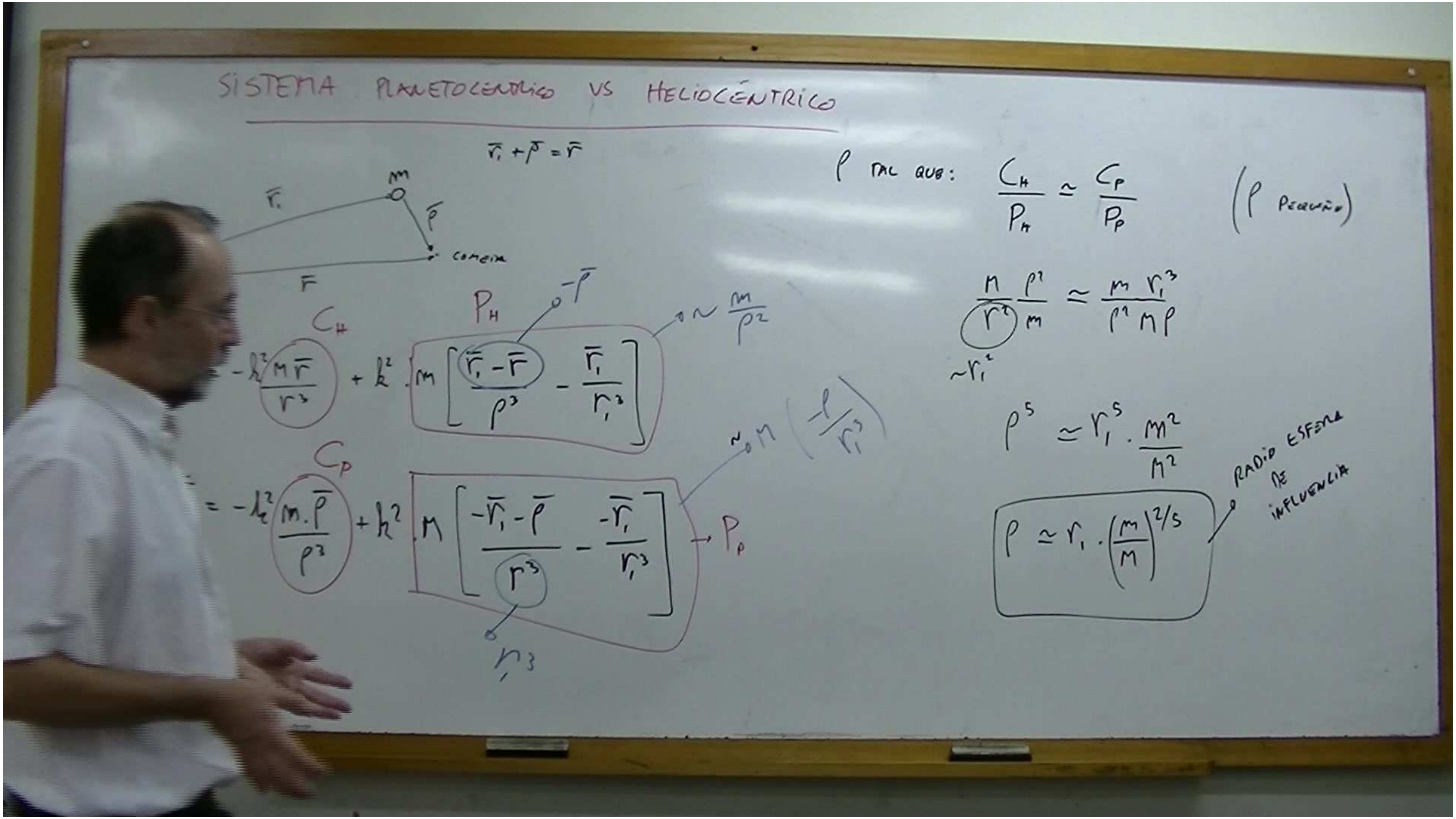

SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
$\bar{r}_1 + \bar{\rho} = \bar{r}$
m
$\bar{r}_1$
$\bar{\rho}$
M
cometa
$\bar{r}$
HELIOCENT.
$\ddot{\bar{r}} = -k^2 M \frac{\bar{r}}{r^3} + k^2 . m \left[\frac{\bar{r}_1 - \bar{r}}{\rho^3} - \frac{\bar{r}_1}{r_1^3} \right]$
PLANETOC.
RA AQUÍ

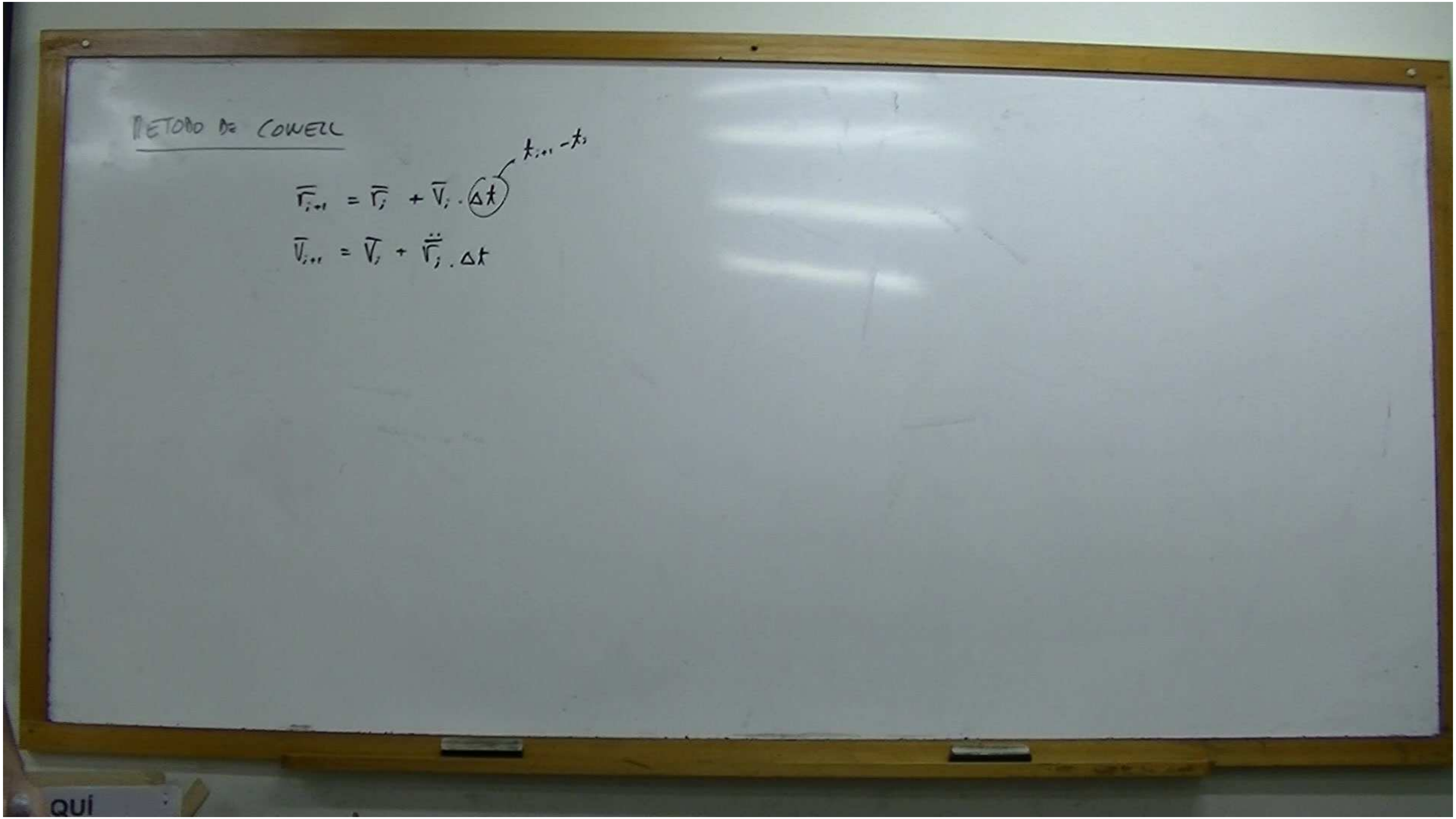

SISTEMA PLANETOCENTRICO VS HELIOCÉNTRICO
$\bar{r}_1 + \bar{\rho} = \bar{r}$
m
$\bar{r}_1$
$\bar{\rho}$
cometa
$\bar{r}$
$\ddot{\bar{r}} = -k^2 M \frac{\bar{r}}{r^3} + k^2 . m \left[\frac{\bar{r}_1 - \bar{r}}{\rho^3} - \frac{\bar{r}_1}{r_1^3} \right]$
$\ddot{\bar{\rho}} = -k^2 \frac{m.\bar{\rho}}{\rho^3} + k^2 . M \left[\frac{-\bar{r}_1 - \bar{\rho}}{} \right.$

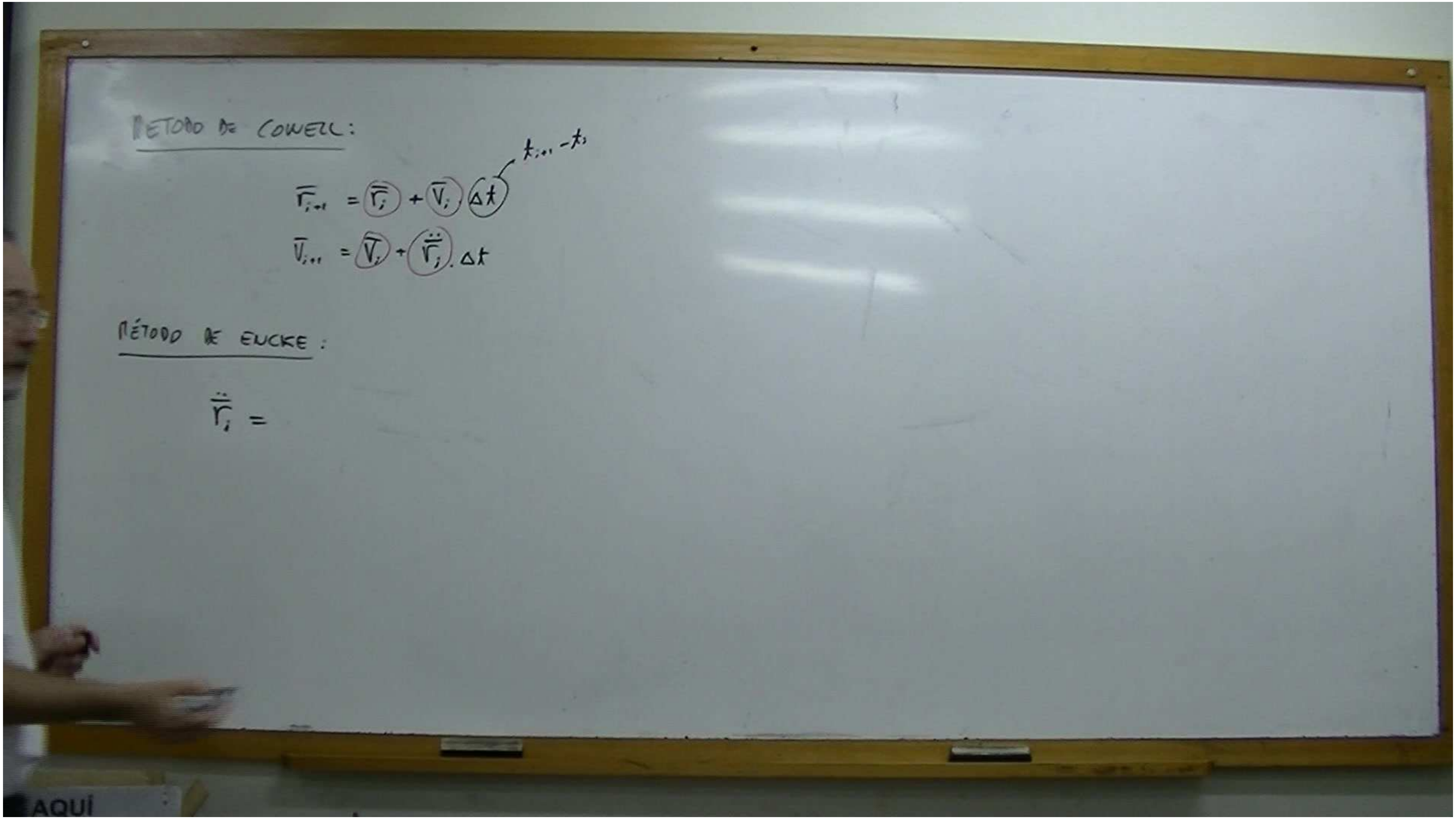

SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
$\bar{r}_1 + \bar{\rho} = \bar{r}$
m
$\bar{r}_1$
$\bar{\rho}$
M
COMETA
$\bar{r}$
HELIOCENT.
$\ddot{\bar{r}} = -k^2 \frac{M \bar{r}}{r^3} + k^2 . m \left[\frac{\bar{r}_1 - \bar{r}}{\rho^3} - \frac{\bar{r}_1}{r_1^3} \right]$
PLANETOC.
$\ddot{\bar{\rho}} = -k^2 \frac{m . \bar{\rho}}{\rho^3} + k^2 . M \left[\frac{-\bar{r}_1 - \bar{\rho}}{r^3} - \frac{-\bar{r}_1}{r_1^3} \right]$
RA AQUÍ


SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
$\bar{r}_1 + \bar{\rho} = \bar{r}$
M
m
$\bar{r}_1$
$\bar{\rho}$
COMETA
$\bar{r}$
HELIOCENT.
C_H
P_H
$-\bar{\rho}$
$\ddot{\bar{r}} = -k^2 \frac{M\bar{r}}{r^3} + k^2 m \left[\frac{\bar{r}_1 - \bar{r}}{\rho^3} - \frac{\bar{r}_1}{r_1^3} \right]$
PLANETOC.
C_P
$\ddot{\bar{\rho}} = -k^2 \frac{m.\bar{\rho}}{\rho^3} + k^2 M \left[\frac{-\bar{r}_1 - \bar{\rho}}{r^3} - \frac{-\bar{r}_1}{r_1^3} \right] \to P_P$
ρ TAL QUE: $\frac{C_H}{P_H} \simeq \frac{C_P}{P_P}$
(ρ PEQUEÑO)
RA AQUÍ


SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
COMETA
HELIOCENT.
PLANETOC.
ρ TAL QUE:
(ρ PEQUEÑO)
RA AQUÍ


SISTEMA PLANETOCÉNTRICO VS HELIOCÉNTRICO
$\bar{r}_1 + \bar{\rho} = \bar{r}$
m
cometa
C_H
P_H
C_P
P_P
ρ TAL QUE: $\frac{C_H}{P_H} \simeq \frac{C_P}{P_P}$
(ρ PEQUEÑO)
$\rho^5 \simeq r_1^5 \cdot \frac{m^2}{M^2}$
$\rho \simeq r_1 \cdot \left(\frac{m}{M}\right)^{2/5}$
RADIO ESFERA DE INFLUENCIA


METODO DE COWELL
$t_{i+1} - t_i$
$\bar{r}_{i+1} = \bar{r}_i + \bar{V}_i \cdot \Delta t$
$\bar{V}_{i+1} = \bar{V}_i + \ddot{\bar{r}}_i \cdot \Delta t$
QUI


MÉTODO DE COWELL:
$t_{i+1} - t_i$
$\bar{r}_{i+1} = \bar{r}_i + \bar{V}_i \Delta t$
$\bar{V}_{i+1} = \bar{V}_i + \ddot{\bar{r}}_i \cdot \Delta t$
MÉTODO DE ENCKE:
$\ddot{\bar{r}}_i =$
AQUÍ

Método de Cowell:

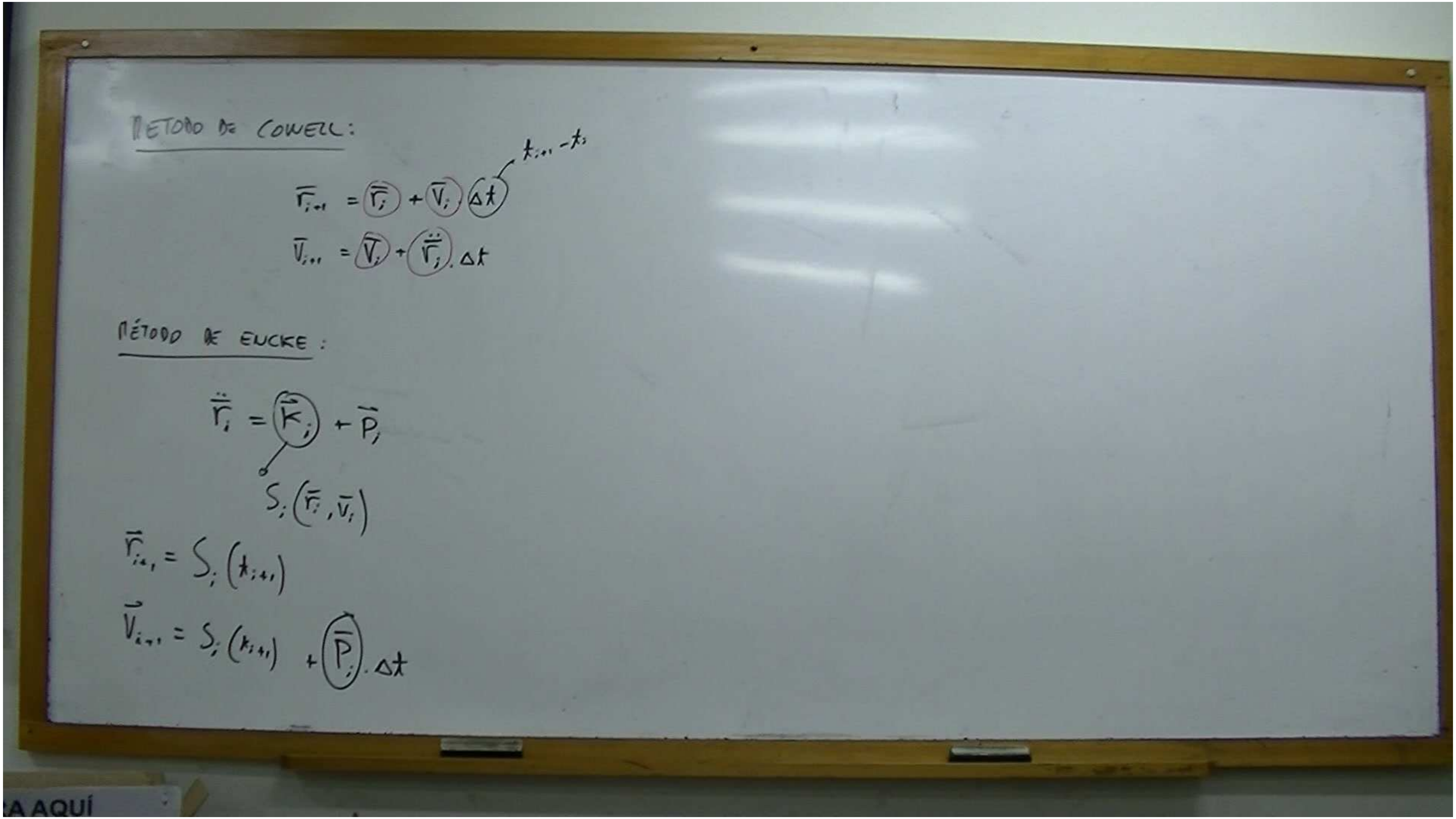
$$\vec{r}_{i+1} = \vec{r}_i + \vec{V}_i \, \overbrace{\Delta t}^{t_{i+1} - t_i}$$

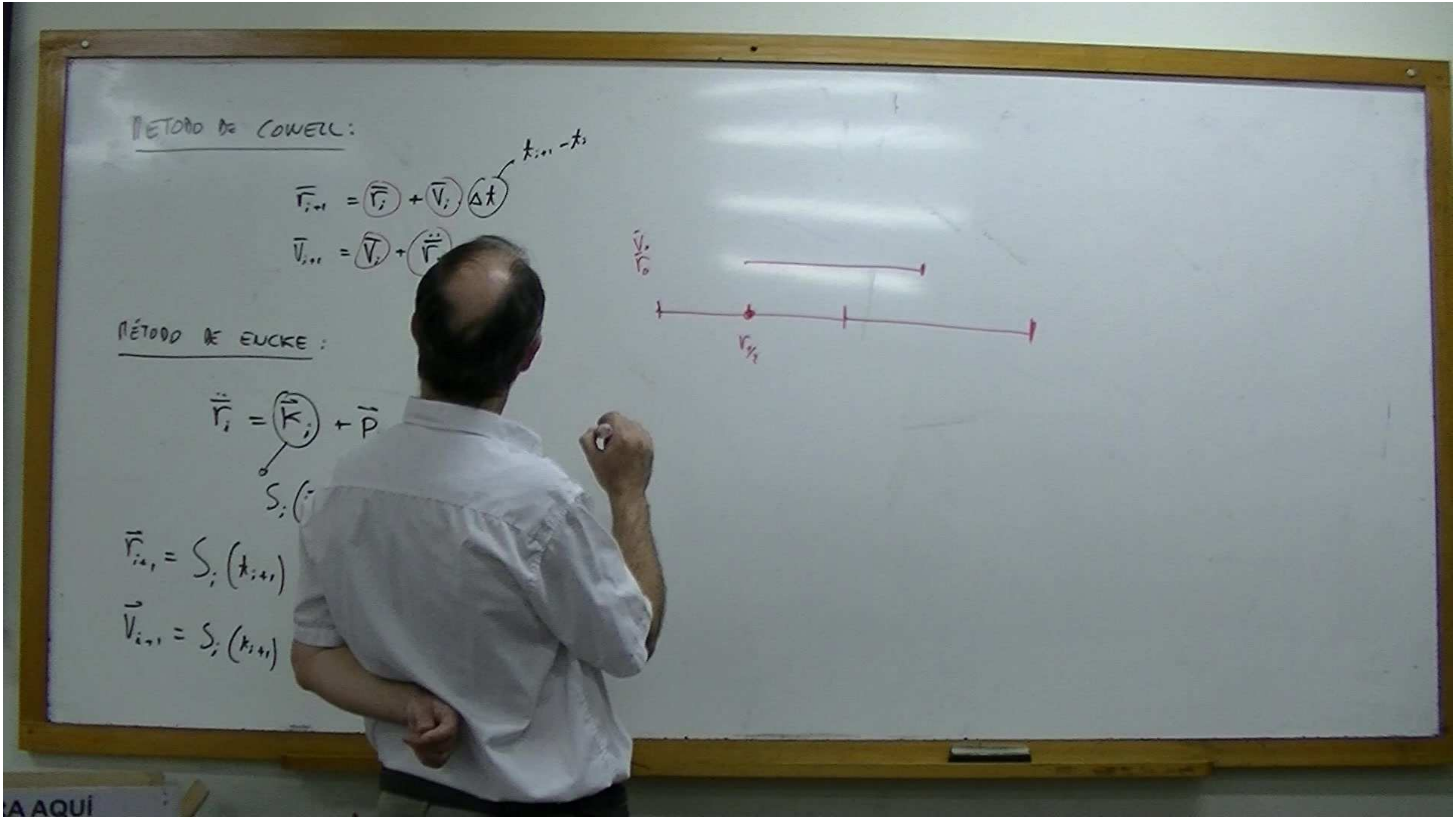
$$\vec{V}_{i+1} = \vec{V}_i + \ddot{\vec{r}}_i \, . \, \Delta t$$

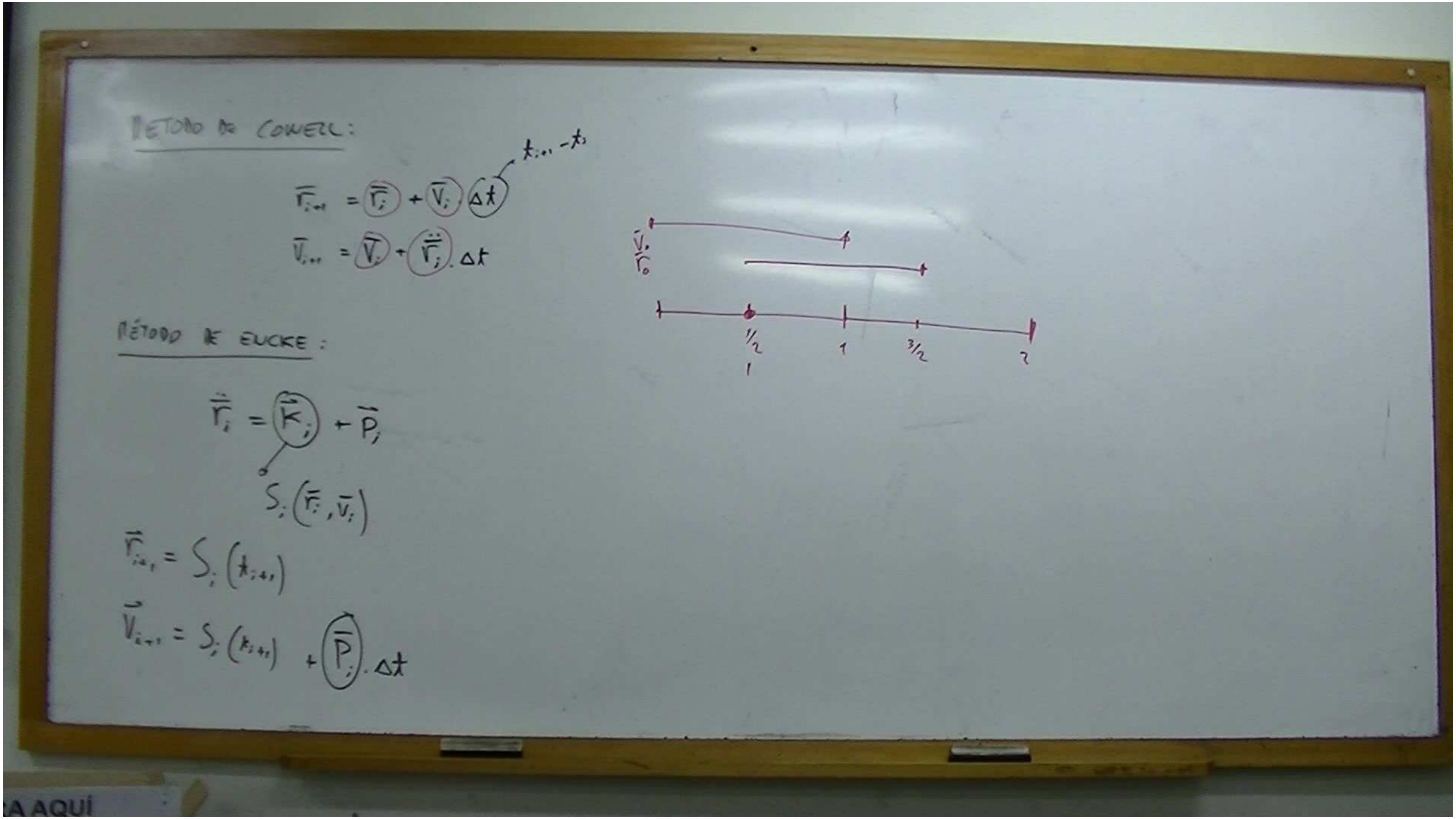
Método de Encke:

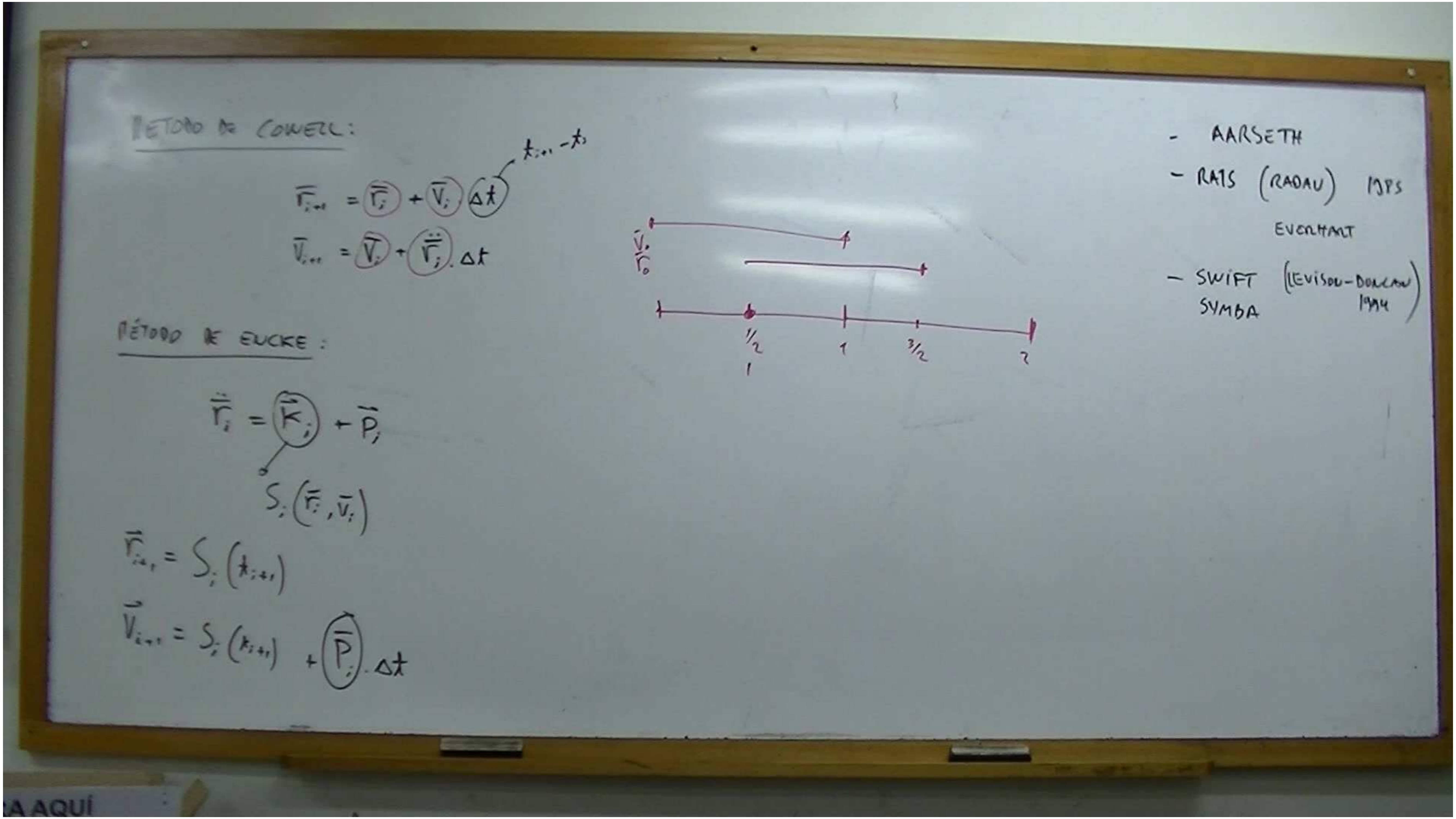
$$\ddot{\vec{r}}_i = \underbrace{\vec{K}_i}_{S_i(\vec{r}_i,\, \vec{v}_i)} + \vec{P}_i$$

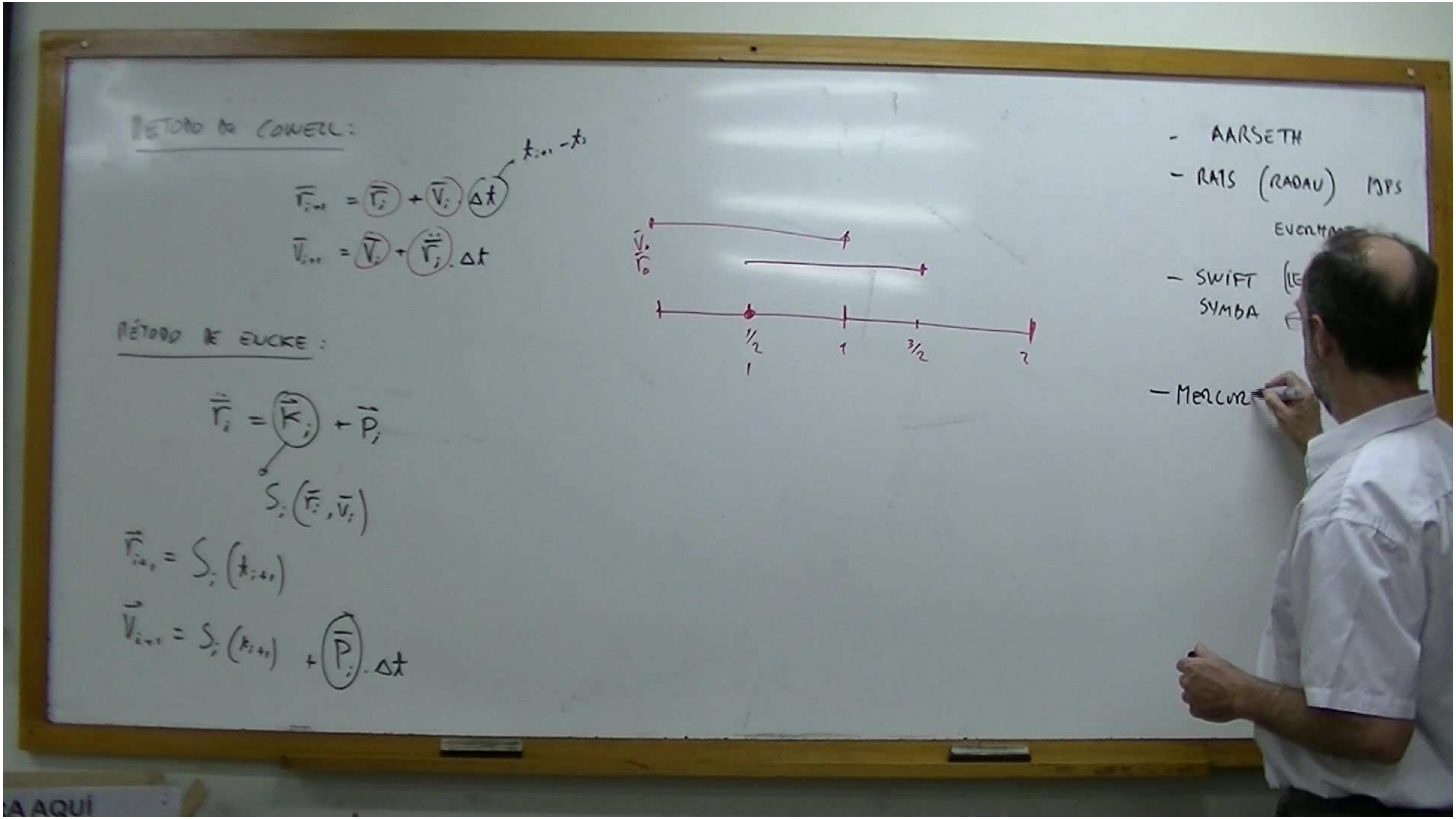
$$\vec{r}_{i+1} = S_i(t_{i+1})$$

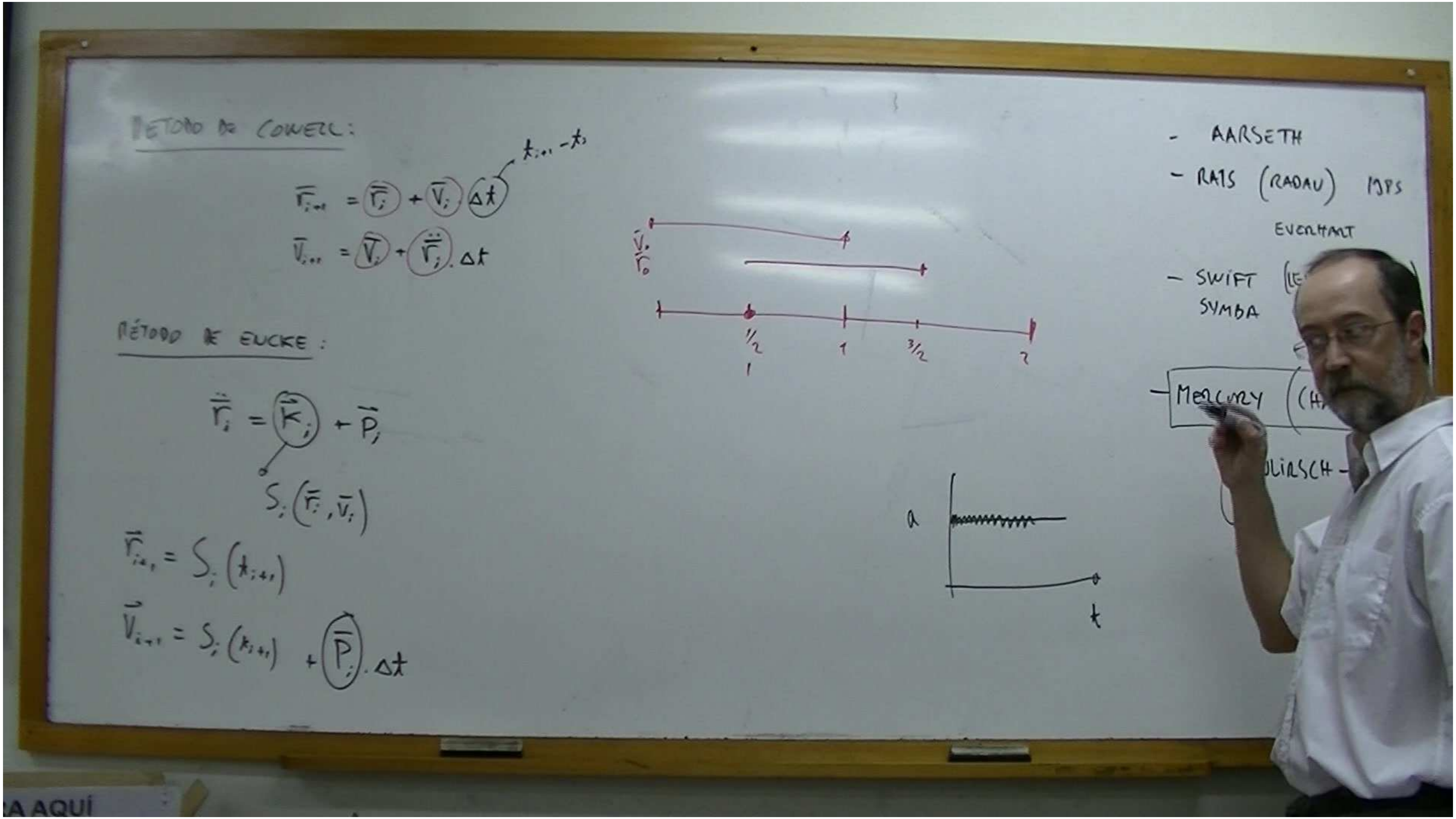

MÉTODO DE COWELL:
$t_{i+1} - t_i$
$\vec{r}_{i+1} = \vec{r}_i + \vec{V}_i \Delta t$
$\vec{V}_{i+1} = \vec{V}_i + \ddot{\vec{r}}_i \cdot \Delta t$
MÉTODO DE ENCKE:
$\ddot{\vec{r}}_i = \vec{K}_i + \vec{P}_i$
$S_i(\vec{r}_i, \vec{v}_i)$
$\vec{r}_{i+1} = S_i(t_{i+1})$
$\vec{V}_{i+1} = S_i(t_{i+1}) + \vec{P}_i \cdot \Delta t$

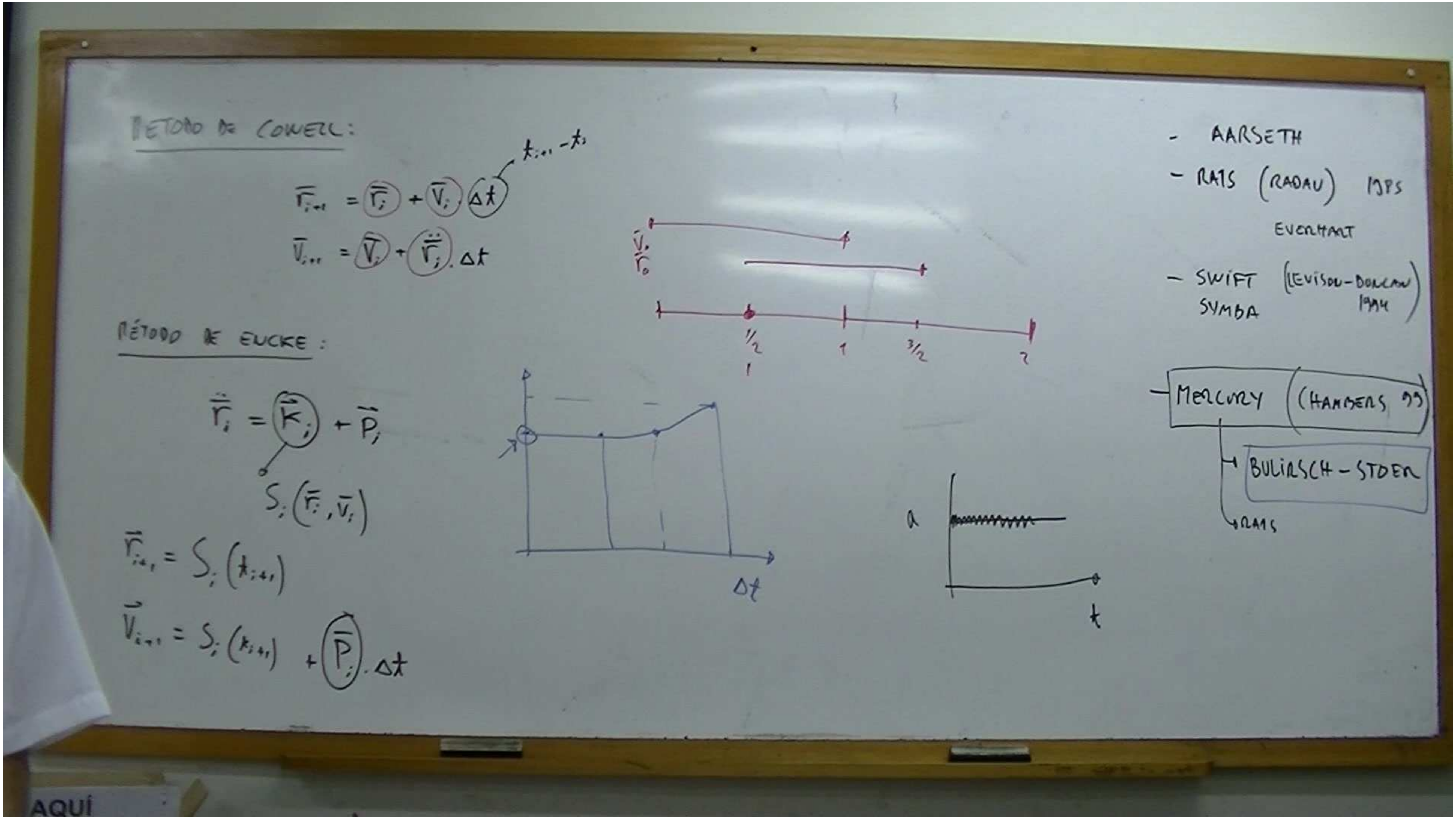

ETODO DE COWELL:
$\vec{r}_{i+1} = \vec{r}_i + \vec{V}_i \Delta t$
$t_{i+1} - t_i$
MÉTODO DE ENCKE:
$\ddot{\vec{r}}_i = \vec{K}_i + \vec{P}$
$\vec{r}_{i+1} = S_i(t_{i+1})$
$\vec{V}_{i+1} = S_i(t_{i+1})$
$\vec{V}_0$
$\vec{r}_0$
$r_{1/2}$

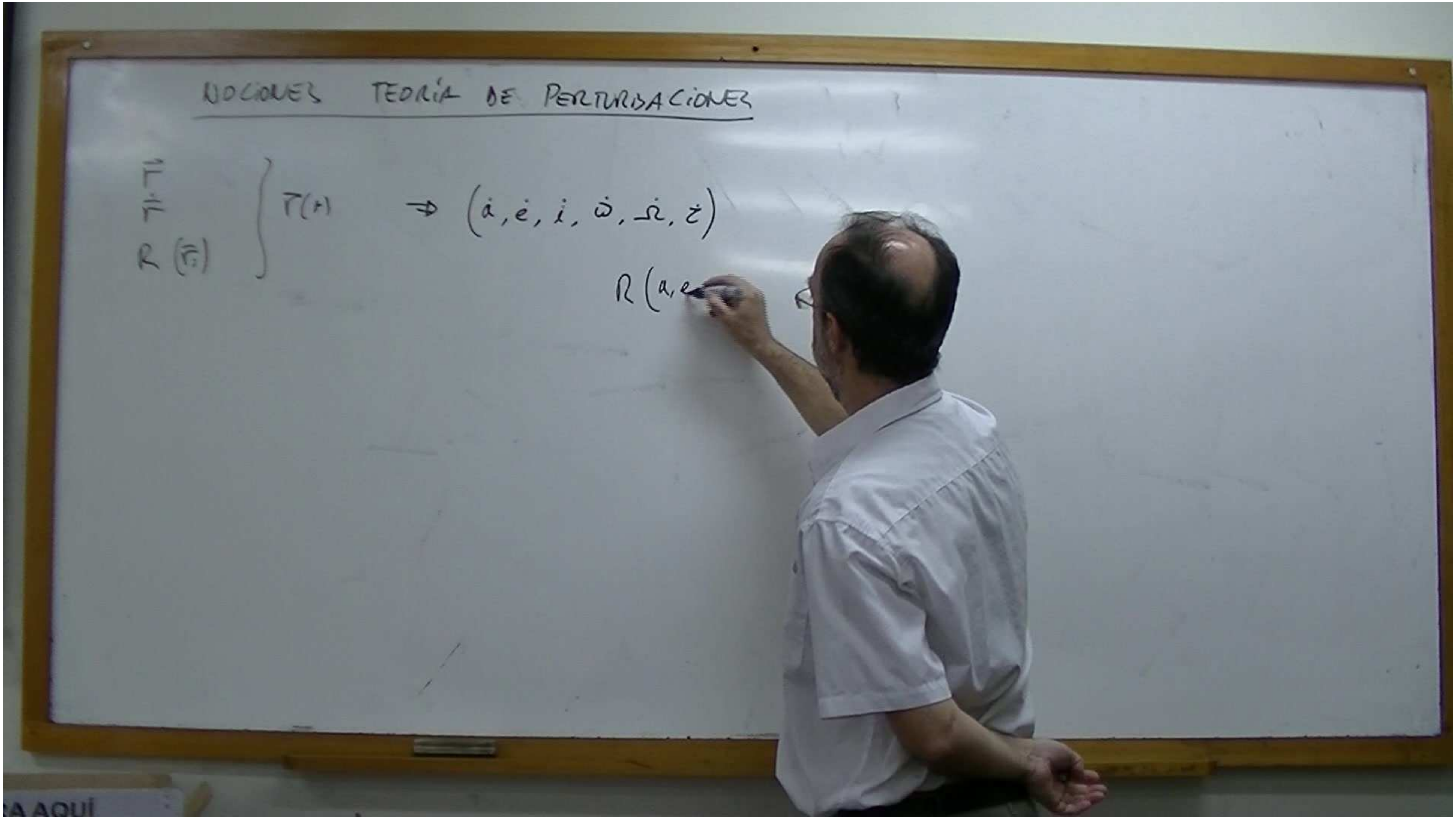

MÉTODO DE COWELL:
$\vec{r}_{i+1} = \vec{r}_i + \vec{V}_i \Delta t$
$t_{i+1} - t_i$
$\vec{V}_{i+1} = \vec{V}_i + \ddot{\vec{r}}_i \cdot \Delta t$
$\vec{V}_0$
$\vec{r}_0$
1/2
1
3/2
2
MÉTODO DE ENCKE:
$\ddot{\vec{r}}_i = \vec{K}_i + \vec{P}_i$
$S_i(\vec{r}_i, \vec{v}_i)$
$\vec{r}_{i+1} = S_i(t_{i+1})$
$\vec{V}_{i+1} = S_i(t_{i+1}) + \vec{P}_i \cdot \Delta t$

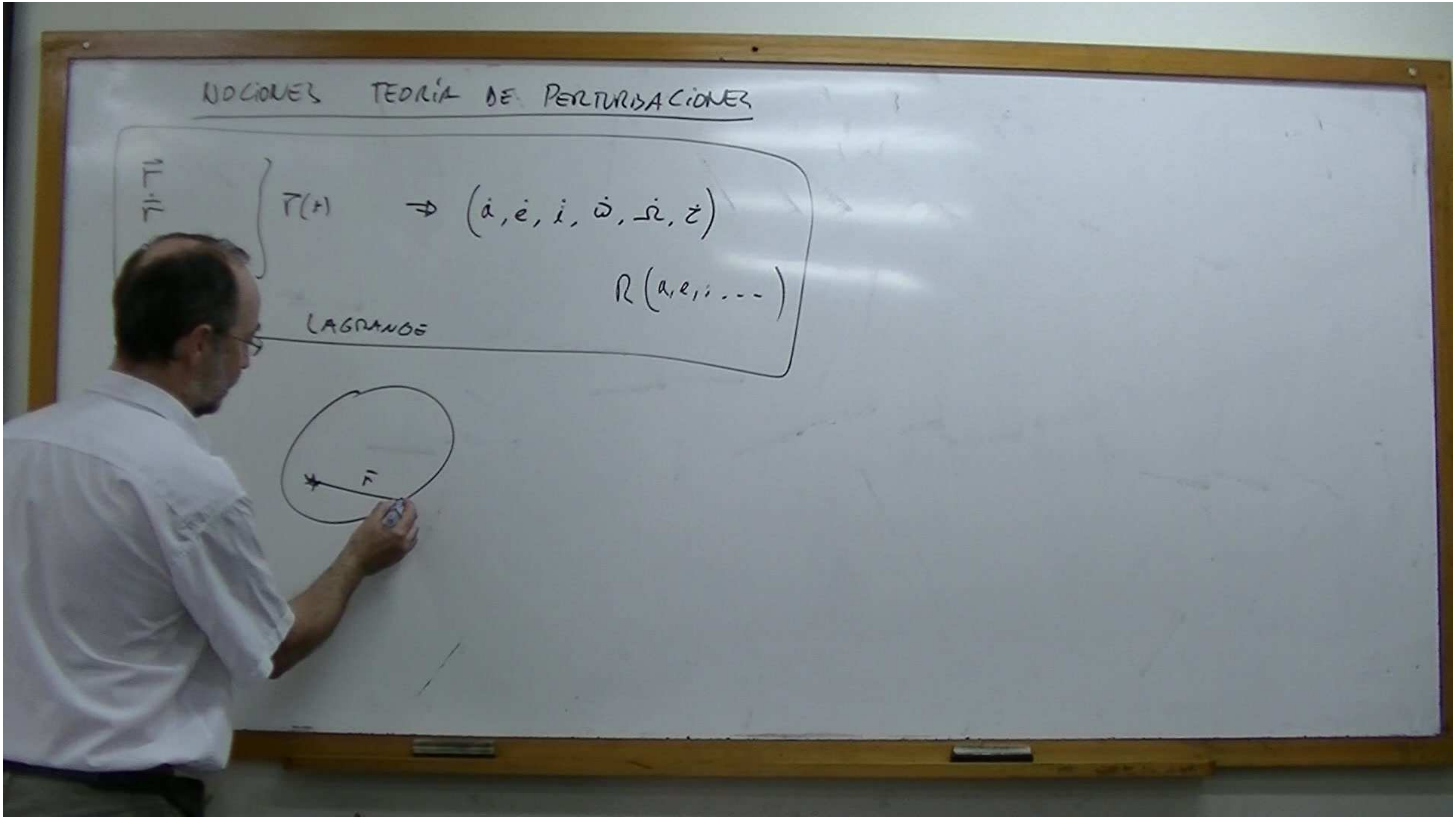

MÉTODO DE COWELL:
$t_{i+1} - t_i$
$\vec r_{i+1} = \vec r_i + \vec V_i \Delta t$
$\vec V_{i+1} = \vec V_i + \ddot{\vec r}_i . \Delta t$
MÉTODO DE ENCKE:
$\ddot{\vec r}_i = \vec K_i + \vec P_i$
$S_i(\vec r_i, \vec v_i)$
$\vec r_{i+1} = S_i(t_{i+1})$
$\vec V_{i+1} = S_i(t_{i+1}) + \vec P_i . \Delta t$
$\vec V_0$
$\vec r_0$
1/2
1
3/2
2
- AARSETH
- RATS (RADAU) 1985
EVERHART
- SWIFT (LEVISON-DUNCAN 1994)
SYMBA

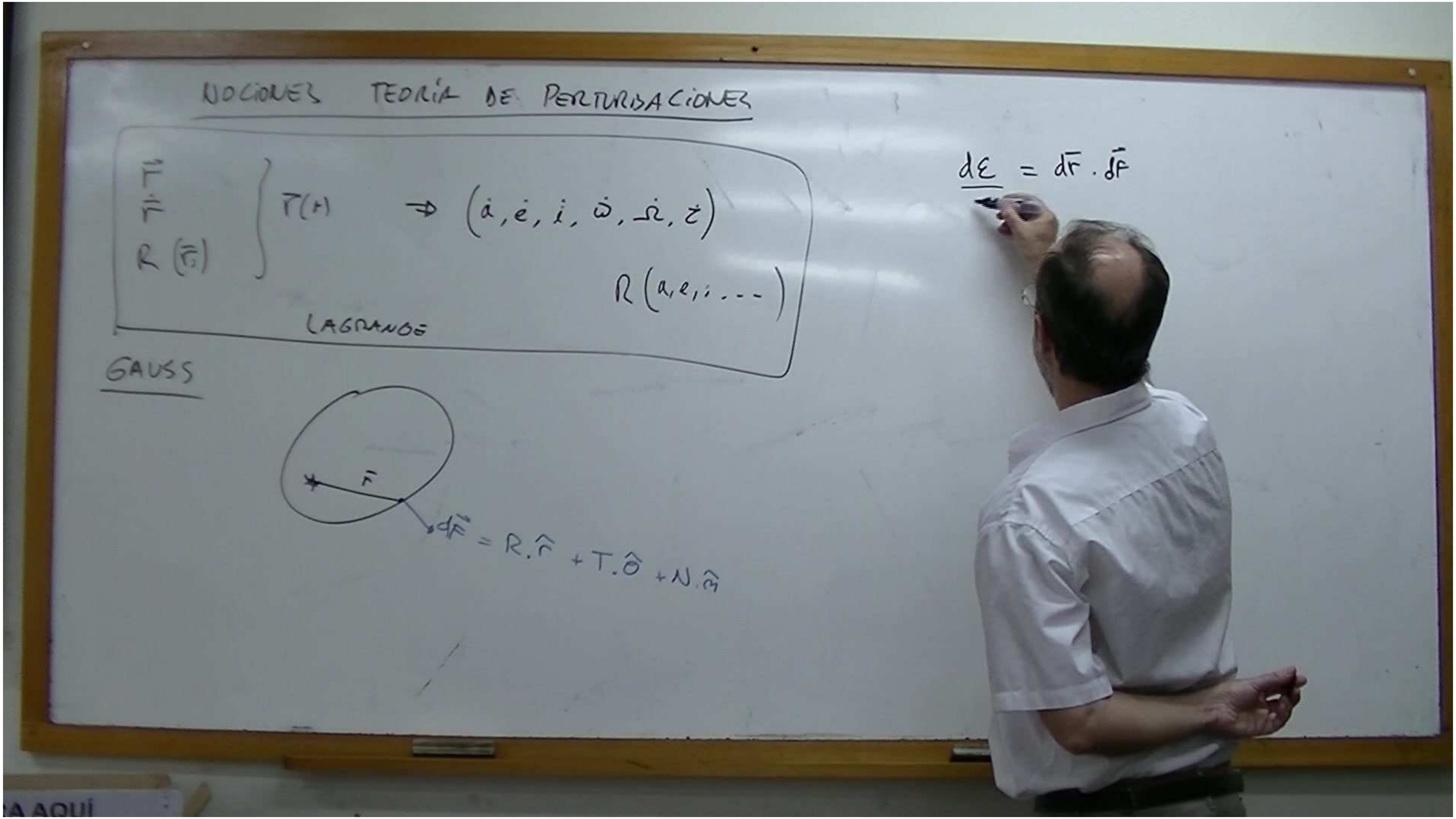

MÉTODO DE COWELL:
$t_{i+1} - t_i$
$\vec{r}_{i+1} = \vec{r}_i + \vec{V}_i \, \Delta t$
$\vec{V}_{i+1} = \vec{V}_i + \ddot{\vec{r}}_i \, . \Delta t$
MÉTODO DE ENCKE:
$\ddot{\vec{r}}_i = \vec{K}_i + \vec{P}_i$
$S_i(\vec{r}_i, \vec{v}_i)$
$\vec{r}_{i+1} = S_i(t_{i+1})$
$\vec{V}_{i+1} = S_i(t_{i+1}) + \vec{P}_i \, . \Delta t$
$\vec{V}_0$
$\vec{r}_0$
1/2
1
3/2
2
- AARSETH
- RATS (RADAU) MPS
- SWIFT
SYMBA

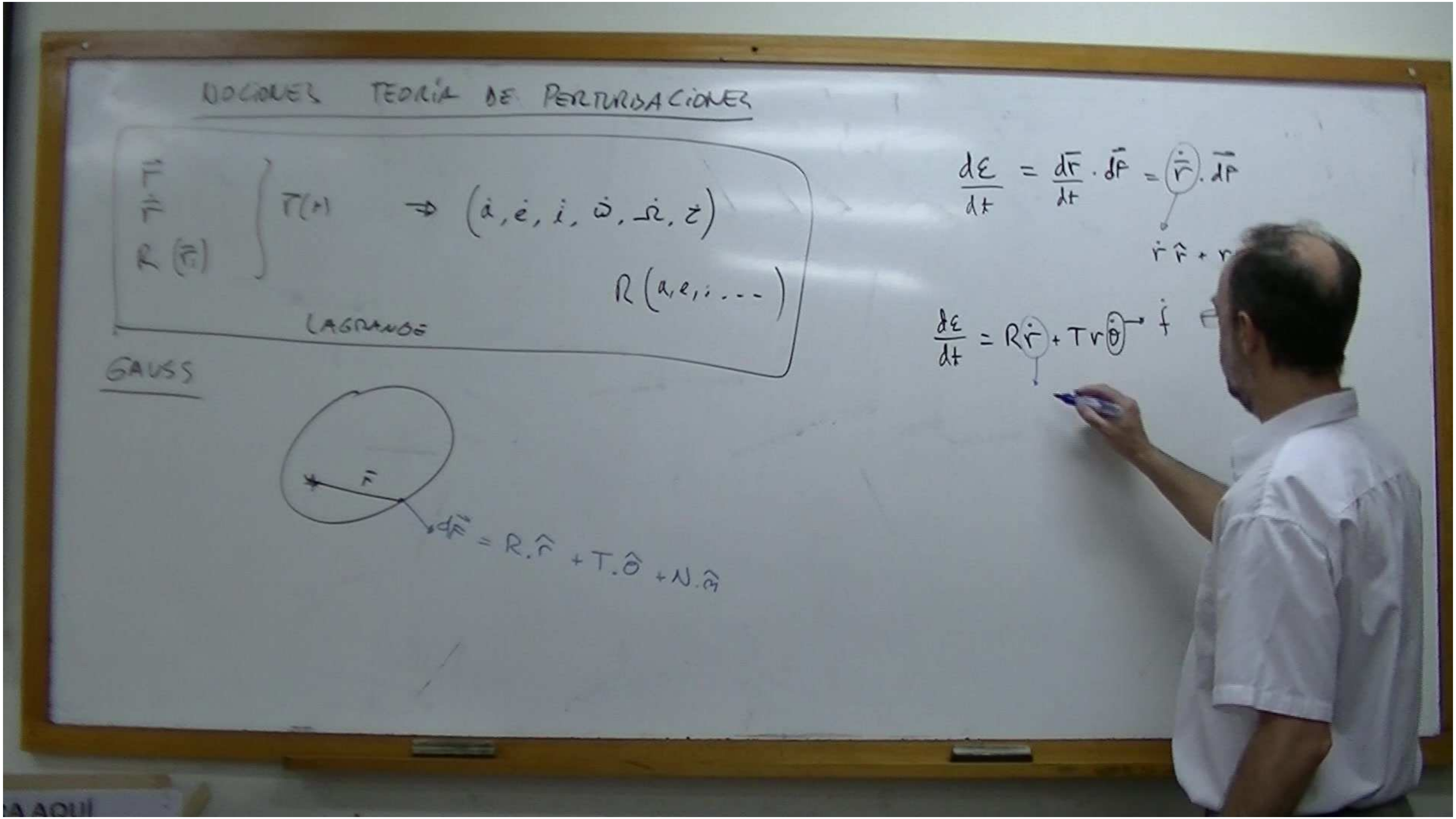

MÉTODO DE COWELL:
MÉTODO DE ENCKE:
AARSETH
RADAU
EVERHART
SWIFT
SYMBA
MERCURY

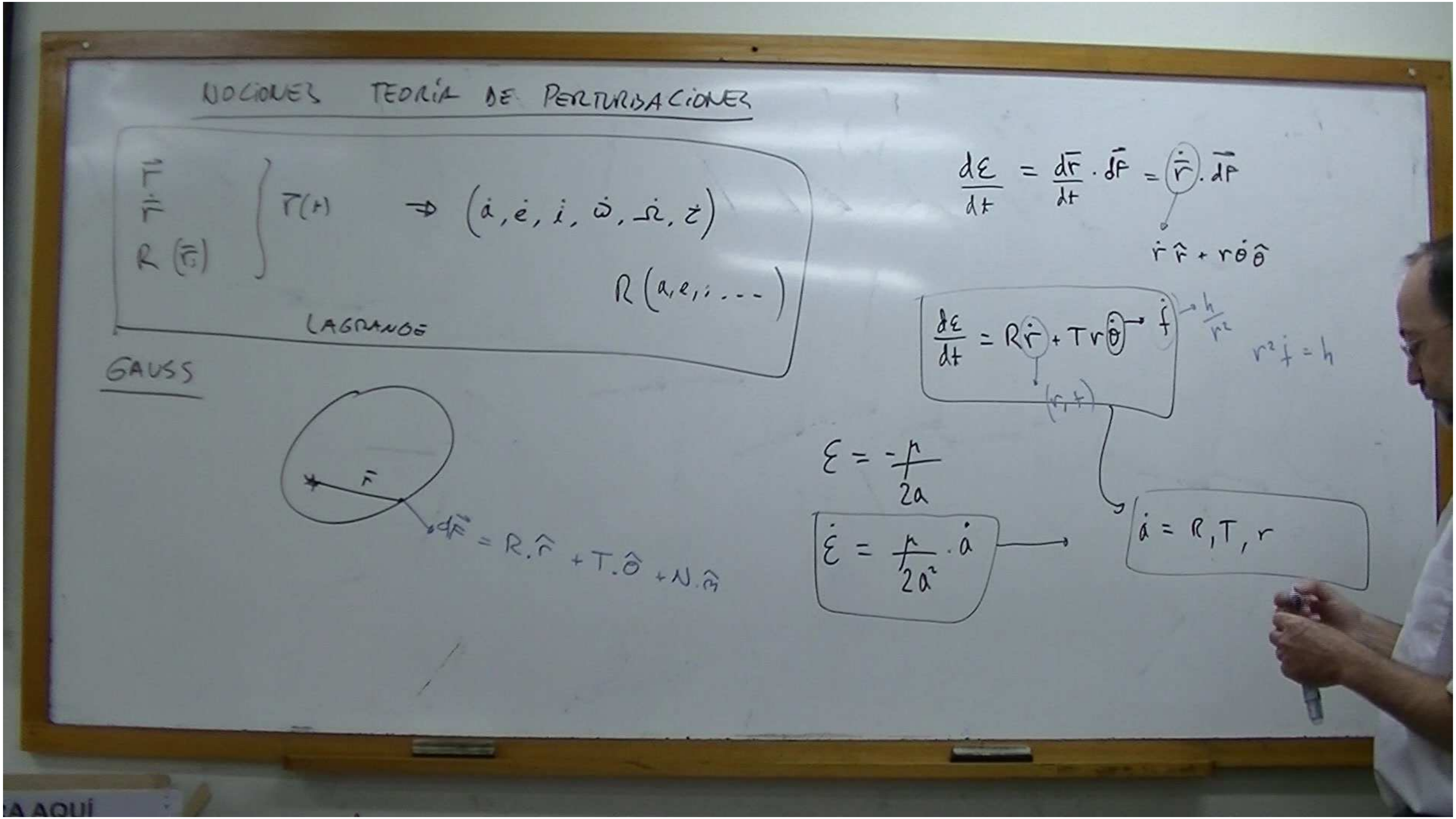

MÉTODO DE COWELL:
MÉTODO DE ENCKE:
AARSETH
RA15 (RADAU) 1985
EVERHART
SWIFT (LEVISON-DUNCAN 1994)
SYMBA
MERCURY (CHAMBERS 99)
BULIRSCH-STOER
RA15
AQUÍ

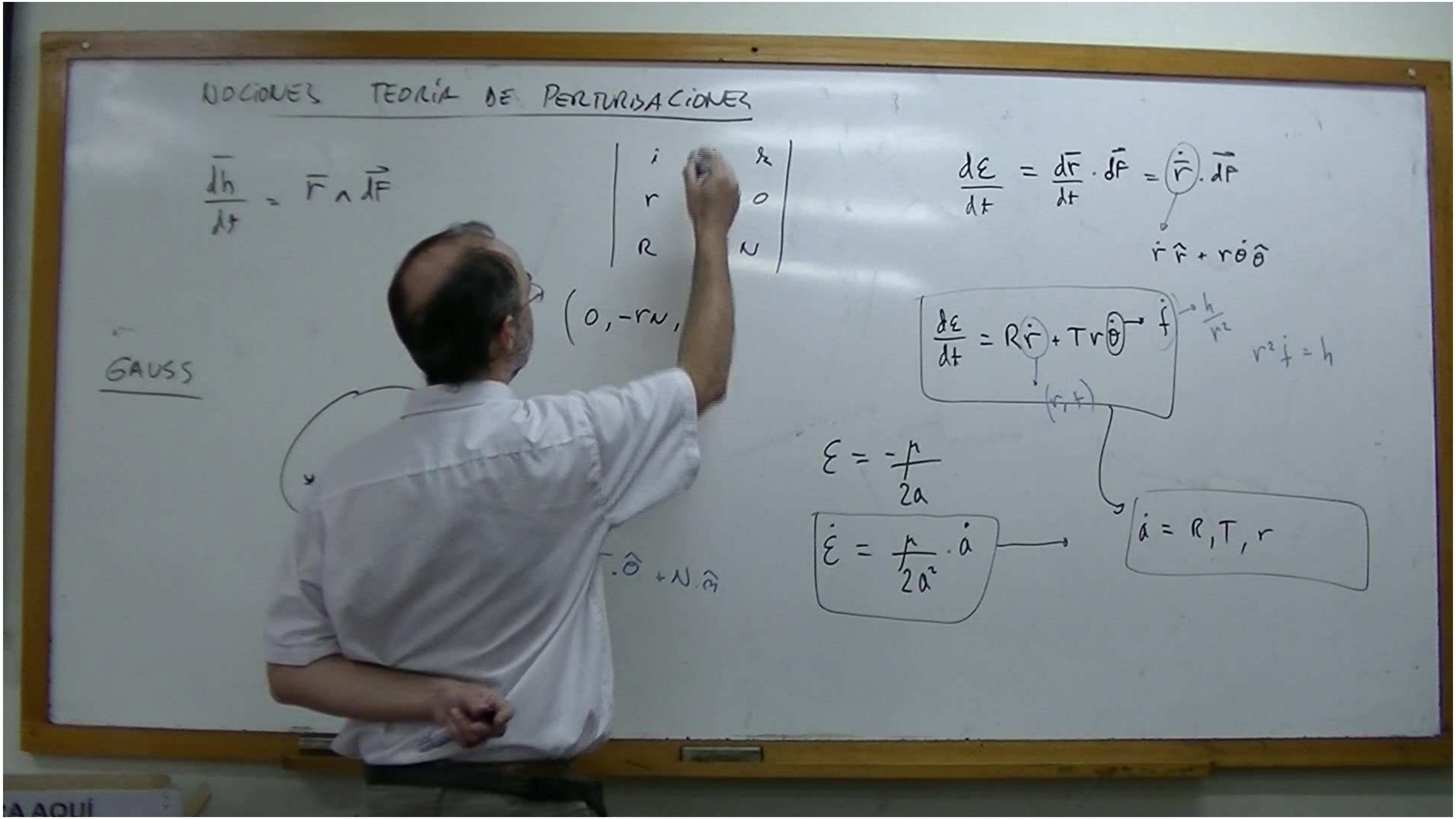

NOCIONES TEORÍA DE PERTURBACIONES
$\vec{r}$
$\dot{\vec{r}}$
$R(\vec{r}_i)$
$\vec{r}(t)$
$\Rightarrow (\dot{a}, \dot{e}, \dot{i}, \dot{\omega}, \dot{\Omega}, \dot{\tau})$
$R(a, e$


NOCIONES TEORÍA DE PERTURBACIONES
$\vec{r}$
$\dot{\vec{r}}$
$\vec{r}(t) \Rightarrow (\dot{a}, \dot{e}, \dot{i}, \dot{\omega}, \dot{\Omega}, \dot{\tau})$
$R(a, e, i, \ldots)$
LAGRANGE
$\vec{r}$

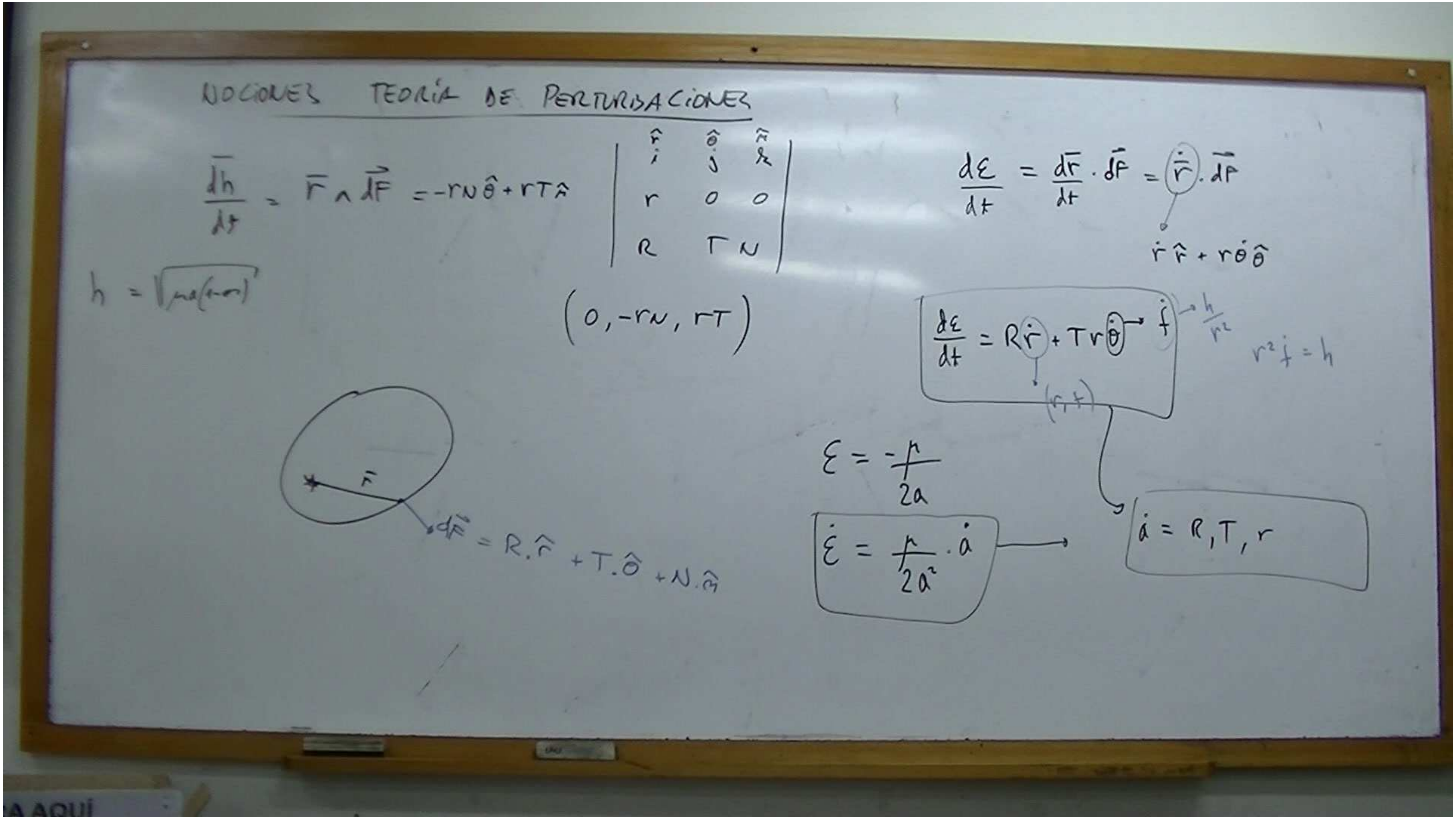

NOCIONES TEORÍA DE PERTURBACIONES
r(t) ⇒ (a, e, i, ω, Ω, τ)
R(a, e, i ...)
LAGRANGE
GAUSS
dF = R.r̂ + T.θ̂ + N.n̂
dE = dr . dF

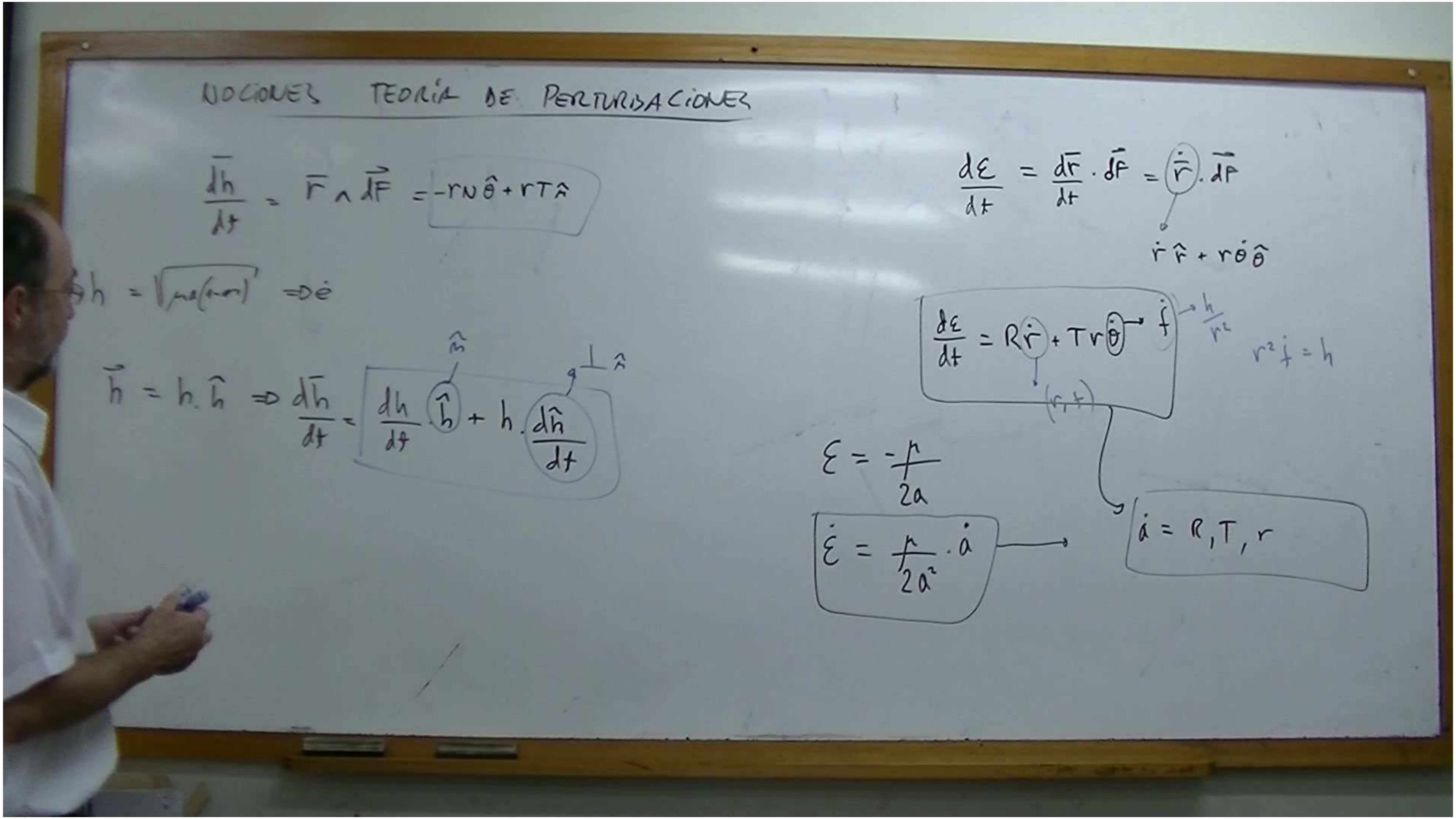

NOCIONES TEORÍA DE PERTURBACIONES
$\vec{F}$
$\dot{\vec{r}}$
$R(\vec{r})$
$T(t) \Rightarrow (\dot{a}, \dot{e}, \dot{i}, \dot{\omega}, \dot{\Omega}, \dot{\tau})$
$R(a, e, i \ldots)$
LAGRANGE
GAUSS
r
$d\vec{F} = R.\hat{r} + T.\hat{\theta} + N.\hat{n}$
$\frac{dE}{dt} = \frac{d\vec{r}}{dt} \cdot d\vec{F} = \dot{\vec{r}} \cdot \vec{dF}$
$\dot{r}\hat{r} + r$
$\frac{dE}{dt} = R\dot{r} + Tr\dot{\theta}$
$\dot{f}$


NOCIONES TEORÍA DE PERTURBACIONES
$\vec{r}$
$\dot{\vec{r}}$
$R(\vec{r})$
$\vec{r}(t) \Rightarrow (\dot{a}, \dot{e}, \dot{i}, \dot{\omega}, \dot{\Omega}, \dot{\tau})$
$R(a, e, i \ldots)$
LAGRANGE
GAUSS
$\vec{r}$
$d\vec{F} = R.\hat{r} + T.\hat{\theta} + N.\hat{n}$
$\frac{d\mathcal{E}}{dt} = \frac{d\vec{r}}{dt} \cdot d\vec{F} = \dot{\vec{r}} \cdot d\vec{F}$
$\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
$\frac{d\mathcal{E}}{dt} = R\dot{r} + Tr\dot{\theta}$
$\dot{f}$
$\frac{h}{r^2}$
$r^2\dot{f} = h$
(r, f)
$\mathcal{E} = -\frac{\mu}{2a}$
$\dot{\mathcal{E}} = \frac{\mu}{2a^2} \cdot \dot{a}$
$\dot{a} = R, T, r$



NOCIONES TEORÍA DE PERTURBACIONES
$\frac{d\bar{h}}{dt} = \bar{r} \wedge d\vec{F}$
$\begin{vmatrix} i & & k \\ r & & 0 \\ R & & N \end{vmatrix}$
$(0, -rN,$
GAUSS
$T.\hat{\theta} + N.\hat{m}$
$\frac{d\varepsilon}{dt} = \frac{d\bar{r}}{dt} . d\vec{F} = \dot{\bar{r}} . d\vec{F}$
$\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
$\frac{d\varepsilon}{dt} = R\dot{r} + Tr\dot{\theta}$
$\dot{f} \to \frac{h}{r^2}$
$r^2 \dot{f} = h$
(r, f)
$\varepsilon = -\frac{\mu}{2a}$
$\dot{\varepsilon} = \frac{\mu}{2a^2} . \dot{a}$
$\dot{a} = R, T, r$


NOCIONES TEORÍA DE PERTURBACIONES
$\frac{d\bar{h}}{dt} = \bar{r} \wedge \vec{dF} = -rN\hat{\theta} + rT\hat{n}$
$\hat{r}\ \hat{\theta}\ \hat{n}$
$\hat{i}\ \hat{j}\ \hat{k}$
$r\ 0\ 0$
$R\ T\ N$
$h = \sqrt{\mu a(1-e^2)}$
$(0, -rN, rT)$
$\frac{d\varepsilon}{dt} = \frac{d\bar{r}}{dt} \cdot \vec{dF} = \dot{\bar{r}} \cdot \vec{dF}$
$\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
$\frac{d\varepsilon}{dt} = R\dot{r} + Tr\dot{\theta}$
$\dot{f}$
$\frac{h}{r^2}$
$r^2\dot{f} = h$
(r, f)
$\varepsilon = -\frac{\mu}{2a}$
$\dot{\varepsilon} = \frac{\mu}{2a^2} \cdot \dot{a}$
$\dot{a} = R, T, r$
$\bar{r}$
$\vec{dF} = R.\hat{r} + T.\hat{\theta} + N.\hat{n}$


NOCIONES TEORÍA DE PERTURBACIONES
$\frac{d\bar{h}}{dt} = \bar{r} \wedge \vec{dF} = -rN\hat{\theta} + rT\hat{r}$
$h = \sqrt{\mu a(1-e^2)} \Rightarrow \dot{e}$
$\vec{h} = h.\hat{h} \Rightarrow \frac{d\bar{h}}{dt} = \frac{dh}{dt}.\hat{h} + h.\frac{d\hat{h}}{dt}$
$\frac{d\varepsilon}{dt} = \frac{d\bar{r}}{dt}.\vec{dF} = \dot{\bar{r}}.\vec{dF}$
$\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
$\frac{d\varepsilon}{dt} = R\dot{r} + Tr\dot{\theta}$
$\dot{f}$
$\frac{h}{r^2}$
$r^2\dot{f} = h$
(r, f)
$\varepsilon = -\frac{\mu}{2a}$
$\dot{\varepsilon} = \frac{\mu}{2a^2}.\dot{a}$
$\dot{a} = R, T, r$

NOCIONES TEORÍA DE PERTURBACIONES

$$\frac{d\bar{h}}{dt} = \bar{r} \wedge d\vec{F} = \boxed{-rN\hat{\theta} + rT\hat{r}}$$

$$\boxed{h = \sqrt{\mu a(1-e^2)}} \quad (*) \Rightarrow \dot{e}$$

$$\vec{h} = h.\hat{h} \Rightarrow \frac{d\bar{h}}{dt} = \boxed{\frac{dh}{dt}\,\overset{\hat{n}}{\hat{h}} + h.\underbrace{\frac{d\hat{h}}{dt}}_{\perp\,\hat{h}}}$$

$$\boxed{\frac{dh}{dt} = r.T} \rightarrow \dot{h}$$

JUNTO A $(*)$ $\Rightarrow$ $\boxed{\dot{e} = T, R \ldots}$

$$\frac{d\mathcal{E}}{dt} = \frac{d\bar{r}}{dt}\cdot d\vec{F} = \dot{\bar{r}}\cdot d\vec{F}$$

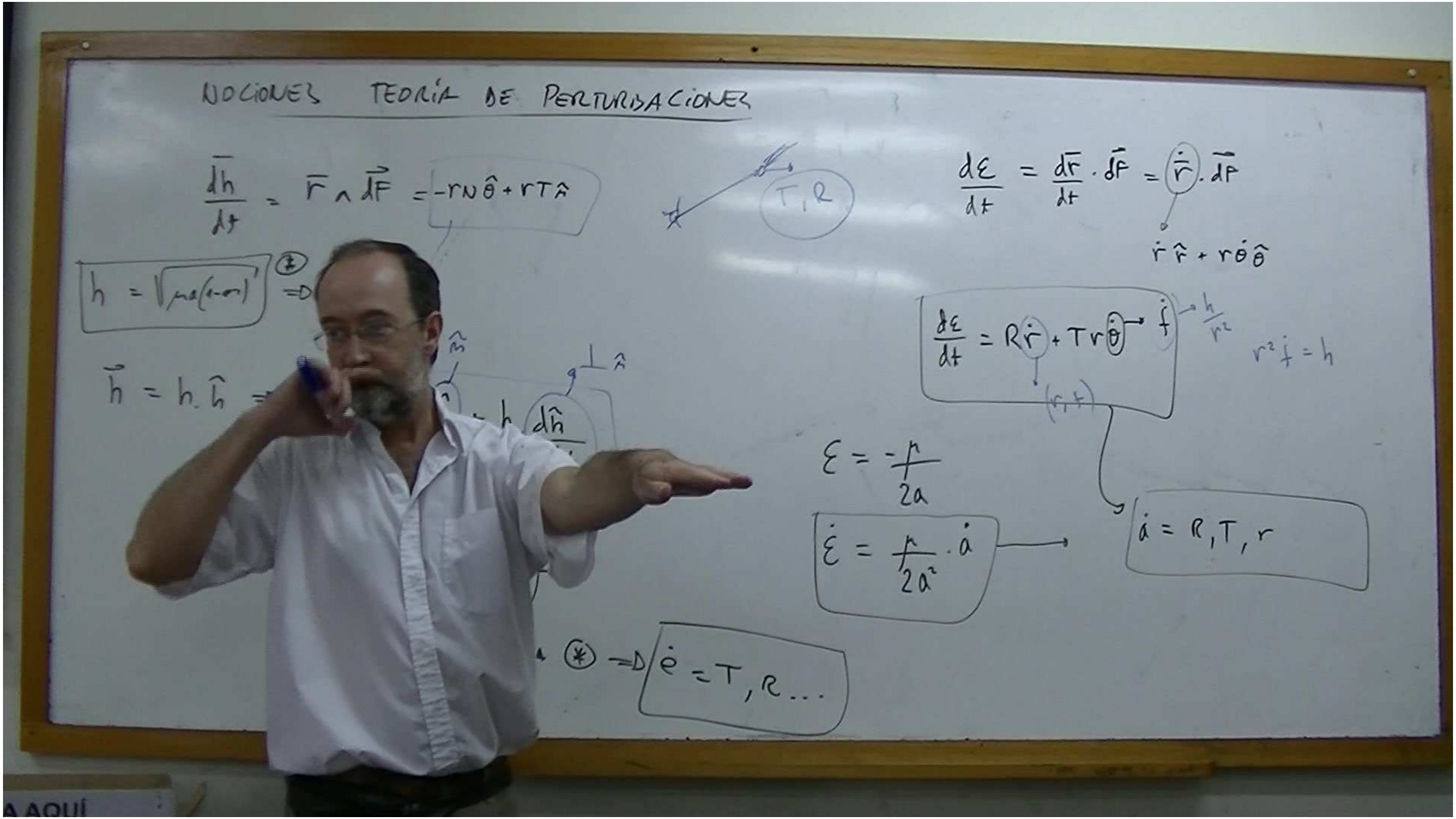
$$\dot{\bar{r}} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$$

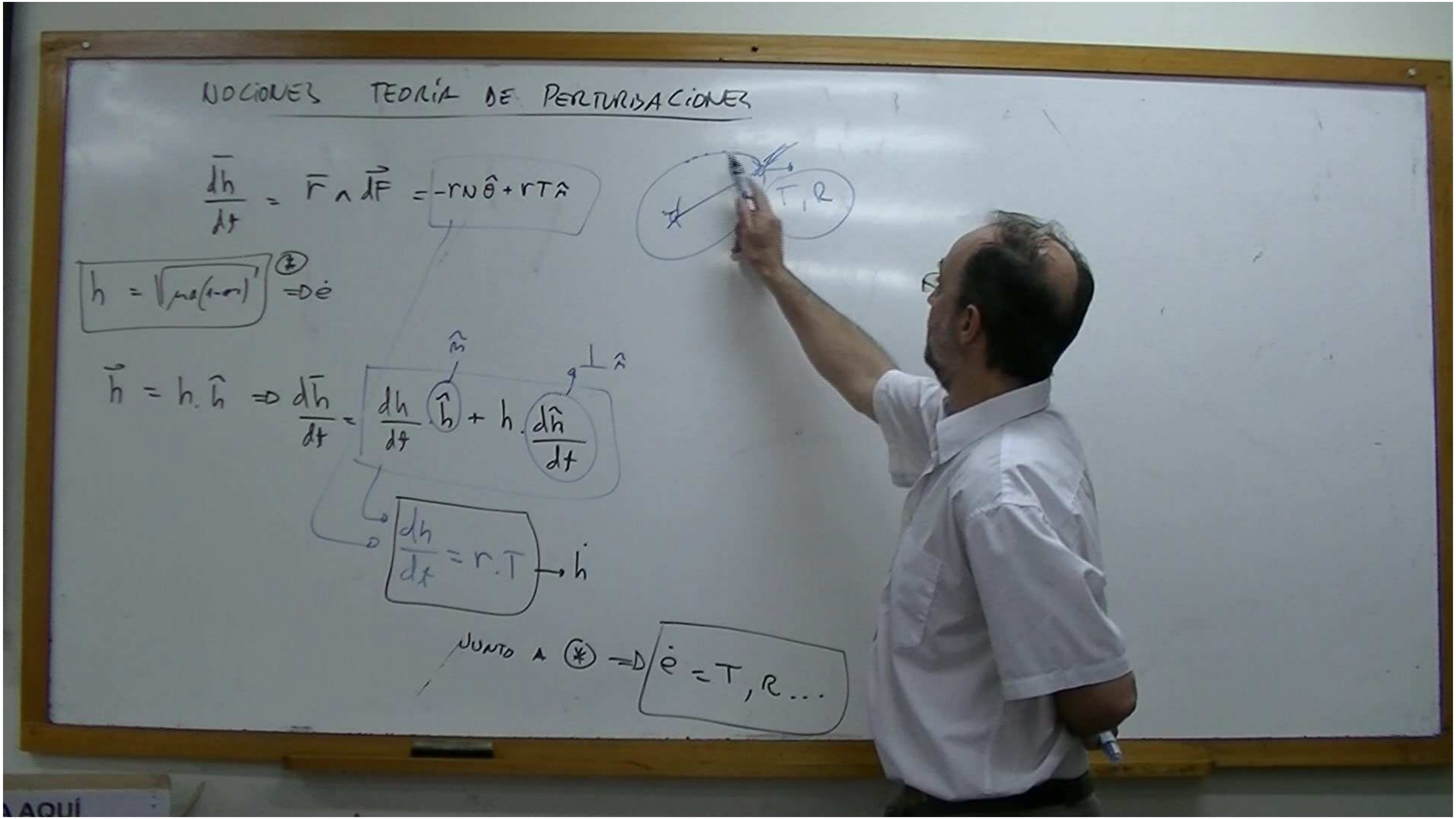
$$\boxed{\frac{d\mathcal{E}}{dt} = R\dot{r} + Tr\dot{\theta}} \qquad \dot{\theta} \rightarrow \dot{f} \rightarrow \frac{h}{r^2} \qquad r^2\dot{f} = h$$

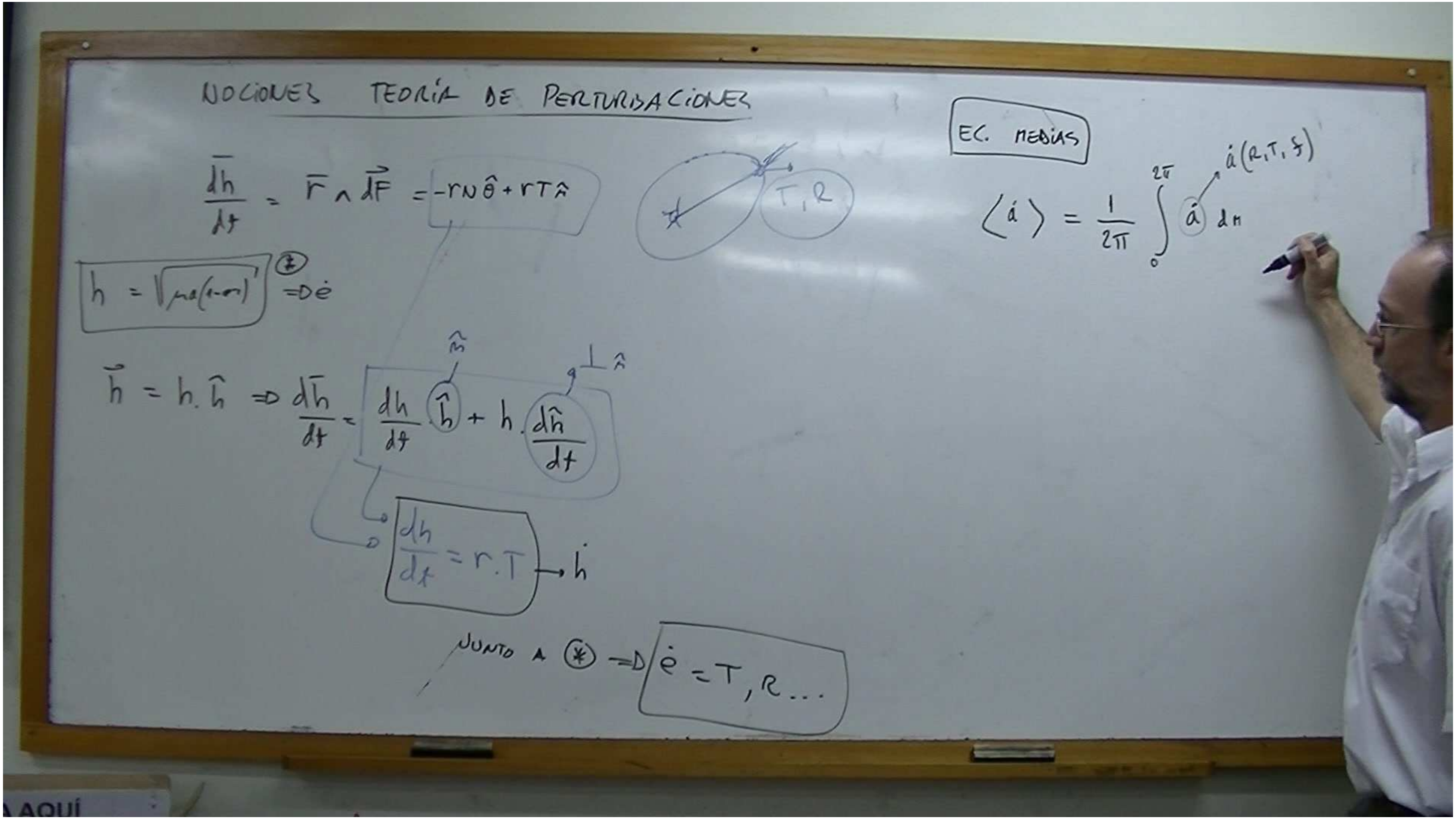
$$\dot{r} \rightarrow (r, f)$$

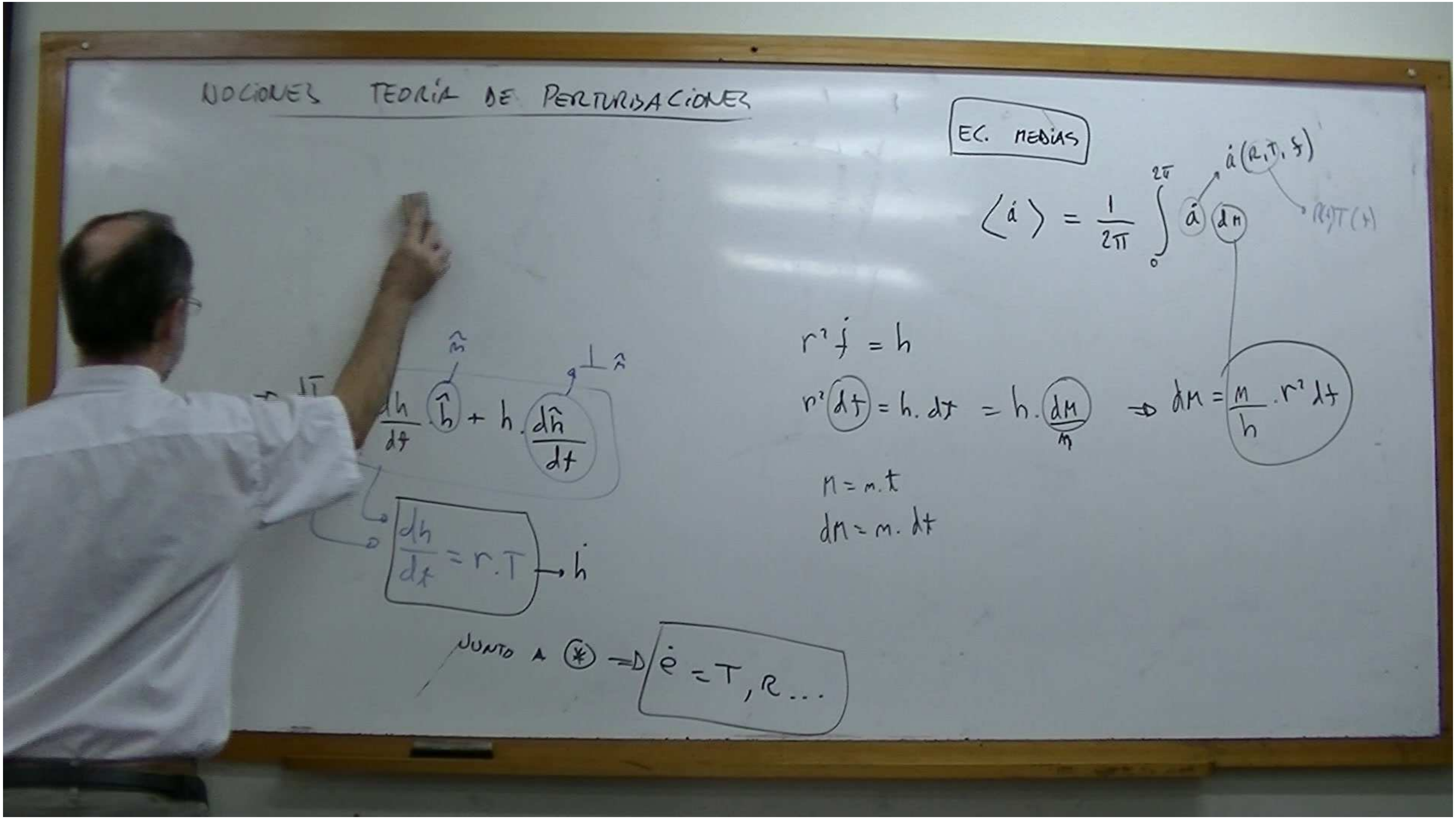
$$\mathcal{E} = -\frac{\mu}{2a}$$

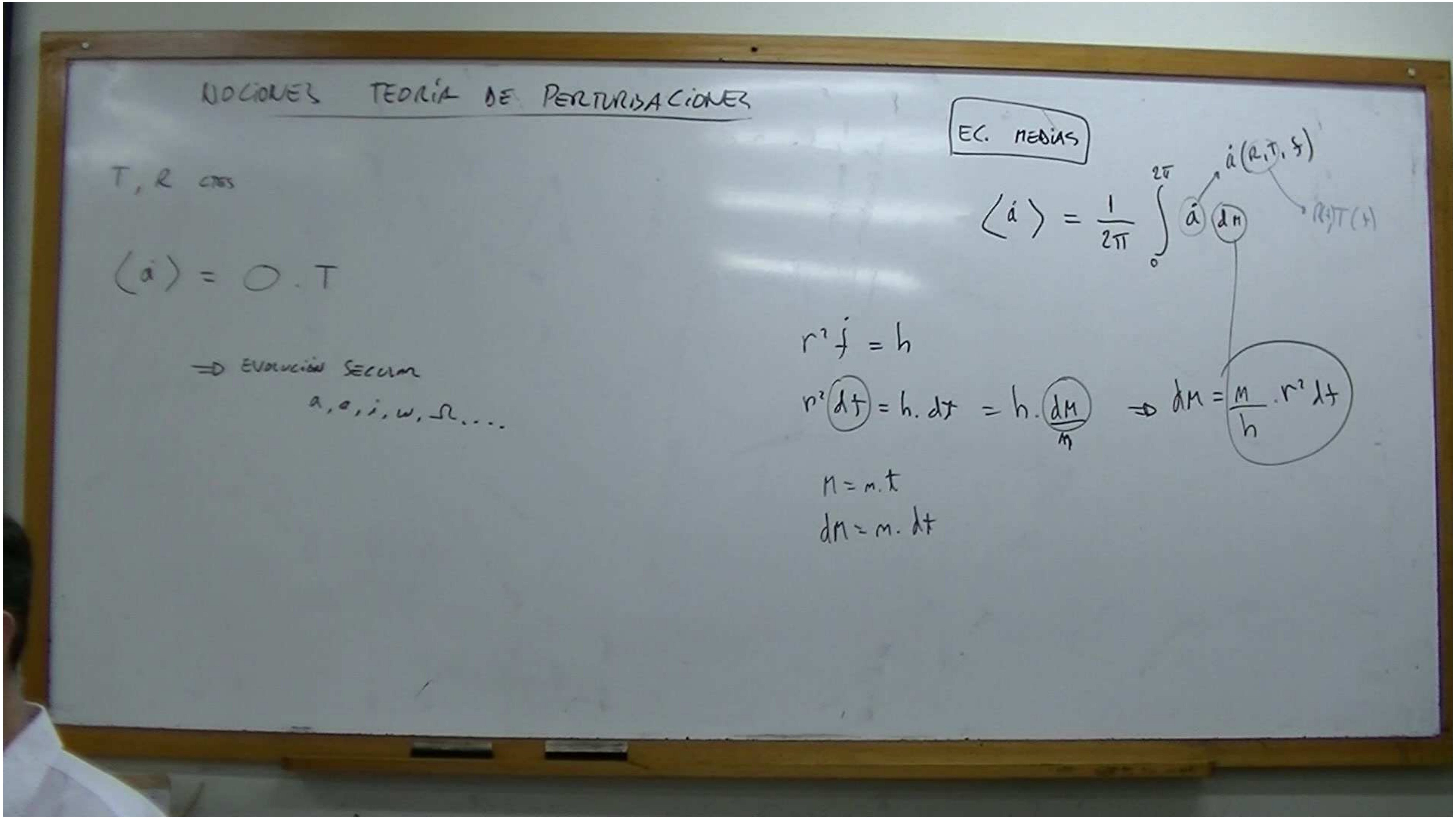
$$\boxed{\dot{\mathcal{E}} = \frac{\mu}{2a^2}\cdot\dot{a}} \rightarrow \boxed{\dot{a} = R, T, r}$$

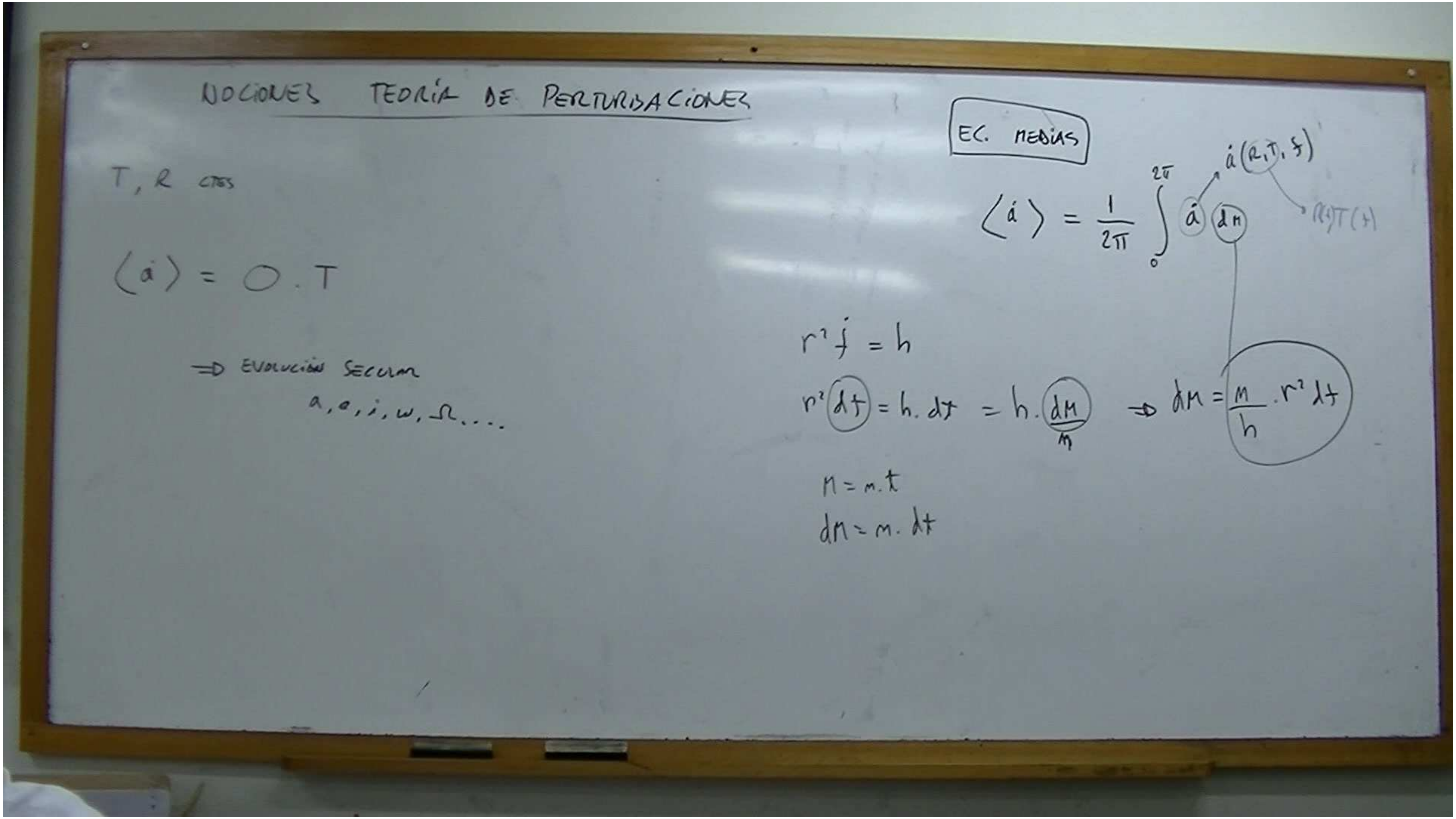

NOCIONES TEORÍA DE PERTURBACIONES
dh/dt = r ∧ dF = -rNθ̂ + rTr̂
T, R
dE/dt = dr/dt · dF = ṙ · dF
ṙr̂ + rθ̇θ̂
dE/dt = Rṙ + Trθ̇
f
h/r²
r²ḟ = h
(r, f)
E = -μ/2a
Ė = μ/2a² · ȧ
ȧ = R, T, r
h = √(μa(1-e²))
h = h ĥ
dĥ
ė = T, R ...

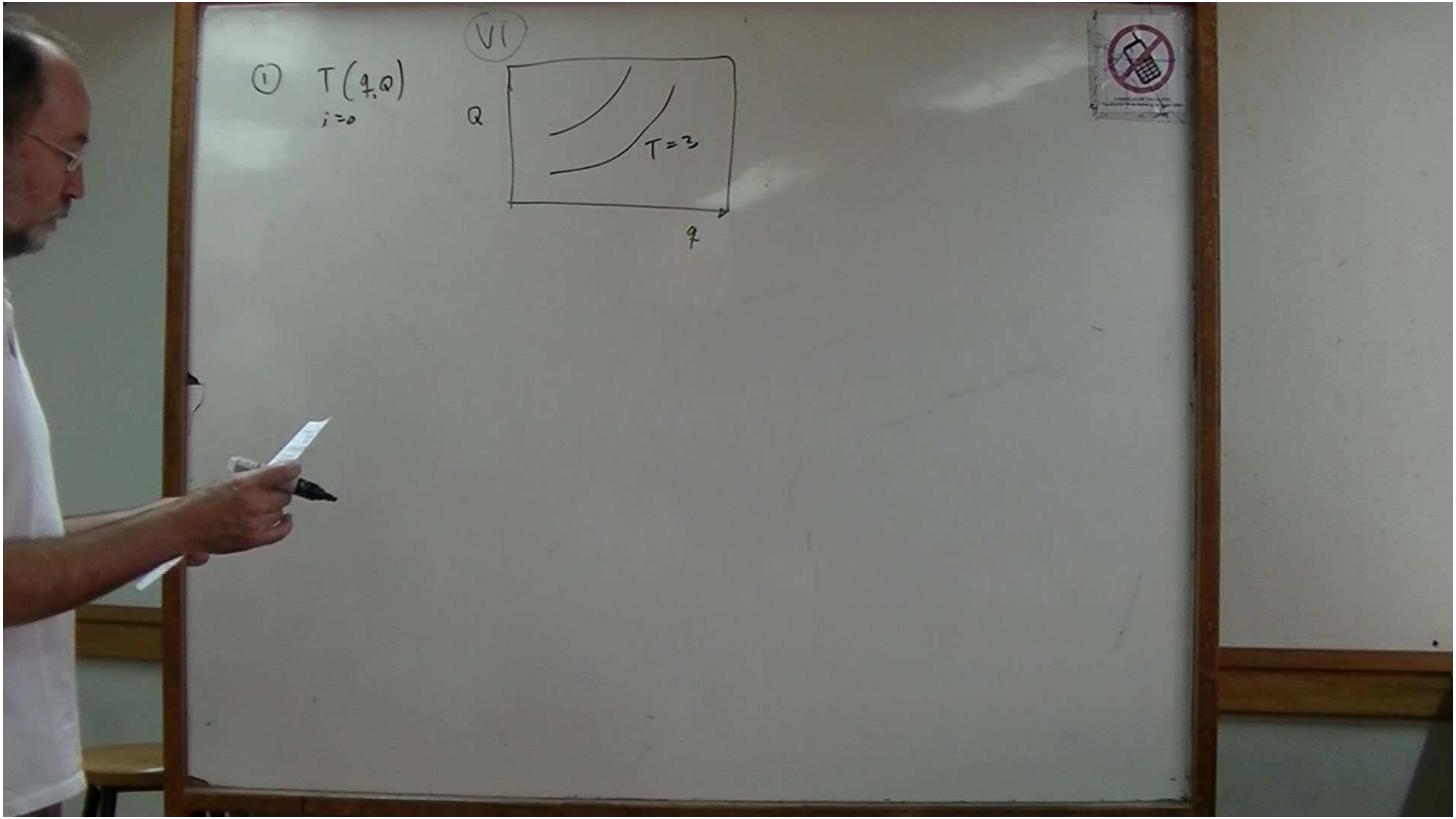

NOCIONES TEORÍA DE PERTURBACIONES
$\frac{d\bar{h}}{dt} = \bar{r} \wedge d\vec{F} = -rN\hat{\theta} + rT\hat{r}$
$h = \sqrt{\mu a(1-e^2)}$ (*) $\Rightarrow \dot{e}$
$\vec{h} = h.\hat{h} \Rightarrow \frac{d\bar{h}}{dt} = \frac{dh}{dt}\hat{h} + h.\frac{d\hat{h}}{dt}$
$\hat{n}$
$\perp \hat{r}$
$\frac{dh}{dt} = r.T \to \dot{h}$
T, R
JUNTO A (*) $\Rightarrow \dot{e} = T, R \ldots$

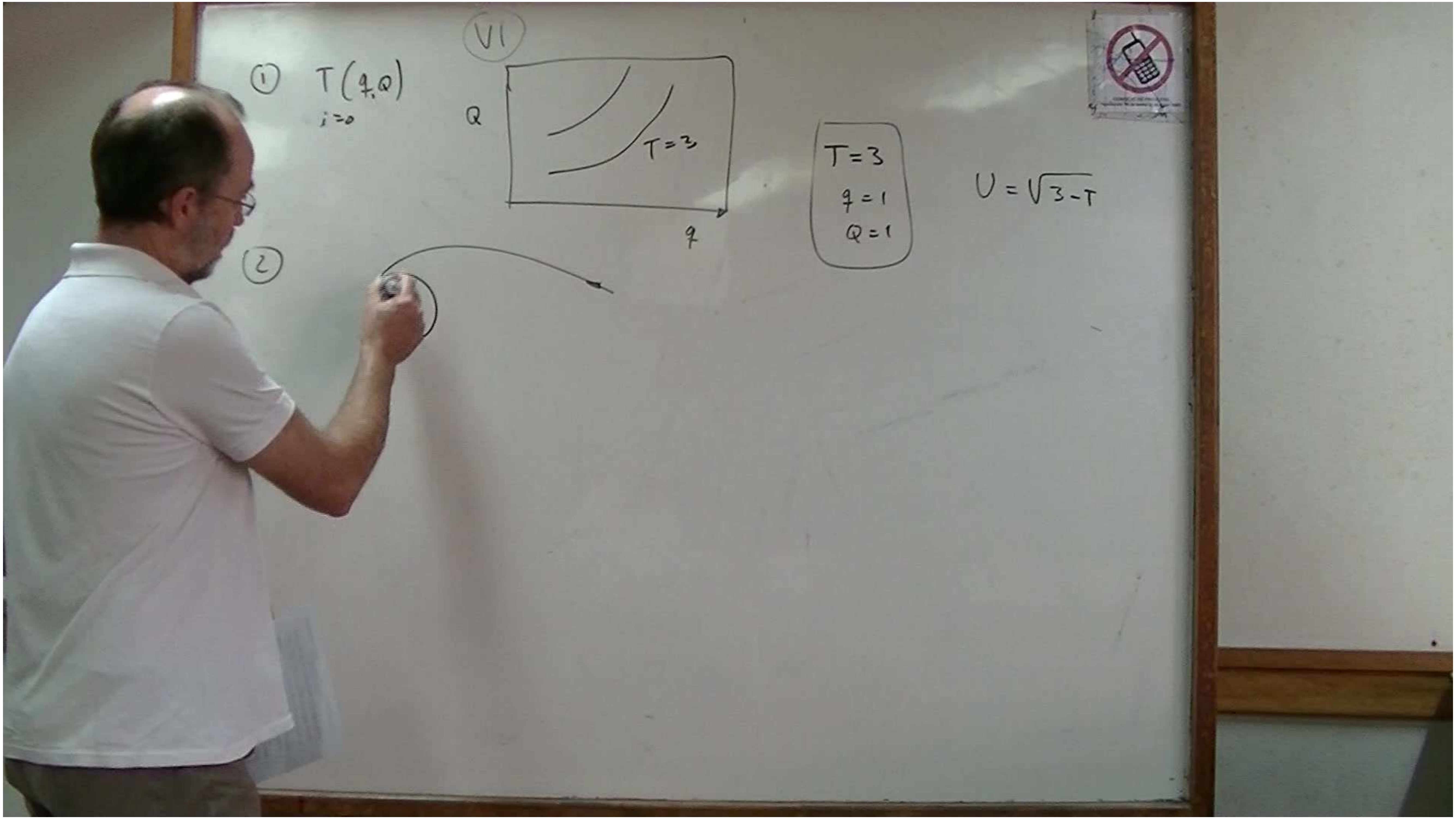

NOCIONES TEORÍA DE PERTURBACIONES
$\frac{d\bar{h}}{dt} = \bar{r} \wedge d\vec{F} = -rN\hat{\theta} + rT\hat{n}$
$h = \sqrt{\mu a(1-e^2)}$ (*) $\Rightarrow \dot{e}$
T, R
$\vec{h} = h.\hat{h} \Rightarrow \frac{d\bar{h}}{dt} = \frac{dh}{dt}.\hat{h} + h.\frac{d\hat{h}}{dt}$
$\hat{n}$
$\perp \hat{n}$
$\frac{dh}{dt} = r.T \rightarrow \dot{h}$
JUNTO A (*) $\Rightarrow \dot{e} = T, R \ldots$
EC. MEDIAS
$\langle \dot{a} \rangle = \frac{1}{2\pi}\int_0^{2\pi} \dot{a}\, dM$
$\dot{a}(R, T, S)$
A AQUÍ

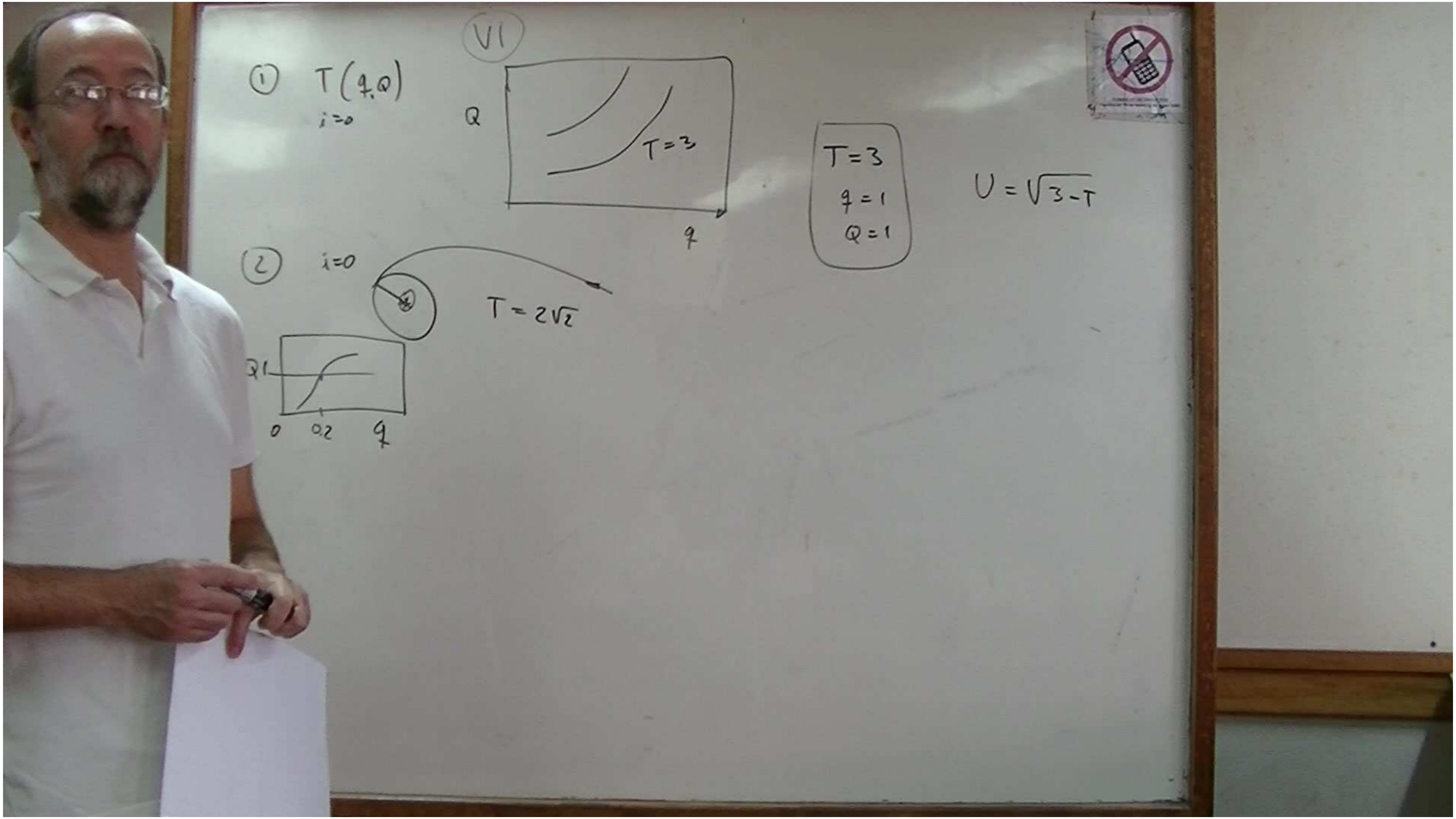

NOCIONES TEORÍA DE PERTURBACIONES
EC. MEDIAS
⟨ȧ⟩ = 1/2π ∫₀^2π ȧ dM
ȧ(R, T, f)
R(f) T(f)
r² ḟ = h
r² df = h. dt = h. dM/n
⇒ dM = n/h . r² df
M = n.t
dM = n. dt
dh/dt . ĥ + h . dĥ/dt
dh/dt = r.T → ḣ
JUNTO A ⊛ ⇒ ė = T, R ...

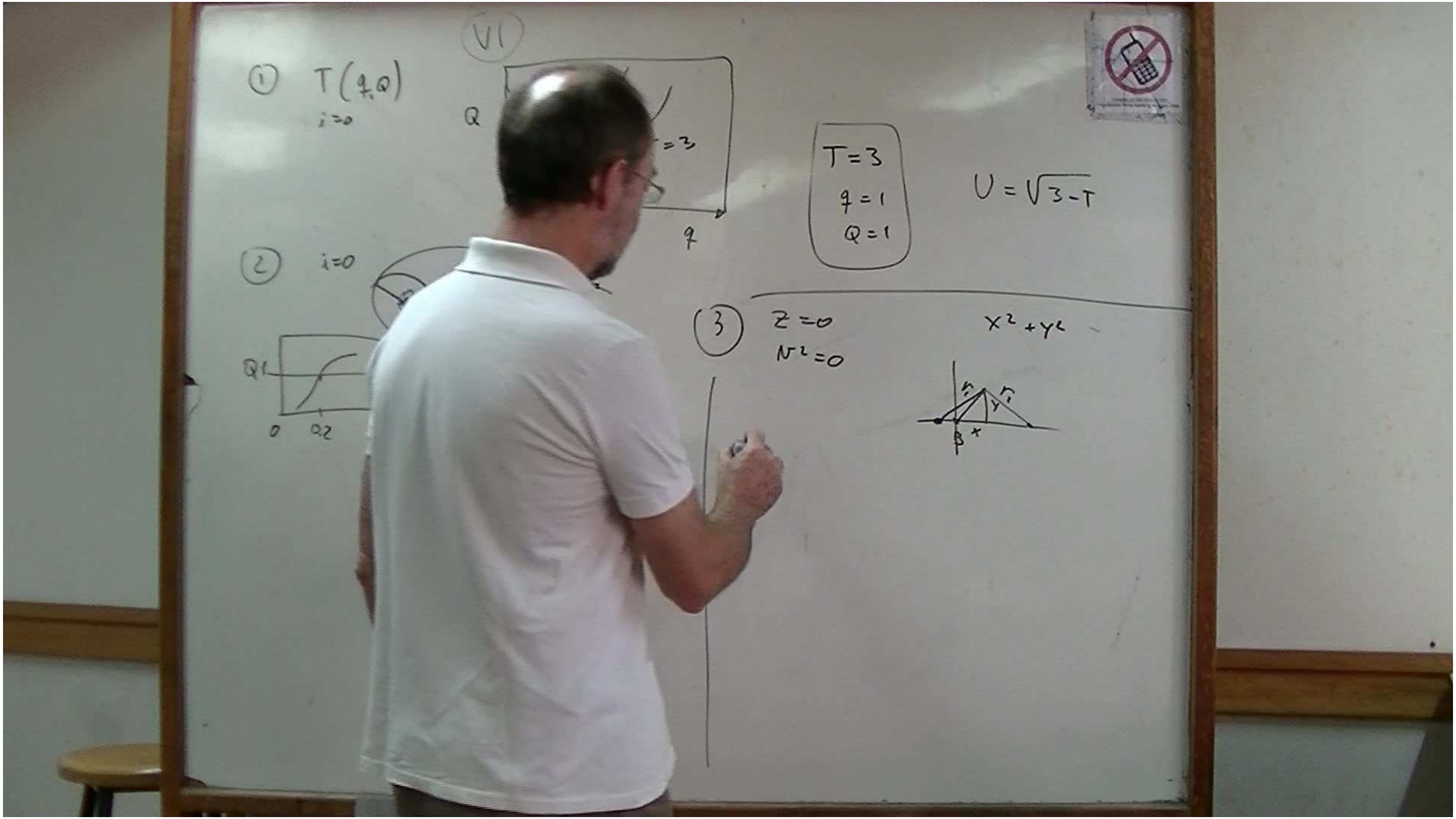

NOCIONES TEORÍA DE PERTURBACIONES
T, R ctes
$\langle \dot{a} \rangle = 0 . T$
⇒ EVOLUCIÓN SECULAR
$a, e, i, \omega, \Omega \ldots$
EC. MEDIAS
$\langle \dot{a} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \dot{a} \, dM$
$\dot{a}(R, T, f)$
$R(t) T(t)$
$r^2 \dot{f} = h$
$r^2 df = h . dt = h . \frac{dM}{n}$
$\Rightarrow dM = \frac{n}{h} . r^2 df$
$M = n . t$
$dM = n . dt$

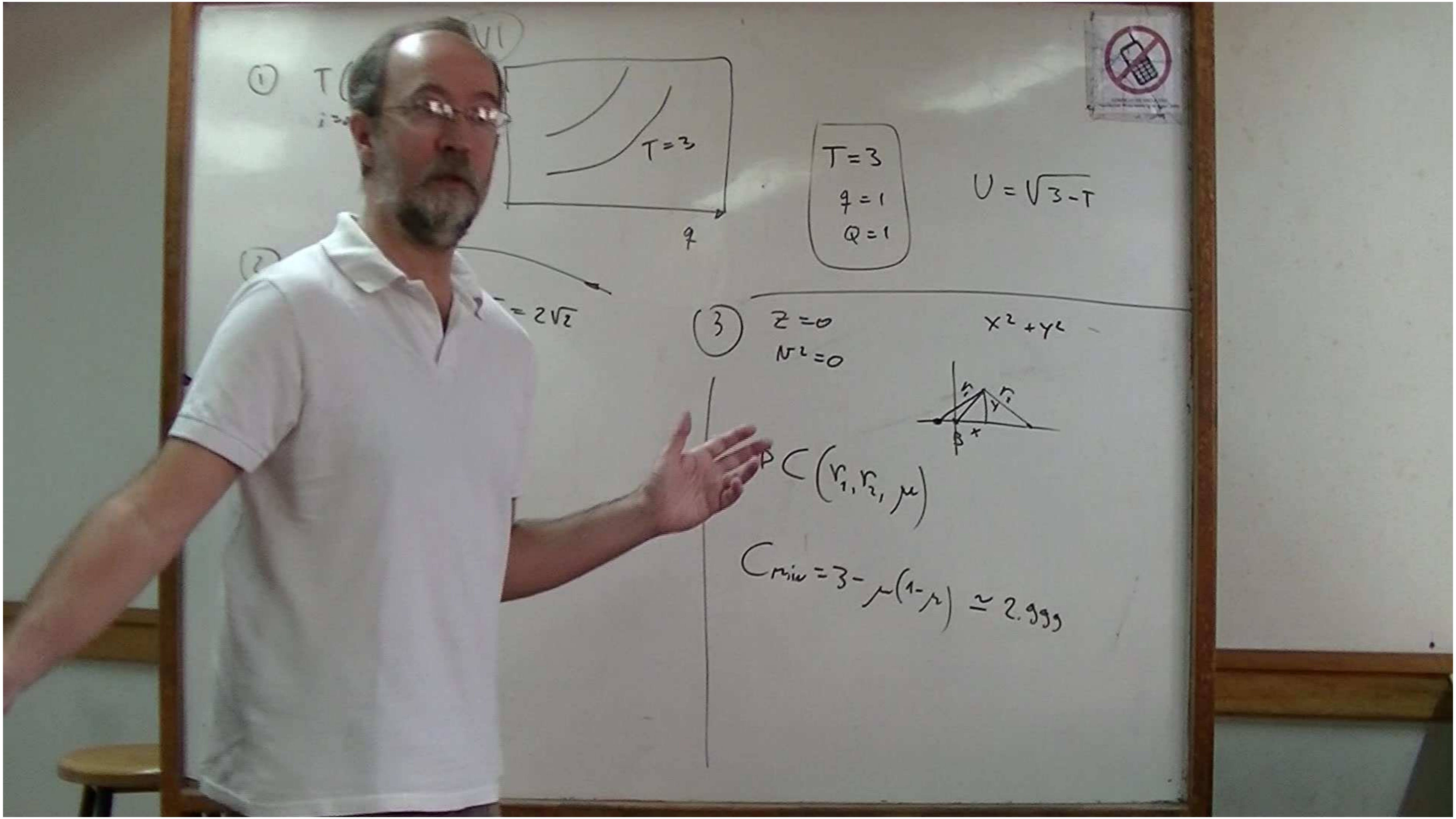

NOCIONES TEORÍA DE PERTURBACIONES
T, R ctes
$\langle \dot{a} \rangle = O \cdot T$
⇒ EVOLUCIÓN SECULAR
$a, e, i, \omega, \Omega \ldots$
EC. MEDIAS
$\langle \dot{a} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \dot{a} \, dM$
$\dot{a}(R, T, f)$
$R(f) T(f)$
$r^2 \dot{f} = h$
$r^2 df = h \cdot dt = h \cdot \frac{dM}{n} \Rightarrow dM = \frac{n}{h} \cdot r^2 df$
$M = n \cdot t$
$dM = n \cdot dt$

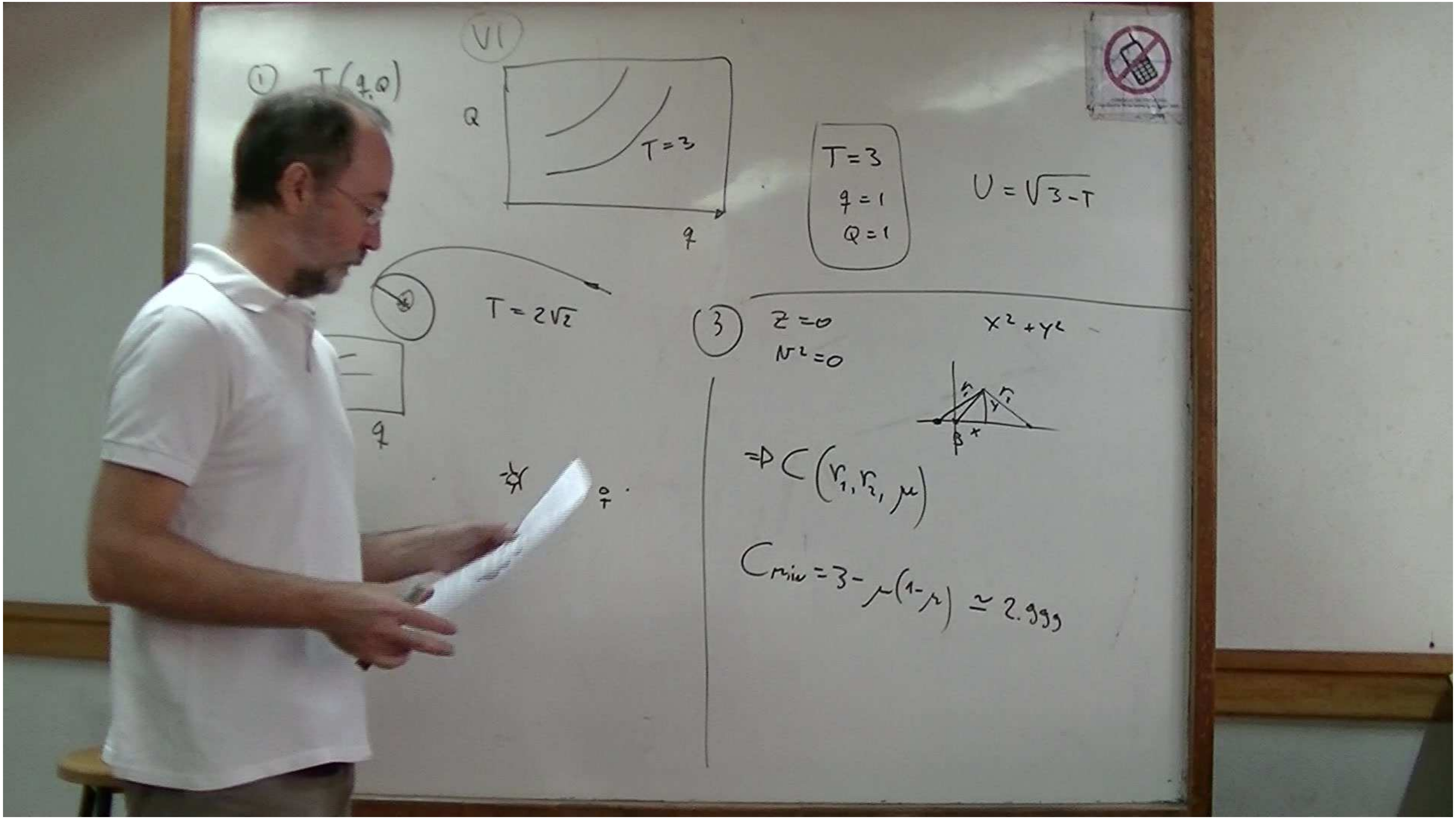

VI
① T(q,Q)
i=0
Q
T=3
q

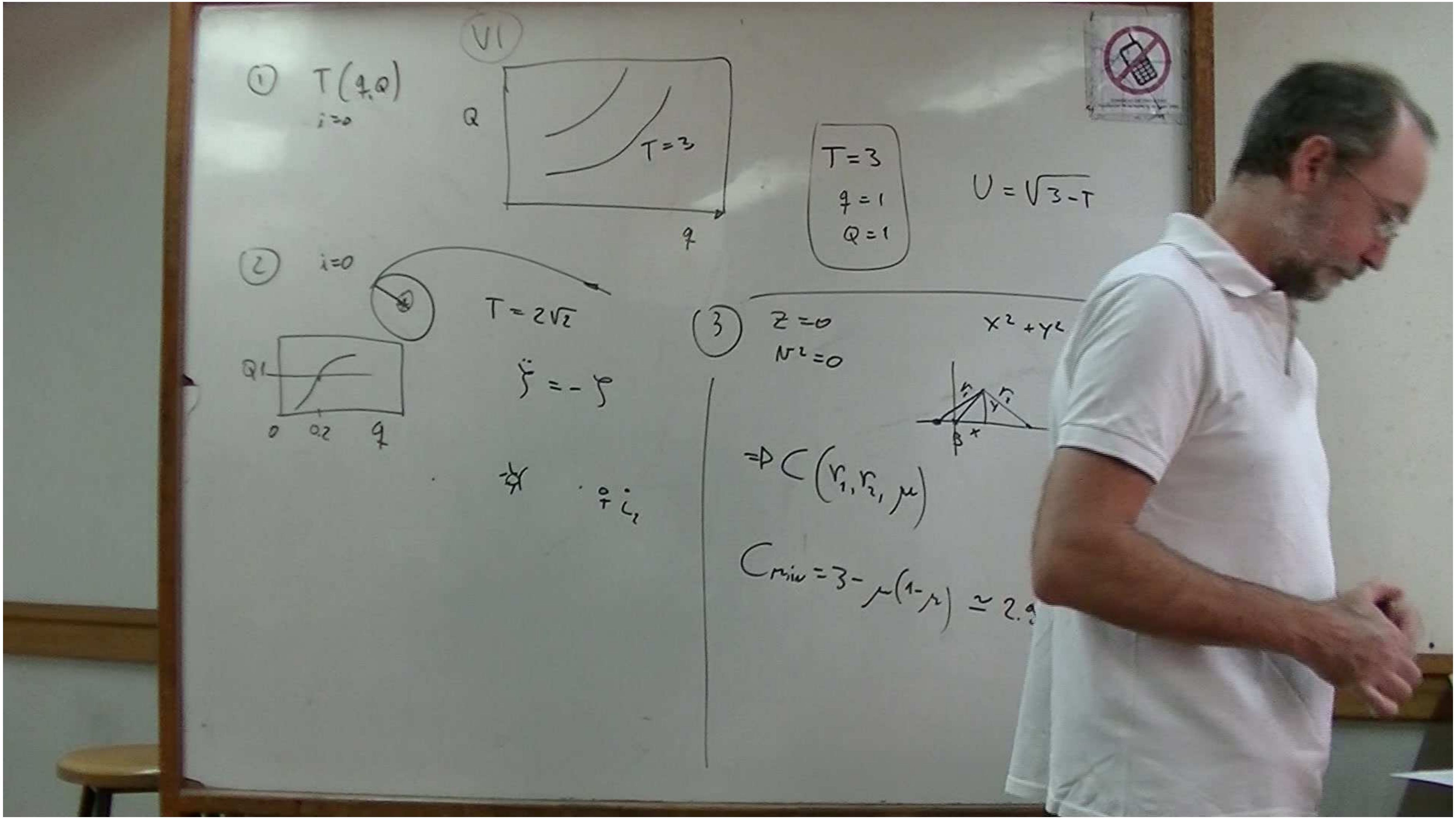

VI
1 T(q,Q)
i=0
Q
T=3
q
T=3
q=1
Q=1
U = √3-T
2

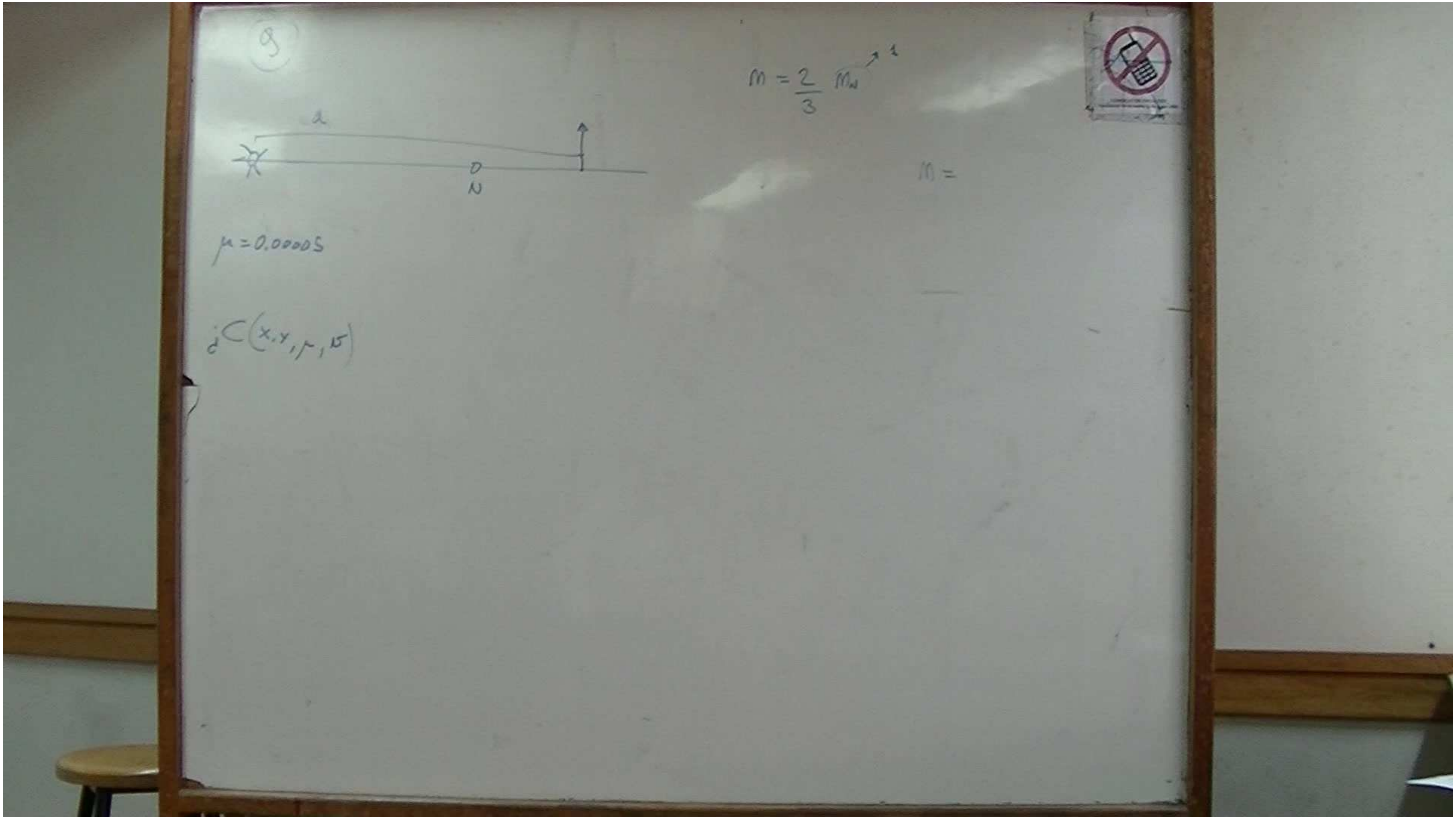

VI
1
T(q,Q)
i=0
Q
T=3
q
2
i=0
T = 2√2
Q1
0
0.2
q
T=3
q=1
Q=1
U = √(3-T)

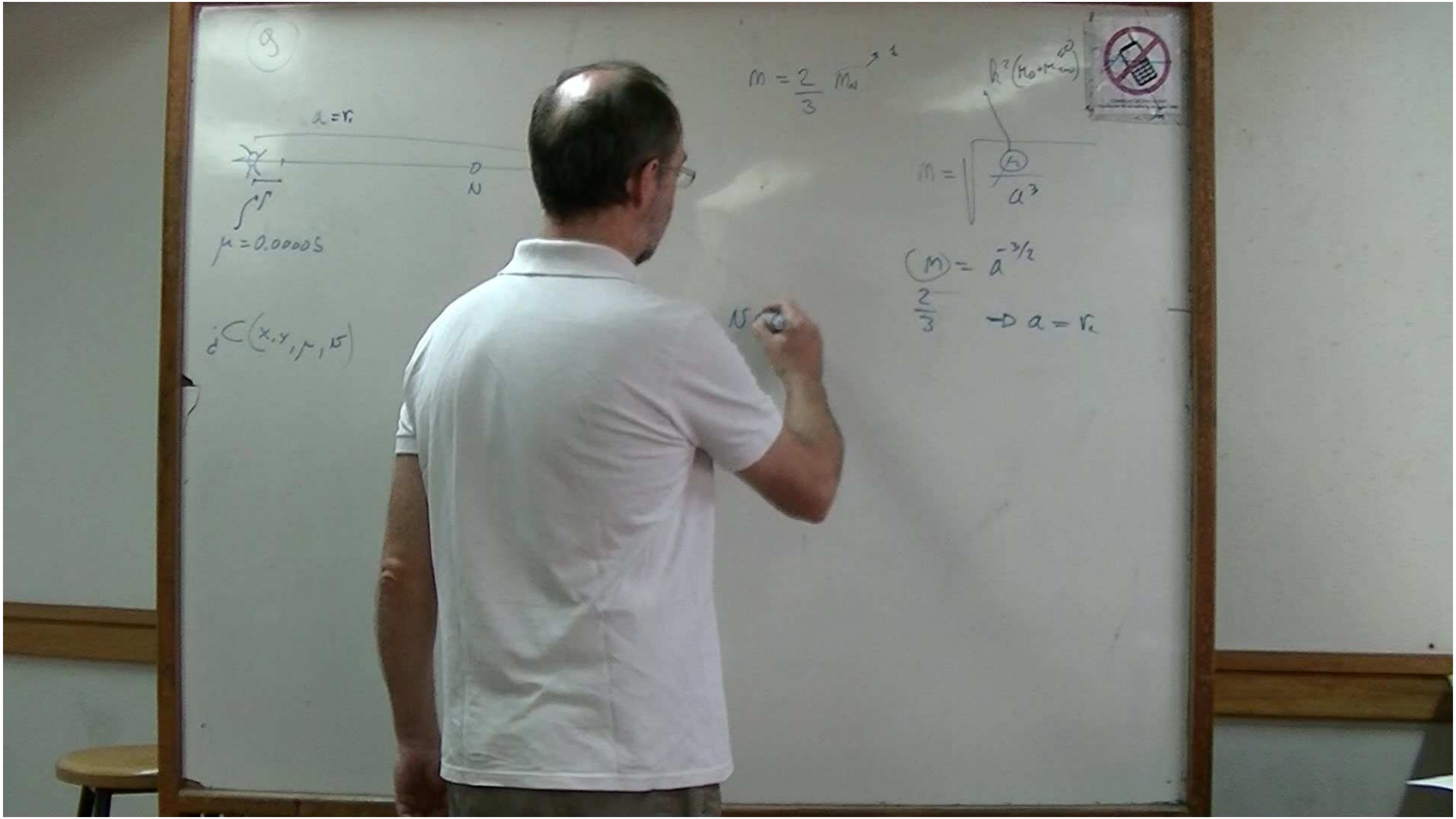

VI
① T(q,Q)
i=0
Q
T=3
q
② i=0
Q1
0
0.2
T=3
q=1
Q=1
U = √3-T
③ Z=0
v² = 0
x² + y²
r
r₁
y
β
x

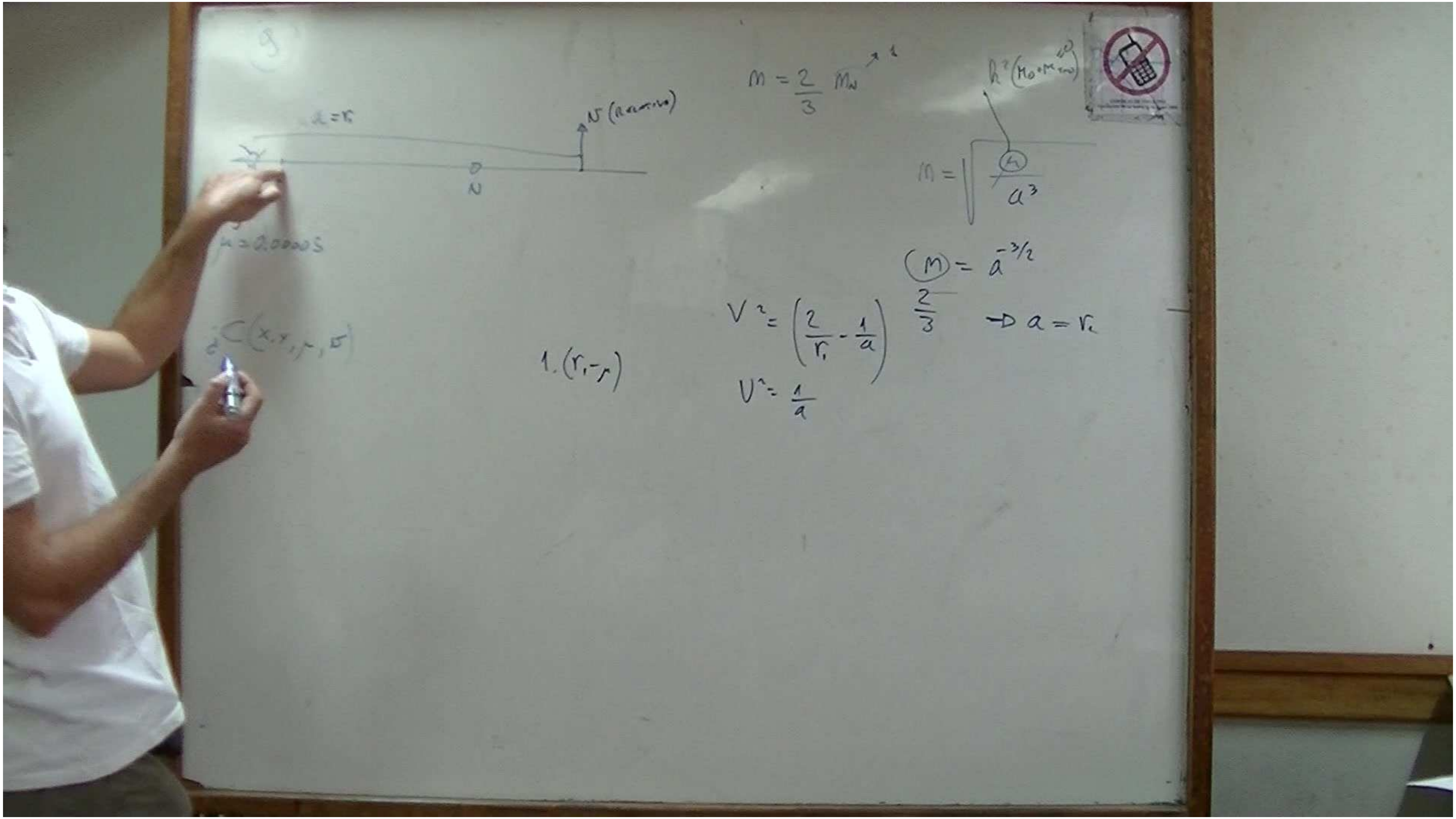

T = 3
q
T = 3
q = 1
Q = 1
U = √3 - T
3
Z = 0
x² + y²
C(r₁, r₂, μ)
C_min = 3 - μ(1 - μ) ≃ 2.99

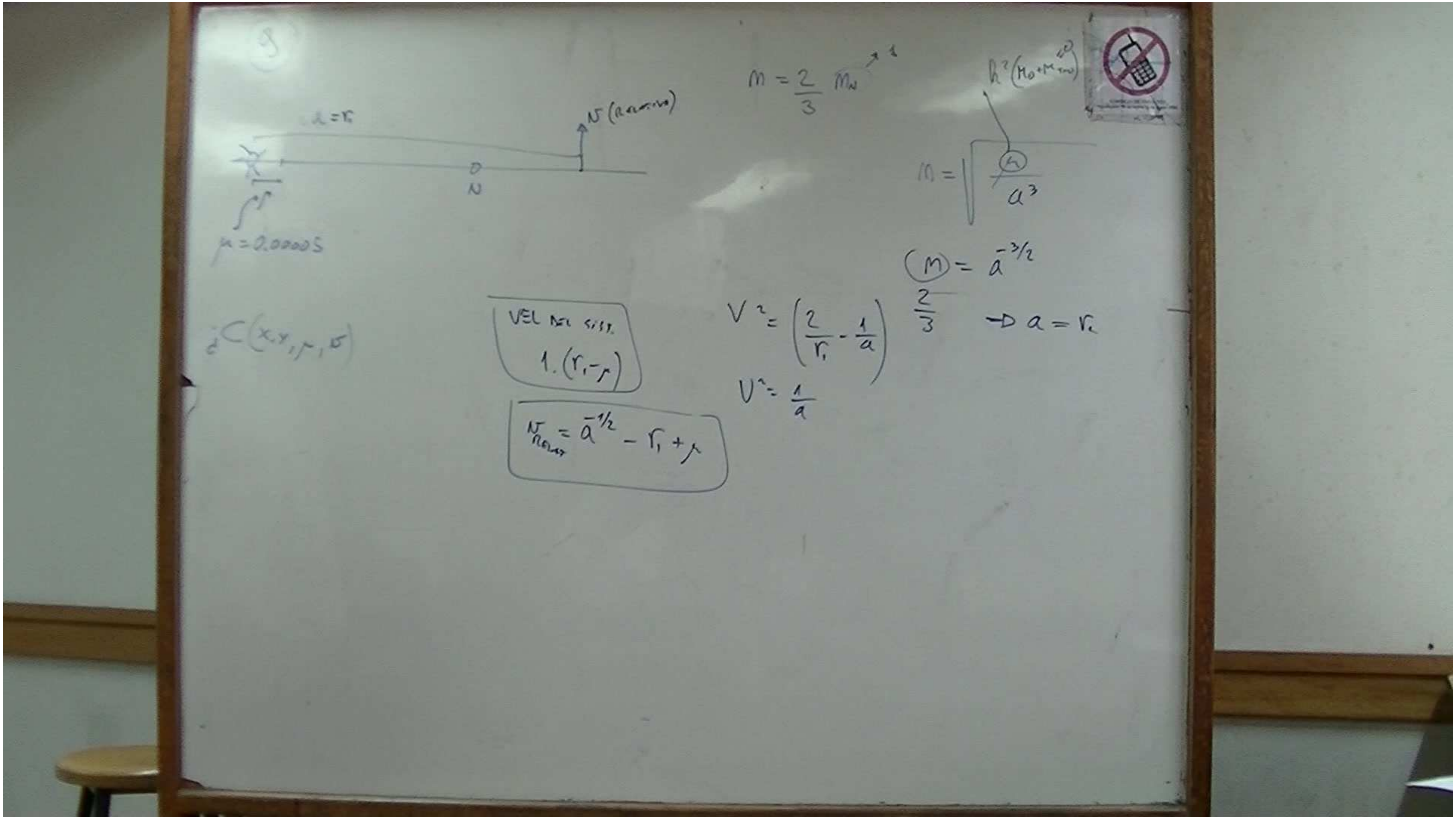

VI
1 T(q,Q)
Q
T = 3
q
T = 3
q = 1
Q = 1
U = √(3 - T)
T = 2√2
3 z = 0
v² = 0
x² + y²
⇒ C(r₁, r₂, μ)
C_min = 3 - μ(1 - μ) ≃ 2.99

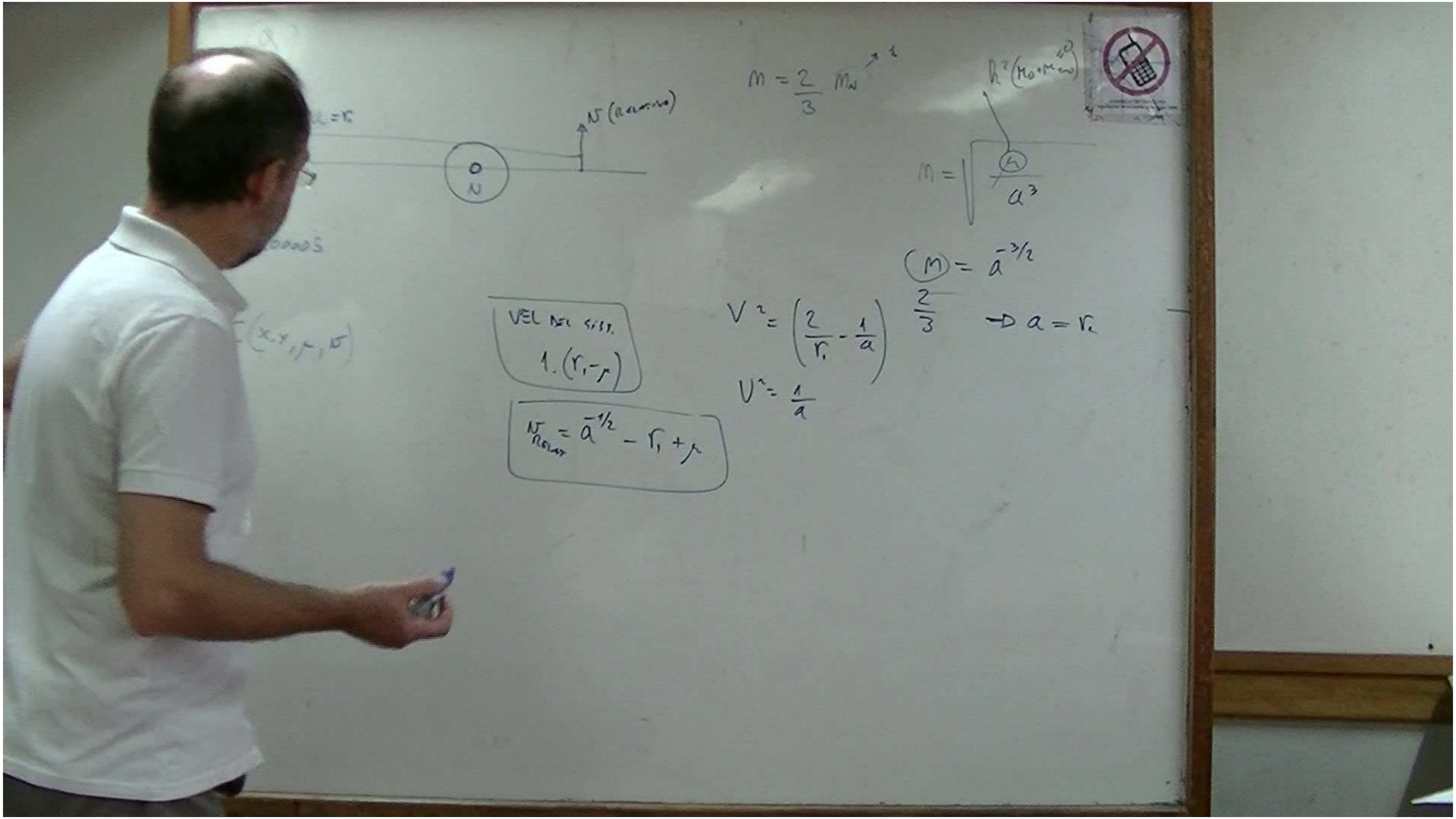

VI
1 T(q,Q)
i=0
Q
T=3
q
T=3
q=1
Q=1
U = √3-T
2 i=0
T = 2√2
0 0.2 q
3 z=0
v²=0
x²+y²
⇒ C(r1, r2, μ)
Cmin = 3 - μ(1-μ) ≃ 2.9

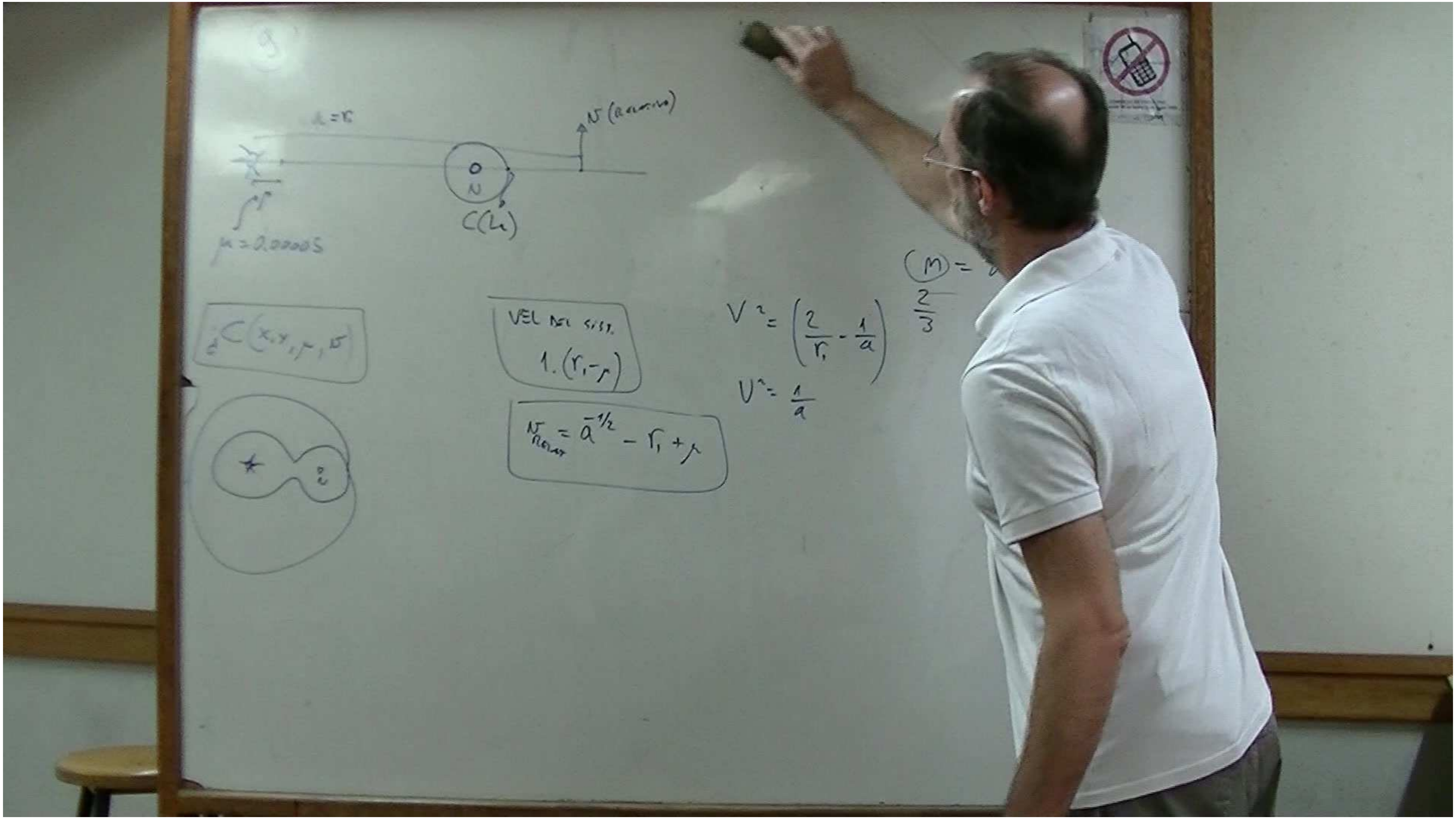

9
a
N
m = 2/3 m
m =
μ = 0.00005
¿C(x,y,μ,v)

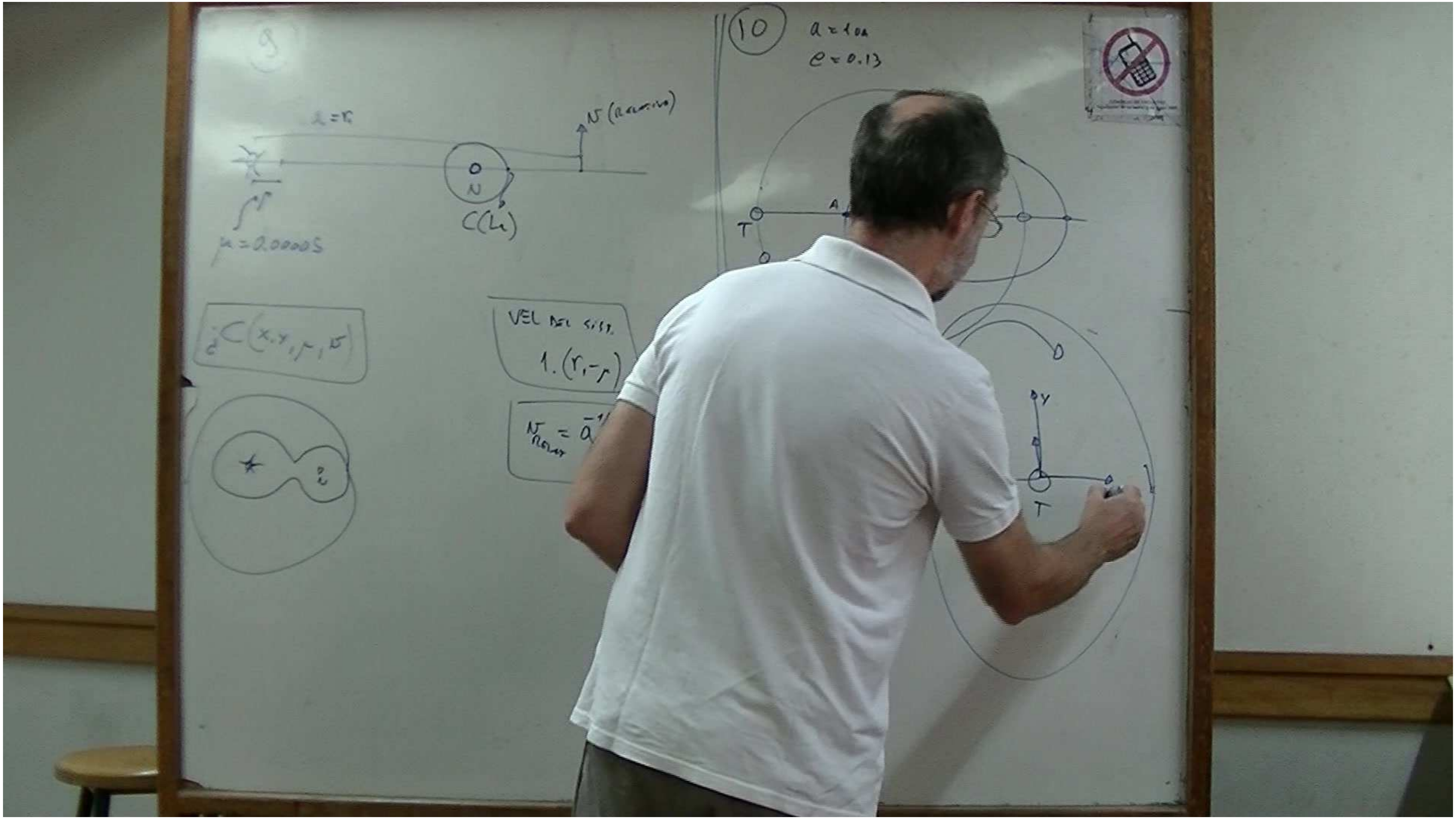

a = r
μ = 0.00005
N
¿C(x, y, μ, v)
n = 2/3 n
a³
a = r

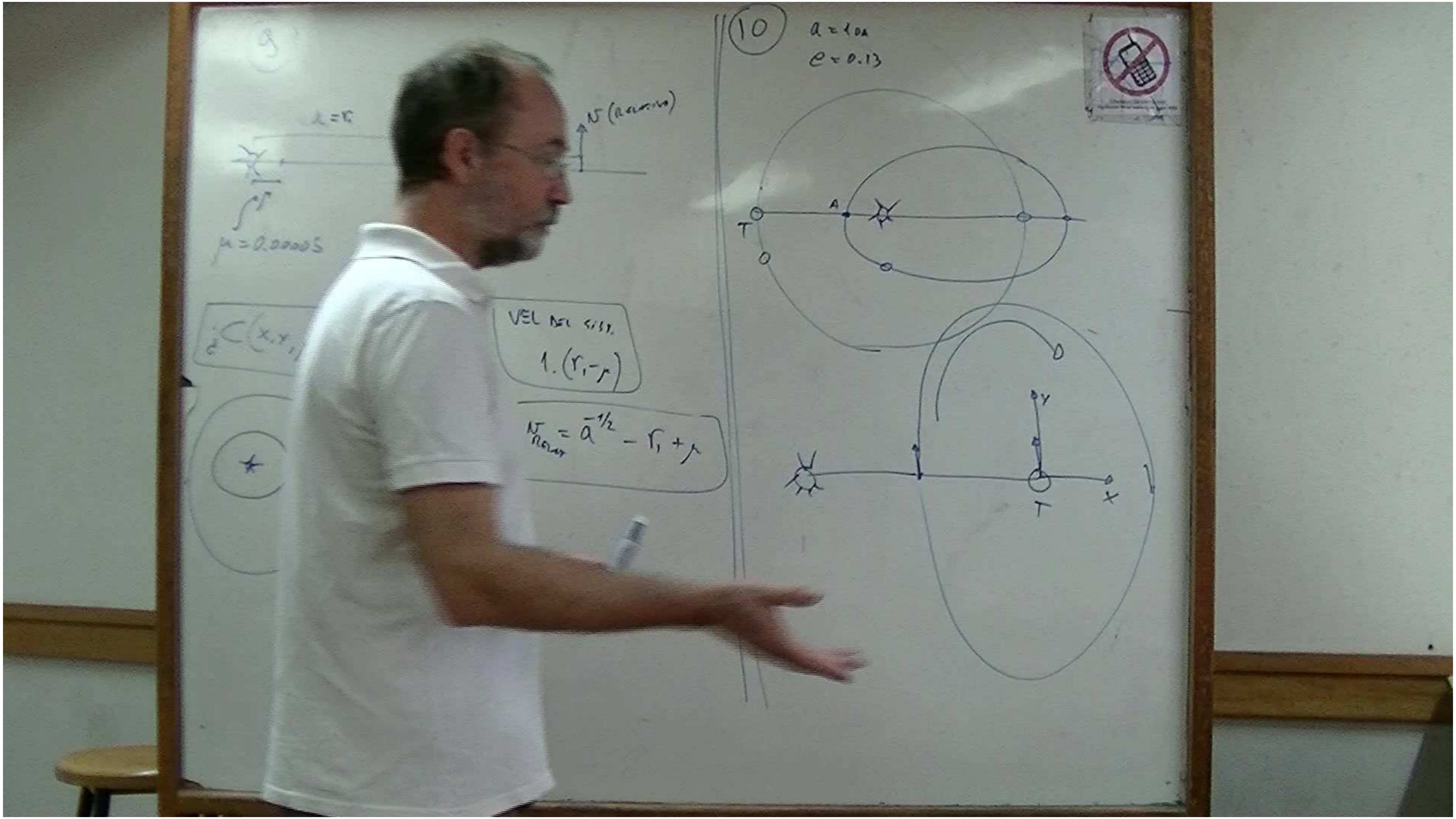

N
1. (r, - r)
V² = (2/r, - 1/a)
V² = 1/a
n = 2/3 n_N
n = √(k/a³)
n = a^-3/2
2/3 → a = r,

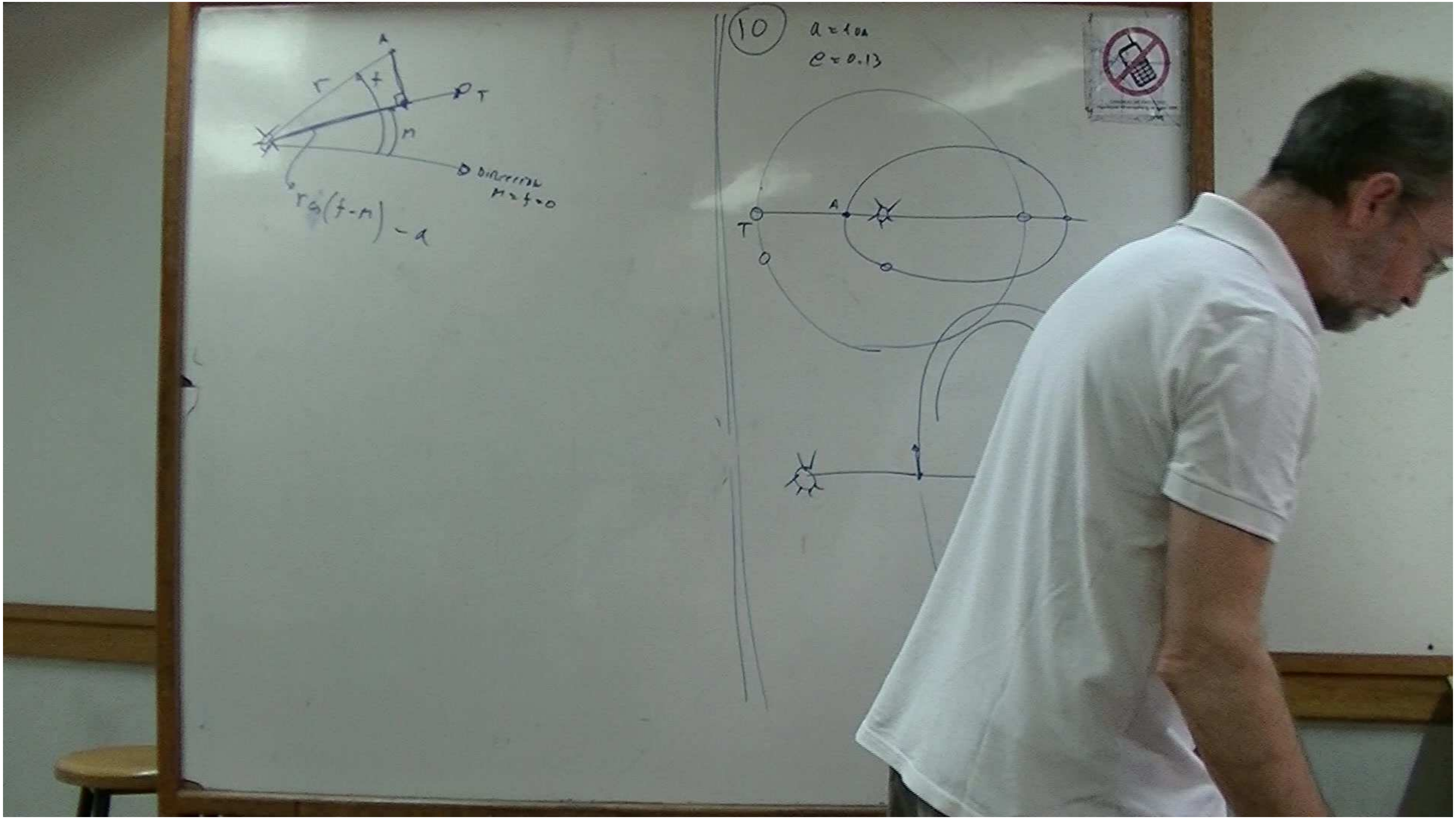

a = r1
N (relativo)
N
μ = 0.00005
¿C(x, y, μ, v)
VEL DEL SIST.
1.(r1 − μ)
N relat = a^(−1/2) − r1 + μ
V² = (2/r1 − 1/a)
V² = 1/a
n = 2/3 nN
n = √(k²(M0+M)/a³)
n = a^(−3/2)
2/3 → a = r1

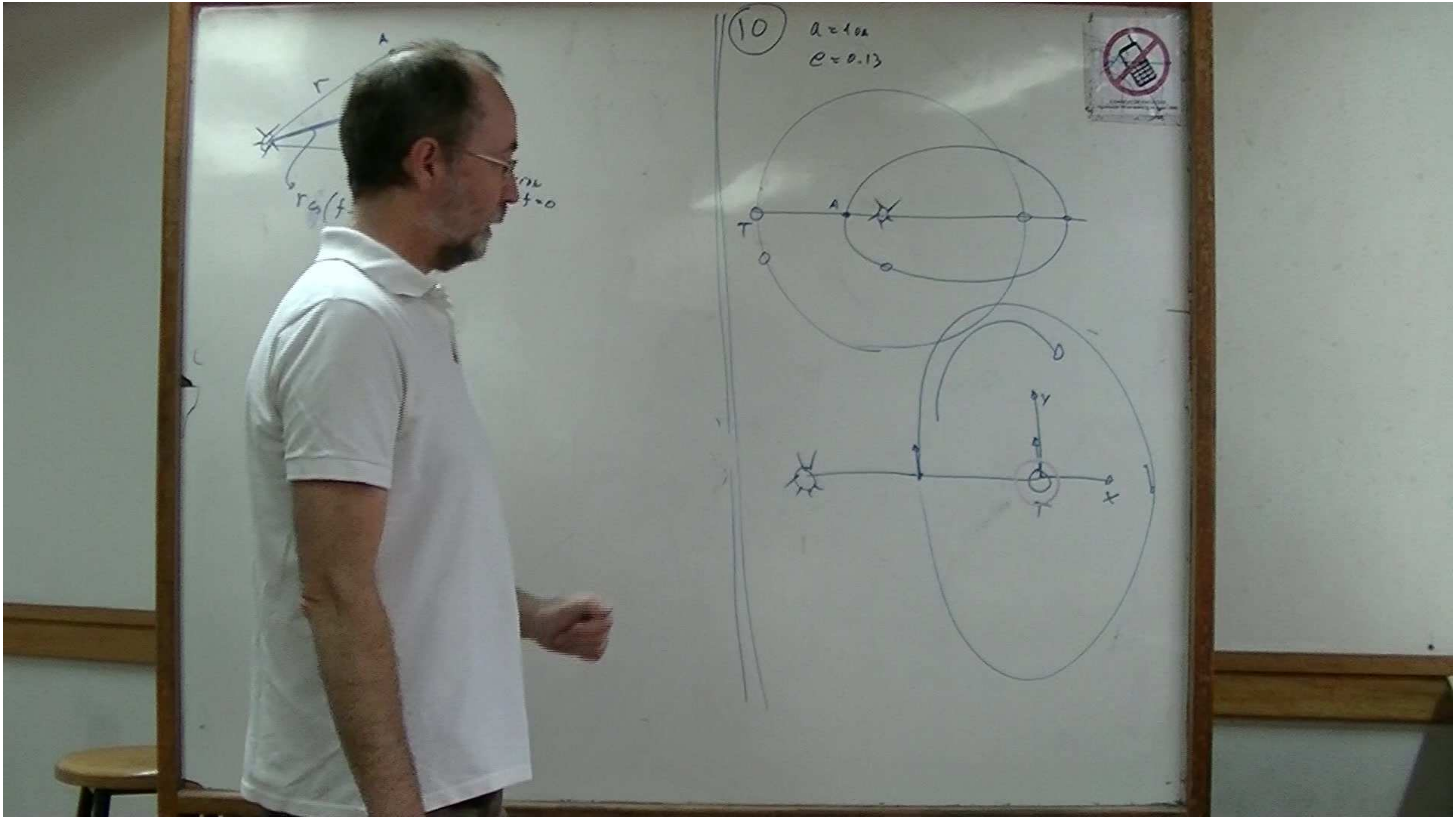

VEL DEL SIST.
1.(r1-μ)
V² = (2/r1 - 1/a)
V² = 1/a
M = 2/3 Mν
M = a^-3/2
2/3 → a = r2

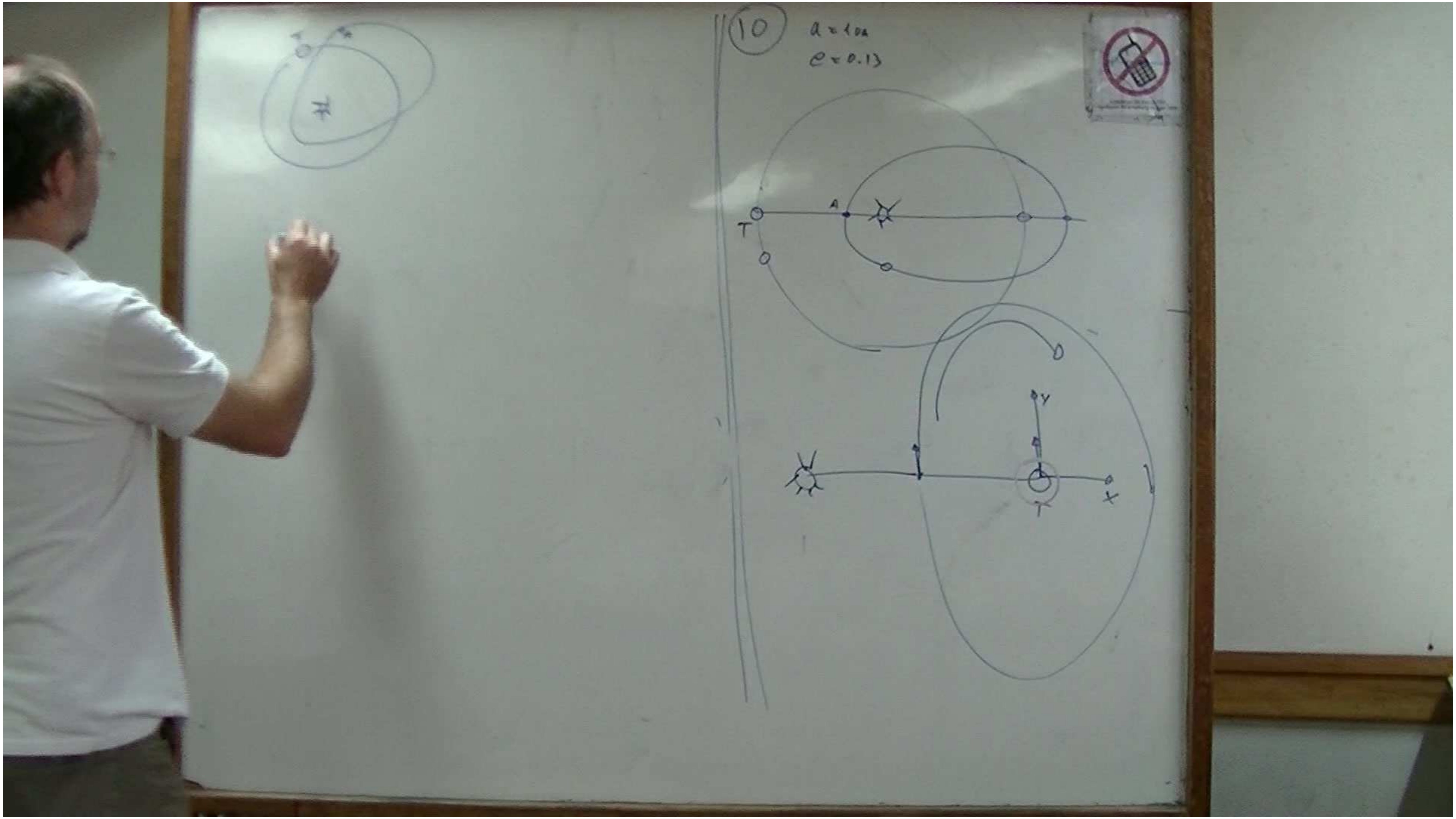


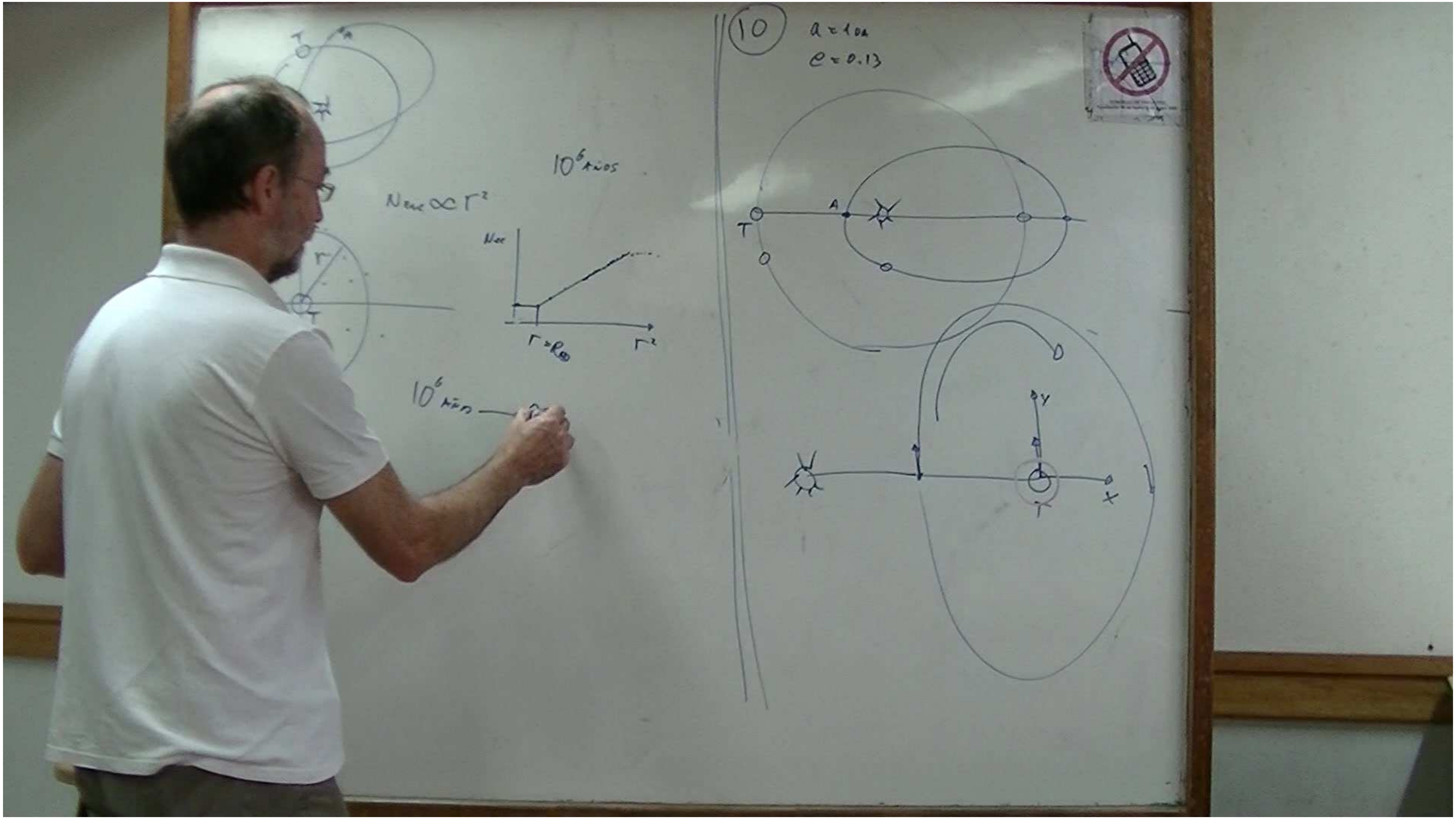


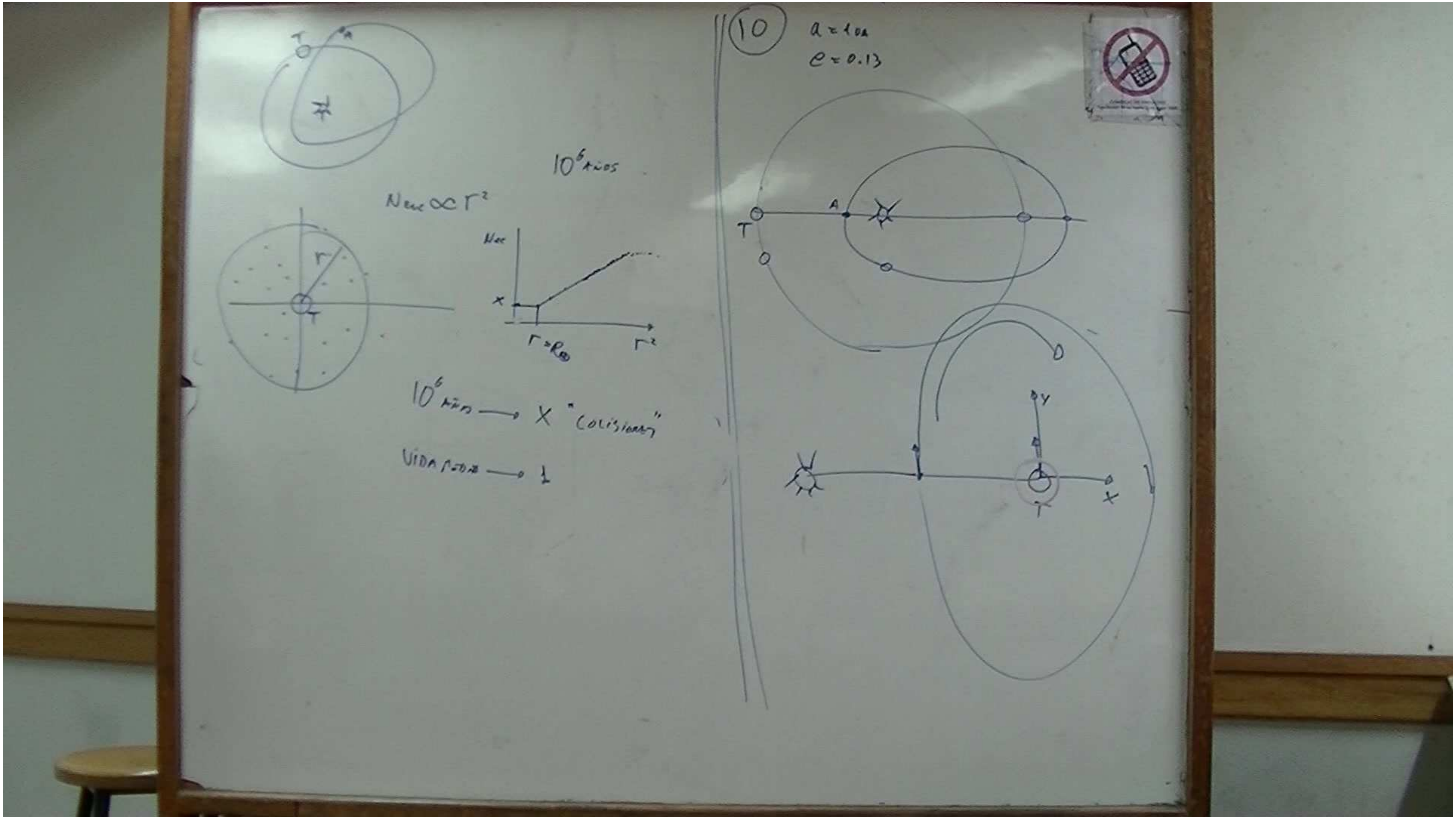

10
e = 0.13
VEL DEL SIST.
μ = 0.00005

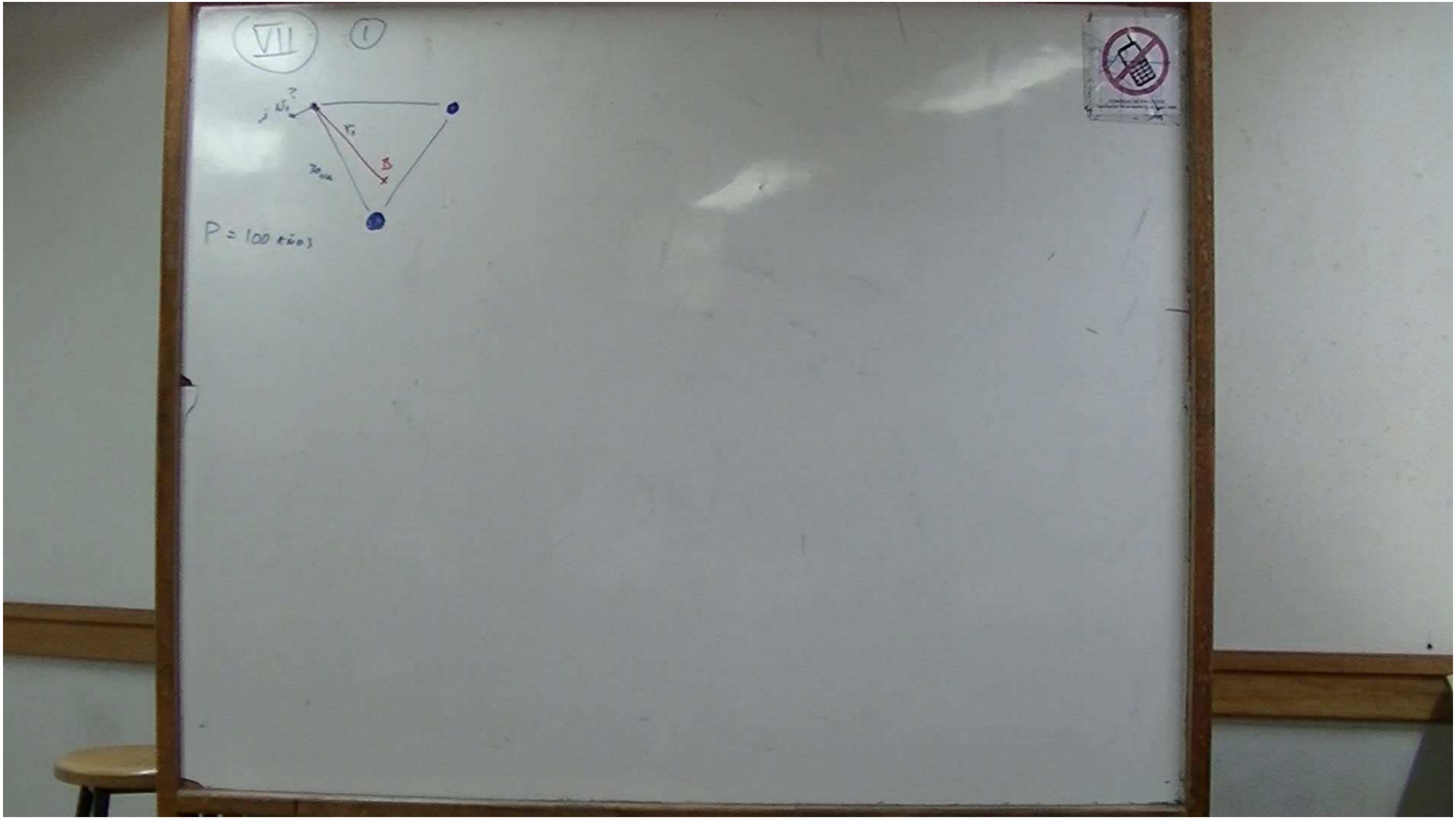

10
a = 1 UA
e = 0.13
r
f
n
T
A
$r \cos(f-n) - a$

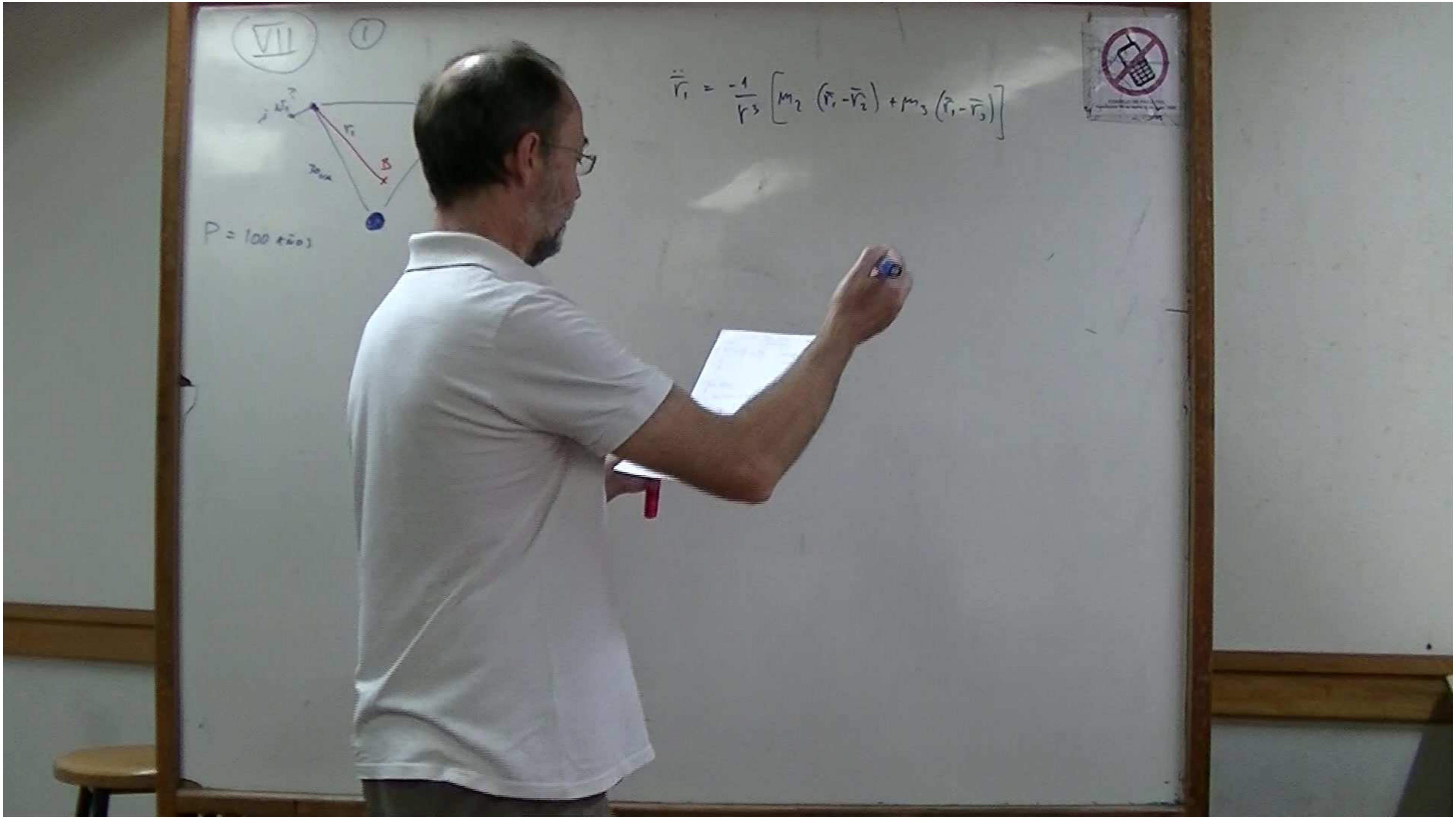

10
e = 0.13
A
T
y
x

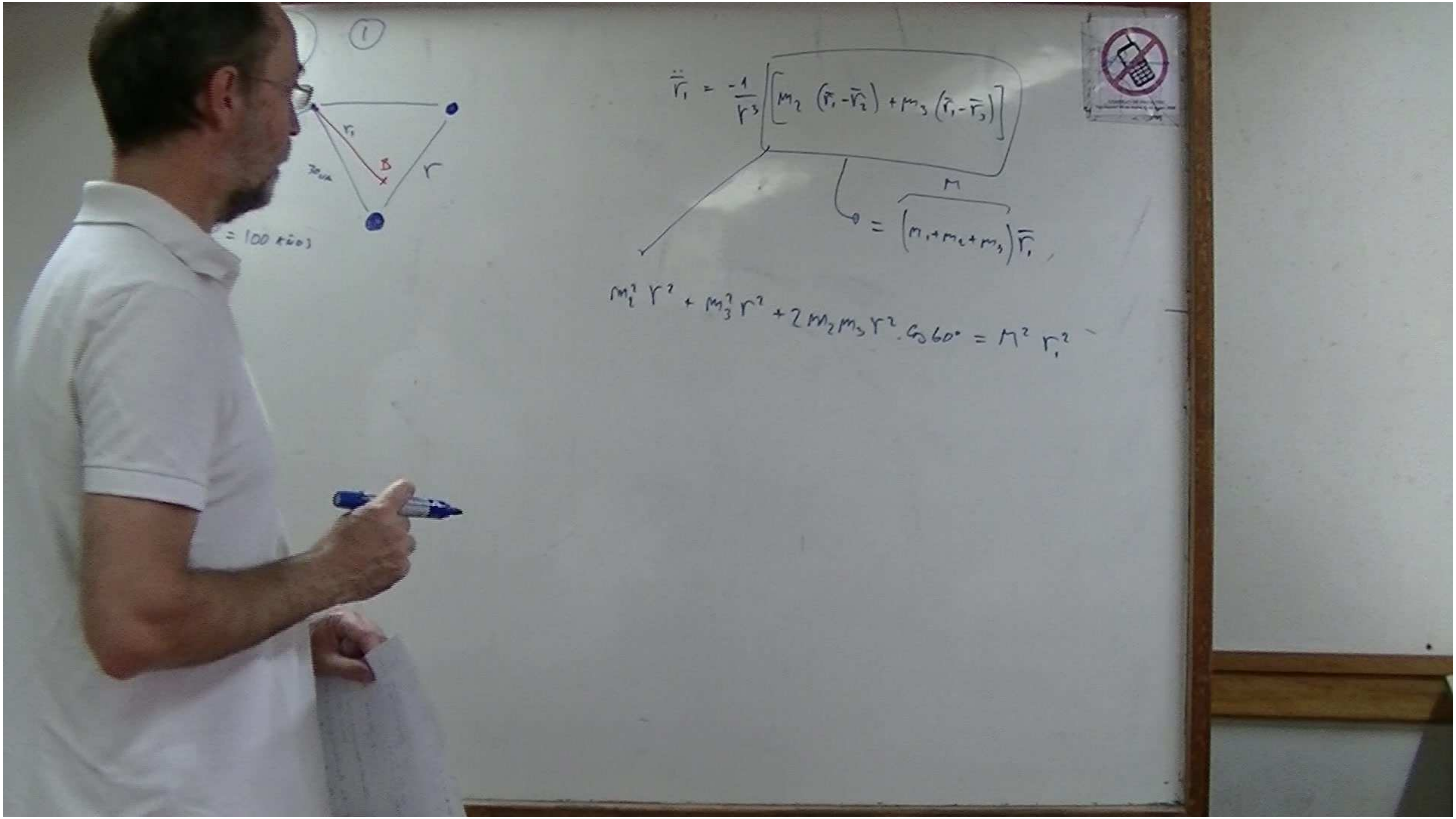

10
a = 1ua
e = 0.13
A
T
Y
X
T

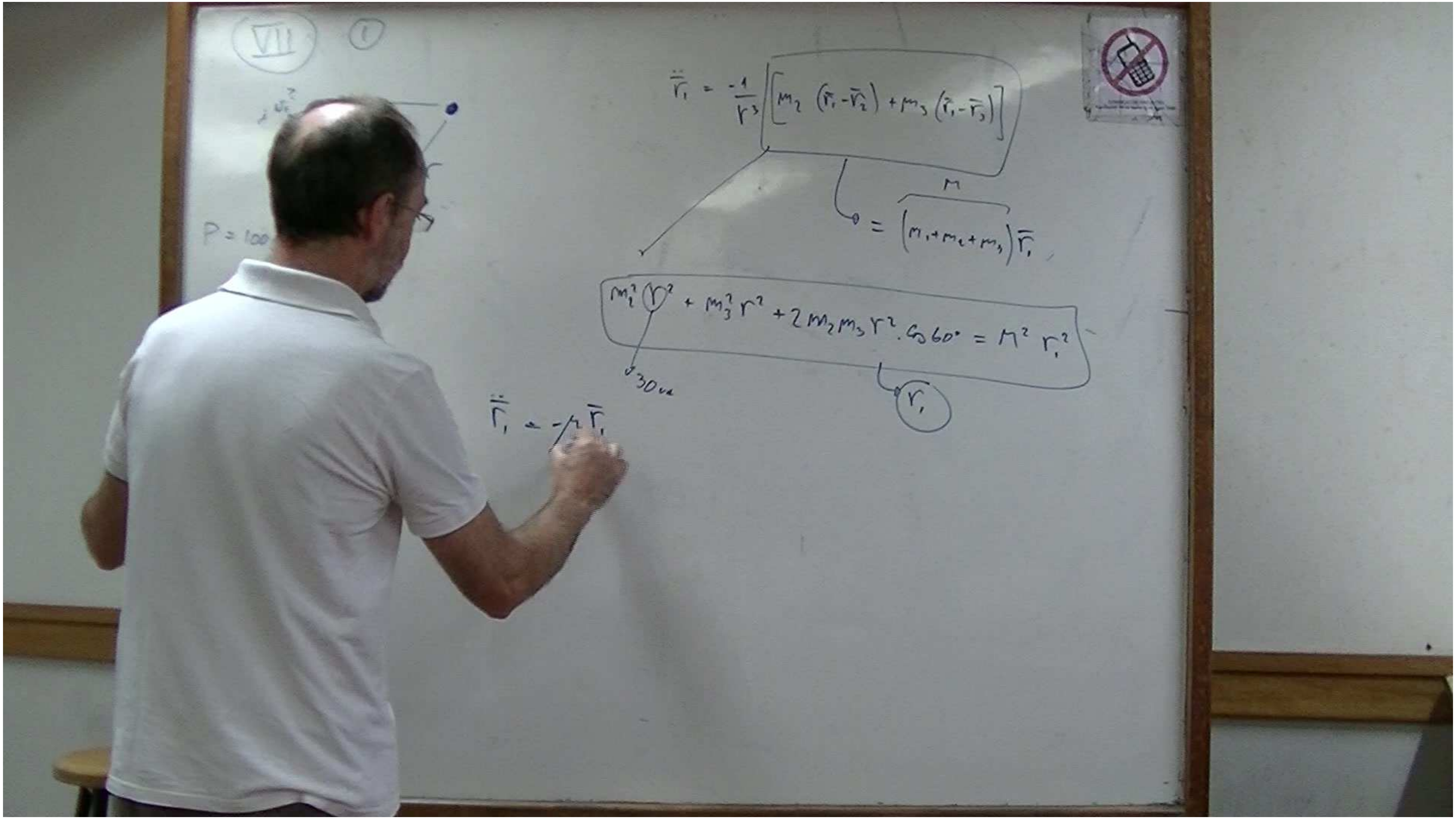

10
a = 1UA
e = 0.13
10^6 años
$N_{enc} \propto r^2$
N_{enc}
r = $R_\oplus$
r^2
10^6 años
A
T
y
x
T

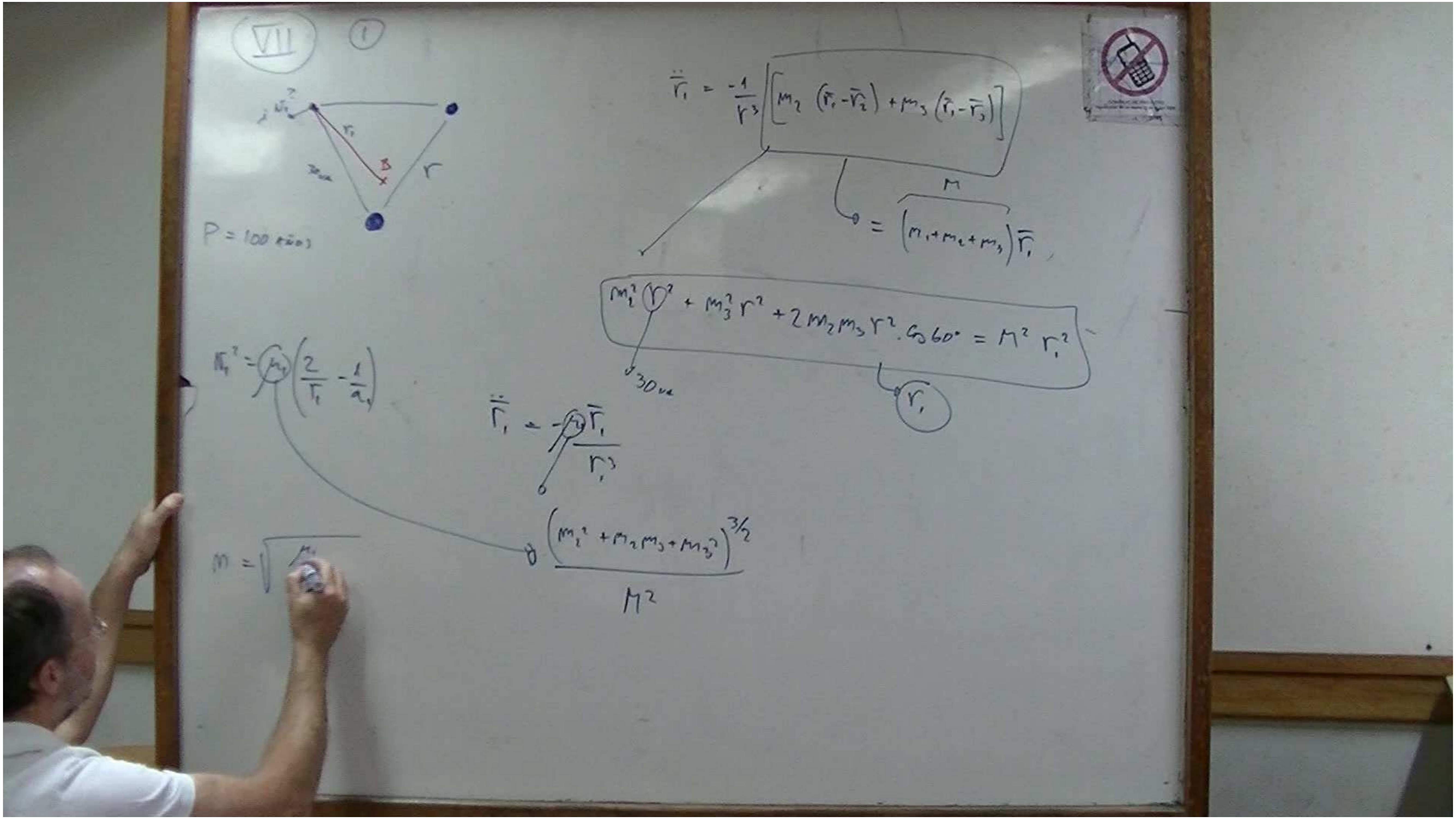

10
a = 1 UA
e = 0.13
10^6 años
N ∝ r^2
r = R
r^2
10^6 años → X "colisiones"
T
A
x
y

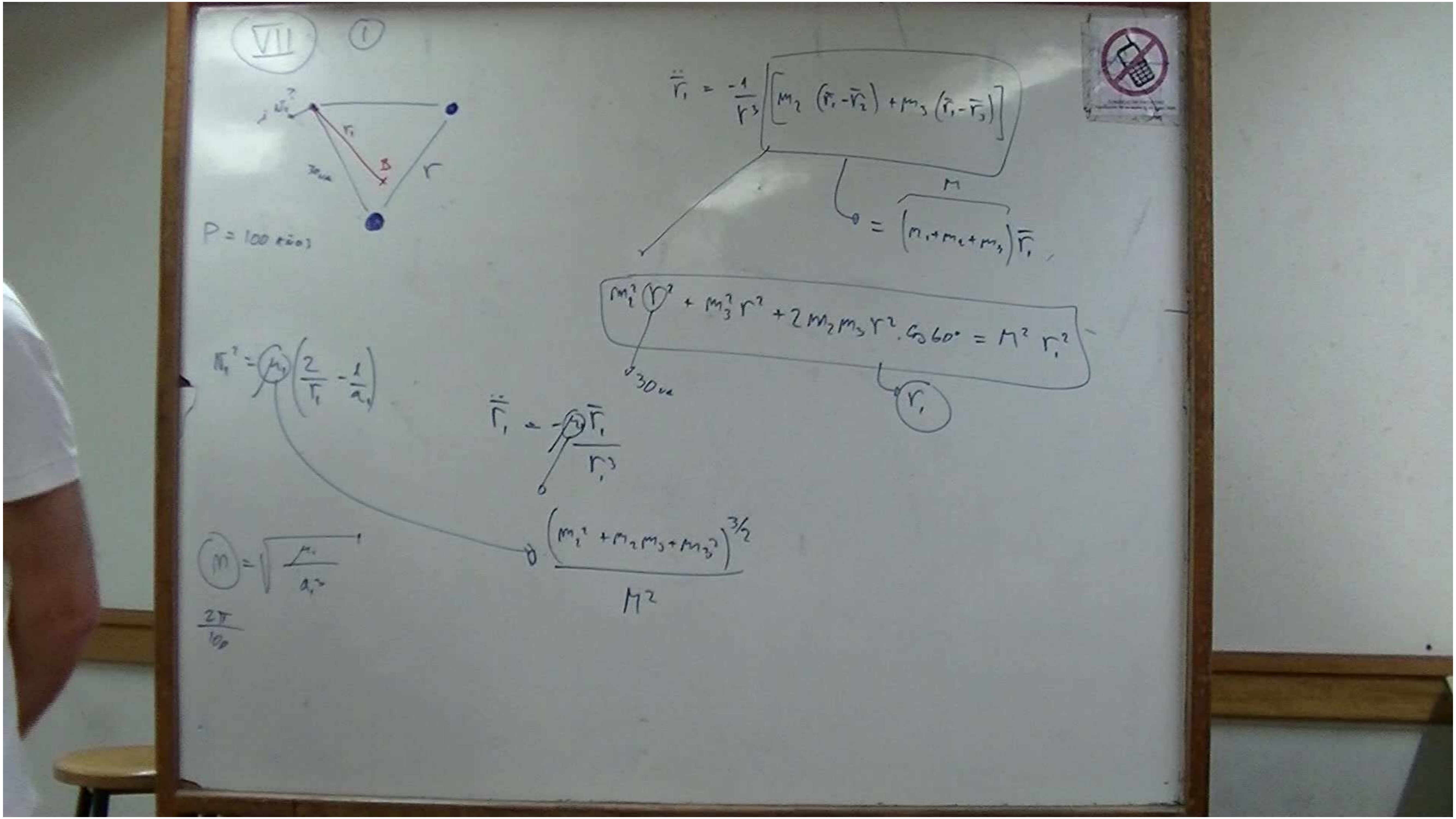

VII
1
P = 100 años

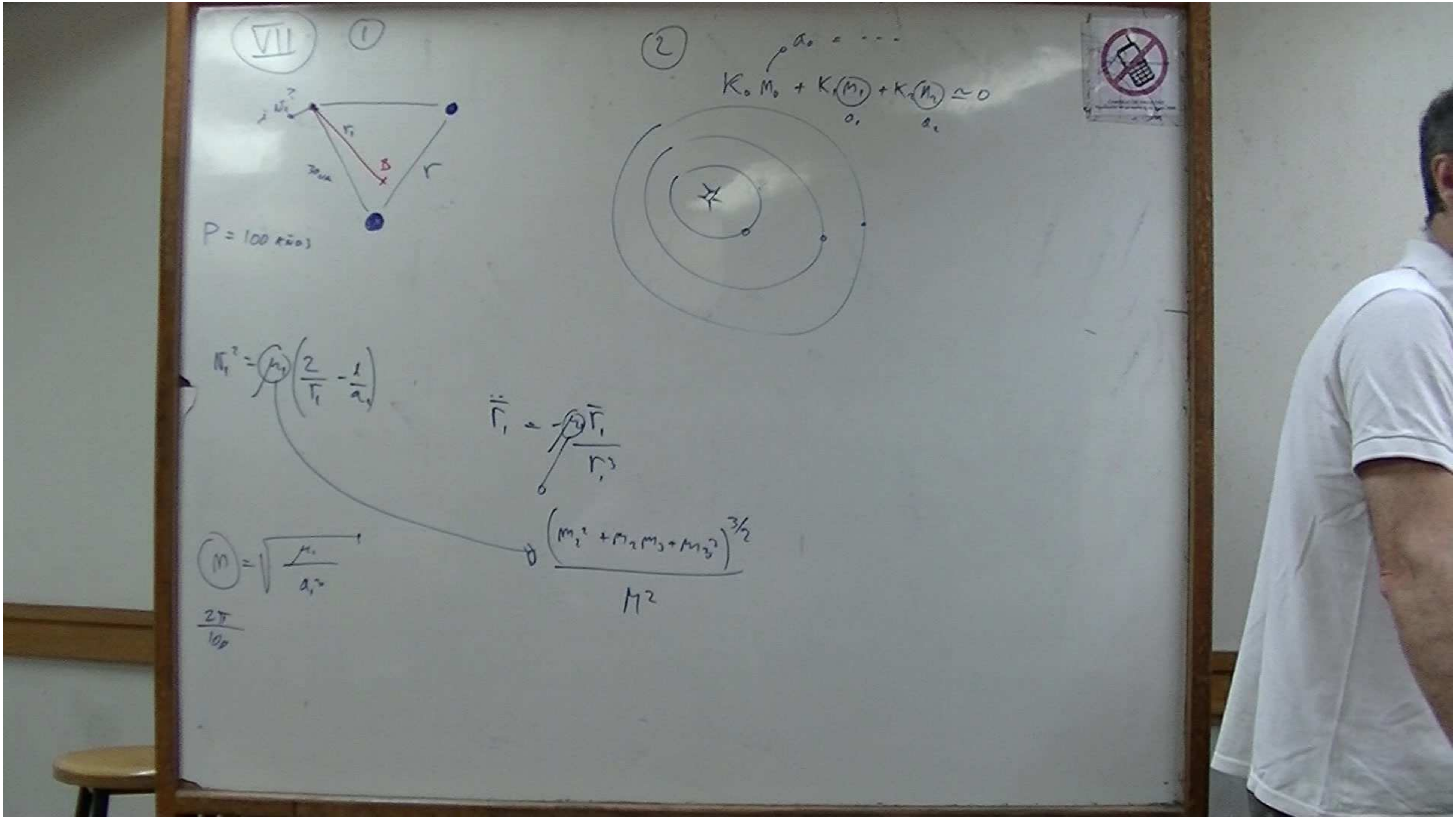

VII
$\ddot{\bar{r}}_1 = \frac{-1}{r^3}\left[m_2(\bar{r}_1-\bar{r}_2)+m_3(\bar{r}_1-\bar{r}_3)\right]$
P = 100 años

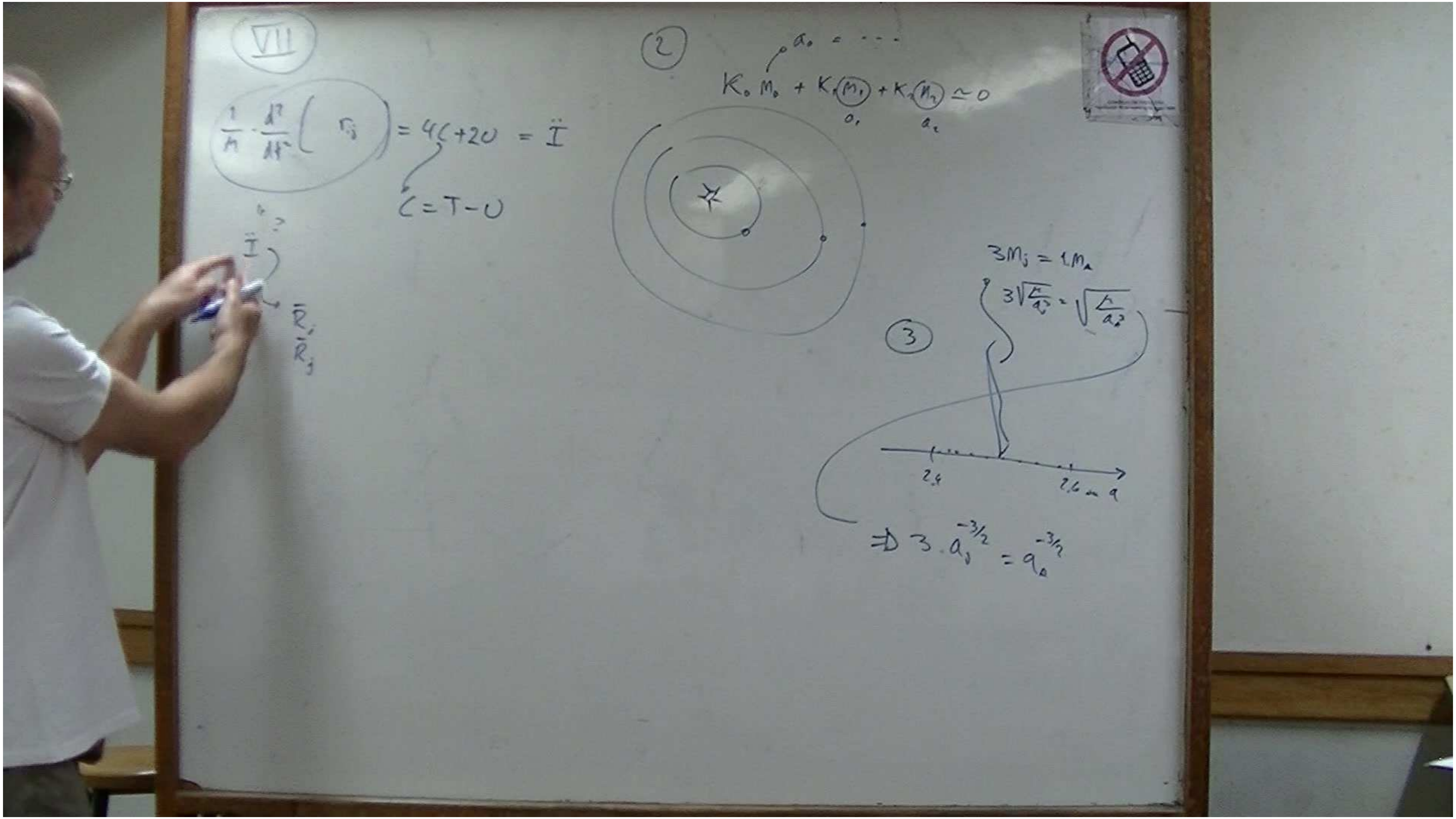

(1)
r_1
B
r
$= 100$ años
$\ddot{\bar r}_1 = -\frac{1}{r^3}\left[m_2(\bar r_1 - \bar r_2) + m_3(\bar r_1 - \bar r_3)\right]$
$= (\overbrace{m_1+m_2+m_3}^{M})\bar r_1$
$m_2^2 r^2 + m_3^2 r^2 + 2 m_2 m_3 r^2 \cdot \cos 60° = M^2 r_1^2$

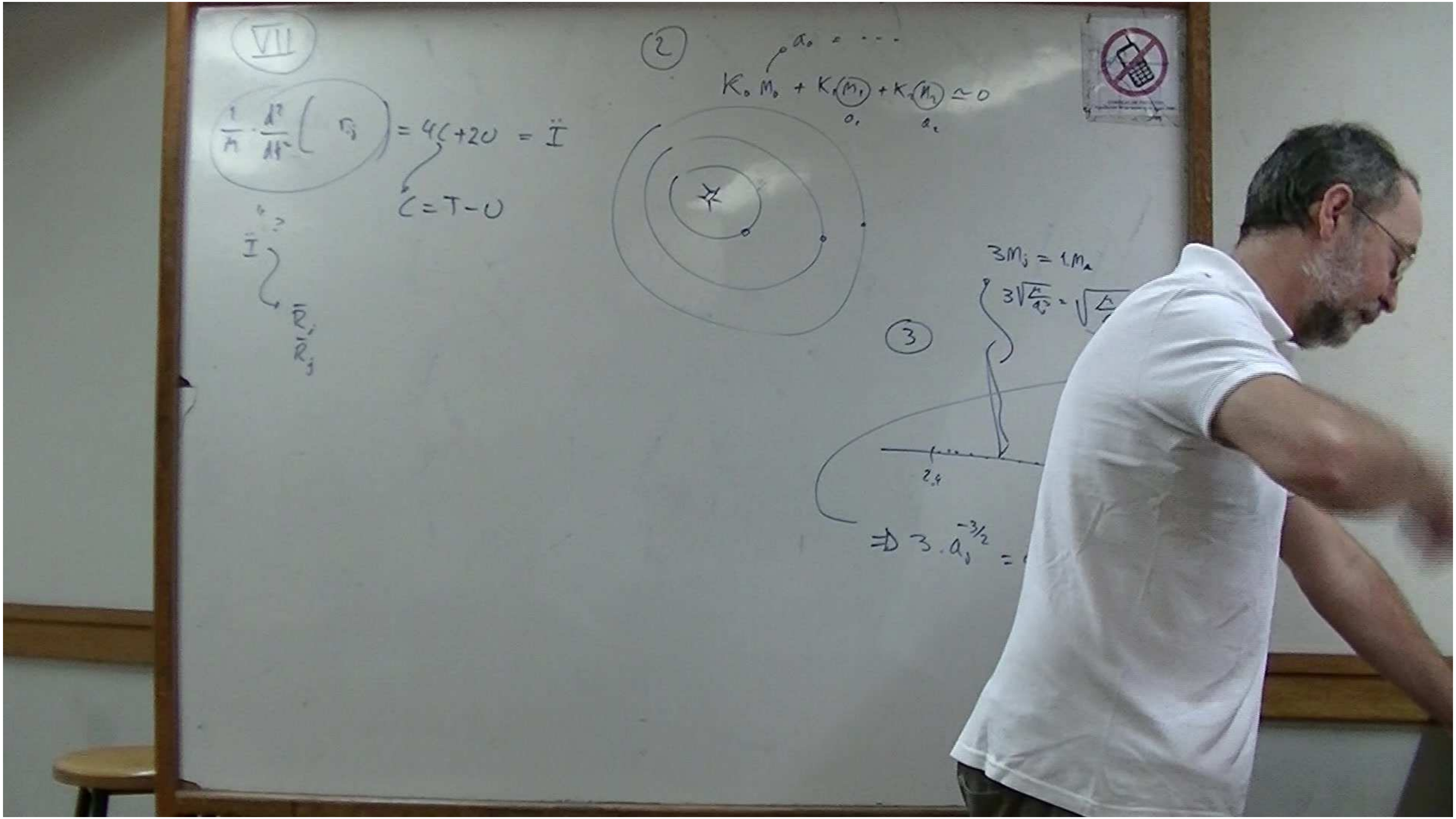

VII
r̈1 = -1/r³ [m2 (r̄1 - r̄2) + m3 (r̄1 - r̄3)]
M
= (m1+m2+m3) r̄1
m2² r² + m3² r² + 2 m2 m3 r² . cos 60° = M² r1²
30 ua
r1


VII
1
r_1
r
30ua
$P = 100$ años
$\ddot{\bar r}_1 = -\frac{1}{r^3}\left[m_2(\bar r_1 - \bar r_2) + m_3(\bar r_1 - \bar r_3)\right]$
$= (\overbrace{m_1 + m_2 + m_3}^{M})\,\bar r_1$
$m_2^2 r^2 + m_3^2 r^2 + 2 m_2 m_3 r^2 \cdot \cos 60° = M^2 r_1^2$
r_1
$v_1^2 = \mu_1\left(\frac{2}{r_1} - \frac{1}{a_1}\right)$
$\ddot{\bar r}_1 = -\mu_1 \frac{\bar r_1}{r_1^3}$
$\frac{(m_2^2 + m_2 m_3 + m_3^2)^{3/2}}{M^2}$
$m = \sqrt{\mu_1}$

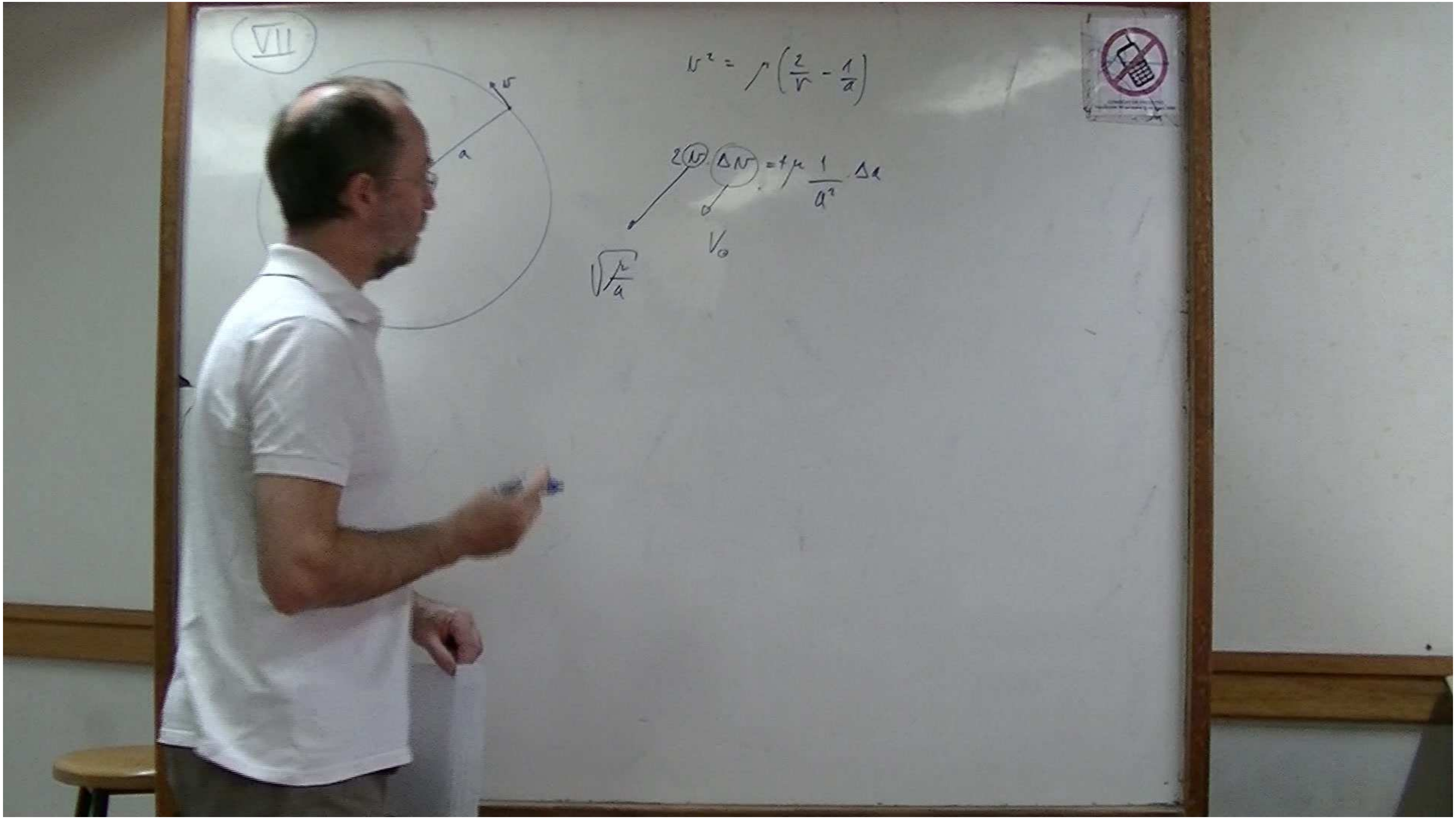

VII
1
P = 100 años
30 ua
r
$\ddot{\bar{r}}_1 = -\frac{1}{r^3}\left[m_2(\bar{r}_1-\bar{r}_2)+m_3(\bar{r}_1-\bar{r}_3)\right]$
$= (m_1+m_2+m_3)\bar{r}_1$
M
$m_2^2 r^2 + m_3^2 r^2 + 2 m_2 m_3 r^2 \cos 60° = M^2 r_1^2$
30 ua
r_1
$\ddot{\bar{r}}_1 = -\mu_1 \frac{\bar{r}_1}{r_1^3}$
$\frac{(m_2^2+m_2 m_3+m_3^2)^{3/2}}{M^2}$
$n = \sqrt{\frac{\mu_1}{a_1^3}}$
$\frac{2\pi}{100}$

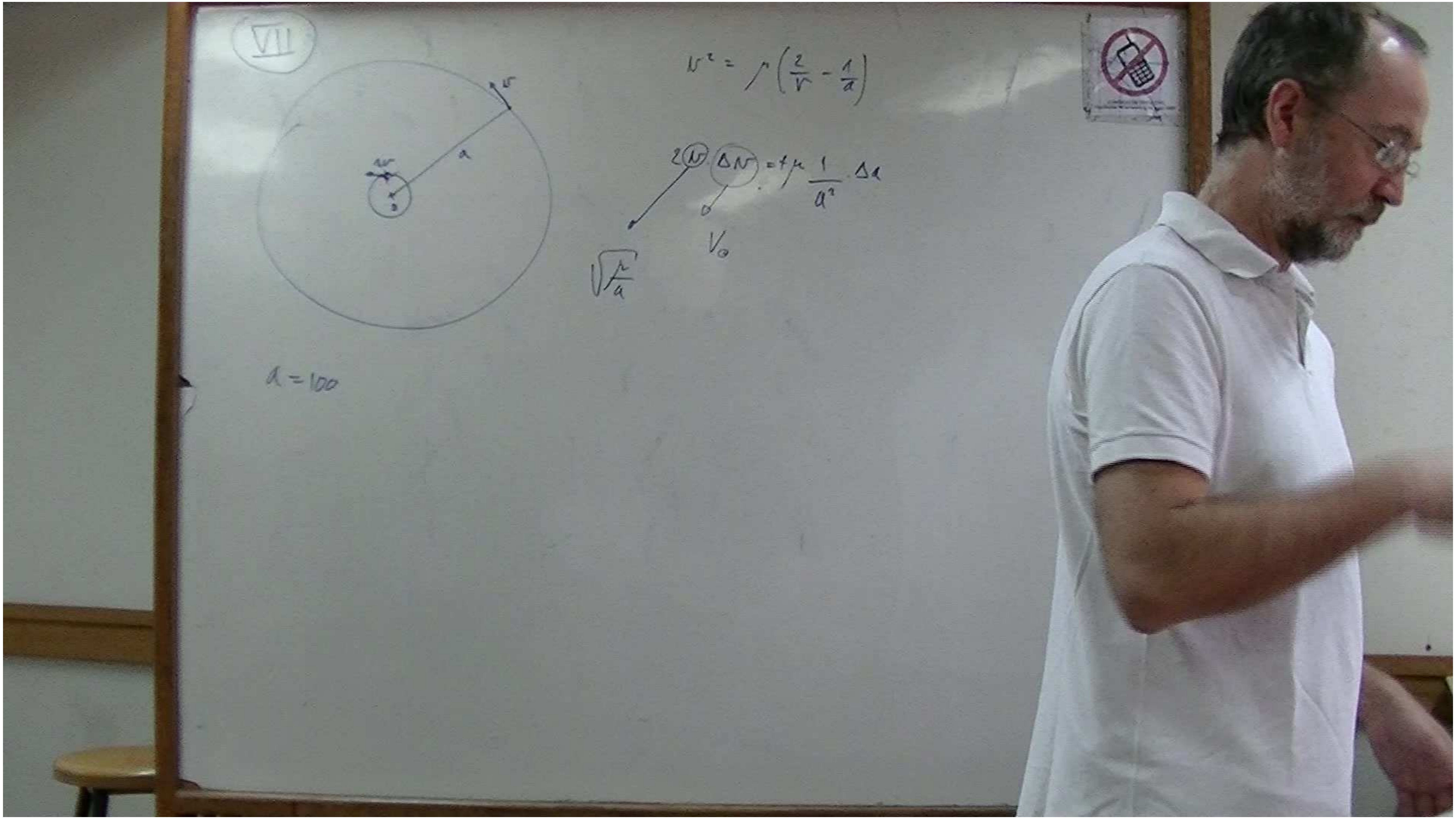

VII
1
2
P = 100 años
r
M²

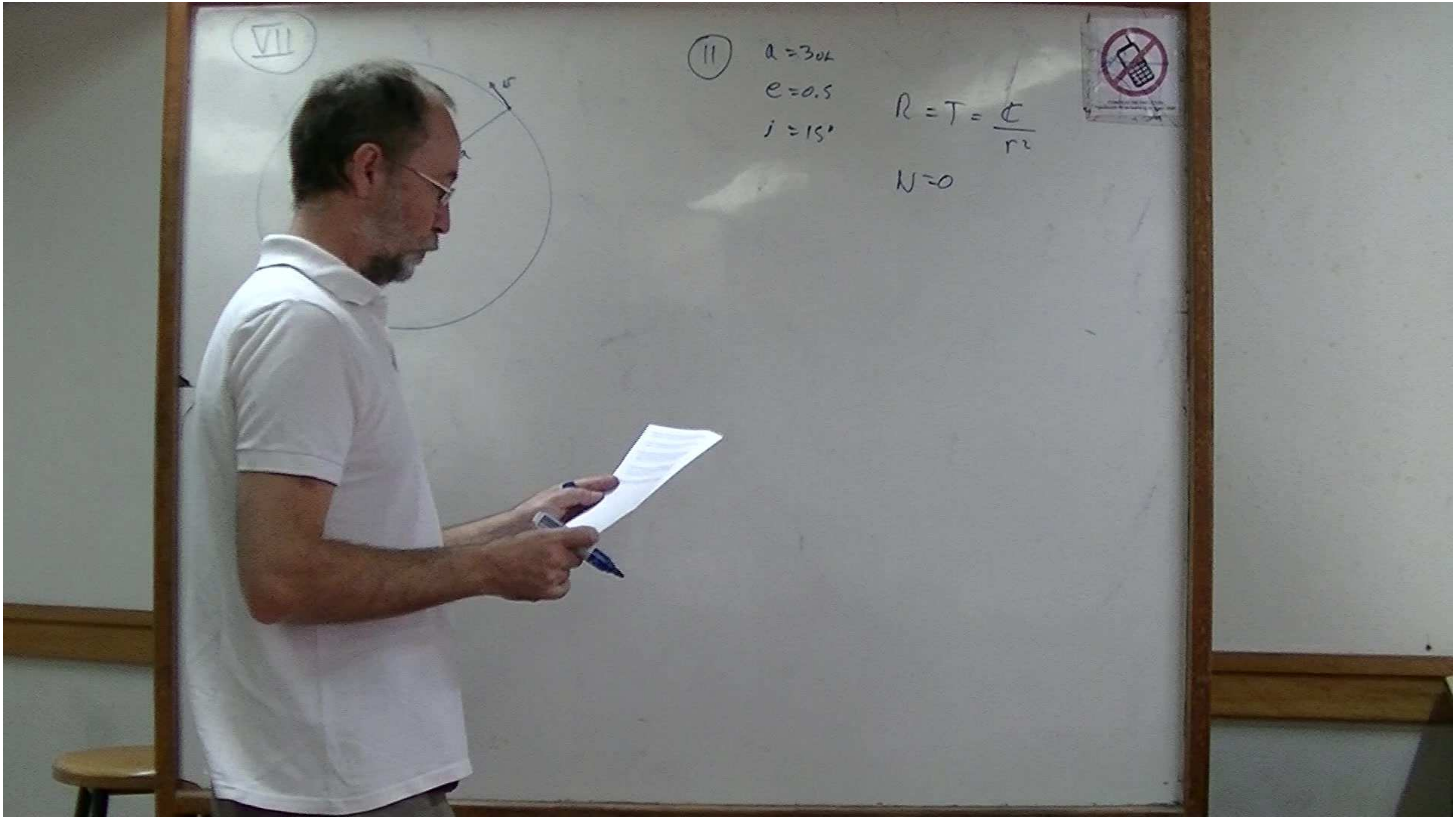

VII
$= 4C + 2U = \ddot{I}$
$C = T - U$
2
$K_0 m_0 + K_1 m_1 + K_2 m_2 \simeq 0$
3
$3 m_j = 1 m_a$
2,4
2,6
a
$\Rightarrow 3 \cdot a_j^{-3/2} = a_a^{-3/2}$

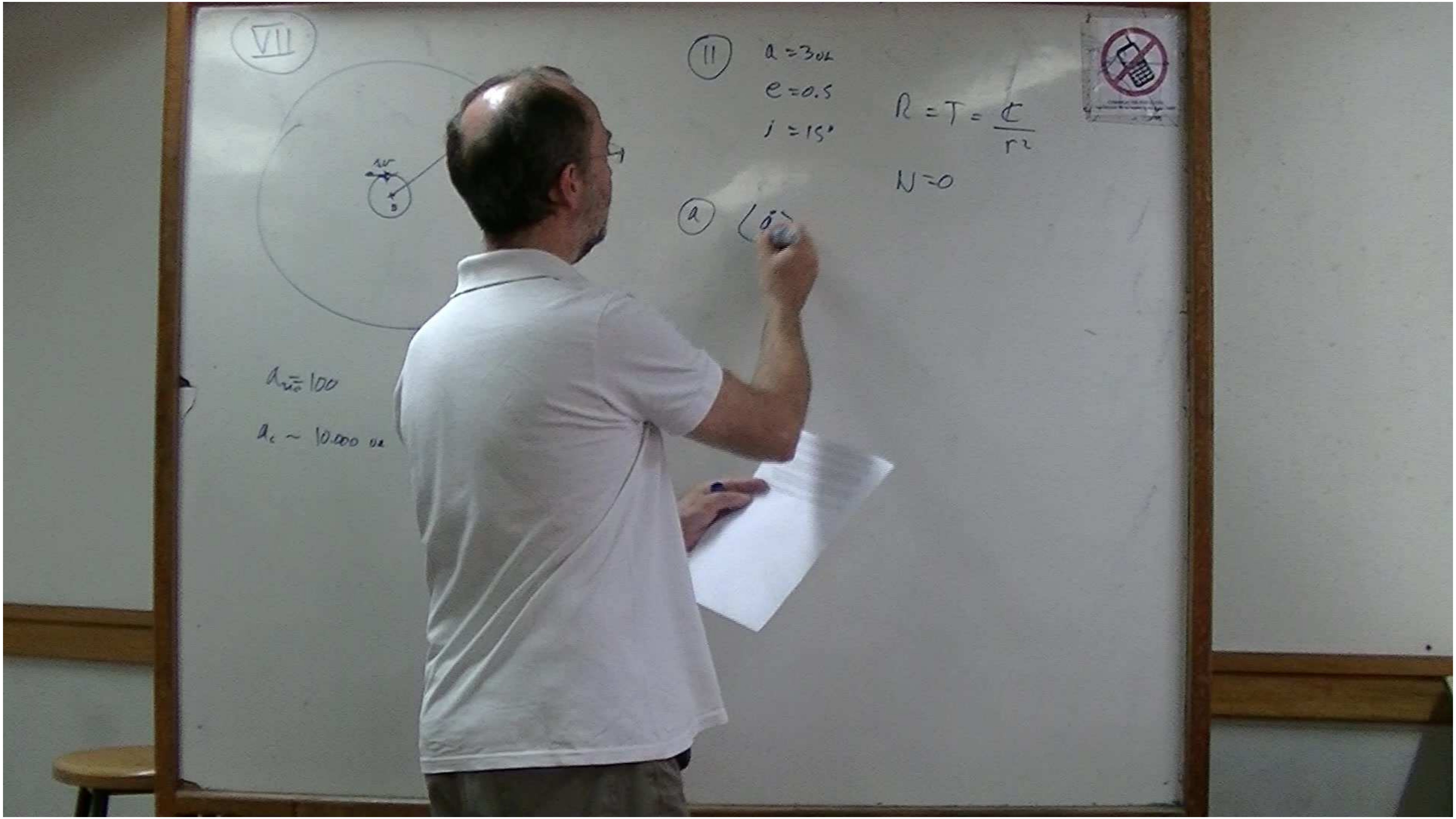

VII
= 4C + 2U = Ï
C = T − U
2
K₀ M₀ + K₁ M₁ + K₂ M₂ ≃ 0
3M_j = 1 M_s
3
2,4

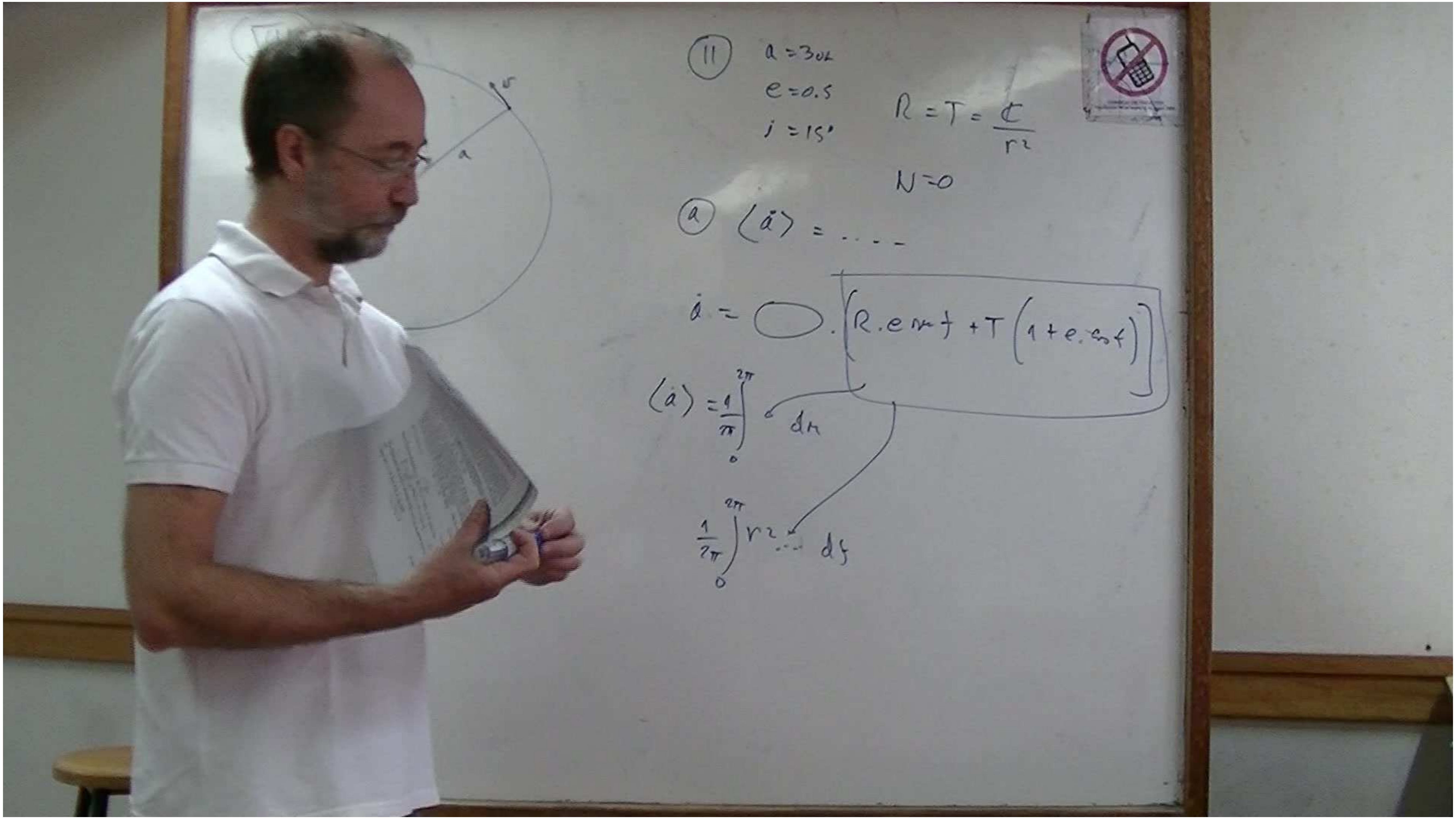

VII
a
$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$
$2v \cdot \Delta v = \mu\,($

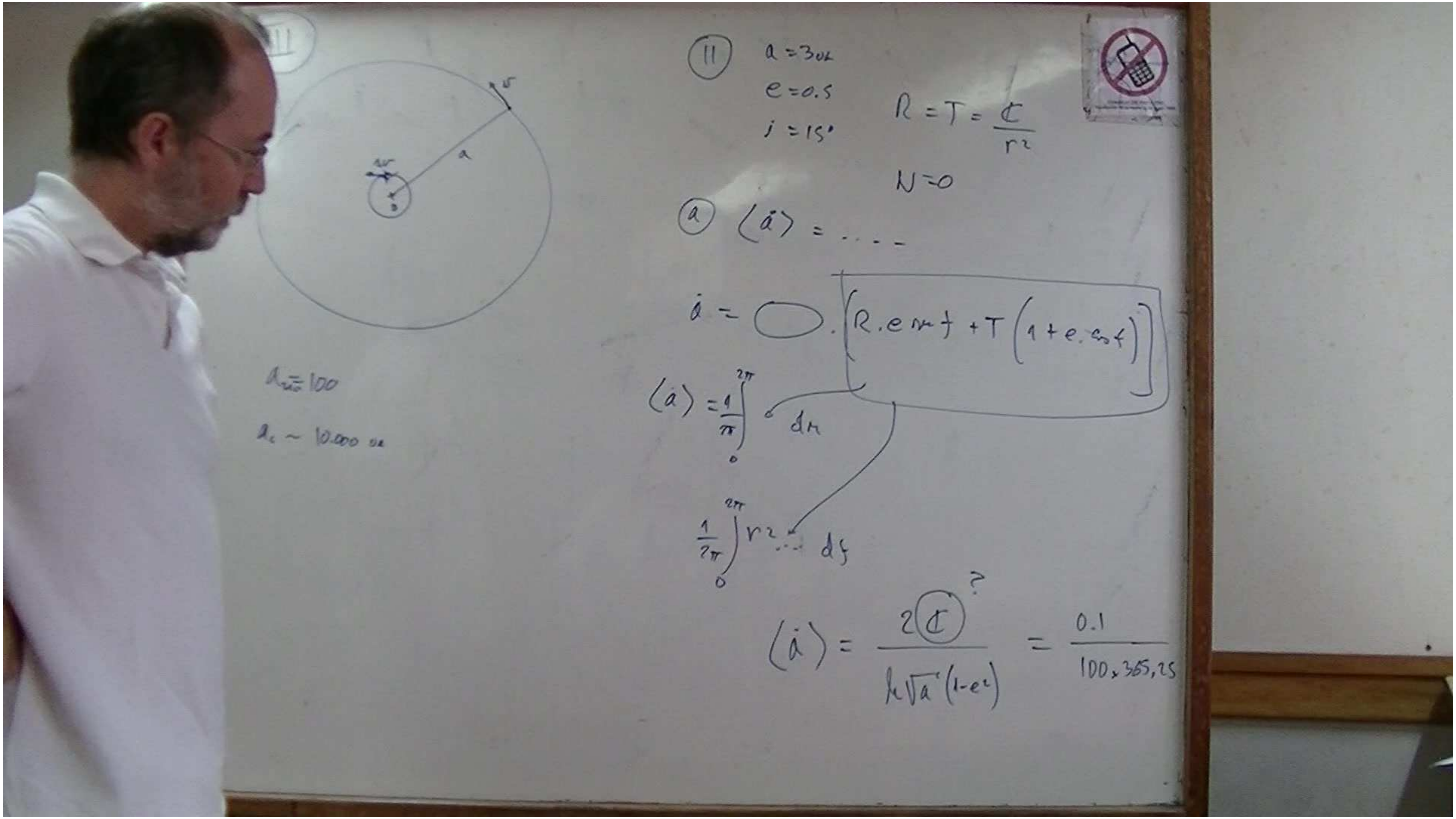

VII
v
a
$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$
$2v \cdot \Delta v = +\mu \frac{1}{a^2} \cdot \Delta a$
$\sqrt{\frac{\mu}{a}}$
V_0

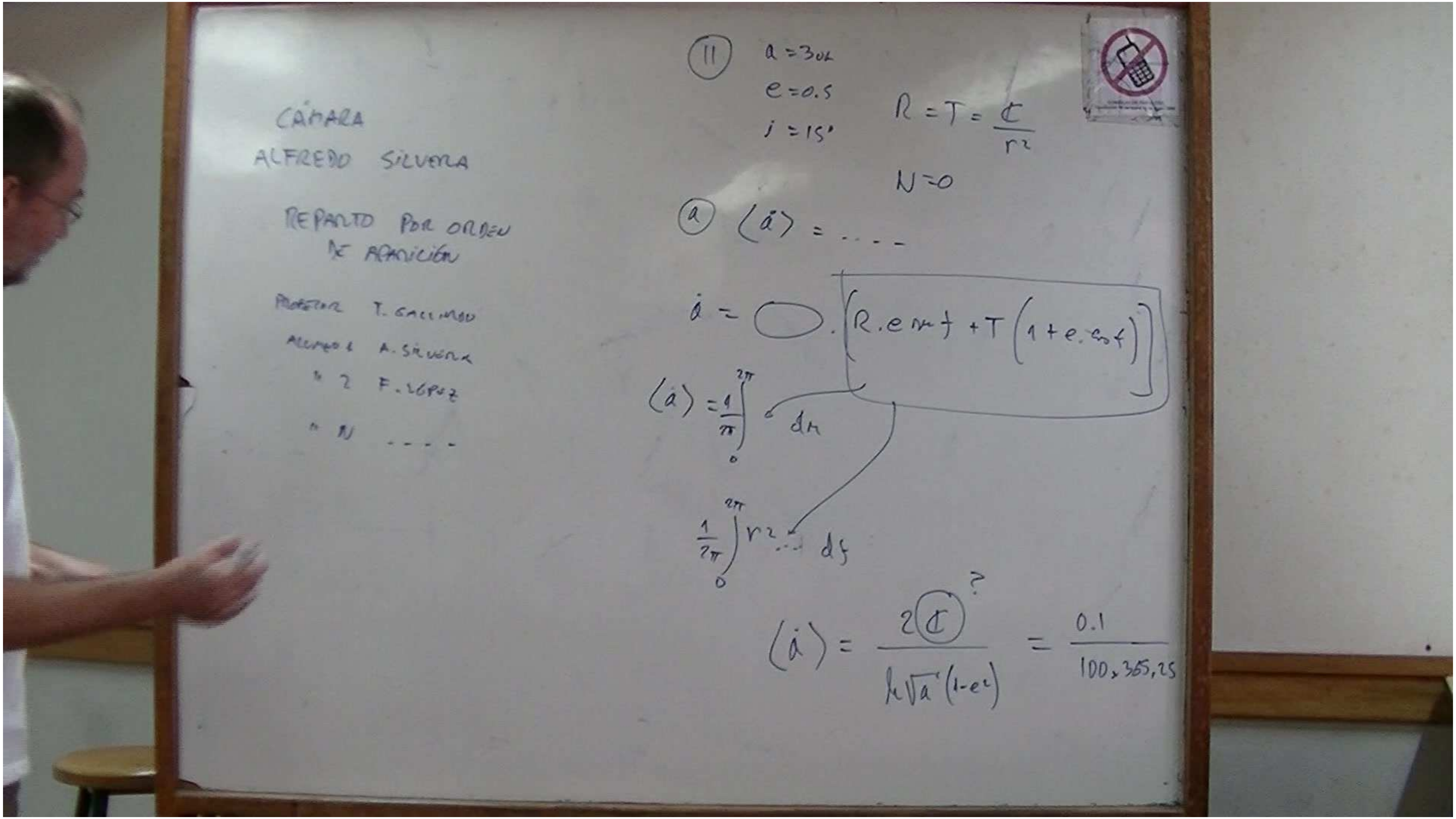

VII
v² = μ (2/r − 1/a)
2v · Δv = +μ 1/a² · Δa
√(μ/a)
V₀
a
a = 100


VII
11
a = 3ul
e = 0.5
i = 15°
R = T = C / r²
N = 0


VII
II
a = 30 ua
e = 0.5
i = 15°
R = T = C / r²
N = 0
a
a ~ 100
a ~ 10.000 ua


11
e = 0.5
i = 15°
R = T = C / r²
N = 0
⟨ȧ⟩ = ...
ȧ = ◯ . [R . e sen f + T (1 + e . cos f)]
⟨ȧ⟩ = 1/2π ∫₀^2π ◦ dM
1/2π ∫₀^2π r² . . . df


11
a = 3ua
e = 0.5
i = 15°
R = T = C / r2
N = 0
a
⟨ȧ⟩ = ...
ȧ = ○ . [R.e sen f + T (1 + e.cos f)]
⟨ȧ⟩ = 1/π ∫ dM
1/2π ∫ r2 ... df
⟨ȧ⟩ = 2C / k√a (1-e2) = 0.1 / 100x365,25
a = 100
ac ~ 10.000 ua


CÁMARA
ALFREDO SILVERA
REPARTO POR ORDEN DE APARICIÓN
PROFESOR T. GALLARDO
ALUMNO 1 A. SILVERA
" 2 F. LÓPEZ
" N ....
11
a = 3ua
e = 0.5
i = 15°
R = T = C / r²
N = 0
a ⟨ȧ⟩ = ...
ȧ = ( ) . [R.e sen f + T (1 + e.cos f)]
⟨ȧ⟩ = 1/π ∫ dM
1/2π ∫ r² ... df
⟨ȧ⟩ = 2C / (k√a (1-e²)) = 0.1 / (100 x 365,25)