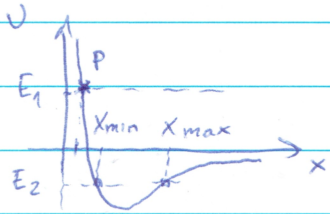
1) a) $F(x) = \dfrac{36}{x^3}\,Nm^3 - \dfrac{9}{x^2}\,Nm^2 \qquad x>0 \qquad M = 1\,kg$

$\Rightarrow U = -\dfrac{9}{x}\,Nm^2 + \dfrac{18}{x^2}\,Nm^3 \qquad \left(F(x) = -\dfrac{dU}{dx}\right)$

$\Rightarrow$



- si $E = E_1 > 0 \Rightarrow E_1 = \dfrac{m\dot{x}^2}{2} + U(x)$
  pero si $\dot{x} > 0 \Rightarrow$ No se detiene
  (si $\dot{x} < 0$ llega hasta P * y luego pasa a ser $\dot{x} > 0$)

- si $E = E_2 < 0 \Rightarrow E_2 = \dfrac{m\dot{x}^2}{2} + U(x)$
  pero en x_{min} y x_{max} es $U(x_{min}) = U(x_{max}) = E_2$
  y en dichos puntos $\dot{x} = 0$
  si la partícula se extendiera de dichos límites sería $\dot{x}^2 < 0$ lo cual no es posible

Al ser el movimiento en una única dimensión debe pasar siempre por cada punto con la misma velocidad dando lugar a un mov. periódico: $x(t) = x(t+T)$ $\left(\dot{x}(t) = \dot{x}(t+T)\right)$

b) $x_0 = 4\,m$ y $\dot{x} = 0{,}5\,\frac{m}{s} \Rightarrow E = -1\,J = -1\,Nm$ y $U(\underset{x_{min}}{3\,m}) = U(\underset{x_{max}}{6\,m}) = -1\,J$

$$\frac{M}{2}\left(\frac{dx}{dt}\right)^2 + U(x) = E \Rightarrow \sqrt{\frac{M}{2}}\int_{t_1}^{t_2}\frac{\frac{dx}{dt'}\,dt'}{\sqrt{E-U(x)}} = \int_{t_1}^{t_2} dt' = t_2 - t_1$$

$$\Rightarrow \sqrt{\frac{M}{2}}\int_{x(t_1)}^{x(t_2)}\frac{dx'}{\sqrt{\frac{9}{x'}\,Nm^2 - \frac{18}{x'^2}\,Nm^3 - 1\,Nm}} \underset{\tilde{x} = \frac{x'}{m}}{=} \sqrt{\frac{1}{2}}\int_{\tilde{x}(t_1)}^{\tilde{x}(t_2)}\frac{\tilde{x}\,d\tilde{x}}{\sqrt{\underbrace{-\tilde{x}^2 + 9\tilde{x} - 18}_{(\tilde{x}-3)(6-\tilde{x})}}}\ \overset{\text{unidades}}{s} = t_2 - t_1$$

si tomamos $t_2 - t_1 = \dfrac{T}{2}$ y $\tilde{x}(t_1) = 3$ $(x(t_1) = 3\,m)$, $\tilde{x}(t_2) = 6$ $(x(t_2) = 6\,m)$ $\Rightarrow \dfrac{T}{2} = \dfrac{1}{\sqrt{2}}\displaystyle\int_3^6 \frac{\tilde{x}\,d\tilde{x}}{\sqrt{(6-\tilde{x})(\tilde{x}-3)}}\,s =$

$$\Rightarrow \boxed{T = \frac{9\pi}{\sqrt{2}}\,s}$$

2) a) $F(r) = -\dfrac{\ell^2}{mr^2}\left(u(r) + \dfrac{d^2u(r)}{d\theta^2}\right) \leftarrow$ Binet

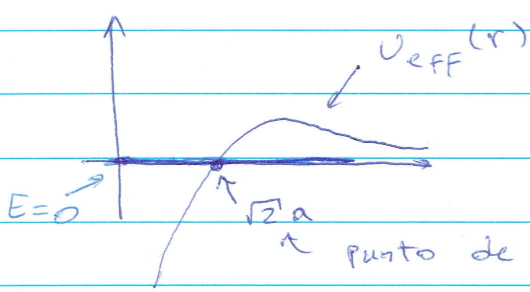
$\dfrac{du}{d\theta} = \dfrac{du}{dr}\dfrac{dr}{d\theta}$ $\qquad u = \dfrac{1}{r} \rightarrow \dfrac{du}{dr} = -\dfrac{1}{r^2} \rightarrow \dfrac{d^2u}{dr^2} = \dfrac{2}{r^3}$

$\dfrac{d^2u}{d\theta^2} = \dfrac{du}{dr}\dfrac{d^2r}{d\theta^2} + \dfrac{d^2u}{dr^2}\left(\dfrac{dr}{d\theta}\right)^2$ $\qquad r = \sqrt{2a^2}\sqrt{\cos(2\theta)}$

$\left(\dfrac{dr}{d\theta}\right)^2 = \dfrac{2a^2\,\overbrace{(1-\cos^2(2\theta))}^{\text{sen}^2(2\theta)}}{\cos(2\theta)}$ $\qquad \dfrac{d^2r}{d\theta^2} = -\sqrt{2a^2}\left\{2\sqrt{\cos(2\theta)} + \dfrac{1}{(\cos(2\theta))^{3/2}} - \sqrt{\cos(2\theta)}\right.$

$\Rightarrow F(r) = -\dfrac{12\,\ell^2 a^4}{m r^7}$

b) $U(r) = -\int F(r)\,dr = -\dfrac{2\ell^2 a^4}{mr^2} \Rightarrow U_{eff} = \dfrac{\ell^2}{2mr^2}\left(1 - \dfrac{4a^4}{r^4}\right)$



si $r_0 = \sqrt{2}a$ y $\vec{V} = v_0\hat{e}_\theta$

$\Rightarrow \dot{r}_0 = 0$

$\Rightarrow E = U_{eff}(r_0) = 0$

c) $\dfrac{\ell}{mr^2} = \dfrac{d\theta}{dt}$

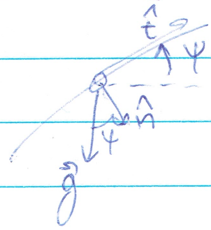
$\Rightarrow \int dt' = \dfrac{m}{\ell}\int r^2(\theta')\dfrac{d\theta'}{dt'}dt' = \dfrac{2ma^2}{\ell}\int \cos(2\theta)\,d\theta$

integrando de $-\frac{\pi}{4}$ a $\frac{\pi}{4}$ es medio período

$\Rightarrow \dfrac{T}{2} = \dfrac{2ma^2}{\ell}\displaystyle\int_{-\pi/4}^{\pi/4}\cos(2\theta)\,d\theta \rightarrow T = \dfrac{2ma^2}{\ell}\,\text{sen}(2\theta)\Big|_{-\pi/4}^{\pi/4} \rightarrow T = \dfrac{4ma^2}{\ell}$

3) a) La ecuación de mov. en coord. intrínsecas se lee:

$$m\left[\ddot{s}\,\hat{t} + \frac{\dot{s}^2}{\rho}\hat{n}\right] = m\vec{g} - \kappa v^{n-1}\vec{v}$$



siendo que $\vec{v} = v\hat{t}$ y $\vec{g} = -g\,sen(\psi)\,\hat{t} + g\cos(\psi)\,\hat{n}$

$\Rightarrow \hat{t})\quad m\ddot{s} = -mg\,sen(\psi) - \kappa v^n \quad (I)$

$\hat{n})\quad m\frac{\dot{s}^2}{\rho} = mg\cos(\psi) \quad (II)$ notando que $v = \dot{s}$ llegamos al resultado

b) $v = \dot{s} \rightarrow \ddot{s} = \frac{dv}{dt} = \frac{dv}{d\psi}\frac{d\psi}{dt} = \dot{\psi}\frac{dv}{d\psi} \quad (III)$

por otra parte

$-\rho\, d\psi = ds \quad (d\psi < 0) \Rightarrow \frac{ds}{d\psi} = -\rho \Rightarrow v = \frac{ds}{dt} = \frac{ds}{d\psi}\frac{d\psi}{dt} = -\rho\dot{\psi} \quad (IV)$

$\Rightarrow$ Haciendo $\frac{I}{II}$ es $\frac{m\ddot{s}}{m\dot{s}^2}\rho = -\tan(\psi) - \frac{\kappa}{mg} v^n \sec(\psi)$

reemplazando (III) y (IV) en el miembro izquierdo es

$-\frac{1}{v}\frac{dv}{d\psi} = -\tan(\psi) - \frac{\kappa}{mg} v^n \sec(\psi)$ que es lo buscado.

c) si $\kappa = 0 \Rightarrow \frac{1}{v}\frac{dv}{d\psi} = \tan(\psi) \Rightarrow \int_\theta^\psi \frac{1}{v}\frac{dv}{d\psi'}d\psi' = \int_\theta^\psi \tan(\psi')\,d\psi'$

$= \ln\left(\frac{v(\psi)}{v(\theta)}\right) = -\ln\left(\frac{\cos(\psi)}{\cos(\theta)}\right)$

$\Rightarrow v\cos(\psi) = v(\theta)\cos(\theta) = cte. \quad (V)$

d) $\frac{dx}{d\psi} = \left[\frac{dx}{dt}\right]\cdot\left(\frac{dt}{d\psi}\right) = \left[v\cos(\psi)\right]\left(\frac{1}{\dot{\psi}}\right) = -\cos(\psi)\rho = -\frac{v^2}{g}$

usando (IV) $\dot{\psi} = -\frac{v}{\rho}$ usando (II) $\frac{v^2}{\rho} = g\cos(\psi)$

sustituyendo con (V) es $\frac{dx}{d\psi} = -\frac{v_0^2\cos^2(\theta)}{g\cos^2(\psi)} \Rightarrow \int_\theta^\psi \frac{dx}{d\psi'}d\psi' = -\frac{v_0^2\cos^2(\theta)}{g}\int_\theta^\psi \frac{d\psi'}{\cos^2(\psi')}$

$\Rightarrow x(\psi) - x(\theta) = -\frac{v_0^2\cos^2(\theta)}{g}\left[\tan(\psi) - \tan(\theta)\right]$ despejando $\tan(\psi)$ es

$\frac{dy}{dx} = \tan(\psi) = \underbrace{\frac{g}{v_0^2\cos^2(\theta)}x}_{A\,x} + \underbrace{\left\{\frac{g\,x_0}{v_0^2\cos^2\theta} + \tan(\theta)\right\}}_{B}$ $\quad \int_{x_0}^{x}\frac{dy}{dx'}dx' = \int_{x_0}^{x}(Ax' + B)\,dx'$

$x(\psi) = x$

$x(\theta) = x_0$

$\Rightarrow \boxed{y(x) = y(x_0) + \underline{A}(x^2 - x_0^2) + B(x - x_0)}$