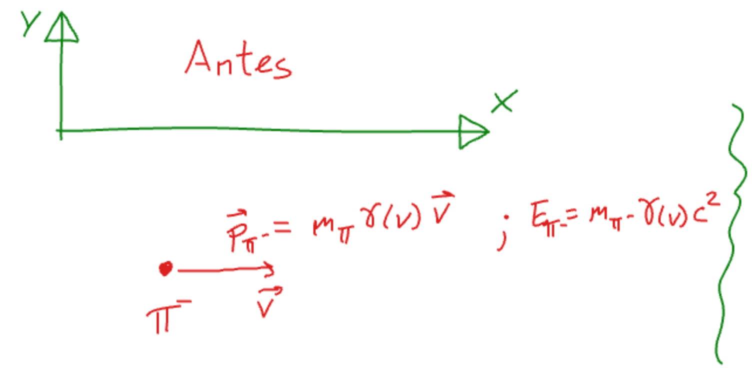
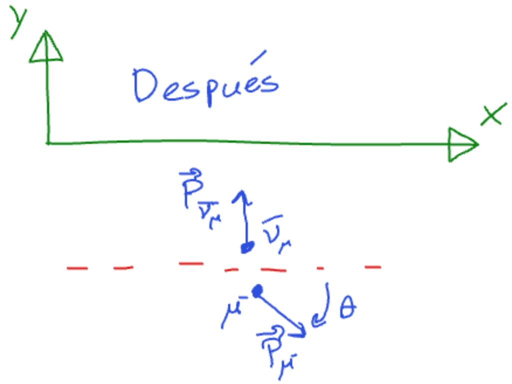
Un pión (π^-) viajando a velocidad v, decae en un muón (μ^-) y un antineutrino muón ($\bar{\nu}_\mu$) [que consideramos de masa despreciable].

Si el neutrino surge a un ángulo de 90° respecto a la dirección inical del movimiento del pión ($\vec{P}_{\bar{\nu}_\mu} \cdot \vec{P}_{\pi^-} = 0$), ¿a qué ángulo sale el muón?



$\vec{P}_{\pi^-} = m_\pi \gamma(v) \vec{v}$; $E_{\pi^-} = m_{\pi^-} \gamma(v) c^2$



El cuadrivector $\tilde{P}_{\text{inicial}} = \tilde{P}_{\pi^-}$

$$\tilde{P}_{\pi^-} = (m_{\pi^-}\gamma(v)c, m_\pi \gamma(v) v, 0, 0)$$

El cuadrivector $\tilde{P}_{\text{final}} = \tilde{P}_{\mu^-} + \tilde{P}_{\bar{\nu}}$

$$\tilde{P}_{\mu^-} = \left(\frac{E_{\mu^-}}{c}, |\vec{P}_\mu| \cos\theta, -|\vec{P}_\mu| \operatorname{sen}\theta, 0\right)$$

$$\tilde{P}_{\bar{\nu}_\mu} = \left(\frac{E_{\bar{\nu}_\mu}}{c}, 0, |\vec{P}_{\bar{\nu}}|, 0\right)$$

Utilizando que para un 4-Vector $\tilde{p} = (p^0, p^1, p^2, p^3)$ es $\tilde{p}^2 = \tilde{p}\cdot\tilde{p} = p^{0^2} - p^{1^2} - p^{2^2} - p^{3^2}$ invariante para el 4-vector momento de una partícula de masa en reposo m es

$$\tilde{p}^2 = \underbrace{m^2 \gamma^2(v) c^2}_{\frac{E}{c}} - \underbrace{m^2\gamma^2(v) v^2}_{|\vec{p}|^2} = m^2c^2\gamma^2(v)\underbrace{[1-\frac{v^2}{c^2}]}_{\gamma^{-2}(v)} = m^2c^2 \Rightarrow \boxed{\tilde{p}^2 = m^2c^2}$$

$$\Rightarrow \boxed{\begin{array}{l} \tilde{p}_{\pi^-}^2 = m_{\pi^-}^2 c^2 \\ \tilde{p}_{\mu^-}^2 = m_{\mu^-}^2 c^2 \\ \tilde{p}_{\bar{\nu}_\mu}^2 = m_{\bar{\nu}_\mu}^2 c^2 \sim 0 \end{array}} \quad \mathrm{I}$$

Planteando la conservación del momento y la energía es $\tilde{P}_{\text{inicial}} = \tilde{P}_{\text{final}}$

$$\Rightarrow \tilde{P}_{\pi^-} = \tilde{P}_{\mu^-} + \tilde{P}_{\bar{\nu}_\mu} \Rightarrow \tilde{P}_{\mu^-} = \tilde{P}_{\pi^-} - \tilde{P}_{\bar{\nu}_\mu}$$

$$\Rightarrow \tilde{p}_{\mu^-}^2 = m_{\mu^-}^2 c^2 = (\tilde{p}_{\pi^-} - \tilde{p}_{\bar{\nu}_\mu})\cdot(\tilde{p}_{\pi^-} - \tilde{p}_{\bar{\nu}_\mu}) = \underbrace{\tilde{p}_{\pi^-}^2}_{m_{\pi^-}^2 c^2} + \underbrace{\tilde{p}_{\bar{\nu}_\mu}^2}_{\simeq 0} - 2\tilde{p}_{\pi^-}\cdot\tilde{p}_{\bar{\nu}_\mu}$$

$$\Rightarrow \boxed{m_{\mu^-}^2 c^2 = m_{\pi^-}^2 c^2 - 2\tilde{p}_{\pi^-}\cdot\tilde{p}_{\bar{\nu}_\mu}} \quad \mathrm{II}$$

recordando:

$$\left.\begin{aligned}\tilde{P}_{\pi^-} &= (m_{\pi^-}\gamma(v)c,\ m_{\pi}\gamma(v)V,\ 0,\ 0)\\ \tilde{P}_{\bar{\nu}_\mu} &= \left(\frac{E_{\bar\nu_\mu}}{c},\ 0,\ |\vec{p}_{\bar\nu}|,\ 0\right)\end{aligned}\right\} \Rightarrow \left.\begin{aligned}&\tilde{P}_{\pi^-}\cdot\tilde{P}_{\bar\nu_\mu} = m_{\pi^-}\gamma(v)c\cdot\frac{E_{\bar\nu_\mu}}{c}\\ &\text{además}\quad E_{\bar\nu}^2 = \underbrace{m_\nu^2c^4}_{\simeq 0} + |\vec{p}_{\bar\nu_\mu}|^2c^2 \approx |\vec{p}_{\bar\nu_\mu}|^2c^2\end{aligned}\right\} \Rightarrow \tilde{P}_{\pi^-}\cdot\tilde{P}_{\bar\nu_\mu} = m_{\pi^-}\gamma(v)\,|\vec{p}_{\bar\nu_\mu}|c$$

$\Rightarrow$ II es:

$$(m_{\pi^-}^2 - m_{\mu^-}^2)c^2 = 2\,\tilde{P}_{\pi^-}\cdot\tilde{P}_{\bar\nu_\mu} = 2\,m_{\pi^-}\gamma(v)\,|\vec{p}_{\bar\nu_\mu}|c$$

$$\Rightarrow \boxed{|\vec{p}_{\bar\nu_\mu}| = \frac{(m_{\pi^-}^2 - m_{\mu^-}^2)c}{2m_{\pi^-}\gamma(v)}} \quad \text{III}$$

De la conservación del momento es

$$\tilde{P}_{\text{inicial}} = (m_\pi\gamma(v)c,\ m_\pi\gamma(v)v,\ 0,\ 0) = \left(\frac{E_{\mu^-}}{c} + \frac{E_{\bar\nu_\mu}}{c},\ |\vec{p}_{\mu^-}|\cos\theta,\ |\vec{p}_{\bar\nu_\mu}| - |\vec{p}_{\mu^-}|\operatorname{sen}\theta,\ 0\right) = \tilde{P}_{\text{final}}$$

$$\Rightarrow \left.\begin{aligned}|\vec{p}_{\mu^-}|\cos\theta &= m_\pi\gamma(v)v\\ |\vec{p}_{\mu^-}|\operatorname{sen}\theta &= |\vec{p}_{\bar\nu_\mu}|\end{aligned}\right\} \Rightarrow \tan\theta = \frac{|\vec{p}_{\bar\nu_\mu}|}{m_\pi\gamma(v)v} \underset{\text{III}}{=} \frac{(m_{\pi^-}^2 - m_{\mu^-}^2)c}{2m_{\pi^-}^2\gamma^2(v)v} \Rightarrow \boxed{\tan\theta = \frac{1 - \frac{m_{\mu^-}^2}{m_{\pi^-}^2}}{2\gamma^2\beta}}$$