Segundo parcial TEM 2022. Ejercicio 1.

$$\vec{E}(r,\theta,\phi,t) = A\,\frac{\operatorname{sen}(\theta)}{r}\left[\cos(kr-wt) - \frac{1}{kr}\operatorname{sen}(kr-wt)\right]\hat{e}_\phi$$

$$= E_\phi\,\hat{e}_\phi$$

a- $\vec{B}$ asociado (de Faraday)

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow$$ miro rotor en esféricas (manual, Griffiths).

$$\nabla\times\vec{E} = \frac{1}{r\operatorname{sen}\theta}\left[\frac{\partial}{\partial\theta}(\operatorname{sen}\theta\,E_\phi)\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}(r E_\phi)\right]\hat{e}_\theta$$

$$\operatorname{sen}\theta\,E_\phi = \operatorname{sen}^2\theta\left[\frac{A}{r}\left[\cos(kr-wt) - \frac{1}{kr}\operatorname{sen}(kr-wt)\right]\right]$$

$$\Rightarrow [\ \ ] = 2\operatorname{sen}\theta\cos\theta\,[\ \ ]$$

$$r E_\phi = A\operatorname{sen}\theta\left[\cos(kr-wt) - \frac{1}{kr}\operatorname{sen}(kr-wt)\right]$$

$$\frac{\partial}{\partial r}\cos(kr-wt) = -k\operatorname{sen}(kr-wt), \quad \frac{\partial}{\partial r}\operatorname{sen}(kr-wt) = k\cos(kr-wt)$$

$$\Rightarrow [\ \ ] = A\operatorname{sen}\theta\left[-k\operatorname{sen}(kr-wt) + \frac{1}{kr^2}\operatorname{sen}(kr-wt) - \frac{1}{r}\cos(kr-wt)\right]$$

$$\Rightarrow \nabla\times\vec{E} = \frac{2A}{r^2}\cos\theta\left(\cos(kr-wt) - \frac{1}{kr}\operatorname{sen}(kr-wt)\right)\hat{e}_r$$

$$+ A\operatorname{sen}\theta\left[\left(\frac{k}{r} - \frac{1}{kr^3}\right)\operatorname{sen}(kr-wt) - \frac{1}{r}\cos(kr-wt)\right]\hat{e}_\theta$$

$$= -\frac{\partial\vec{B}}{\partial t}$$

Integro en t para obtener $\vec{B}$ (a menos de una CTE).

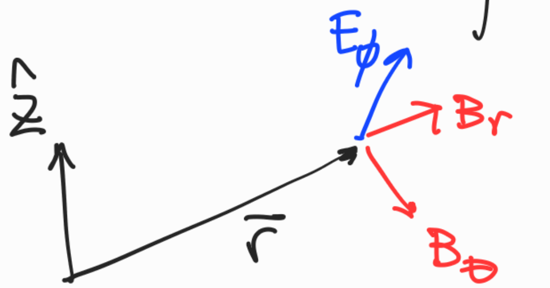
$$\int \cos(kr-\omega t)\,dt = -\frac{1}{\omega}\sin(kr-\omega t) \quad \text{y} \quad \int \sin(kr-\omega t)\,dt = \frac{1}{\omega}\cos(kr-\omega t)$$

$$\Rightarrow \vec{B}(\vec{r},t) = \frac{-2A}{\omega}\frac{\cos\theta}{r^2}\left(\sin(kr-\omega t)+\frac{1}{kr}\cos(kr-\omega t)\right)\hat{e}_r$$

$$-\frac{A\sin\theta}{\omega r}\left(\left(\frac{1}{kr^2}-k\right)\cos(kr-\omega t)+\frac{1}{r}\sin(kr-\omega t)\right)\hat{e}_\theta$$

$$\vec{B}(\vec{r},t) = B_r\,\hat{e}_r + B_\theta\,\hat{e}_\theta$$

$$\vec{E}(\vec{r},t) = E_\phi\,\hat{e}_\phi$$



$$\hat{e}_\phi \times \hat{e}_\theta = -\hat{e}_r$$

b- Vector de Poynting: densidad de flujo de energía

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}E_\phi\,\hat{e}_\phi \times \left(B_r\,\hat{e}_r + B_\theta\,\hat{e}_\theta\right)$$

$$\vec{S} = \frac{1}{\mu_0}\left[-E_\phi B_\theta\,\hat{e}_r - E_\phi B_r\,\hat{e}_\theta\right]$$

$$E_\phi B_\theta = \frac{A^2\sin^2\theta}{\mu_0 r^2\omega}\left[\cos^2(u)\left(\frac{1}{kr^2}-k\right) + \frac{1}{kr}\left(\frac{1}{kr^2}-k\right)\sin u\cos u + \frac{1}{kr}\sin u\cos u - \frac{1}{kr^2}\sin^2 u\right]$$

componente radial de $\vec{S}$

$u = kr-\omega t$

Término que contribuye a la radiación (parte c)

Términos que NO contribuyen a la radiación c- Términos de $\vec{E}$ y $\vec{B}$ que contribuyen a la radiación:

$$\frac{dU}{dt} = \lim_{r\to\infty} \left(\vec{S}\cdot\hat{r}\, r^2\right) d\Omega$$

$\vec{E}_{rad} \propto \frac{1}{r}$, $\vec{B}_{rad} \propto \frac{1}{r}$

$$\vec{E}_{rad} = \frac{A \operatorname{sen}\theta}{r} \cos(u)\, \hat{e}_\phi, \quad u = kr - wt$$

$$\vec{B}_{rad} = -\frac{A \operatorname{sen}\theta}{r} \frac{k}{\omega} \cos(u)\, \hat{e}_\theta$$

d- los términos radiales de $\vec{S}$ que no son de "radiación" son:

$$\text{"}S_{NO\ RAD}\text{"} = \frac{A^2 \operatorname{sen}^2\theta}{\mu_0 r^2 \omega} \left\{ \frac{-1}{kr^2}\left(\cos^2 u - \operatorname{sen}^2 u\right) + \left(-\frac{2}{r} + \frac{1}{k^2 r^3}\right)\operatorname{sen} u \cos u \right\}$$

$u = kr - wt$

Ahora: $\int_0^T \cos^2 u\, dt = \int_0^T \operatorname{sen}^2 u\, dt = \frac{1}{2}$ $\Longrightarrow$ $\int_0^T \{\ \}\, dt = 0$

y $\int_0^T \operatorname{sen} u \cos u\, dt = 0$

la integral dé CERO: No contribuyen.

e- Tensor de tensiones de Maxwell T^{ij} sólo campos de radiación

$$\vec{E}_{rad} = \frac{A \operatorname{sen}\theta}{r} \cos(\underbrace{kr-\omega t}_{\mu})\,\hat{e}_\phi \quad , \quad E^2 = \frac{A^2 \operatorname{sen}^2\theta \cos^2(\mu)}{r^2}$$

$$\vec{B}_{rad} = -\frac{A \operatorname{sen}\theta\, k}{\omega r} \cos(\underbrace{kr-\omega t}_{\mu})\,\hat{e}_\theta \quad , \quad B^2 = \frac{A^2 \operatorname{sen}^2\theta \cos^2(\mu)}{c^2 r^2}$$

$\omega = kc$

Tenemos $E^2 = \frac{B^2}{c^2}$ $\quad c^2 = \frac{1}{\mu_0 \varepsilon_0}$

En coordenadas ortonormales:

$$T^{ij} = \varepsilon_0\left(E^i E^j - \frac{1}{2}\delta^{ij} E^2\right) + \frac{1}{\mu_0}\left(B^i B^j - \frac{1}{2}\delta^{ij} B^2\right)$$

$i,j = r,\theta,\phi$ en esféricas. Sólo E_ϕ y $B_\theta \neq 0 \Rightarrow$ $T^{ij} = 0$ si $i \neq j$

Ahora

$$T^{rr} = -\frac{\varepsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2 = -\frac{1}{2}\left(\varepsilon_0 + \frac{1}{\mu_0 c^2}\right) E^2 = -\varepsilon_0 E^2$$

$$T^{\theta\theta} = -\frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 = -\frac{1}{2}\left(\varepsilon_0 - \frac{1}{\mu_0 c^2}\right) E^2 = 0$$

$$T^{\phi\phi} = \frac{\varepsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2 = \frac{1}{2}\left(\varepsilon_0 - \frac{1}{\mu_0 c^2}\right) E^2 = 0$$

$$T = \begin{pmatrix} -\varepsilon_0 E^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

El término da lugar a una "presión" de radiación en la dirección radial.

Reviso esta cuentita:

$$\left\{ \frac{A \operatorname{sen}\theta}{\omega r}\left(\left(\frac{1}{kr^2} - k\right)\cos(kr-\omega t) + \frac{1}{r}\operatorname{sen}(kr-\omega t)\right) \right.$$

$$\left. * \; A\,\frac{\operatorname{sen}(\theta)}{r}\left[\cos(kr-\omega t) - \frac{1}{kr}\operatorname{sen}(kr-\omega t)\right] \right\} = E_\phi B_\theta$$

$$-E_\phi B_\theta = \frac{A^2 \operatorname{sen}^2\theta}{\omega r^2}\left[\left(\frac{-1}{kr^2} + k\right)\cos^2 u - \frac{1}{r}\operatorname{sen} u \cos u\right.$$

$$\left. + \frac{1}{kr}\left(\frac{1}{kr^2} - k\right)\cos u \operatorname{sen} u + \frac{\operatorname{sen}^2 u}{kr^2}\right]$$

OR

$$= -\frac{A^2 \operatorname{sen}^2\theta}{\omega r^2}\left\{\frac{1}{kr^2}\left(\cos^2 u - \operatorname{sen}^2 u\right) - k\cos^2 u \right.$$

$$\left. + \operatorname{sen} u \cos u \underbrace{\left[\frac{1}{r} - \frac{1}{kr}\left(\frac{1}{kr^2} - k\right)\right]}_{\left(\frac{2}{r} - \frac{1}{k^2 r^3}\right)}\right\}$$