Pert. dep. tiempo

$$H(t) = H_0 + H'(t)$$

$\hookrightarrow$ perturbación $\quad H'(t) = \lambda V(t)$

$\lambda \hookrightarrow$ "pequeño", adimensional

Desarrollo en λ (ó H')

H_0 : conocido $|\phi_n\rangle$, E_n^0 $\qquad H_0|\phi_n\rangle = E_n^0|\phi_n\rangle$

$$|\Psi(t)\rangle = \sum_n c_n(t)\, e^{-iE_n^0 t/\hbar}|\phi_n\rangle \;\rightarrow\; \text{ec. Schrodinger}$$

$$\boxed{i\hbar \dot{c}_m = \lambda \sum_n c_n(t)\, e^{i(E_m^0 - E_n^0)t/\hbar}\langle\phi_m|V(t)|\phi_n\rangle}$$

inicialmente $t=0$ $\quad |\Psi(0)\rangle = |\phi_k\rangle$ único estado ocupado $\rightarrow k$

$$c_n(0) = \delta_{nk}$$

1^{er} orden λ (H') : $\quad m \neq k \quad i\hbar\dot{c}_m = e^{i(E_m^0 - E_k^0)t/\hbar}\,\lambda\,\langle\phi_m|V(t)|\phi_k\rangle$

Sol. 1^{er} orden en la pert.

$$\boxed{c_m(t) = \frac{\lambda}{i\hbar}\int_0^t dt'\, e^{i\omega_{mk}t'}\, V_{mk}(t')}, \quad \omega_{mk} = \frac{E_m^0 - E_k^0}{\hbar}$$

$$V_{mk} = \langle\phi_m|V|\phi_k\rangle$$

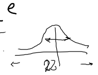
$$P_n(t) = P(k \rightarrow n, (0,t)) = |\langle\phi_n|\Psi(t)\rangle|^2 = |c_n(t)|^2$$

$$\sum_n P_n(t) = \sum_n |c_n(t)|^2 = \sum_n \langle\phi_n|\Psi\rangle\langle\phi_n|\Psi\rangle^* = \sum_n \langle\phi_n|\Psi\rangle\langle\Psi|\phi_n\rangle$$

$$= \sum_n \langle\Psi|\phi_n\rangle\langle\phi_n|\Psi\rangle = \langle\Psi|\Big(\underbrace{\sum_n |\phi_n\rangle\langle\phi_n|}_{=\mathbb{1}}\Big)|\Psi\rangle$$

$$= \langle\Psi|\Psi\rangle = 1$$

ej osc. armn cargado c/ campo el. $\lambda V(t) = qEx\, e^{-t^2/\tau^2}$



$|0\rangle \rightarrow |n\rangle$

$t = -\infty \quad |\Psi(t)\rangle = |0\rangle$

$t = +\infty \quad |\Psi(+\infty)\rangle = |n\rangle$?

$\hat{x} \rightarrow a, a^\dagger \rightarrow$ conectado $|0\rangle$ con $|1\rangle$

$$P_1 = \frac{\pi}{2}\, e^{-\omega^2\tau^2/2}\,\frac{q^2E^2\tau^2}{m\hbar\omega} \qquad P_n = 0, \; n = 2, 3, \ldots$$

$\tau \rightarrow \infty \quad P_1 \rightarrow 0 \quad$ pert. adiabática : no hay transición importante

Potencial periódico

$$V(t) = M e^{\mp i\omega t} \qquad \left(V \text{ hermítico} : V = Me^{-i\omega t} + M^\dagger e^{+i\omega t}\right)$$

$M \hookrightarrow$ indep. t

$$c_m(t) = \frac{1}{i\hbar}\langle\phi_m|M|\phi_k\rangle \int_0^t dt'\, e^{i\omega_{mk}t'} e^{\mp i\omega t'} \quad \text{ó } (M^\dagger, e^{+i\omega t'})$$

$$\int_0^t \ldots = \frac{e^{i(\omega_{mk}\mp\omega)t} - 1}{i(\omega_{mk}\mp\omega)} = e^{i(\omega_{mk}\mp\omega)t/2}\,\frac{\operatorname{sen}(\omega_{mk}\mp\omega)t/2}{(\omega_{mk}\mp\omega)/2}$$

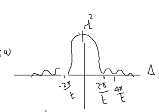
$$P_m(t) = |c_m(t)|^2 = \frac{1}{\hbar}|\langle\phi_m|M|\phi_k\rangle|^2\left(\frac{\operatorname{sen}(\omega_{mk}\mp\omega)t/2}{(\omega_{mk}\mp\omega)/2}\right)^2$$

$E_m - E_k \mp \omega\hbar$: donde esto se anule (denominador) $\Rightarrow$ prob. va a ser dominante / resonantes.

$k \rightarrow m$: $\quad E_m = E_k \pm \omega\hbar$ $\quad$ ($+$: absorción cuanto $\hbar\omega$; $-$: emisión $\hbar\omega$)

Tiempos largos de acción de la pert.

$$F(t) = \frac{4}{\Delta^2}\operatorname{sen}^2 \Delta t/2 \qquad \Delta = \omega_{mk} \mp \omega$$



$$\int_{-\infty}^{\infty} f(\Delta)\,\frac{4}{\Delta^2}\operatorname{sen}^2\frac{\Delta t}{2}\,d\Delta \underset{t \text{ grande}}{=} f(0)\int_{-\infty}^{\infty}\frac{4}{\Delta^2}\operatorname{sen}^2\frac{\Delta t}{2} = 2t f(0)\underbrace{\int_{-\infty}^{\infty} du\,\frac{\operatorname{sen}^2 u}{u^2}}_{\pi} = 2\pi t f(0)$$

$$\frac{4}{\Delta^2}\operatorname{sen}^2\frac{t\Delta}{2} \rightarrow 2\pi t\,\delta(\Delta) = 2\pi\hbar t\,\delta(E_m^0 - E_k^0 \mp \hbar\omega)$$

$$\Gamma_{k\rightarrow m} = \frac{P(k\rightarrow m, t)}{t} = \frac{2\pi}{\hbar}|\langle\phi_m|M|\phi_k\rangle|^2\,\delta(E_m^0 - E_k^0 \mp \hbar\omega)$$

Regla oro Fermi