# Prob. de ionización por C. Eléctrico

Átomo hidrógeno, estado base: $e^-$, $E_0 = -13.6\,eV = -\frac{\mu e^4}{2\hbar^2} = -\frac{e^2}{2a}$

$a = \frac{\hbar^2}{\mu e^2} = \frac{\hbar}{\mu c \alpha}$, $\alpha \simeq \frac{1}{137}$ cte estructura fina, $\mu \simeq m_e$ $\mu = 1.00054\, m_e$

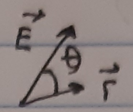
$\simeq 5.29 \times 10^{-11}\, m$

$e^-$: del estado base al continuo: $\frac{\hbar^2 k^2}{2\mu} = -\frac{\mu e^4}{2\hbar^2} + \hbar\omega$ ⊛

$\hbar\omega \to$ fotón/C.E.

$E_n = -\frac{Z e^2}{2\hbar^2 a}$ átomo hidrogenoide carga $Z$

$\vec{E} = 2E_0 \operatorname{sen}\omega t\, \hat{E}_0 = iE_0\left(e^{-i\omega t} - e^{i\omega t}\right)$, $H' = -\vec{D}\cdot\vec{E} = e\vec{E}\cdot\vec{r}$




$H' = ieE_0 r\cos\theta\left(e^{-i\omega t} - e^{i\omega t}\right)$

$e^{-i\omega t} \to$ abs, "$M^+$" in last class

$e^{i\omega t} \to$ emision

$\rho(E)$: box $\vec{k}$ / $k_x = 2\pi n_x/L \Rightarrow \left(\frac{L}{2\pi}\right)^3 dk_x\, dk_y\, dk_z$ states in the range $dk_x\, dk_y\, dk_z$

$|\vec{k}|$ fixed by energy conservation ⊛

$$\rho(k)\, dE_k = \left(\frac{L}{2\pi}\right)^3 k^2\, dk\, d\Omega_k, \quad d\Omega_k = \operatorname{sen}\theta_k\, d\theta_k\, d\varphi_k$$

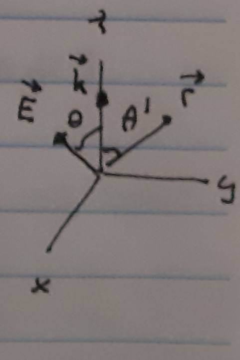
$$E_k = \frac{\hbar^2 k^2}{2\mu} \Rightarrow dE_k = \frac{\hbar^2 k\, dk}{\mu}, \quad k = |\vec{k}| \Rightarrow \rho(k) = \frac{\mu L^3}{8\pi^3 \hbar^2} k\, d\Omega_k$$

density of states per energy in direction $(\theta_k, \varphi_k)$ in $d\Omega_k$

$$\psi_0 = (\pi a^3)^{-1/2} e^{-r/a}, \quad \psi_k = L^{-3/2} e^{i\vec{k}\cdot\vec{r}}$$

$$\langle k | M^+ | 0 \rangle = \int d^3r\, L^{-3/2} e^{-i\vec{k}\cdot\vec{r}}\, iE_0 e r\cos\theta'' (\pi a^3)^{-1/2} e^{-r/a}$$

abs.




$$\cos\theta'' = \hat{E}\cdot\hat{r} = (s\theta, 0, c\theta)\cdot(s\theta' c\varphi', s\theta' s\varphi', c\theta')$$
$$= s\theta s\theta' c\varphi' + c\theta c\theta'$$

$s\theta s\theta' c\varphi'$: vanishes when $\varphi'$ of $\vec{r}$ is integrated

$d^3r = r^2 dr\, d\theta' d\varphi'$

$$\langle k|M^1|0\rangle = \frac{2\pi i E_0 e\cos\theta}{(\pi a^3 L^3)^{1/2}} \int_0^\infty dr\, r^3 e^{-r/a} \int_0^\pi d\theta'\, e^{-ikr\cos\theta'} \cos\theta' \operatorname{sen}\theta'$$

$$\hookrightarrow \int_{-1}^{1} du\, u e^{-ikru} = \frac{2i}{(kr)^2}(kr\cos kr - \operatorname{sen} kr)$$

$$= \frac{2\pi i E_0 e\cos\theta}{(\pi a^3 L^3)^{1/2}} \frac{2i}{k^2} \int_0^\infty dr\, r\, e^{-r/a}(kr\cos kr - \operatorname{sen} kr)$$

$$\hookrightarrow \frac{1}{k^2}\int_0^\infty du\, u e^{-u/ka}(u\cos u - \operatorname{sen} u) =$$

$$2(ka)^3\left(-\frac{1}{(1+k^2a^2)^2} + \frac{1-3k^2a^2}{(1+k^2a^2)^3}\right)$$

$$= -\frac{8(ka)^5}{(1+k^2a^2)^3}$$

$$\Rightarrow + \frac{32\pi E_0 e\cos\theta\,(ka)^5}{(\pi a^3 L^3)^{1/2} k^4 (1+k^2a^2)^3}$$

$$w = d\Gamma_k = \frac{2\pi}{\hbar}|\langle k|M^1|0\rangle|^2 \rho(k) = \frac{2\pi}{\hbar}\left(\frac{32\pi E_0 e\cos\theta\,(ka)^5}{(\pi a^3 L^3)^{1/2} k^4 (1+k^2a^2)^3}\right)^2 \frac{\mu L^3}{8\pi^3\hbar^2} k\, d\Omega_k$$

$$\Rightarrow w = d\Gamma_k = \frac{256\,\mu}{\pi\hbar^3}\frac{E_0^2 e^2 k^3 a^7}{(1+k^2a^2)^6}\cos^2\theta\, d\Omega_k$$

$$\boxed{\Gamma = \int d\Gamma_k = \frac{1024}{3\hbar^3}\mu E_0^2 e^2 \frac{k^3 a^7}{(1+k^2a^2)^6}}, \quad \int \cos^2\theta\, d\Omega = 4\pi/3$$

↳ integrate all directions

when is the computation ok? 1) interaction with atom (not $e^-$!)

$$\Rightarrow \lambda_\gamma \gg a \text{ or } k_\gamma a \ll 1.$$

2) free $e^-$ (~~not~~ eigenstate of $1/r$ same of free) : $\gamma$ and $e^-$ energy much higher than $\frac{\mu e^4}{2\hbar^2}$ (*) $\Rightarrow \frac{\hbar^2 k_e^2}{2\mu} \simeq \hbar\omega = \hbar c k_\gamma \gg \frac{\mu e^4}{2\hbar^2} \Rightarrow k_\gamma a \gg \frac{\mu e^4}{2\hbar^3 c}\frac{\hbar^2}{\mu e^2} = \frac{e^2}{2\hbar c} = \frac{\alpha}{2} = \frac{1}{274}$

$$\Rightarrow \boxed{1 \gg k_\gamma a \gg \frac{1}{274}}, \text{ besides } k_e^2 a^2 = \frac{c k_\gamma a^2}{\hbar} = 2k_\gamma a/\alpha \gg 2\cdot 2\alpha\frac{1}{\alpha} \gg 1! \Rightarrow \Gamma = \frac{1024}{3}\frac{\mu E_0^2 e^2}{k^9 a^5}$$