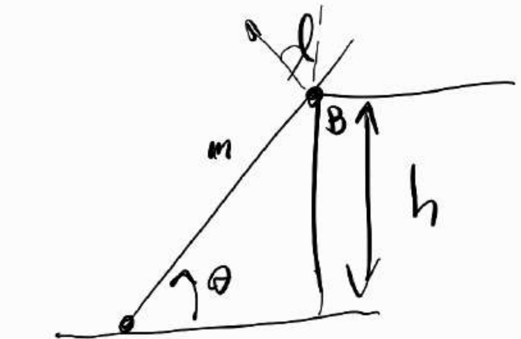


a, b)

$$-mg + N_A + N_B \cos\theta = 0 \quad \Rightarrow \quad N_A = mg\left(1 - \frac{\ell}{2h}\operatorname{sen}\theta\cos^2\theta\right)$$

$$N_B \operatorname{sen}\theta = F_{roz}$$

$$-mg\frac{\ell}{2}\cos\theta + N_B\frac{h}{\operatorname{sen}\theta} = 0 \Rightarrow N_B = \frac{mg\ell}{2h}\operatorname{sen}\theta\cos\theta$$

$$\frac{\cancel{mg}\ell}{2h}\operatorname{sen}^2\theta\cos\theta = F_{roz} \leq f N_A = f\cancel{mg}\left(1 - \frac{\ell}{2h}\operatorname{sen}\theta\cos^2\theta\right)$$

$$\Rightarrow \quad f \geq \frac{\ell}{2h}\operatorname{sen}\theta\cos\theta \frac{\operatorname{sen}\theta}{1 - \left(\frac{\ell}{2h}\operatorname{sen}\theta\cos\theta\right)\cos\theta}$$

$$f \geq \frac{\ell}{2h}\,\frac{1}{2}\,\frac{\frac{1}{\sqrt{2}}}{1 - \frac{\ell}{4h}\frac{1}{\sqrt{2}}} = \frac{1}{\frac{4\sqrt{2}\,h}{\ell} - 1} = \frac{1}{2\sqrt{2}-1} \leftarrow f_b$$

---

c)

$$-2mg + N_A + N_B\cos\theta = 0 \quad , \quad N_B \operatorname{sen}\theta = F_{roz}$$

$$-mg\left(\frac{\ell}{2} + x\right)\cos\theta + N_B\frac{h}{\operatorname{sen}\theta} = 0 \Rightarrow N_B = \frac{mg}{h}\left(\frac{\ell}{2} + x\right)\cos\theta\operatorname{sen}\theta$$

$$\Rightarrow N_A = mg\left(2 - \frac{(\ell + 2x)}{2h}\operatorname{sen}\theta\cos^2\theta\right)$$

$$\Rightarrow F_{roz} = \frac{mg}{2h}(\ell + 2x)\cos\theta\operatorname{sen}^2\theta \leq f N_A = f mg\left(2 - \frac{(\ell + 2x)}{2h}\operatorname{sen}\theta\cos^2\theta\right)$$

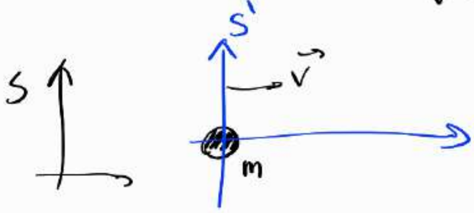
$$\cos\theta\operatorname{sen}^2\theta \leq f\left[\frac{4h}{\ell + 2x} - \operatorname{sen}\theta\cos^2\theta\right] \Rightarrow x \leq \frac{2fh}{\cos\theta\operatorname{sen}\theta\left[\operatorname{sen}\theta + f\cos\theta\right]} - \ell$$

$$h = \frac{\ell}{2} \Rightarrow x \leq \ell\left[\frac{\frac{1}{2\sqrt{2}-1}}{\frac{\sqrt{3}}{4}\left[\frac{\sqrt{3}}{2} + \frac{1}{2(2\sqrt{2}-1)}\right]}\right] = \frac{8\ell}{\sqrt{3}\left(\sqrt{3}(2\sqrt{2}-1) + 1\right)} \approx \frac{8}{7{,}2}\ell \Longrightarrow$$

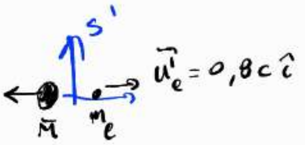
puede subir todo el tablón

2)

$\vec{V} = \underbrace{0{,}05c}_{V}\,\hat{\imath}$



a)



$\Rightarrow u_{ex} = \frac{u'_{ex}+V}{1+\frac{u'_{ex}V}{c^2}} = \frac{0{,}85c}{1{,}04} \approx 0{,}817c$, $\vec{u}_e = u_{ex}\,\hat{\imath}$

$$\left.\begin{array}{l} m_e\gamma(u'_e) + \bar{M}\gamma(u'_{\bar M}) = m \\ m_e\gamma(u'_e)u'_e + \bar{M}\gamma(u'_{\bar M})u'_{\bar M} = 0 \end{array}\right\} \Rightarrow m_e\gamma(u'_e)(u'_e - u'_{\bar M}) + m\,u'_{\bar M} = 0$$

$$\Rightarrow \boxed{\frac{-m_e\gamma(u'_e)u'_e}{m - m_e\gamma(u'_e)} = u'_{\bar M}} \Rightarrow \boxed{u_{\bar M} = \frac{u'_{\bar M}+V}{1+\frac{u'_{\bar M}V}{c^2}} = \frac{V - \frac{m_e\gamma(u'_e)u'_e}{m - m_e\gamma(u'_e)}}{1 - \frac{V m_e\gamma(u'_e)u'_e}{c^2(m - m_e\gamma(u'_e))}}}$$

b)

$$\left.\begin{array}{l} u_{ex} = \frac{u'_{ex}+V}{1+\frac{u'_{ex}V}{c^2}} = 0{,}05c = V \\ u_{ey} = \frac{u'_{ey}}{\gamma(V)\left(1+\frac{u'_{ex}V}{c^2}\right)} = \frac{0{,}8c}{\gamma(V)} \end{array}\right\} \Rightarrow \vec{u}_e = 0{,}05c\,\hat{\imath} + \frac{0{,}8c}{\gamma(V)}\,\hat{\jmath}$$

$$m_e\underbrace{\gamma(u'_e)}c^2 + \bar{M}\gamma(u'_{\bar M})c^2 = m_e\frac{5}{3}c^2 + c^2\bar{M}\gamma(u'_{\bar M}) = mc^2 \Rightarrow \bar{M}\gamma(u'_{\bar M}) = m - m_e\frac{5}{3}$$

$$m_e\gamma(u'_e)u'_{ex} + \bar{M}\gamma(u'_{\bar M})u'_{\bar M x} = 0 \Rightarrow u'_{\bar M x} = -\frac{m_e\gamma(u'_e)u'_{ex}}{m - \frac{5}{3}m_e} = 0$$

$$\Rightarrow \boxed{u_{\bar M x} = \frac{u'_{\bar M x}+V}{1+\frac{u'_{\bar M x}V}{c^2}} = V = 0{,}05c}$$

$$m_e\gamma(u'_e)u'_{ey} + \bar{M}\gamma(u'_{\bar M})u'_{\bar M y} = 0 \Rightarrow u'_{\bar M y} = -\frac{m_e\gamma(u'_e)u'_{ey}}{m - m_e\frac{5}{3}}$$

$$\Rightarrow \boxed{u_{\bar M y} = \frac{u'_{\bar M y}}{\gamma(V)\left(1+\frac{V u'_{\bar M x}}{c^2}\right)} = -\frac{m_e\overbrace{\gamma(u'_e)}^{\frac{5}{3}}u'_{ey}}{\gamma(V)\left(m - m_e\frac{5}{3}\right)} = \frac{m_e}{\left(\frac{3}{5}m - m_e\right)}\cdot\frac{0{,}8c}{\gamma(V)}}$$

c)

$u_{ey} = 0{,}8c$

$u_{ex} = 0$

$$m\gamma(V)c^2 = m_e\gamma(u_e)c^2 + \bar{M}\gamma(u_{\bar M})c^2 \Rightarrow \bar{M}\gamma(u_{\bar M}) = m\gamma(V) - m_e\gamma(u_e)$$

$$m\gamma(V)V = \bar{M}\gamma(u_{\bar M})u_{\bar M x} \Rightarrow u_{\bar M x} = \frac{m\gamma(V)V}{m\gamma(V) - m_e\gamma(u_e)}$$

$$0 = m_e\gamma(u_e)u_{ey} + \bar{M}\gamma(u_{\bar M})u_{\bar M y} \Rightarrow u_{\bar M y} = -\frac{m_e\gamma(u_e)u_{ey}}{m\gamma(V) - m_e\gamma(u_e)}$$

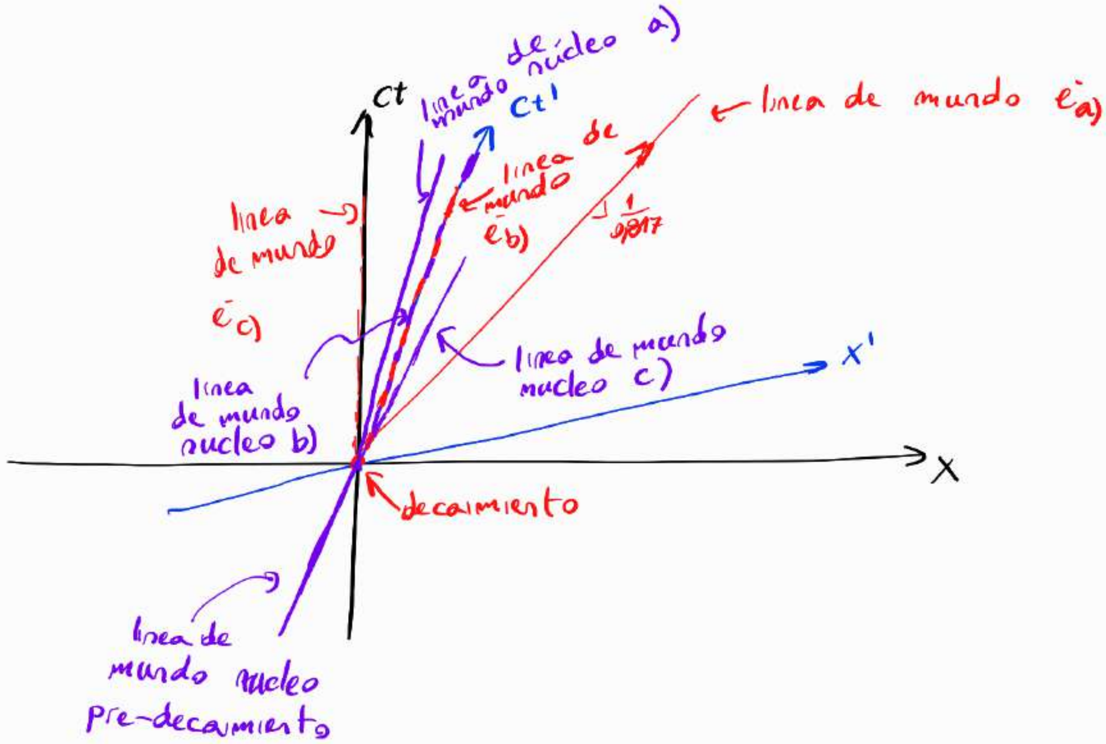
$$u_{e_x} = \frac{u'_{e_x} + V}{1 + \frac{u'_{e_x} V}{c^2}} = 0 \Rightarrow u'_{e_x} = -V$$

$$u_{e_y} = \frac{u'_{e_y}}{\gamma(V)\left(1 - \frac{u'_{e_x} V}{c^2}\right)} = \frac{u'_{e_y}}{\gamma\left(1 - \frac{V^2}{c^2}\right)} \Rightarrow u'_{e_y} = \frac{u_{e_y}}{\gamma(V)} = \frac{0{,}8c}{\gamma(V)}$$

$$\Rightarrow \theta = \operatorname{arctg}(0{,}625\,\gamma(V))$$

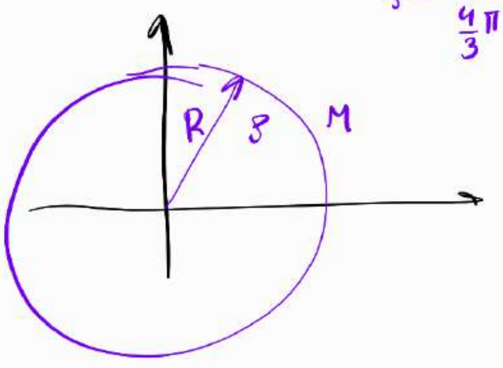
$$\Rightarrow \theta \approx 0{,}0625 \text{ rad}$$

d)



3)

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$



La fuerza actuando sobre la bala es

$$\vec{F}(\vec{r}) = -\frac{GMm}{r^2}\hat{e}_r \quad \text{si} \quad r \geq R$$

o $\vec{F}(\vec{r}) = -G\frac{M'(r)m}{r^2}\hat{e}_r$ si $r \leq R$

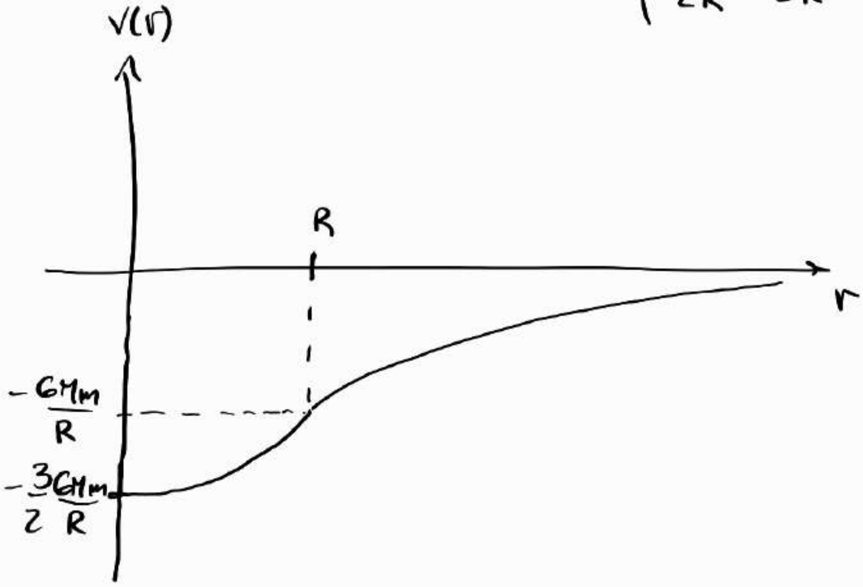
siendo $M'(r)$ la masa encerrada en una esfera de radio r

$$M'(r) = \rho\,\frac{4}{3}\pi r^3 = \frac{M}{\frac{4}{3}\pi R^3}\,\frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$$

$$\Rightarrow \vec{F}(\vec{r}) = -GMm \begin{cases} \frac{\hat{e}_r}{r^2} & \text{si } R \leq r \\ \frac{r}{R^3}\hat{e}_r & \text{si } 0 < r \leq R \end{cases}$$

$$V(\vec{r}) = -\int_\infty^r \vec{F}(\vec{r})\cdot d\vec{r} = \begin{cases} \int_\infty^r \frac{GMm}{r^2}\,dr = -\frac{GMm}{r} & \text{si } R \leq r \\ \int_\infty^R \frac{GMm}{r^2}\,dr + \int_R^r \frac{GMm}{R^3}\,r\,dr = -\frac{GMm}{R} + \frac{GMm}{2R^3}(r^2 - R^2) & \text{si } r \leq R \end{cases}$$

es decir $\quad V(\vec{r}) = -GMm \begin{cases} \dfrac{1}{r} & \text{si } R \leq r \\[2ex] \dfrac{3}{2R} - \dfrac{r^2}{2R^3} & \text{si } r \leq R \end{cases}$



b) $L_o = m\sqrt{\dfrac{GMR}{32}}$, $E = -\dfrac{5GMm}{4R}$

$$E = \frac{m\dot{r}^2}{2} + \frac{L_o^2}{2mr^2} + V(r)$$

puntos de retorno son $\dot{r}=0$

$$\Rightarrow -\frac{5GMm}{4R} = \frac{GMRm}{64\, r_\pm^2} + V(r_\pm)$$

$$\Rightarrow V(r_\pm) = -GMm\left[\frac{5}{4R} + \frac{R}{64 r_\pm^2}\right]$$

$$\Rightarrow \frac{5}{4R} + \frac{R}{64 r_\pm^2} = \begin{cases} \dfrac{1}{r_\pm} & \text{con } R \leq r \\[2ex] \dfrac{3}{2R} - \dfrac{r_\pm^2}{2R^3} & \text{con } r \leq R \end{cases}$$

si $r \geq R \Rightarrow \dfrac{5}{4} r_\pm^2 + \dfrac{R^2}{64} - R r_\pm = 0 \Rightarrow r_\pm = \dfrac{R \pm R\sqrt{1-\frac{5}{64}}}{\frac{5}{2}} = 2R\left(\dfrac{1}{5} \pm \dfrac{\sqrt{59}}{40}\right)$

$\Rightarrow \ < R$ absurdo

$\Rightarrow$ si $r \leq R \quad \dfrac{5}{4R} + \dfrac{R}{64 r_\pm^2} = \dfrac{3}{2R} - \dfrac{r_\pm^2}{2R^3} \Rightarrow \dfrac{5}{4} R^2 r_\pm^2 + \dfrac{R^4}{64} = \dfrac{3}{2} R^2 r_\pm^2 - \dfrac{r_\pm^4}{2}$

$$\Rightarrow r_\pm^4 - \frac{R^2 r_\pm^2}{2} + \frac{R^4}{32} = 0 \Rightarrow r_\pm^2 = \frac{\frac{R^2}{2} \pm \frac{R^2}{2}\sqrt{1-\frac{1}{2}}}{2} = \frac{R^2}{4}\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \boxed{r_\pm = \frac{R}{2}\sqrt{\frac{\sqrt{2}\pm 1}{\sqrt{2}}} = R\sqrt{\frac{2\pm\sqrt{2}}{8}}} \Rightarrow \begin{matrix} r_+ < R \\ r_- < R \end{matrix}$$

la bola siempre está dentro

c) $$(L_0 = mr^2\dot{\theta})$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\dot{\theta} = \frac{dr}{d\theta}\frac{L_0}{mr^2}$$

$$\Rightarrow -\frac{5}{4}\frac{\cancel{GMmR}}{R^2} = \frac{\cancel{GMmR}\,r^2}{2R^4} - \cancel{GMm}\frac{3\cancel{R}}{2R^2} + \left(\frac{dr}{d\theta}\right)^2 \frac{\cancel{GMRm}}{64r^4} + \frac{\cancel{GMRm}}{64r^2}$$

$$\Rightarrow \left(\frac{dr}{d\theta}\right)^2 = 64r^4\left[\frac{3}{2R^2} - \frac{5}{4R^2} - \frac{r^2}{2R^4} - \frac{1}{64r^2}\right]$$

$$\Rightarrow \left(\frac{dr}{d\theta}\right)^2 = r^4\left[\frac{16}{R^2} - 32\frac{r^2}{R^4} - \frac{1}{r^2}\right]$$

$$\Rightarrow \frac{dr}{d\theta} = r^2\sqrt{\frac{16}{R^2} - 32\frac{r^2}{R^4} - \frac{1}{r^2}} = \frac{r^3}{R^2}\sqrt{-32 + 16\frac{R^2}{r^2} - \frac{R^4}{r^4}}$$

$$\Rightarrow \boxed{d\theta = \frac{\frac{R^2}{r^3}dr}{\sqrt{-32 + 16\frac{R^2}{r^2} - \frac{R^4}{r^4}}}}$$

e) $$x = \frac{R^2}{r^2} \Rightarrow dx = -2\frac{R^2 dr}{r^3}$$

$$\Rightarrow -2\int d\theta = \int \frac{dx}{\sqrt{-32+16x-x^2}}$$

$$\Rightarrow 2(\theta_0 - \theta) = \frac{1}{\sqrt{1}}\left[\arcsen\left(\frac{2\frac{R^2}{r^2} - 16}{\sqrt{128}}\right) - \underbrace{\arcsen\left(\frac{2\frac{R^2}{r_0^2} - 16}{\sqrt{128}}\right)}_{2\delta_0}\right]$$

$$\Rightarrow \boxed{\sqrt{\frac{2}{\sqrt{128}\,\mathrm{sen}\left(2(\theta_0 + \delta_0 - \theta)\right) + 16}}\; R = r(\theta)}$$