Introducción a la Complejidad y las Redes Complejas

Andrés Pomi

Curso de Biofísica

1. Primera aproximación a la noción de complejidad:

- Variedad

Variedad según William Ross Ashby

<u>Definición</u>: Dado un sistema compuesto por q elementos

$$S = \langle e_1, e_2, \ldots, e_q \rangle,$$

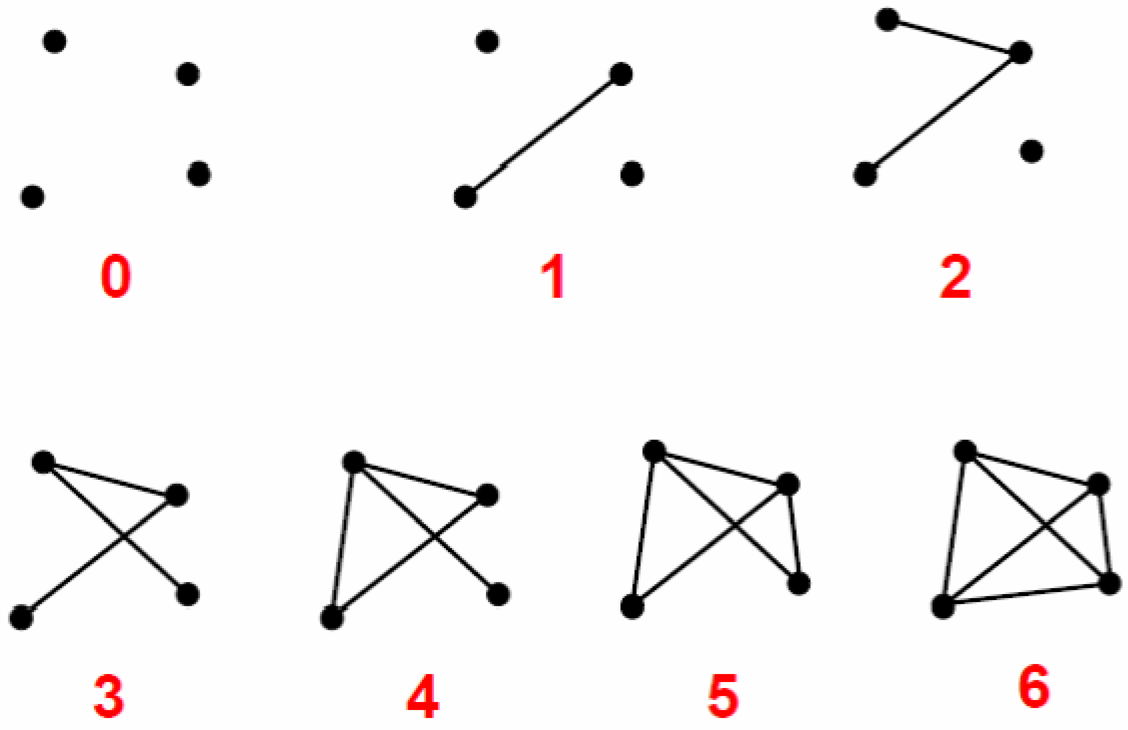
la variedad V se define como: V(S) = Número de elementos de S.

<u>Una definición de la Complejidad C de un sistema S</u>:

Complejidad = Variedad $\qquad$ C(S) = V(S)

El problema de la complejidad de las redes o "grafos"

¿Cuál de los siguientes grafos es más complejo?



Variedad de Grafos

Caso de grafo de N nodos y r conexiones.

Fórmula general:

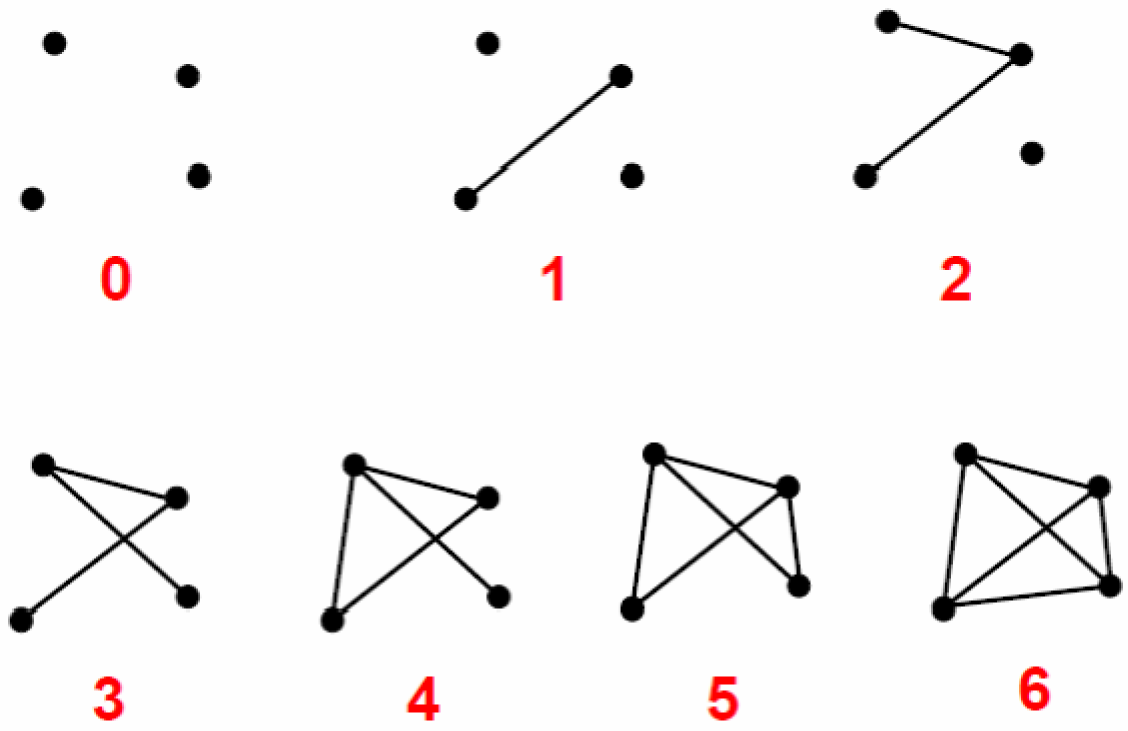
$$V(N,r) = \binom{\frac{N(N-1)}{2}}{r} \quad \text{(Recordar que } \binom{n}{m} = \frac{n!}{m!(m-n)!} \text{)}$$

Ejemplo: V(N, r) para N = 4

r	V(r)
0	1
1	6
2	15
3	20
4	15
5	6
6	1

Entonces....

¿Cuál de los siguientes grafos es más complejo?



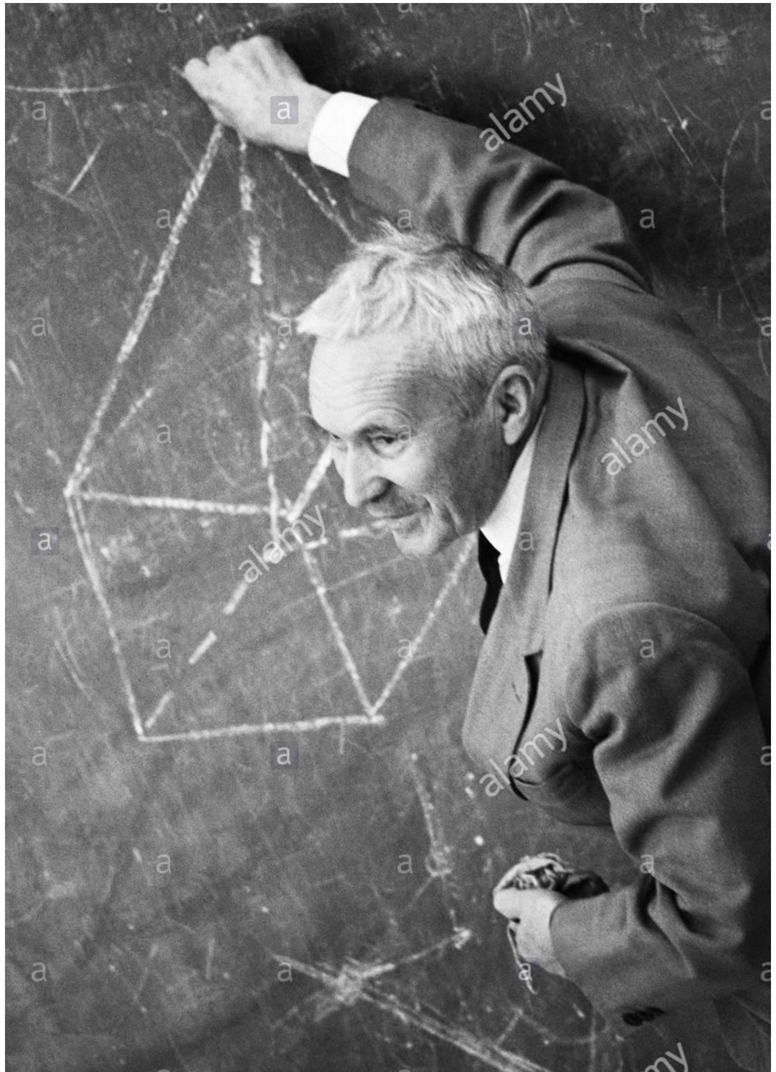
Para N = 4, el grafo más complejo es el que tiene 3 conexiones.

Explicación:

(a) Esta conectividad requiere una más detallada descripción de qué nodos se conectan entre sí.

(b) La descripción compacta de estos detalles consume más tiempo.

Complejidad algorítmica de Kolmogórov



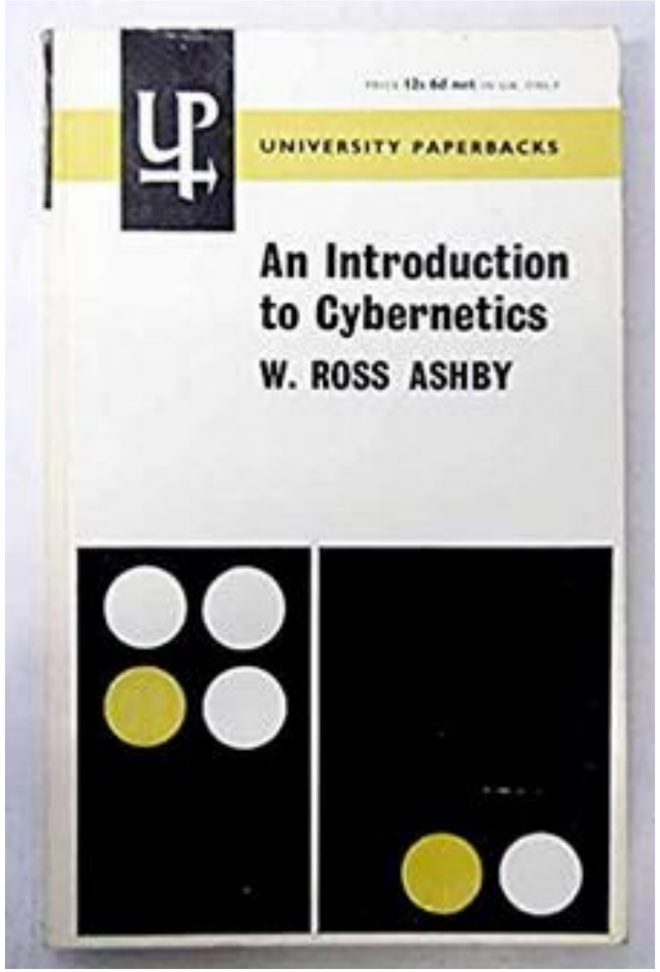
La proyección en el tiempo C(S, t) es una medida alternativa de complejidad

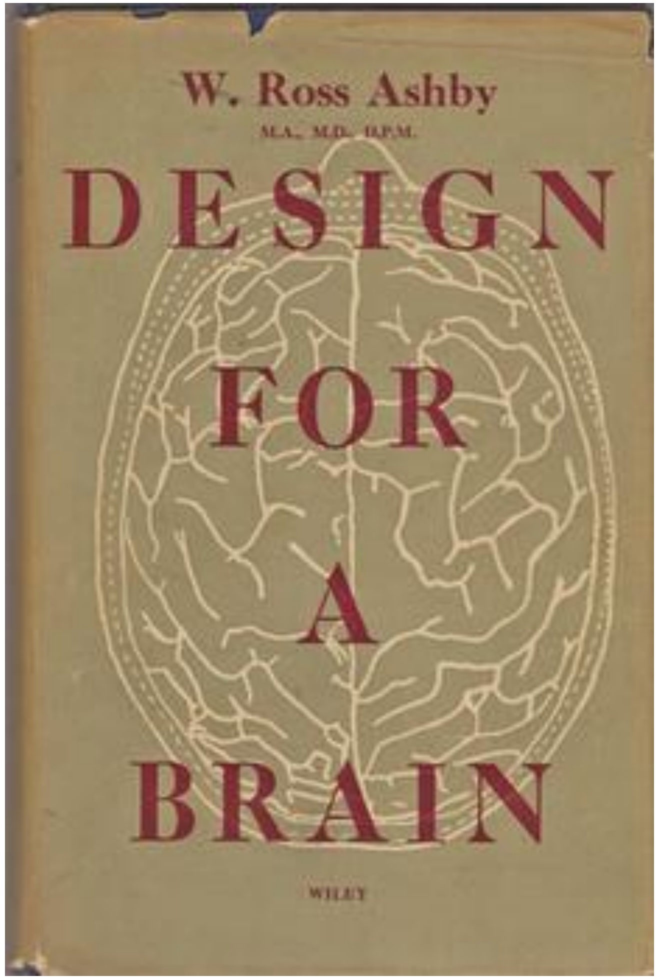
Complejidad de un Sistema $\propto$ Longitud de la descripción más compacta de ese Sistema

(Esta es una medida emparentada a la de "Complejidad algorítmica" de Kolmogorof).

En general es: $C(S, t) \propto V(S)$

Ashby y el Teorema de la variedad necesaria

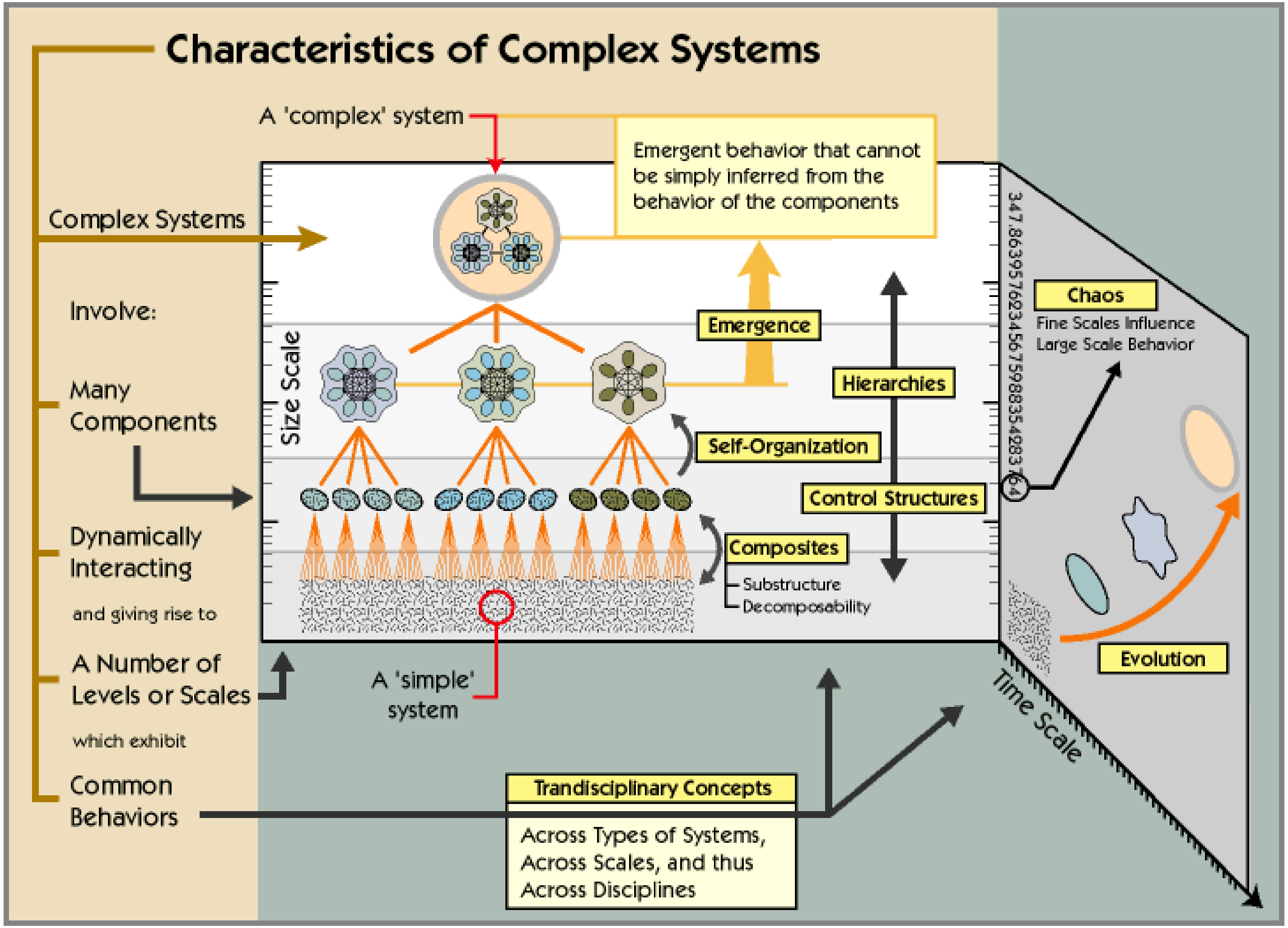




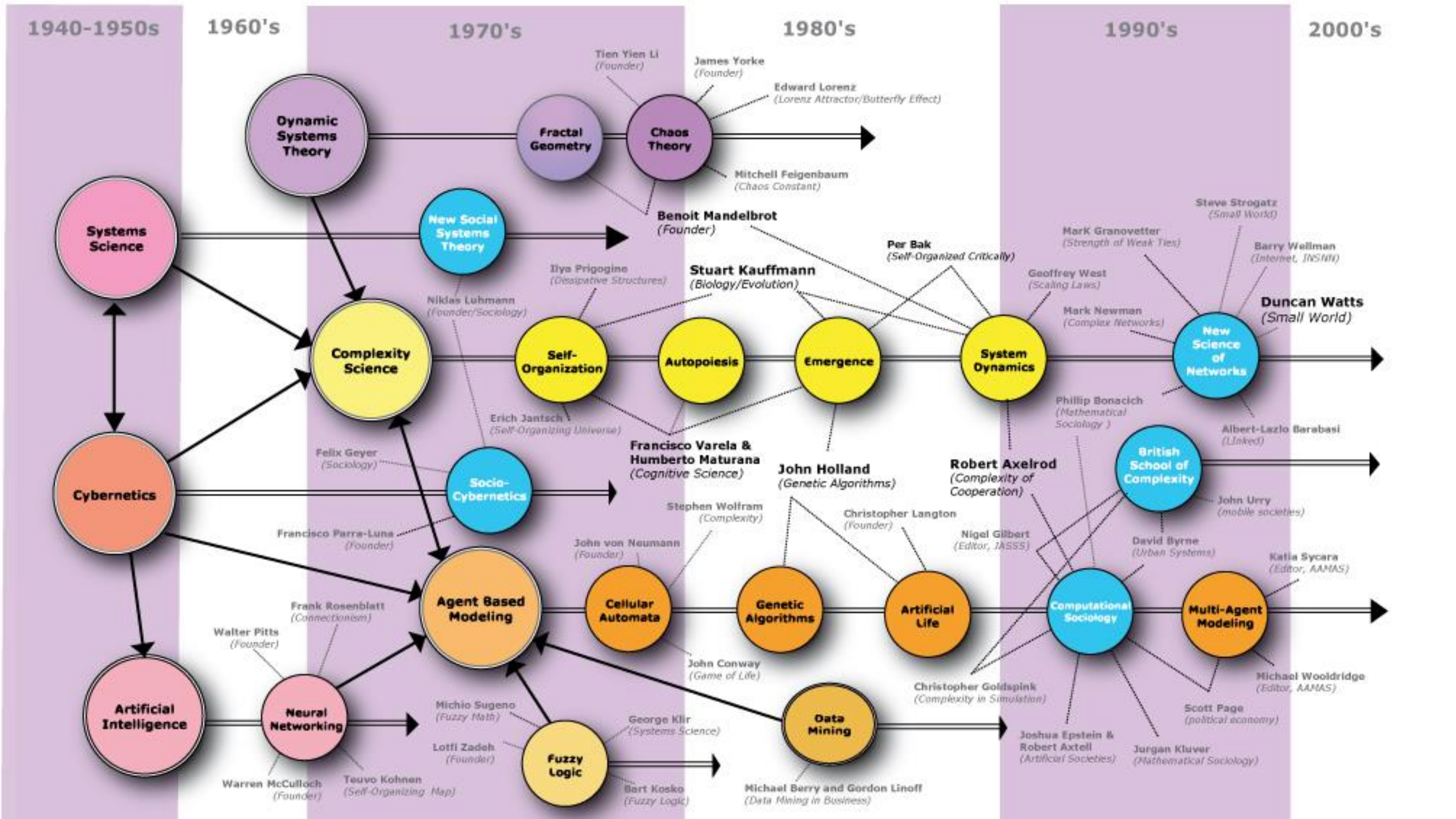
2. Características de los “sistemas complejos”

¿Existen características comunes a los distintos sistemas complejos?

- Santa Fe Institute - Nuevo México (1984)

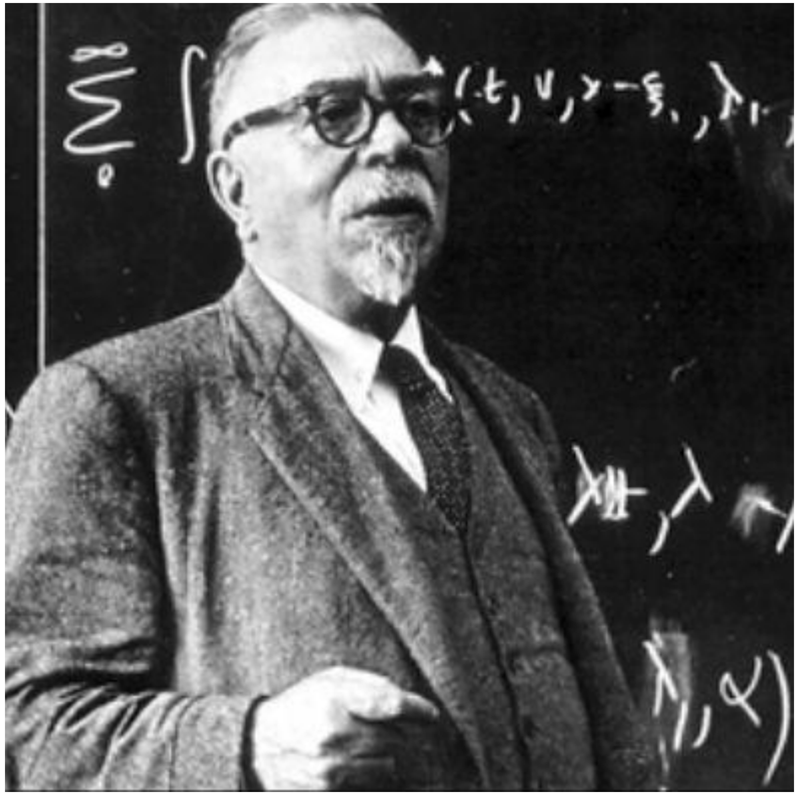


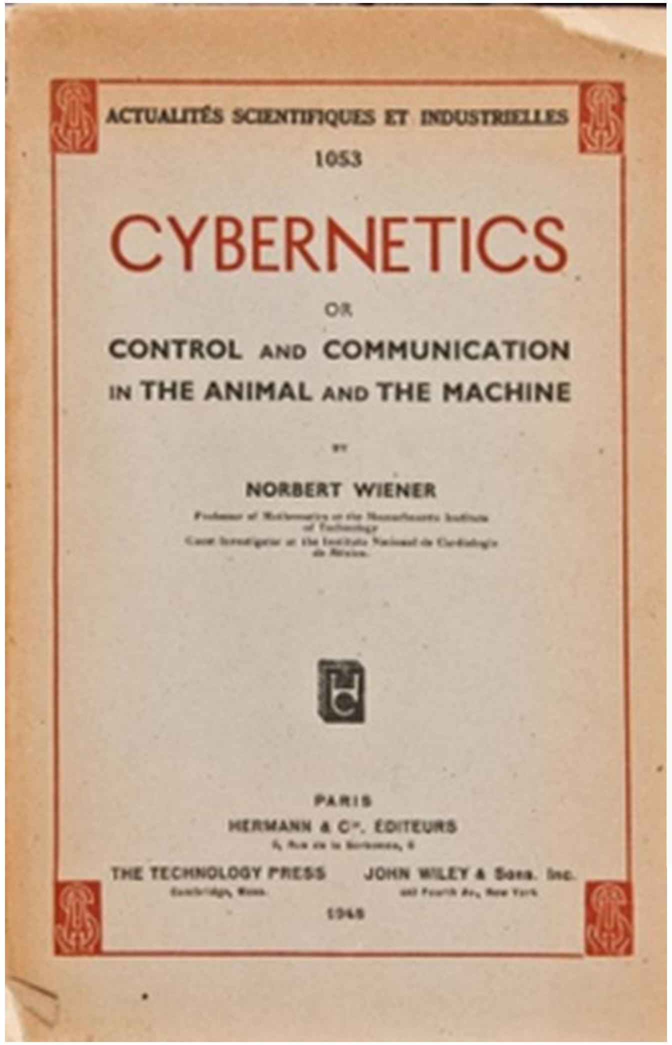
Characteristics of Complex Systems
A 'complex' system
Emergent behavior that cannot be simply inferred from the behavior of the components
Complex Systems
Involve:
Many Components
Dynamically Interacting
and giving rise to
A Number of Levels or Scales
which exhibit
Common Behaviors
Size Scale
Emergence
Hierarchies
Self-Organization
Control Structures
Composites
Substructure
Decomposability
Chaos
Fine Scales Influence Large Scale Behavior
347.86395762345675988354283764
Evolution
Time Scale
A 'simple' system
Trandisciplinary Concepts
Across Types of Systems, Across Scales, and thus Across Disciplines

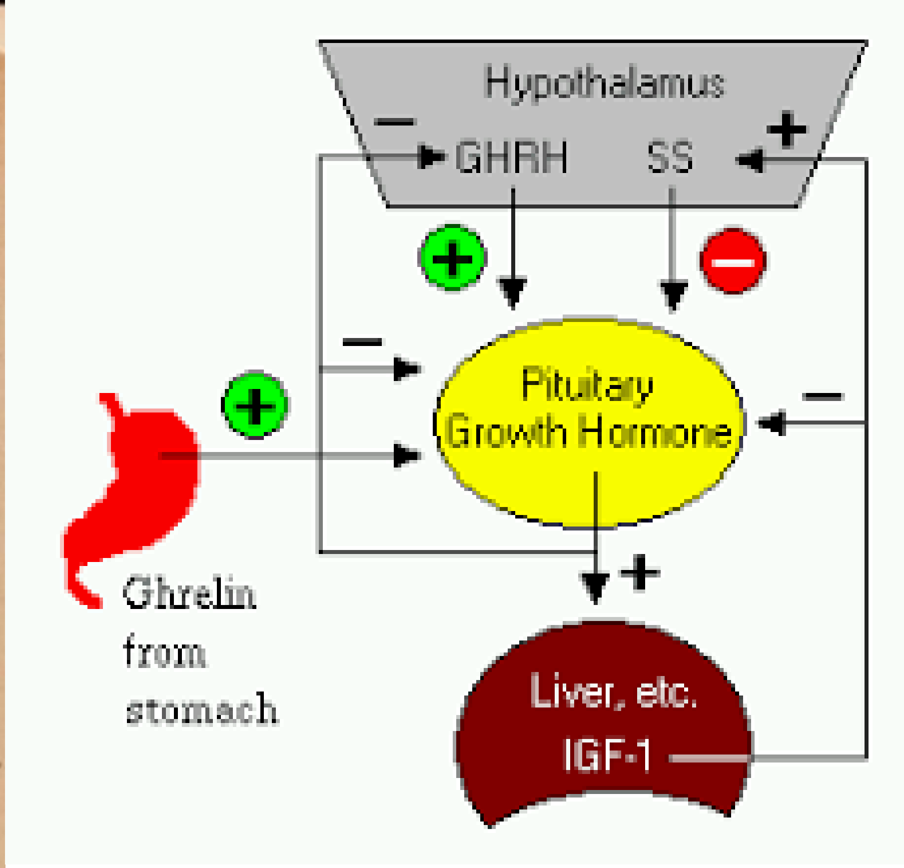


1940-1950s
1960's
1970's
1980's
1990's
2000's
Dynamic Systems Theory
Fractal Geometry
Chaos Theory
Tien Yien Li (Founder)
James Yorke (Founder)
Edward Lorenz (Lorenz Attractor/Butterfly Effect)
Mitchell Feigenbaum (Chaos Constant)
Benoit Mandelbrot (Founder)
Systems Science
New Social Systems Theory
Niklas Luhmann (Founder/Sociology)
Complexity Science
Ilya Prigogine (Dissipative Structures)
Stuart Kauffmann (Biology/Evolution)
Per Bak (Self-Organized Critically)
Self-Organization
Autopoiesis
Emergence
System Dynamics
Geoffrey West (Scaling Laws)
Mark Granovetter (Strength of Weak Ties)
Steve Strogatz (Small World)
Barry Wellman (Internet, INSNN)
Duncan Watts (Small World)
Mark Newman (Complex Networks)
New Science of Networks
Phillip Bonacich (Mathematical Sociology)
Albert-Lazlo Barabasi (Linked)
Erich Jantsch (Self-Organizing Universe)
Francisco Varela & Humberto Maturana (Cognitive Science)
John Holland (Genetic Algorithms)
Robert Axelrod (Complexity of Cooperation)
British School of Complexity
John Urry (mobile societies)
Felix Geyer (Sociology)
Cybernetics
Socio-Cybernetics
Francisco Parra-Luna (Founder)
Stephen Wolfram (Complexity)
Christopher Langton (Founder)
Nigel Gilbert (Editor, JASSS)
David Byrne (Urban Systems)
Katia Sycara (Editor, AAMAS)
John von Neumann (Founder)
Frank Rosenblatt (Connectionism)
Walter Pitts (Founder)
Agent Based Modeling
Cellular Automata
Genetic Algorithms
Artificial Life
Computational Sociology
Multi-Agent Modeling
John Conway (Game of Life)
Christopher Goldspink (Complexity in Simulation)
Michael Wooldridge (Editor, AAMAS)
Scott Page (political economy)
Artificial Intelligence
Neural Networking
Michio Sugeno (Fuzzy Math)
George Klir (Systems Science)
Lotfi Zadeh (Founder)
Fuzzy Logic
Data Mining
Joshua Epstein & Robert Axtell (Artificial Societies)
Jurgan Kluver (Mathematical Sociology)
Warren McCulloch (Founder)
Teuvo Kohnen (Self-Organizing Map)
Bart Kosko (Fuzzy Logic)
Michael Berry and Gordon Linoff (Data Mining in Business)

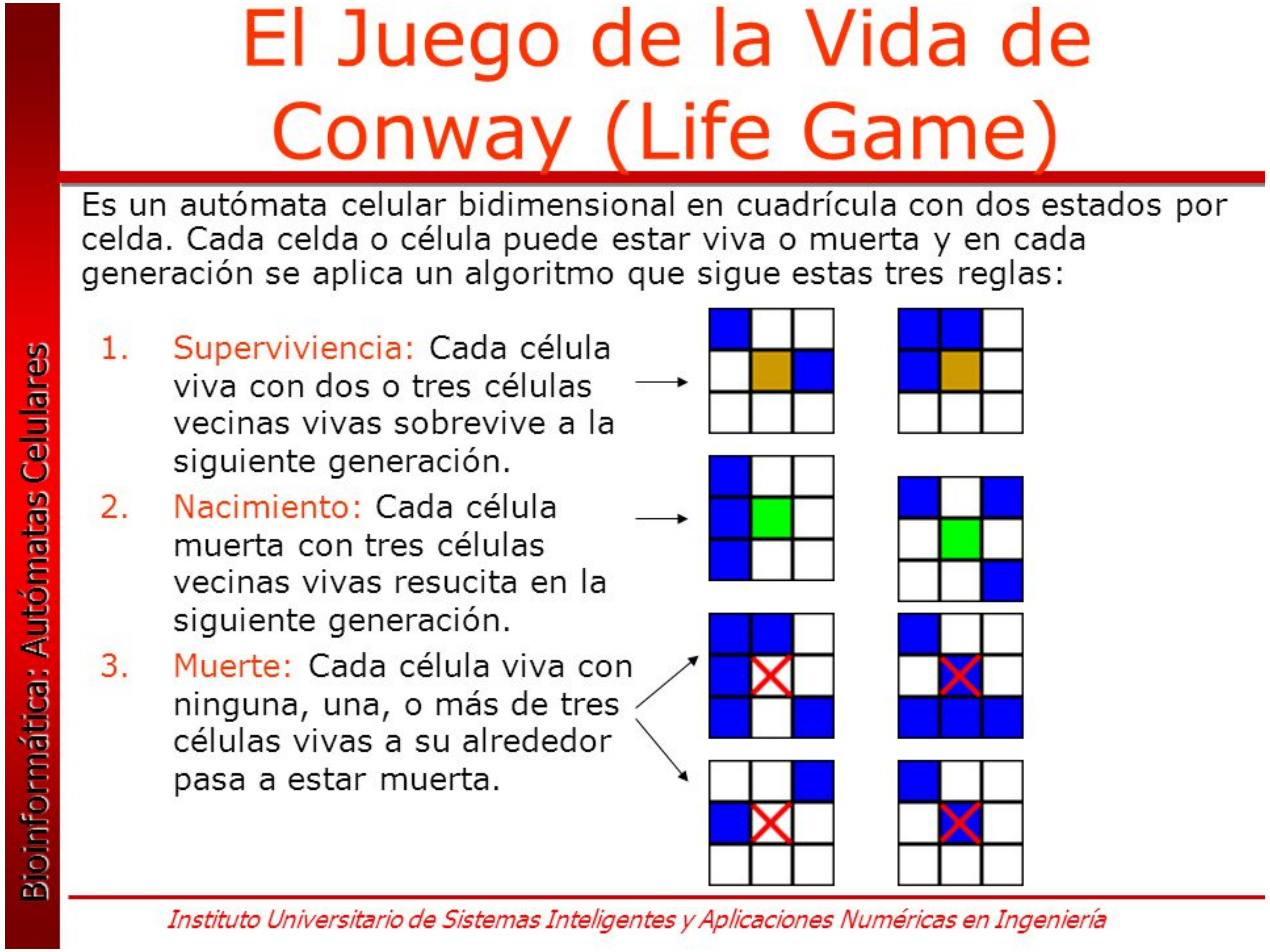
Noción de estructura, jerarquía y capacidad de control (ejemplos de la fisiología)

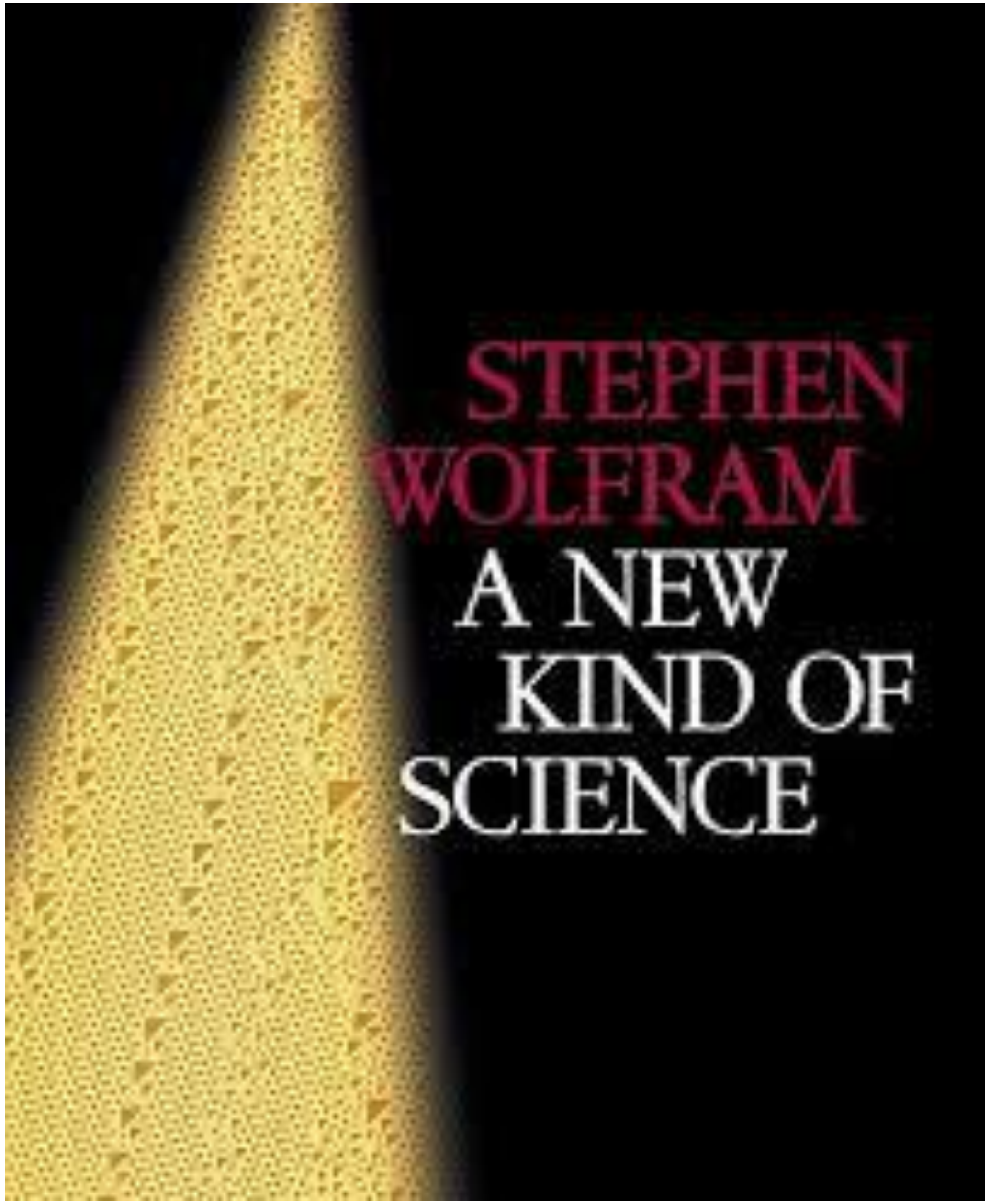




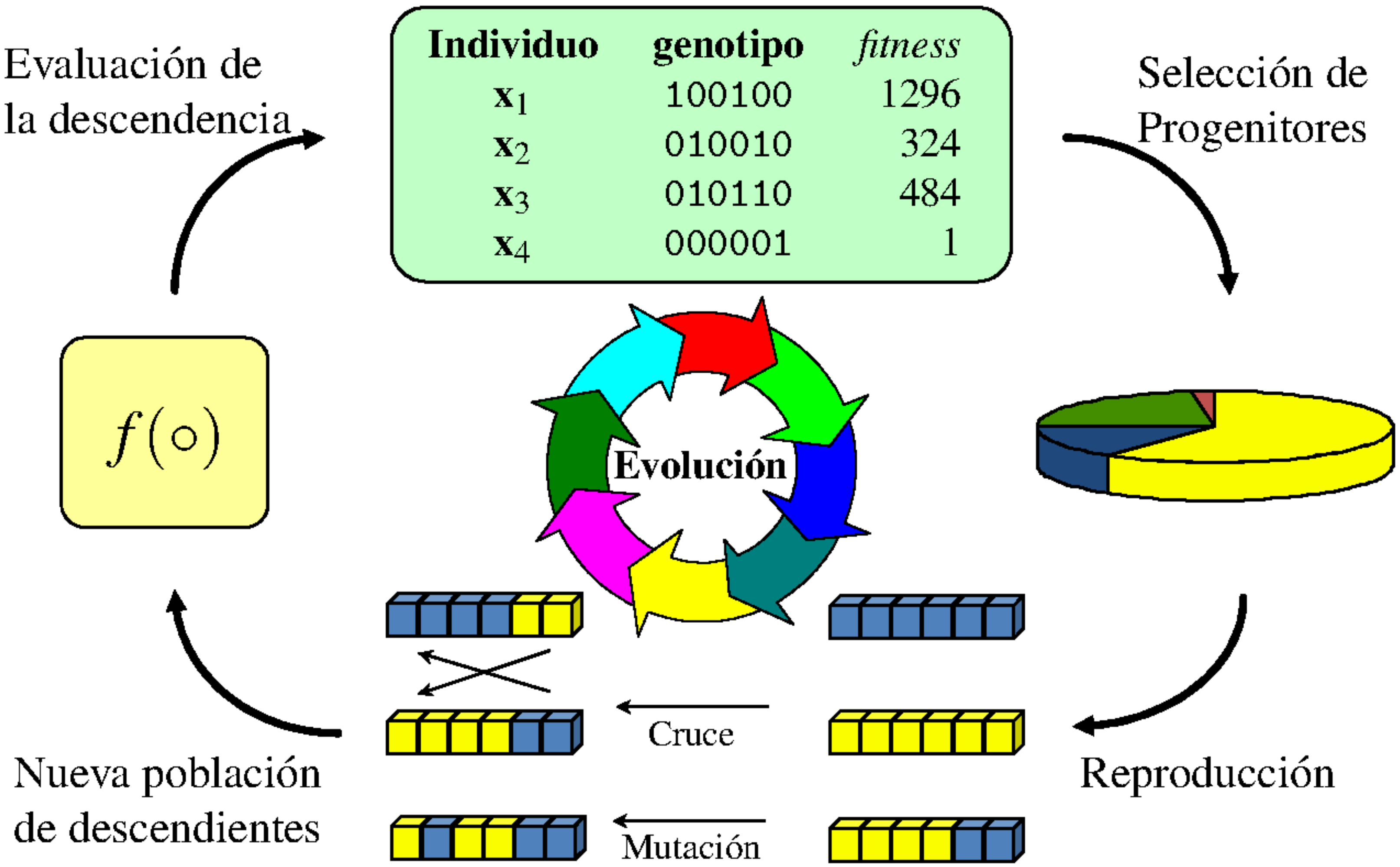


Autómatas celulares





Algoritmos genéticos



- Ruletas de Ashby

(Design for a Brain)

1000 ruletas –magno acierto (MA)

Juego 1 – Se lanzan las mil a la vez

Juego 2 – Se lanza una hasta el rojo y se pasa a la siguiente….

Juego 3 – Se lanzan las mil, se retienen las rojas, se lanzan las que quedan, etc

[un lanzamiento por segundo]

¿Cuánto tiempo se tarda hasta MA?

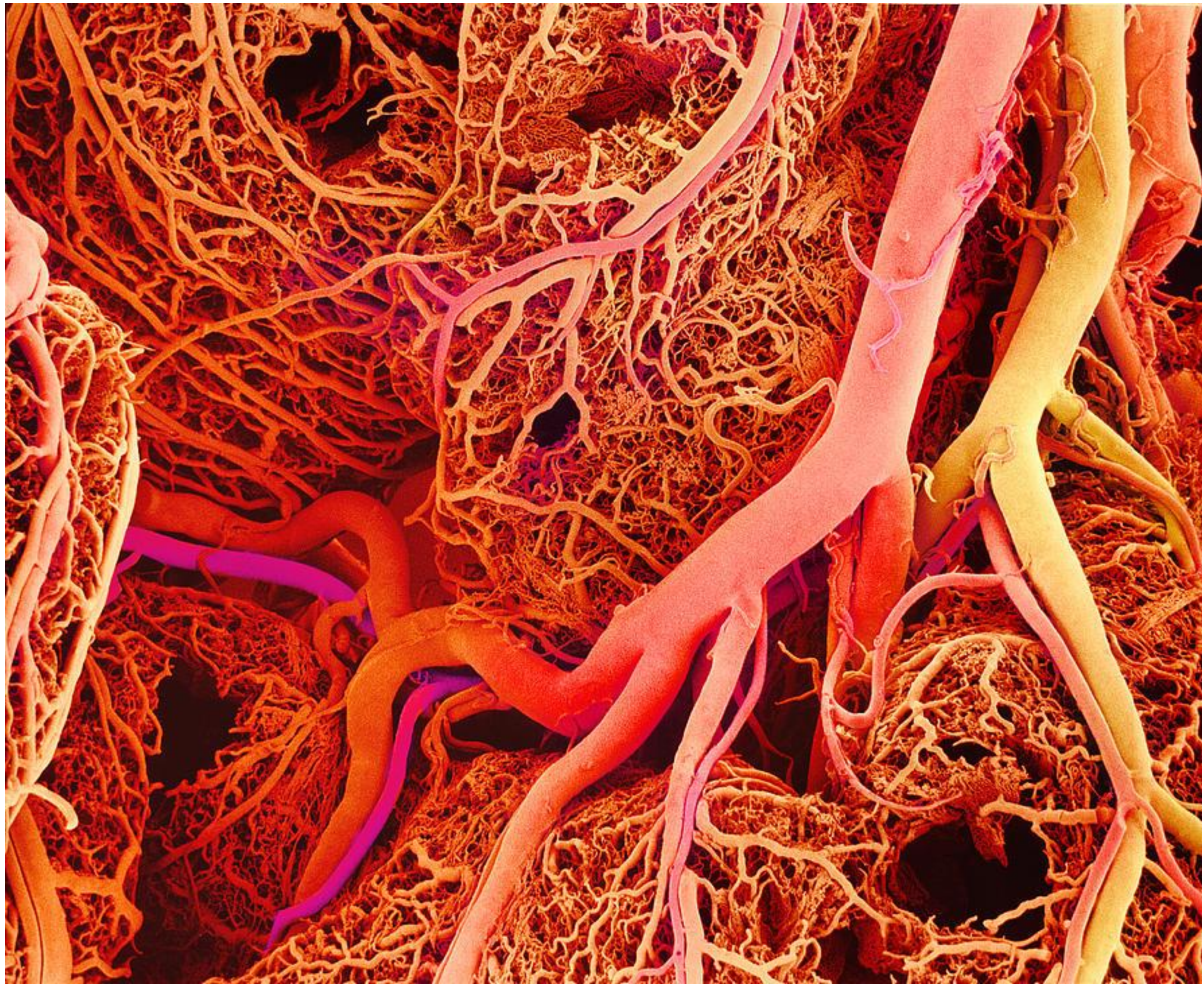
- Juego 1 - 10^{293} años

- Juego 2 - 33 minutos

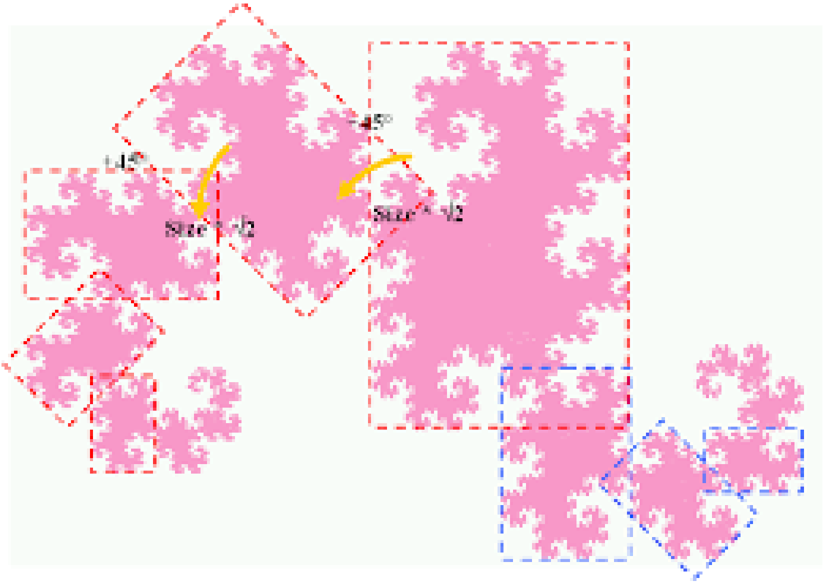
- Juego 3 - 11 segundos!

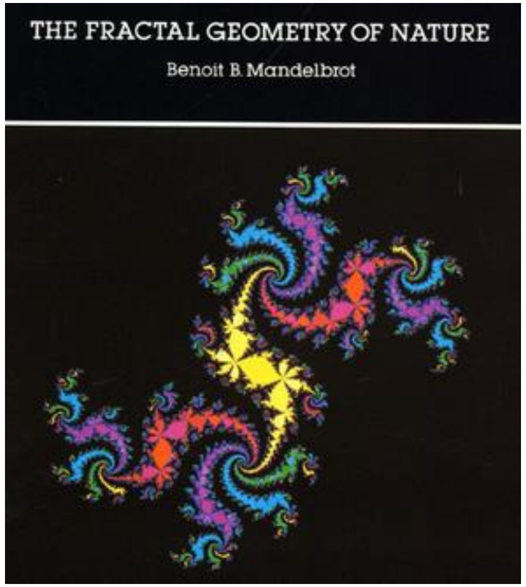
Moraleja: Las metas aparentemente más improbables pueden ser alcanzadas en muy pequeños tiempos si se retienen los aciertos parciales.

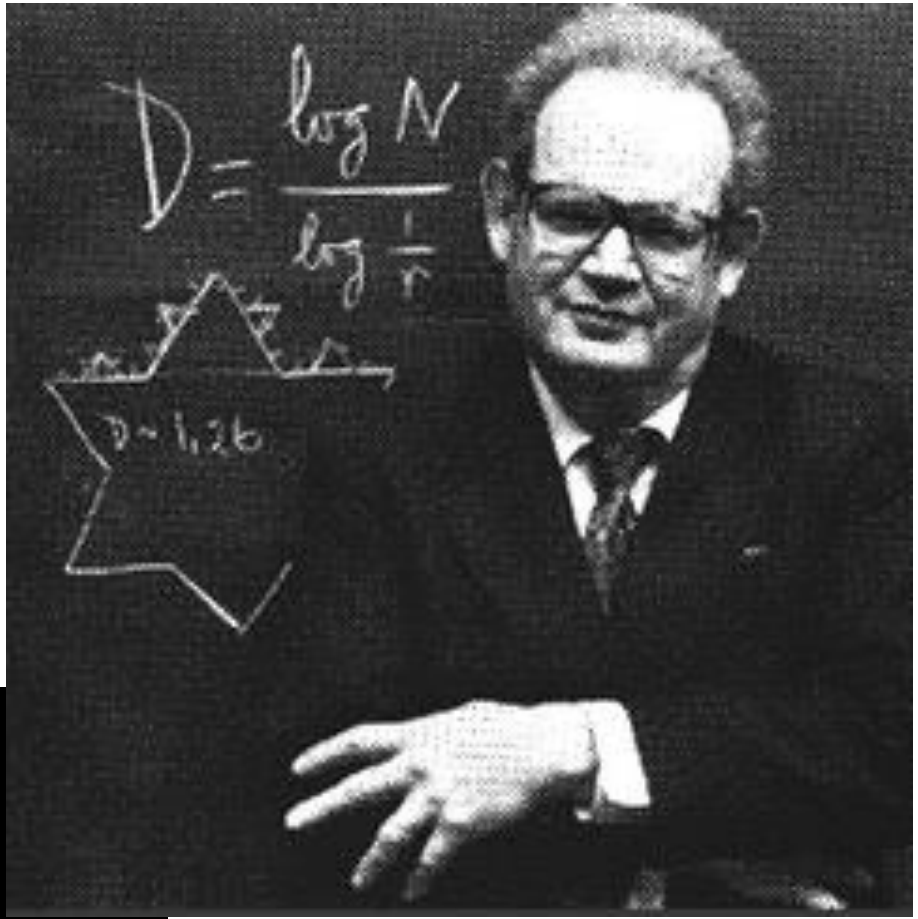
Complejidad, dimensión fractálica y autosimilaridad



Benoît Mandelbrot

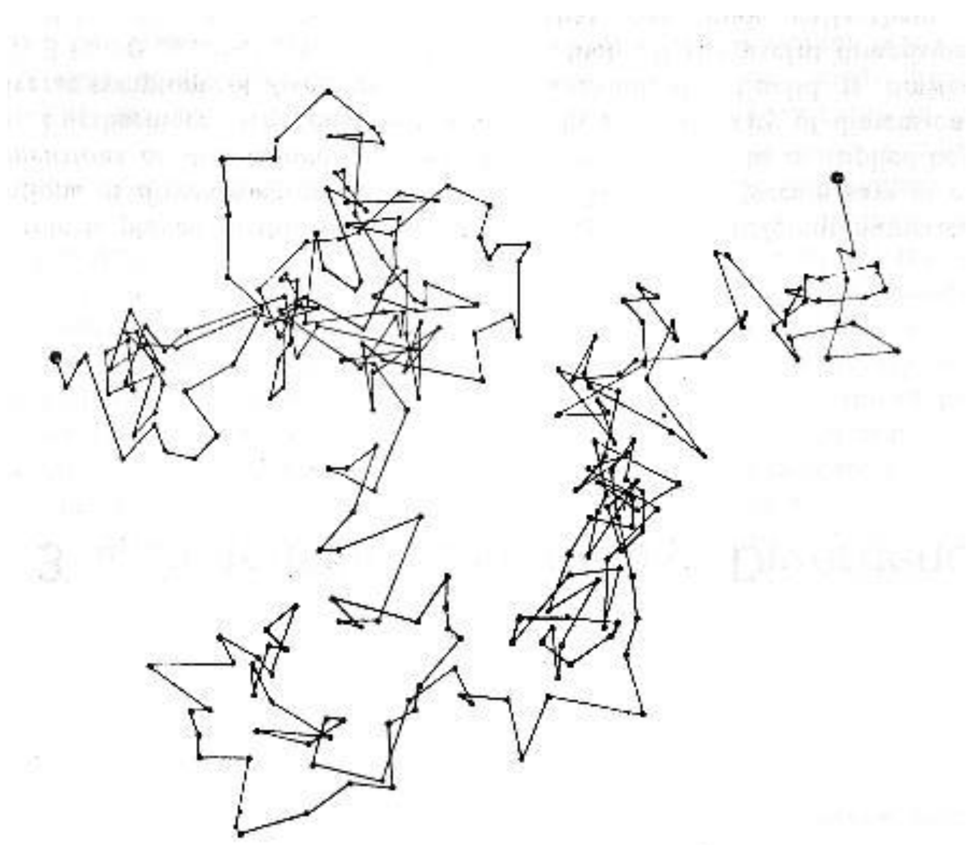


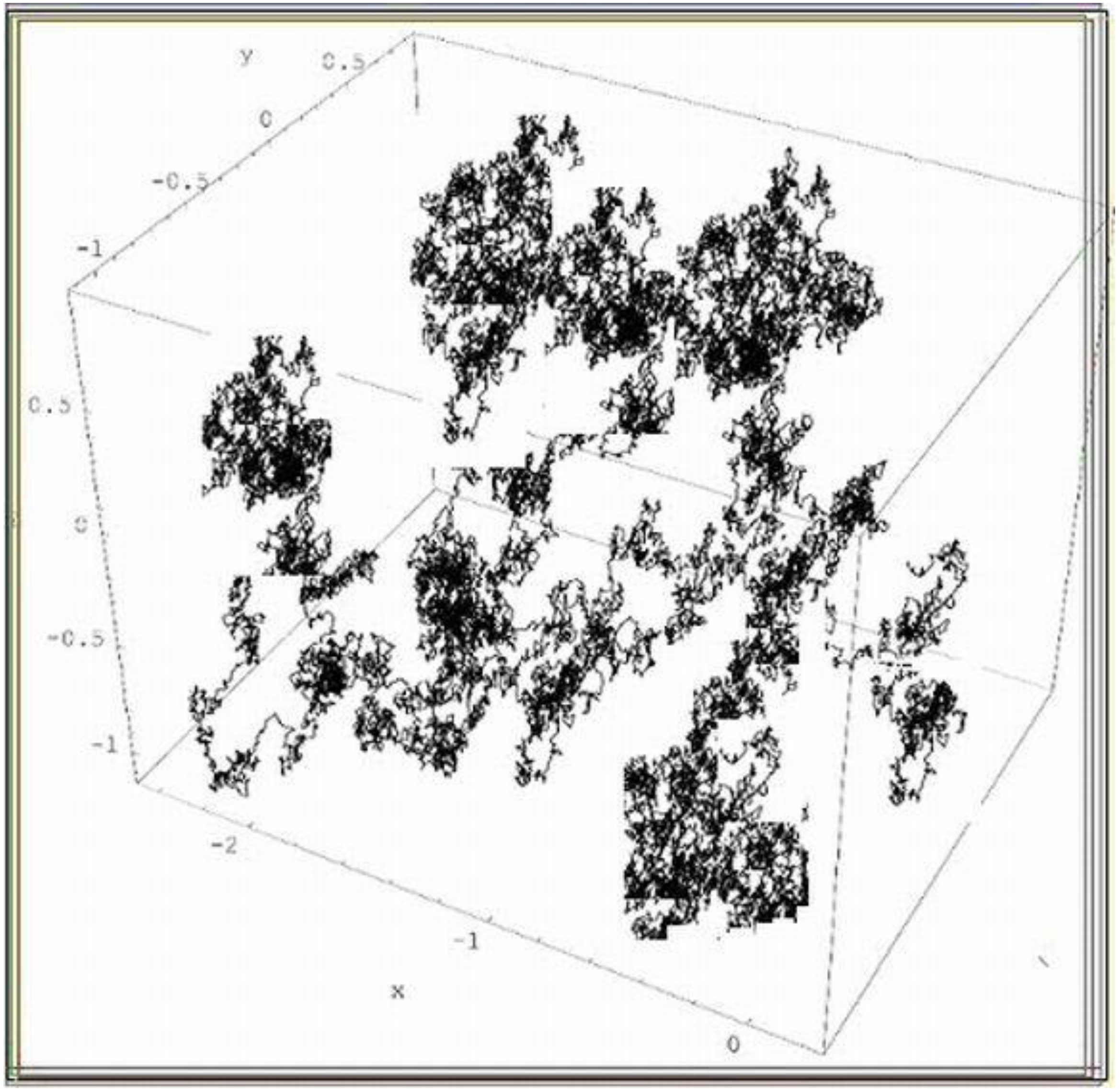




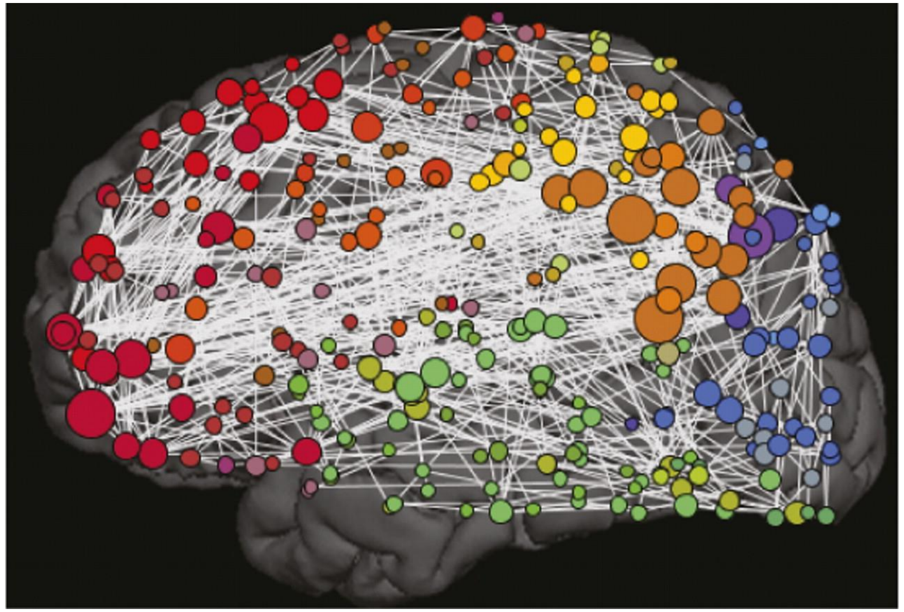
(1924-2010)

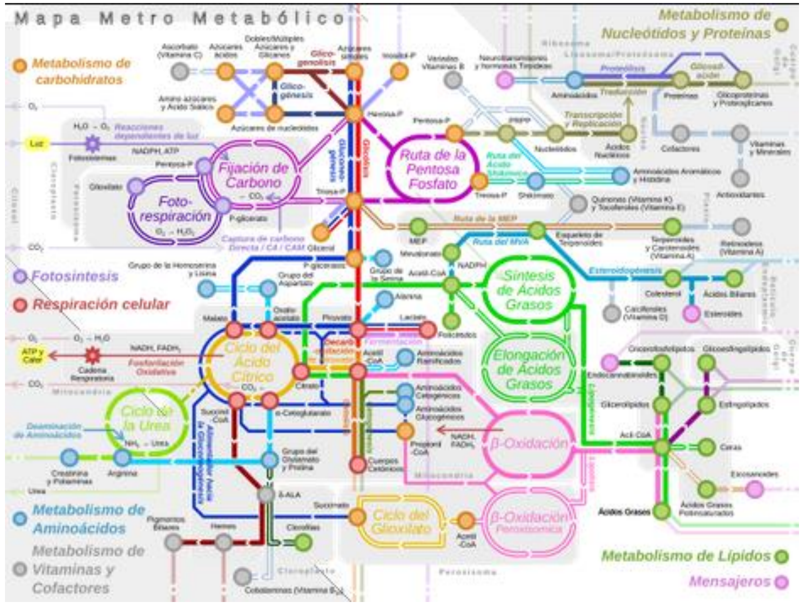
Movimiento browniano



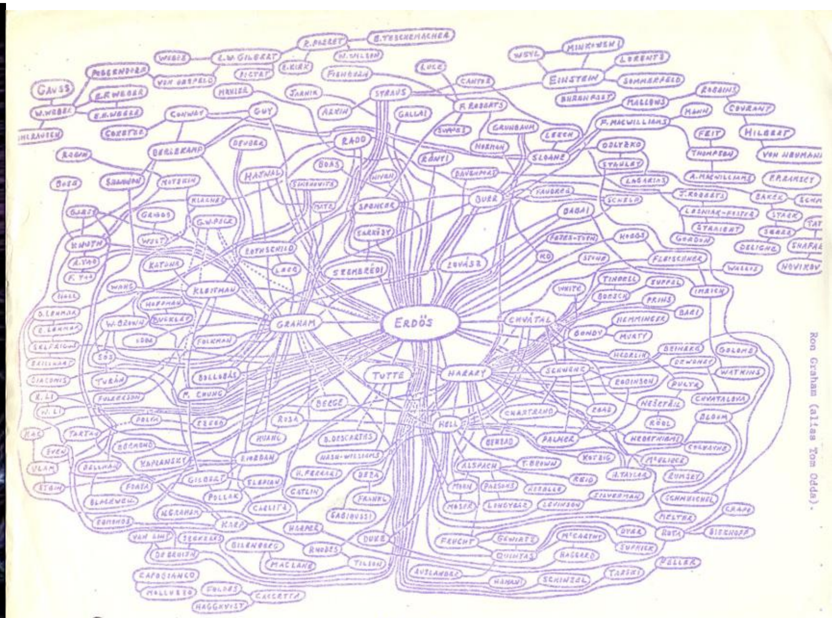


3. Redes complejas





Watts & Strogatz (1999)



1998

typically slower than $\sim 1\,km\,s^{-1}$) might differ significantly from what is assumed by current modelling efforts[27]. The expected equation-of-state differences among small bodies (ice versus rock, for instance) presents another dimension of study; having recently adapted our code for massively parallel architectures (K. M. Olson and E.A, manuscript in preparation), we are now ready to perform a more comprehensive analysis.

The exploratory simulations presented here suggest that when a young, non-porous asteroid (if such exist) suffers extensive impact damage, the resulting fracture pattern largely defines the asteroid's response to future impacts. The stochastic nature of collisions implies that small asteroid interiors may be as diverse as their shapes and spin states. Detailed numerical simulations of impacts, using accurate shape models and rheologies, could shed light on how asteroid collisional response depends on internal configuration and shape, and hence on how planetesimals evolve. Detailed simulations are also required before one can predict the quantitative effects of nuclear explosions on Earth-crossing comets and asteroids, either for hazard mitigation[28] through disruption and deflection, or for resource exploitation[29]. Such predictions would require detailed reconnaissance concerning the composition and internal structure of the targeted object. □

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Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

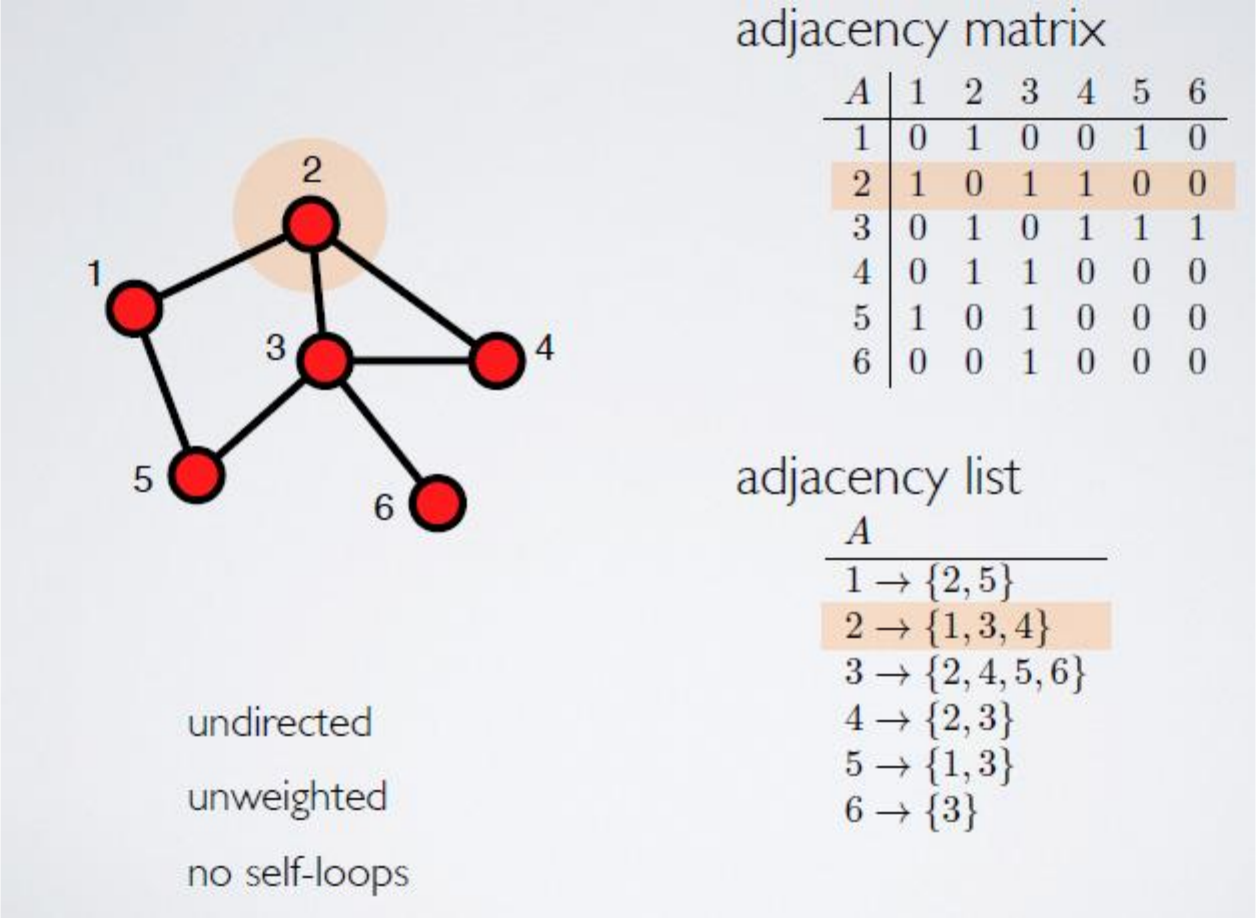
Networks of coupled dynamical systems have been used to model biological oscillators[1–4], Josephson junction arrays[5,6], excitable media[7], neural networks[8–10], spatial games[11], genetic control networks[12] and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either completely regular or completely random. But many biological, technological and social networks lie somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks 'rewired' to introduce increasing amounts of disorder. We find that these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them 'small-world' networks, by analogy with the small-world phenomenon[13,14] (popularly known as six degrees of separation[15]). The neural network of the worm *Caenorhabditis elegans*, the power grid of the western United States, and the collaboration graph of film actors are shown to be small-world networks. Models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In particular, infectious diseases spread more easily in small-world networks than in regular lattices.

To interpolate between regular and random networks, we consider the following random rewiring procedure (Fig. 1). Starting from a ring lattice with n vertices and k edges per vertex, we rewire each edge at random with probability p. This construction allows us to 'tune' the graph between regularity ($p = 0$) and disorder ($p = 1$), and thereby to probe the intermediate region $0 < p < 1$, about which little is known.

We quantify the structural properties of these graphs by their characteristic path length $L(p)$ and clustering coefficient $C(p)$, as defined in Fig. 2 legend. Here $L(p)$ measures the typical separation between two vertices in the graph (a global property), whereas $C(p)$ measures the cliquishness of a typical neighbourhood (a local

La matemática de las redes

- Entonces, (1) cuando hablamos de una red hablamos de un sistema real de componentes interconectados.
- Pero además, en una segunda acepción, (2) cuando hablamos de “redes”, hablamos de una abstracción de este sistema real: un objeto matemático denominado GRAFO.



Redes de mundo pequeño

(small world networks)

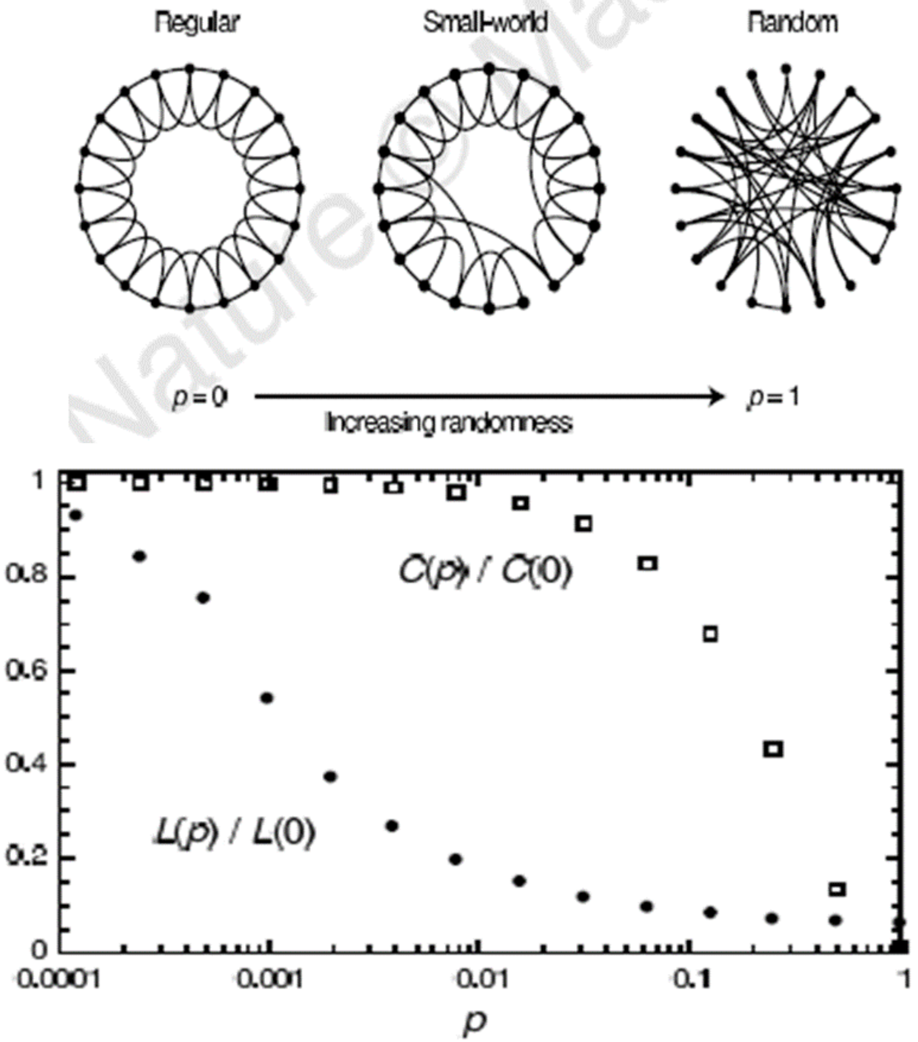


Table 1 Empirical examples of small-world networks

	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05

Characteristic path length L and clustering coefficient C for three real networks, compared to random graphs with the same number of vertices (n) and average number of edges per vertex (k). (Actors: $n = 225{,}226$, $k = 61$. Power grid: $n = 4{,}941$, $k = 2.67$. *C. elegans*: $n = 282$, $k = 14$.) The graphs are defined as follows. Two actors are joined by an edge if they have acted in a film together. We restrict attention to the giant connected component[16] of this graph, which includes ~90% of all actors listed in the Internet Movie Database (available at http://us.imdb.com), as of April 1997. For the power grid, vertices represent generators, transformers and substations, and edges represent high-voltage transmission lines between them. For *C. elegans*, an edge joins two neurons if they are connected by either a synapse or a gap junction. We treat all edges as undirected and unweighted, and all vertices as identical, recognizing that these are crude approximations. All three networks show the small-world phenomenon: $L \gtrsim L_{random}$ but $C \gg C_{random}$.

La distribución del grado de los nodos

(scale-free networks)

Co. This is also observed in Fig. 3A.

For ALO and ALO/STO barriers, a predominant tunneling of s-character electrons (see arrow in Fig. 2B) is the usual explanation of the positive polarization (*6–8*). The rapid drop with bias (Fig. 3B) is similar to what has been observed in most junctions with ALO barriers, and completely different from what is obtained when the tunneling is predominantly by d-character electrons (Fig. 3A). The origin of this rapid decrease of the TMR at relatively small bias has never been clearly explained. This is roughly consistent with the energy dependence of the DOS induced by sp-d bonding effects on the first atomic layer of ALO in the calculation of Nguyen-Mahn *et al.* (*8*) for the Co-ALO interface. But Zhang *et al.* (*13*) have also shown that a large part of the TMR drop can be attributed to the excitation of spin waves.

The experiments reported here and in several recent publications (*3, 4*) demonstrate the important role of the electronic structure of the metal-oxide interface in determining the spin polarization of the tunneling electrons. The negative polarization for the Co-STO interface has been ascribed to d-d bonding effects between Al and Ti (*4*). This interpretation is similar to

Emergence of Scaling in Random Networks

Albert-László Barabási* and Réka Albert

Systems as diverse as genetic networks or the World Wide Web are best described as networks with complex topology. A common property of many large networks is that the vertex connectivities follow a scale-free power-law distribution. This feature was found to be a consequence of two generic mechanisms: (i) networks expand continuously by the addition of new vertices, and (ii) new vertices attach preferentially to sites that are already well connected. A model based on these two ingredients reproduces the observed stationary scale-free distributions, which indicates that the development of large networks is governed by robust self-organizing phenomena that go beyond the particulars of the individual systems.

The inability of contemporary science to describe systems composed of nonidentical elements that have diverse and nonlocal interactions currently limits advances in many disciplines, ranging from molecular biology to computer science (*1*). The difficulty of describing these systems lies partly in their topology: Many of them form rather complex networks whose vertices are the elements of the system and whose edges represent the interactions between them. For example, liv-

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Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA.

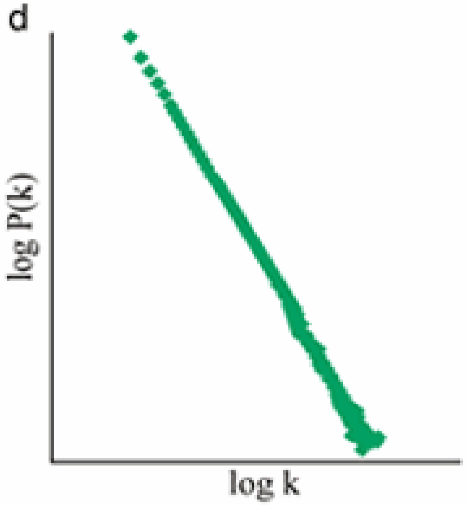
*To whom correspondence should be addressed. E-mail: alb@nd.edu

La distribución de conexiones

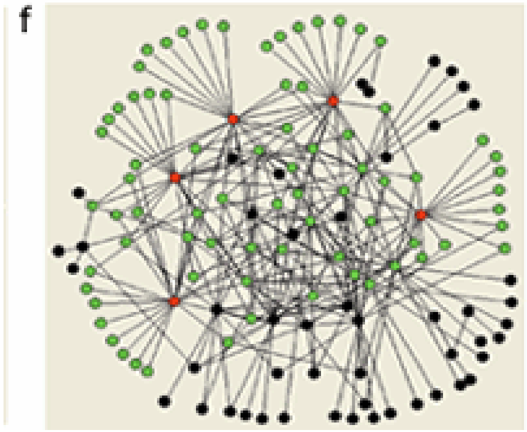
En muchas redes se observa que la distribución del número de conexiones de cada nodo no tiene un valor característico (por eso se las llama <u>scale-free</u>).

La distribución sigue una ley de potencias (al hacer un grafo log-log queda como una recta).

La mayoría de los nodos tienen un grado bajo y hay unos pocos, llamados “hubs”, que tienen muchísimas conexiones.

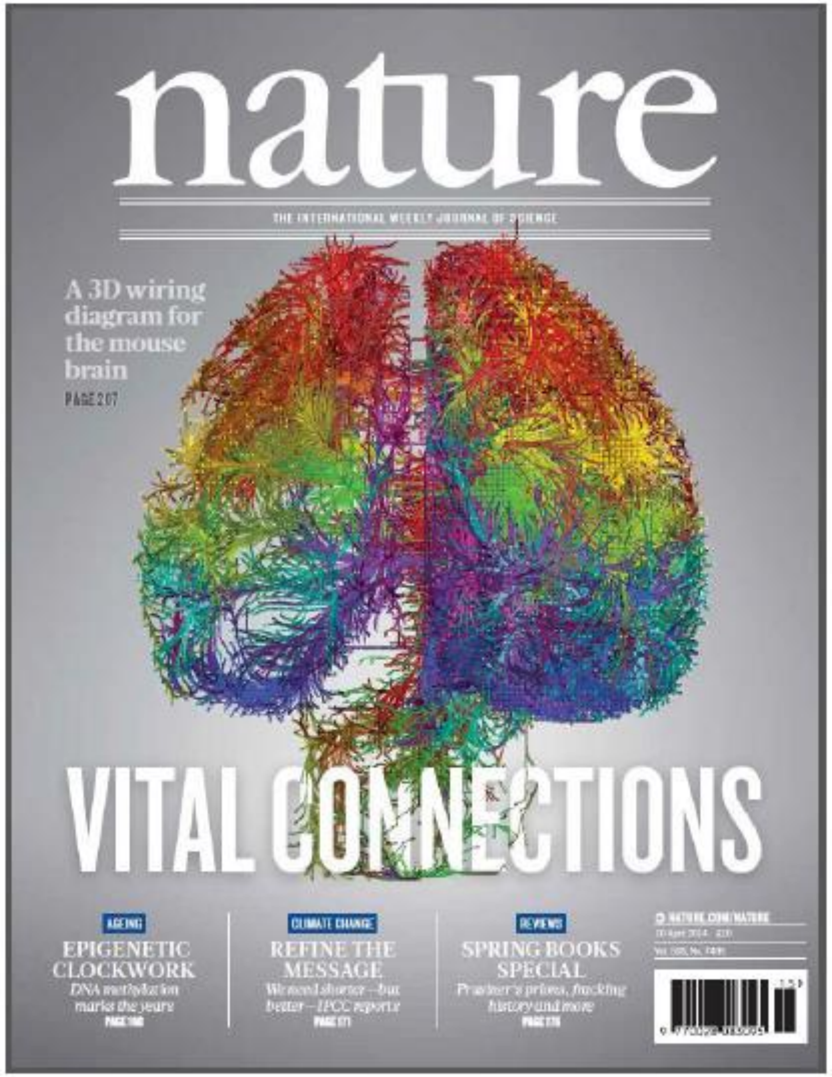


$$P(k) \approx k^{-\gamma}$$



4. La ciencia de las redes

- Detrás de un sistema complejo hay una red compleja.
- Comprender los sistemas complejos es comprender las redes detrás de ellos.
- Existen características comunes a los sistemas complejos, que pueden ser estudiadas
- Proporciona un lenguaje y herramientas de estudio y caracterización comunes, independientes de las disciplinas
- Tuvieron un ‘Desarrollo epidémico’
- Tienen gran impacto en nuestra vida y sociedades actuales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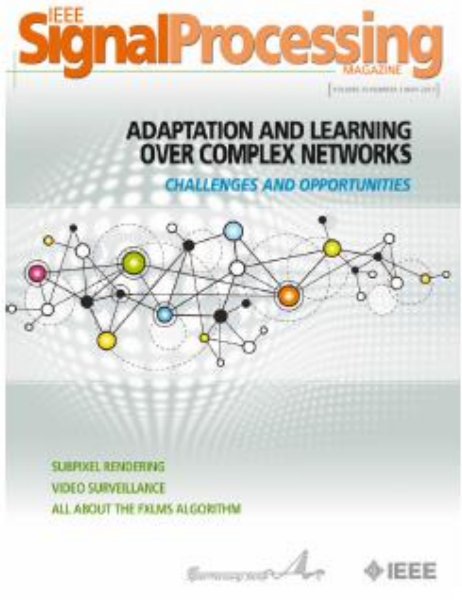
Complex Systems and Networks
AAAS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ALL ABOUT THE FALMS ALGORITHM
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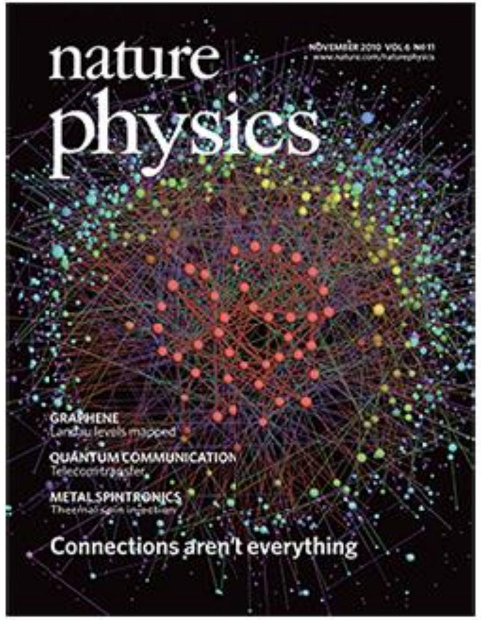
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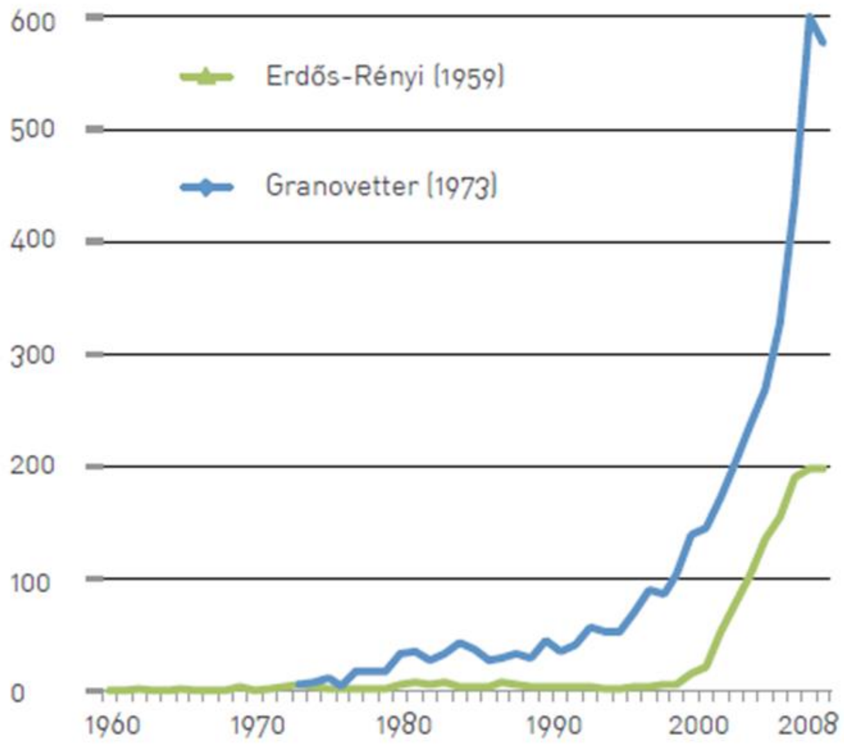
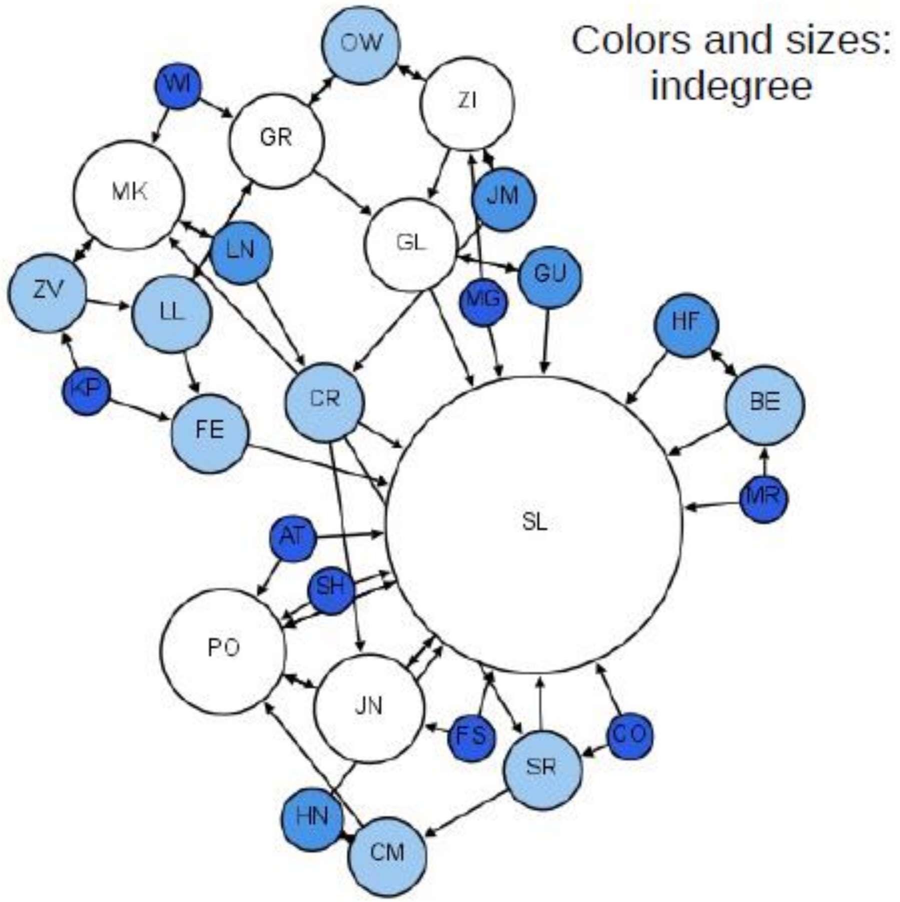
GRAPHENE
QUANTUM COMMUNICATION
METAL SPINTRONICS
Connections aren't everything


Erdős-Rényi (1959)
Granovetter (1973)
600
500
400
300
200
100
0
1960
1970
1980
1990
2000
2008

Redes sociales

Moreno's sociograms

- Early 1930s
- Children in 2nd grade
- Who would you like to sit with?

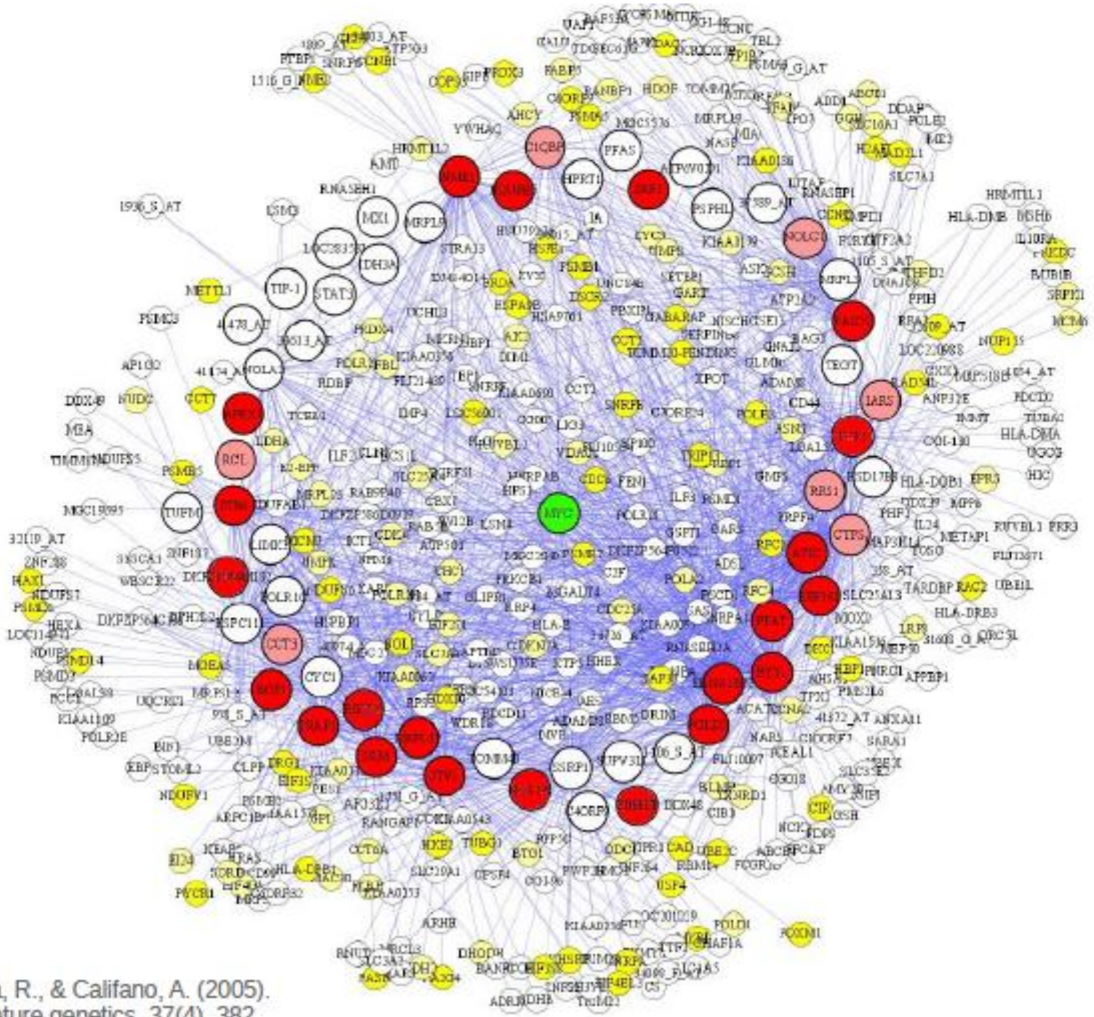


http://www.martingrandjean.ch/social-network-analysis-visualization-morenos-sociograms-revisited/

Redes biológicas

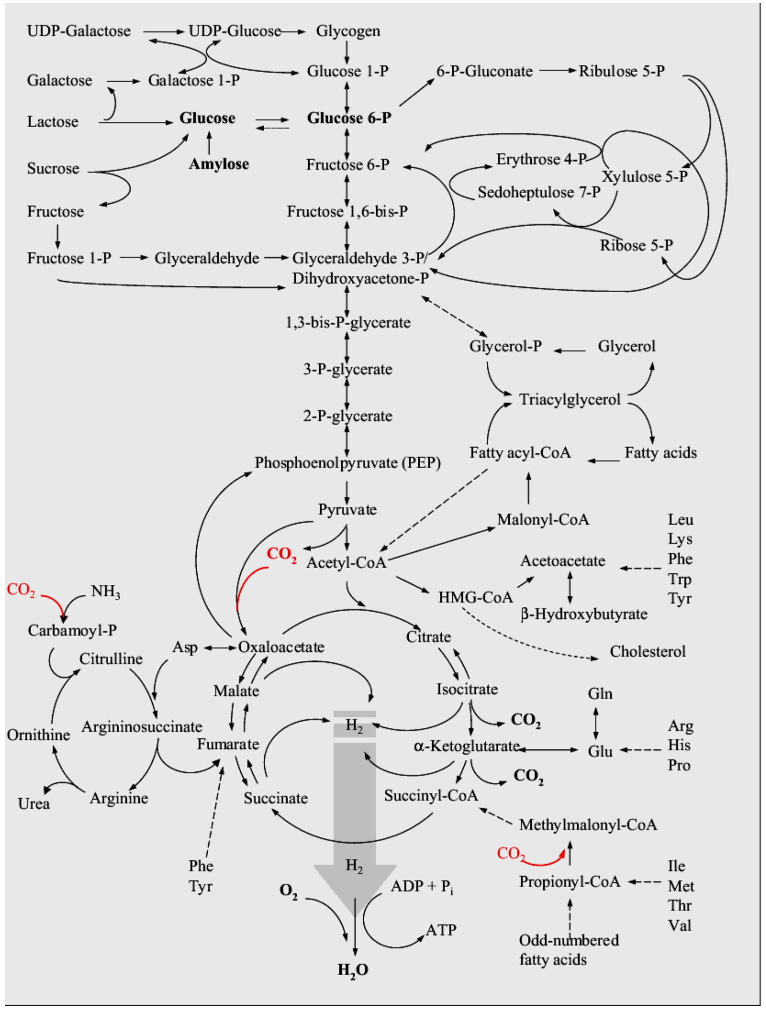
Co-expression of Genes

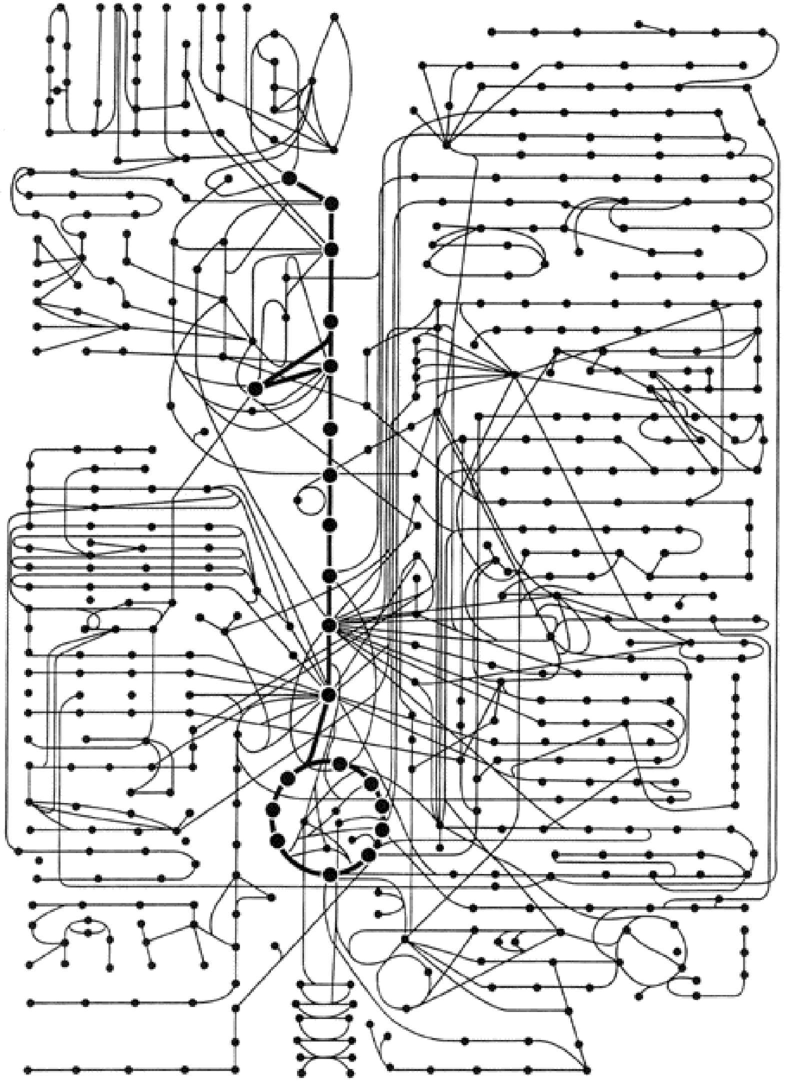
|V|=500 in this plot



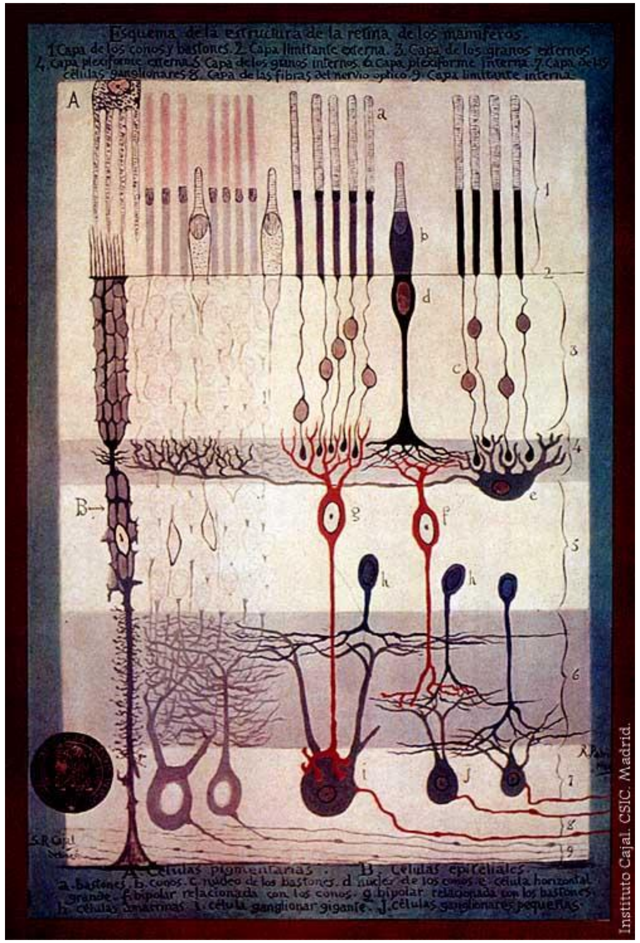
Basso, K., Margolin, A. A., Stolovitzky, G., Klein, U., Dalla-Favera, R., & Califano, A. (2005). Reverse engineering of regulatory networks in human B cells. Nature genetics, 37(4), 382.

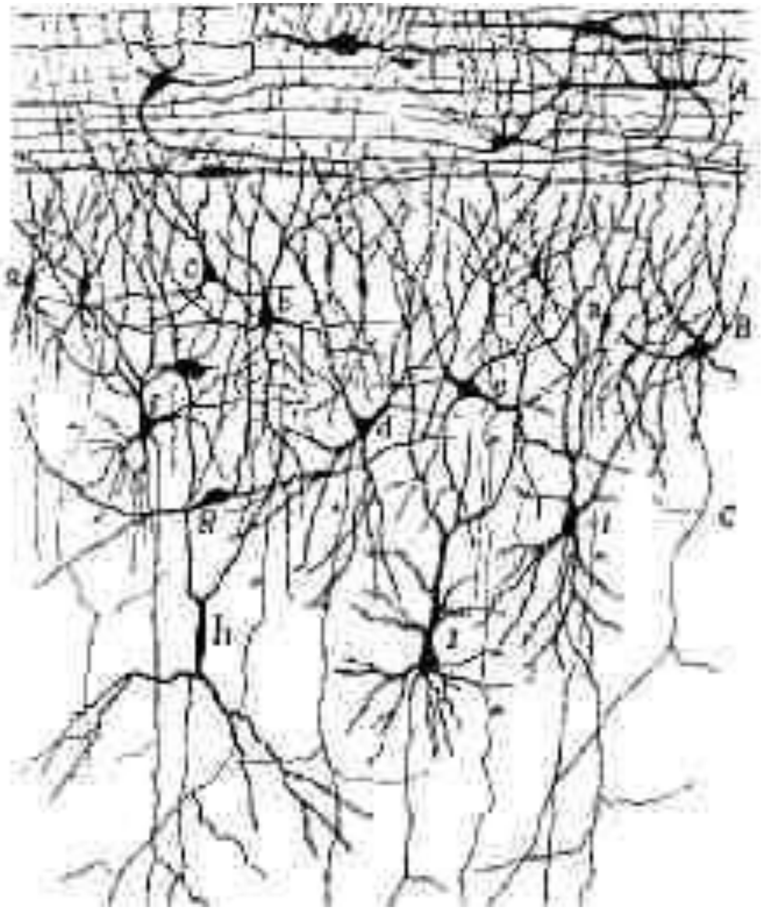
Las redes metabólicas "reales"



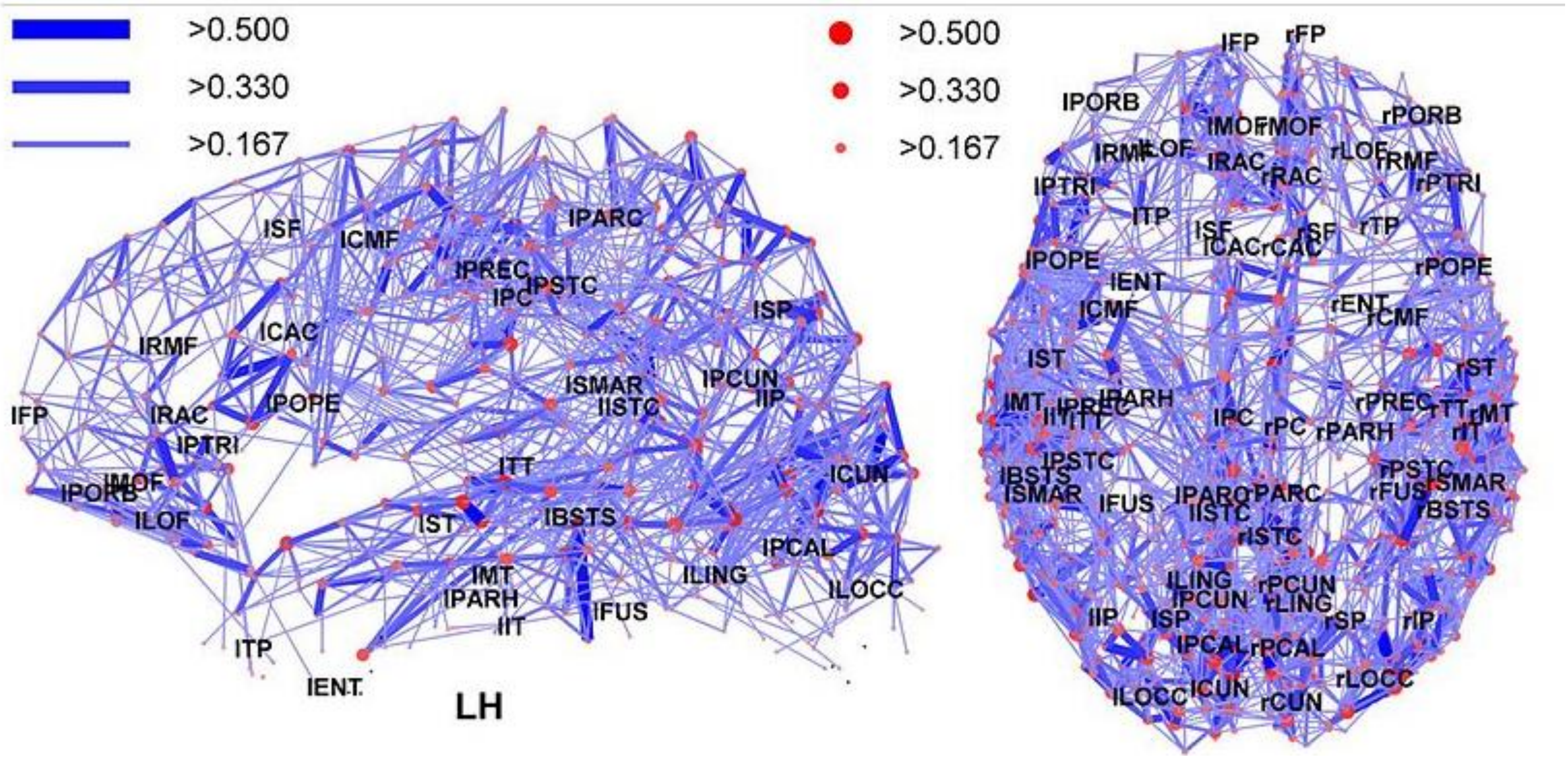


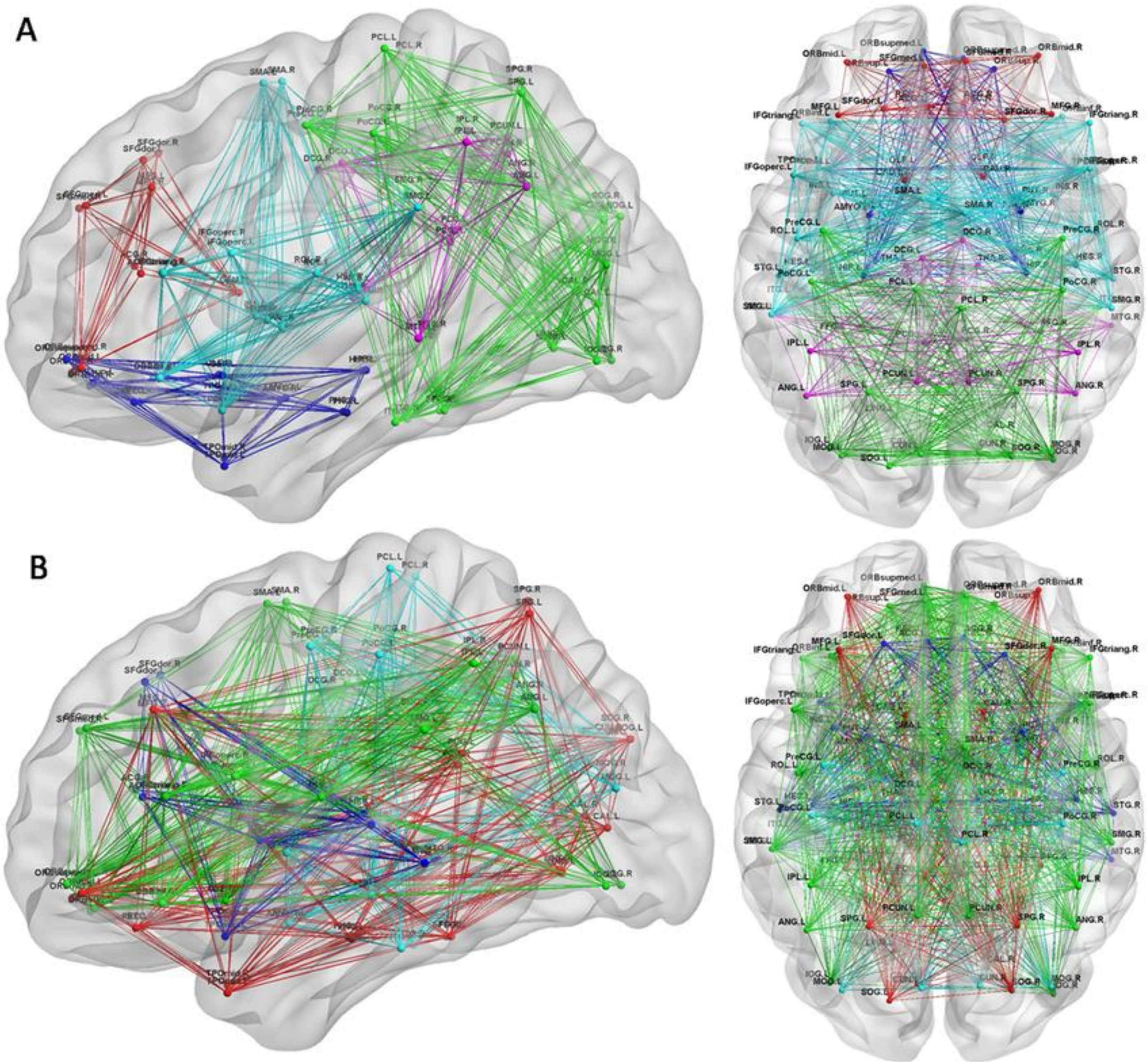
Redes neuronales de Santiago Ramón y Cajal





Estudios de la conectividad de los cerebros




A
B

Otros temas –Fragilidad de las redes

Julio de 2000: La fragilidad de las redes complejas

obtained from the whole map. The entropy is then defined as

$$S(D) = -\int f(D)\log[f(D)]\,\mathrm{d}D \qquad (5)$$

where the integral is taken over all values of D, that is, from 0 to 2π. The use of D, rather than ϕ itself, to define entropy is one way of accounting for the lack of translation invariance of ϕ, a problem that was missed in previous attempts to quantify phase entropy[16]. A uniform distribution of D is a state of maximum entropy (minimum information), corresponding to gaussian initial conditions (random phases). This maximal value of $S_{max} = \log(2\pi)$ is a characteristic of gaussian fields. As the system evolves, it moves into states of greater information content (that is, lower entropy). The scaling of S with clustering growth displays interesting properties[5], establishing an important link between the spatial pattern and the physical processes driving clustering growth. This phase information is a unique 'fingerprint' of gravitational instability, and it therefore also furnishes statistical tests of the presence of any initial non-gaussianity[17–19]. □

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Error and attack tolerance of complex networks

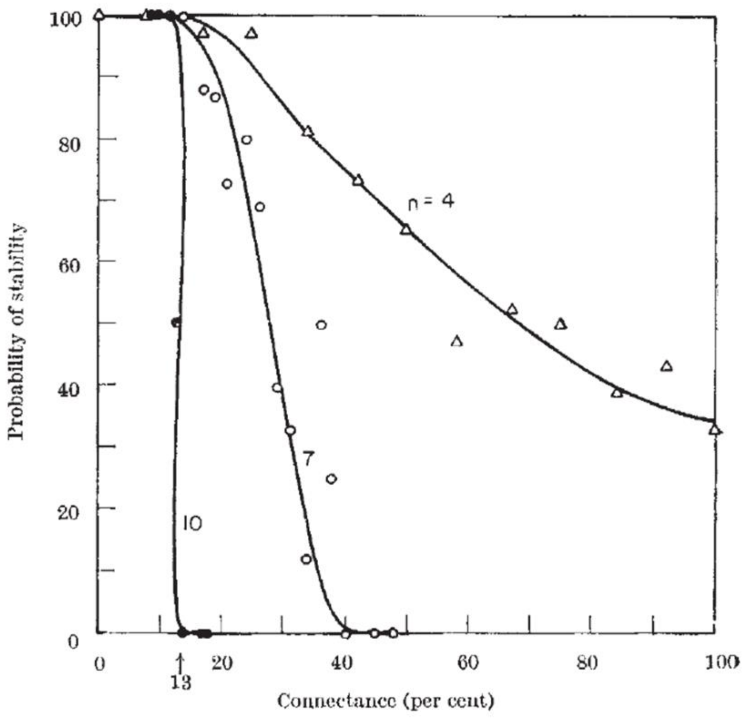
Réka Albert, Hawoong Jeong & Albert-László Barabási

Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA

Many complex systems display a surprising degree of tolerance against errors. For example, relatively simple organisms grow, persist and reproduce despite drastic pharmaceutical or environmental interventions, an error tolerance attributed to the robustness of the underlying metabolic network[1]. Complex communication networks[2] display a surprising degree of robustness: although key components regularly malfunction, local failures rarely lead to the loss of the global information-carrying ability of the network. The stability of these and other complex systems is often attributed to the redundant wiring of the functional web defined by the systems' components. Here we demonstrate that error tolerance is not shared by all redundant systems: it is displayed only by a class of inhomogeneously wired networks,

Procesos dinámicos en redes

- Gardner & Ashby, 1970



Contención epidémica en la red de transporte aéreo. La imagen muestra las redes después de que el 33,3% de los enlaces se eliminaran considerando la importancia de los enlaces en la epidemia. / A. Arer