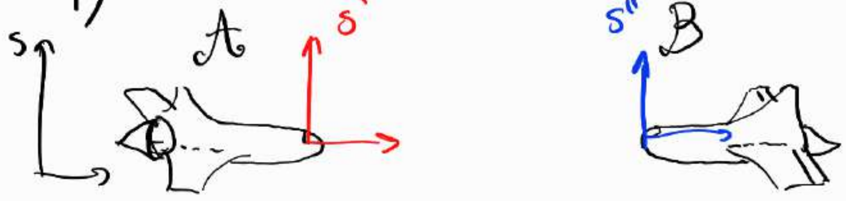
1)



a) E_1: nariz A - nariz B

E_2: nariz A - cola B

E_3: misil A

E_4: cola A - nariz B

E_5: cola A - cola B

$$\boxed{\begin{matrix} x'_{E_1} = x''_{E_1} = 0 \\ ct'_{E_1} = ct''_{E_1} = 0 \end{matrix}}$$

$$\begin{matrix} x'_{E_2} = 0 \\ x''_{E_2} = L_0 \end{matrix} \qquad \begin{matrix} x''_{E_2} = \gamma(x'_{E_2} + \beta ct'_{E_2}) \to \boxed{\dfrac{L_0}{\gamma\beta} = ct'_{E_2}} \\ ct''_{E_2} = \gamma(ct'_{E_2} + \beta x'_{E_2}) \end{matrix}$$

$$\hookrightarrow \boxed{ct''_{E_2} = \frac{L_0}{\beta}}$$

por simetría con E_2 $\quad \left.\begin{matrix} x''_{E_4} = 0 \\ x'_{E_4} = -L_0 \end{matrix}\right\} \Rightarrow \boxed{ct''_{E_4} = \dfrac{L_0}{\gamma\beta}}$ y $\boxed{ct'_{E_4} = \dfrac{L_0}{\beta}}$

$$\boxed{\begin{matrix} x'_{E_5} = -L_0 \\ x''_{E_5} = L_0 \end{matrix}} \qquad \begin{matrix} x''_{E_5} = \gamma(x'_{E_5} + \beta ct'_{E_5}) \longrightarrow \boxed{ct'_{E_5} = \dfrac{L_0}{\beta}\left(\dfrac{1+\gamma}{\gamma}\right) = ct''_{E_5}} \\ ct''_{E_5} = \gamma(ct'_{E_5} + \beta x'_{E_5}) \longrightarrow ct''_{E_5} = \gamma L_0\left[\dfrac{1+\gamma}{\gamma\beta} - \beta\right] \end{matrix}$$

$$= \frac{L_0}{\beta}(\underbrace{1+\gamma-\gamma\beta^2}_{\parallel})$$

$$1+\gamma(1-\beta^2) = 1 + \frac{1}{\gamma} = \frac{1+\gamma}{\gamma}$$

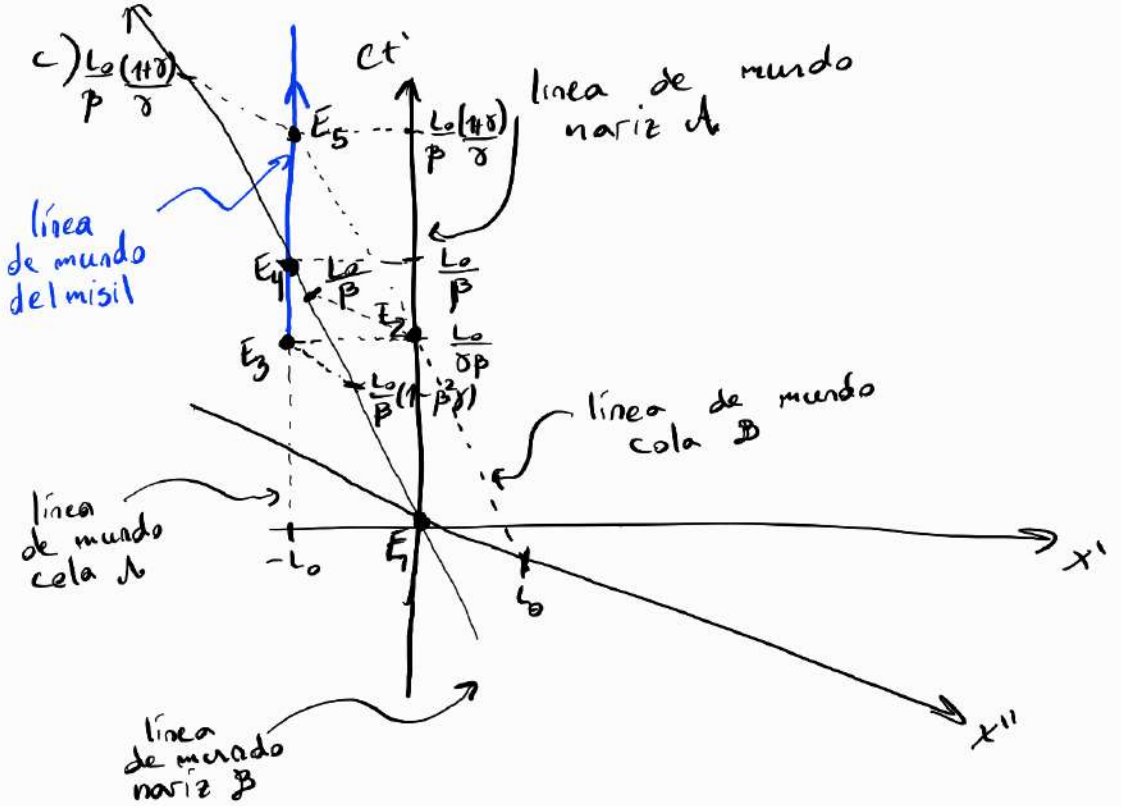
$$\left.\begin{matrix} \boxed{x'_{E_3} = -L_0} \\ \boxed{ct'_{E_3} = ct'_{E_2} = \dfrac{L_0}{\gamma\beta}} \end{matrix}\right\} \Rightarrow \begin{matrix} x''_{E_3} = \gamma(x'_{E_3} + \beta ct'_{E_3}) \to \boxed{x''_{E_3} = L_0(1-\gamma) < 0} \; (*) \\ ct''_{E_3} = \gamma(ct'_{E_3} + \beta x'_{E_3}) \to \boxed{ct''_{E_3} = \dfrac{L_0}{\gamma\beta}[\gamma - \beta^2\gamma^2]} \end{matrix}$$

pero $\gamma - \beta^2\gamma^2 \underset{\substack{\uparrow \\ \text{xq' } \gamma > 1}}{<} \gamma - \beta^2\gamma = \gamma(1-\beta^2) = \dfrac{1}{\gamma} < 1 \Rightarrow \boxed{ct''_{E_3} < ct''_{E_4}}$ (**)

por (*) vemos que incluso, según B, el disparo errará. Esto tambien se deduce de (**) ya que se dispara el cohete, según B, antes de que la cola de A pase por la nariz de B

b) midiendo el largo del cohete B desde S' vemos que, en $ct' = 0$ la cola de B está en: $0 = \gamma(ct''_{cola,B} - \beta x''_{cola,B}) \to ct''_{cola,B} = L_0\beta$

y $L'_B = x'_{cola,B} = \gamma(x''_{cola,B} - \beta ct''_{cola,B}) = \gamma L_0(1-\beta^2) \Rightarrow \boxed{L'_B = \dfrac{L_0}{\gamma}}$



2) a) $W_{F_{el}} = eE\Delta x \overset{*}{=} 3m_0c^2 \Rightarrow \Delta x = \frac{3m_0c^2}{eE} = 0{,}1533\ \text{m}$

b) $K = m_0c^2(\gamma-1) = 3m_0c^2 \Rightarrow \gamma = 4$

$\Rightarrow$ Visto desde el referencial del electrón es $\Delta x' = \frac{\Delta x}{\gamma} \simeq 0{,}0383\,\text{m}$

c) $\vec{F} \parallel \vec{v}$

$$eE = m_0\gamma^3(v)\frac{dv}{dt}$$

$$\int_0^T dt\, eE = \int_0^T m_0\gamma^3(v)\frac{dv}{dt}\,dt \Rightarrow eET = m_0\int_0^{v(T)}\frac{dv'}{\left(1-\frac{v'^2}{c^2}\right)^{3/2}} \underset{\frac{v}{c}=\operatorname{sen}\theta}{=} m_0\int_0^{\operatorname{arcsen}(\frac{v}{c})}\frac{c\cos\theta\, d\theta}{(1-\operatorname{sen}^2)^{3/2}} = m_0c\int_0^{\operatorname{arcsen}(\frac{v}{c})}\frac{d\theta}{\cos^2\theta}$$

pero $\frac{1}{\cos^2\theta} = \frac{\operatorname{sen}^2\theta+\cos^2\theta}{\cos^2\theta} = \frac{d}{d\theta}\left(\frac{\operatorname{sen}\theta}{\cos\theta}\right)$

$\Rightarrow eET = m_0c\frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$ pero $\frac{v}{c} \overset{(**)}{=} 1-\frac{1}{\gamma^2} \Rightarrow eET = m_0c\left(\gamma-\frac{1}{\gamma}\right)$

$$\Rightarrow T = \frac{m_0c}{eE}\left(\gamma-\frac{1}{\gamma}\right) = \frac{m_0c^2}{eE\Delta x}\frac{\Delta x}{c}\left(\gamma-\frac{1}{\gamma}\right) \overset{(*)}{=} \frac{\Delta x}{c}\frac{m_0c^2}{3m_0c^2}\left(\gamma-\frac{1}{\gamma}\right) \simeq 6{,}39\times10^{-10}\,\text{s}$$

d)

$$\tilde{P}_0 = \tilde{P}_{e^-} + \tilde{P}_{e^+} = (m_0 c\gamma,\ m_0 v\gamma, 0, 0) + (m_0 c, 0, 0, 0)$$

$$\tilde{P}_f = \tilde{P}_{\gamma_1} + \tilde{P}_{\gamma_2} = \left(\frac{E_{\gamma_1}}{c}, P_{\gamma_1}\cos\theta, P_{\gamma_1}\operatorname{sen}\theta, 0\right) + \left(\frac{E_{\gamma_2}}{c}, P_{\gamma_2}\cos\phi, -P_{\gamma_2}\operatorname{sen}\phi, 0\right)$$

$$P_\gamma \equiv P_{\gamma_1} = \frac{E_{\gamma_1}}{c} = \frac{E_{\gamma_2}}{c} = P_{\gamma_2} \Rightarrow P_\gamma \operatorname{sen}\theta = P_\gamma \operatorname{sen}\phi \Rightarrow \theta = \phi$$

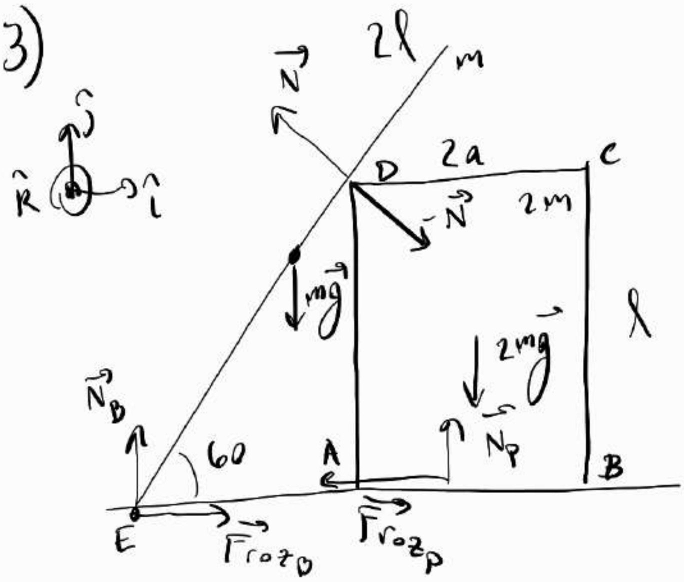
$$2\frac{E_\gamma}{c} = m_0 c(\gamma+1) \Rightarrow \boxed{E_\gamma = \frac{5}{2} m_0 c^2 \approx 1{,}28\ \text{MeV}}$$

$$\Rightarrow P_\gamma = 1{,}28\ \frac{\text{MeV}}{c}$$

e) Además $m_0 v\gamma = 2P_\gamma\cos\theta \Rightarrow m_0 v\gamma = 2\,\frac{5}{2} m_0 c\cdot\cos\theta \Rightarrow \theta = \arccos\left(\frac{\gamma v}{5c}\right) \overset{(**)}{=} \arccos\left(\frac{\gamma - \frac{1}{\gamma}}{5}\right)$

$$\Rightarrow \theta = \arccos\left(\frac{15}{20}\right) \approx 0{,}723\ \text{rad}.$$

3)



1ra cardinal

Barra

$$0 = -mg\hat{j} + N_B\hat{j} + F_{roz_B}\hat{i} - N\operatorname{sen}(60)\hat{i} + N\cos(60)\hat{j}$$

$$\hookrightarrow \boxed{mg = N_B + N\cos(60)\quad (I)}$$

$$\boxed{F_{roz_B} = N\operatorname{sen}(60)\quad (II)}$$

Placa

$$0 = -2mg\hat{j} + N_p\hat{j} - F_{roz_p}\hat{i} + N\operatorname{sen}(60)\hat{i} - N\cos(60)\hat{j}$$

$$\hookrightarrow \boxed{2mg = N_p - N\cos(60)\quad (III)}$$

$$\boxed{F_{roz_p} = N\operatorname{sen}(60)\quad (IV)}$$

2da cardinal

Barra

$$0 = \vec{\tau}_{N,E} + \cancel{\vec{\tau}_{N_B,E}} + \vec{\tau}_{m\vec{g},E} + \cancel{\vec{\tau}_{F_{roz},E}} = \frac{N\ell}{\operatorname{sen}(60)}\hat{k} - mg\,\ell\cos(60)\hat{k}$$

$$\hookrightarrow \boxed{N = mg\operatorname{sen}(60)\cos(60)\quad (V)}$$

Placa

$$0 = \vec{\tau}_{2m\vec{g},A} + \vec{\tau}_{N_p,A} + \vec{\tau}_{F_{roz_p},A} + \vec{\tau}_{-N,A} = N_p x\,\hat{k} - 2mga\,\hat{k} - N\ell\operatorname{sen}(60)\hat{k}$$

$$\hookrightarrow \boxed{N_p x = 2mga + N\ell\operatorname{sen}(60)\quad (VI)}$$

Combinando (V) y (I) es

$$\boxed{mg\left(1-\operatorname{sen}(60)\cos^2(60)\right)=N_B} \quad (*)$$

combinando (V) y (III) es

$$\boxed{mg\left(2+\operatorname{sen}(60)\cos^2(60)\right)=N_P} \quad (**)$$

combinando (V) y (II) es

$$0\leqslant F_{roz_B}=mg\operatorname{sen}^2(60)\cos(60)\leqslant f N_B \overset{(*)}{=} f mg\left(1-\operatorname{sen}(60)\cos^2(60)\right)$$

$\operatorname{sen}(60)=\dfrac{\sqrt{3}}{2}$

$\cos(60)=\dfrac{1}{2}$

$$\Rightarrow \boxed{f\geqslant\frac{3}{8-\sqrt{3}}}\quad (C1)$$

De lo contrario la barra desliza

combinando (V) y (IV) es

$$0\leqslant F_{roz_P}=mg\operatorname{sen}^2(60)\cos(60)\leqslant f N_P \overset{(**)}{=} f mg\left(2+\operatorname{sen}(60)\cos^2(60)\right)$$

$$\Rightarrow \boxed{f\geqslant\frac{3}{16+\sqrt{3}}}\quad (C2)$$

De lo contrario el bloque desliza

si se cumple (C1) también se cumple (C2).

combinando (V), (VI) y (**) es:

$$mg\left(2+\operatorname{sen}(60)\cos^2(60)\right)x=2mga+mg\ell\operatorname{sen}^2(60)\cos(60)$$

$$\Rightarrow\quad 0\leqslant x=\frac{2a+\frac{3\ell}{8}}{2+\frac{\sqrt{3}}{8}}=\frac{16a+3\ell}{16+\sqrt{3}}\leqslant 2a$$

siempre se cumple, la placa no rotará antihorario

$$\Rightarrow \boxed{\ell\leqslant\frac{16+2\sqrt{3}}{3}a}$$

Para que la placa no rote en sentido horario.