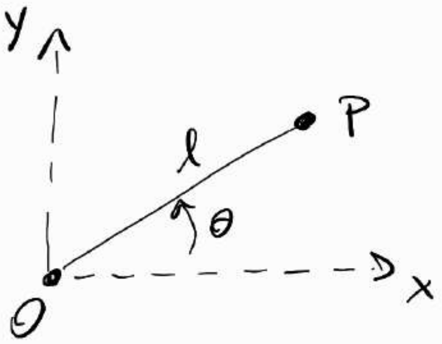
1)



a y b, método 1) anotemos al evento O en el origen de coordenadas espaciotemporales de S $\Rightarrow$ $(x_0, y_0, z_0, ct_0) = (0,0,0,0)$

y P es tal que $(x_p, y_p, z_p, ct_p) = (l\cos\theta,\ l\,\text{sen}\,\theta,\ 0,\ c\frac{l}{c})$

ya que recorre una distancia l a velocidad c.

Supongamos que S' tiene su origen espaciotemporal coincidente con S. De esa forma es

$$x'_E = \gamma(x_E - \beta c t_E)$$
$$y'_E = y_E$$
$$z'_E = z_E$$
$$ct'_E = \gamma(ct_E - \beta x_E)$$

para un evento E cualquiera.

$$\Rightarrow \quad x'_p = \gamma(l\cos\theta - \beta l)$$
$$y'_p = l\,\text{sen}\,\theta$$
$$z'_p = 0$$
$$ct'_p = \gamma(l - \beta l\cos\theta)$$

$$\Rightarrow \tau' = \frac{1}{c}(ct'_p - ct'_0) = \gamma\frac{l}{c}(1-\beta\cos\theta)$$
$$\Rightarrow l' = c\tau' = \gamma l(1-\beta\cos\theta)$$

b, método 2) $l' = \sqrt{(x'_p - x'_0)^2 + (y'_p - y'_0)^2 + (z'_p - z'_0)^2} = l\sqrt{\gamma^2(\cos\theta - \beta)^2 + \underbrace{\text{sen}^2\theta}_{1-\cos^2\theta}}$

$$\Rightarrow l' = l\sqrt{\gamma^2\cos^2\theta - 2\gamma^2\cos\theta\,\beta + \gamma^2\beta^2 + 1 - \cos^2\theta}$$
$$= l\sqrt{\underbrace{(\gamma^2-1)}_{\frac{1-1+\beta^2}{1-\beta^2} = \frac{\beta^2}{1-\beta^2} = \beta^2\gamma^2}\cos^2\theta - 2\gamma^2\cos\theta\,\beta + \underbrace{1+\gamma^2\beta^2}_{\frac{1-\beta^2+\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2}}$$

$$\Rightarrow \boxed{l' = l\gamma\sqrt{\beta^2\cos^2\theta - 2\beta\cos\theta + 1} = \gamma l(1-\beta\cos\theta)}$$

c, método 1)

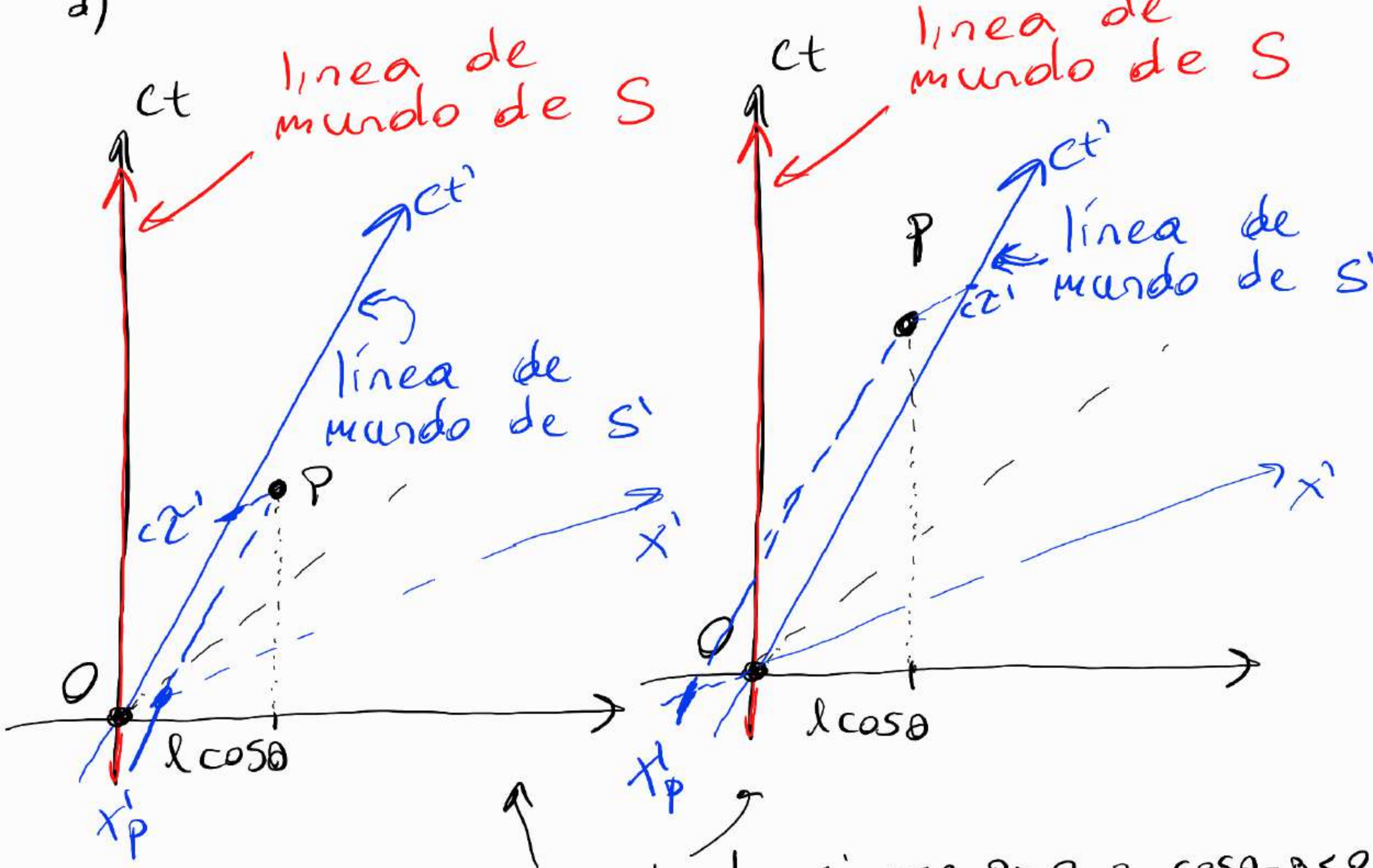
$$u_x = c\cos\theta$$
$$u_y = c\,\mathrm{sen}\,\theta$$
$$u'_x = \frac{x'_P - x'_O}{\tau'} = \frac{\gamma l(\cos\theta - \beta)}{\gamma \frac{l}{c}(1-\beta\cos\theta)} = c\,\frac{\cos\theta - \beta}{1-\beta\cos\theta}$$
$$u'_y = \frac{y'_P - y'_O}{\tau'} = \frac{l\,\mathrm{sen}\,\theta}{\frac{\gamma l}{c}(1-\beta\cos\theta)} = c\,\frac{\mathrm{sen}\,\theta}{\gamma(1-\beta\cos\theta)}$$

c, método 2)

$$u_x = c\cos\theta$$
$$u_y = c\,\mathrm{sen}\,\theta$$

$$u'_x = \frac{u_x - \beta c}{1 - \frac{u_x\beta}{c}} = c\,\frac{\cos\theta-\beta}{1-\beta\cos\theta}$$
$$u'_y = \frac{u_y}{\gamma\left(1-\frac{u_x\beta}{c}\right)} = c\,\frac{\mathrm{sen}\,\theta}{\gamma(1-\beta\cos\theta)}$$

d)



une u otro dependiendo si $\cos\theta-\beta>0$ o $\cos\theta-\beta<0$

2) En el referencial del pión π^+,

$$\tilde{P}'_{inicial} = \tilde{P}'_{\pi^+} = (m_{\pi^+}c, 0, 0, 0)$$

da igual la dirección

$$\tilde{P}'_{final} = \tilde{P}'_{\nu} + \tilde{P}'_{\mu^+} = (|P'_\nu|, P'_\nu, 0, 0) + \left(\frac{E'_{\mu^+}}{c}, P'_{\mu^+}, 0, 0\right)$$

$$\tilde{P}'_{inicial} = \tilde{P}'_{final} \Rightarrow P'_{\mu^+} = -P'_\nu \quad (*)$$

$$E'_\nu = |P'_\nu| c$$

$$(m_{\pi^+}c)^2 = |P'_\nu|^2 + \left(\frac{E'_{\mu^+}}{c^2}\right)^2 + 2|P'_\nu| \frac{E'_{\mu^+}}{c}$$

pero $\quad \frac{E'_{\mu^+}}{c} = \sqrt{m_{\mu^+}^2 c^2 + (P'_{\mu^+})^2} \overset{(*)}{=} \sqrt{m_{\mu^+}^2 c^2 + |P'_\nu|^2}$

$$\Rightarrow (m_{\pi^+}c)^2 \overset{*}{=} |P'_\nu|^2 + m_{\mu^+}^2 c^2 + |P'_\nu|^2 + 2|P'_\nu| \sqrt{m_{\mu^+}^2 c^2 + (P'_{\mu^+})^2}$$

$$\Rightarrow \frac{(m_{\pi^+}^2 - m_{\mu^+}^2)c^2}{2} = |P'_\nu| \Big(\underbrace{|P'_\nu| + \sqrt{m_{\mu^+}^2 c^2 + |P'_\nu|^2}}_{|P'_\nu| + \frac{E'_{\mu^+}}{c} = m_{\pi^+}c} \Big)$$

$$\Rightarrow \boxed{\frac{m_{\pi^+}^2 - m_{\mu^+}^2}{2 m_{\pi^+}} c^2 = |P'_\nu| c = E'_\nu \simeq 29{,}9 \text{ MeV}}$$

b y c) En el referencial del laboratorio es

$$\tilde{P}_{inicial} = \tilde{P}_{\pi^+} = \left(\frac{E_{\pi^+}}{c}, P_{\pi^+}, 0, 0\right)$$

$$\tilde{P}_{final} = \tilde{P}_\nu + \tilde{P}_{\mu^+} = (|P_\nu|, P_\nu \cos\theta, P_\nu \operatorname{sen}\theta, 0)$$
$$+ \left(\frac{E_{\mu^+}}{c}, P_{\mu^+}\cos\alpha, -P_{\mu^+}\operatorname{sen}\alpha, 0\right)$$

$\Rightarrow P_\nu \operatorname{sen}\theta = P_{\mu^+} \operatorname{sen}\alpha, \quad P_\nu \cos\theta + P_{\mu^+}\cos\alpha = P_{\pi^+}, \quad \frac{E_{\pi^+}}{c} = |P_\nu| + \frac{E_{\mu^+}}{c}$

$$\Rightarrow \quad E_\nu = |P_\nu| c$$

pero

$$|P_{\mu^+}|^2 = P_{\mu^+}^2 \operatorname{sen}^2\alpha + P_{\mu^+}^2 \cos^2\alpha = \underbrace{P_\nu^2 \operatorname{sen}^2\theta}_{P_{\mu^+}^2 \operatorname{sen}^2\alpha} + \underbrace{P_\nu^2 \cos^2\theta + P_{\pi^+}^2 - 2P_\nu P_{\pi^+}\cos\theta}_{P_{\mu^+}^2 \cos^2\alpha}$$

Es decir

$$P_{\mu^+}^2 = P_\nu^2 + P_{\pi^+}^2 - 2P_\nu P_{\pi^+}\cos\theta$$

$$\left.\begin{aligned} P_{\mu^+}^2 &= \frac{E_{\mu^+}^2}{c^2} - m_{\mu^+}^2 c^2 \\ \frac{E_{\mu^+}}{c} &= \frac{E_{\pi^+}}{c} - |P_\nu| \end{aligned}\right\} \Rightarrow P_{\mu^+}^2 = \frac{E_{\pi^+}^2}{c^2} + |P_\nu|^2 - 2\frac{E_{\pi^+}}{c}|P_\nu| - m_{\mu^+}^2 c^2$$

$$\Rightarrow \quad \frac{E_{\pi^+}^2}{c^2} + \cancel{|P_\nu|^2} - 2\frac{E_{\pi^+}}{c}|P_\nu| - m_{\mu^+}^2 c^2 = P_\nu^2 + P_{\pi^+}^2 - 2P_\nu P_{\pi^+}\cos\theta$$

$$= \cancel{P_\nu^2} + \frac{E_{\pi^+}^2}{c^2} - m_{\pi^+}^2 c^2 - 2P_\nu P_{\pi^+}\cos\theta$$

$$\Rightarrow \quad (m_{\pi^+}^2 - m_\mu^2)c^2 = 2|P_\nu|\left(\frac{E_{\pi^+}}{c} - P_{\pi^+}\cos\theta\right)$$

$$P_{\pi^+}^2 = \frac{E_{\pi^+}^2}{c^2} - m_{\pi^+}^2 c^2 = \frac{E_{\pi^+}^2}{c^2}\left(1 - \frac{m_{\pi^+}^2 c^4}{E_{\pi^+}^2}\right)$$

$$\Rightarrow \quad \frac{(m_{\pi^+}^2 - m_\mu^2)c^2}{2\frac{E_{\pi^+}}{c}\left(1 - \underbrace{\sqrt{1 - \frac{m_{\pi^+}^2 c^4}{E_{\pi^+}^2}}}_{1 - \frac{m_{\pi^+}^2 c^4}{2E_{\pi^+}^2}}\cos\theta\right)} = P_\nu$$

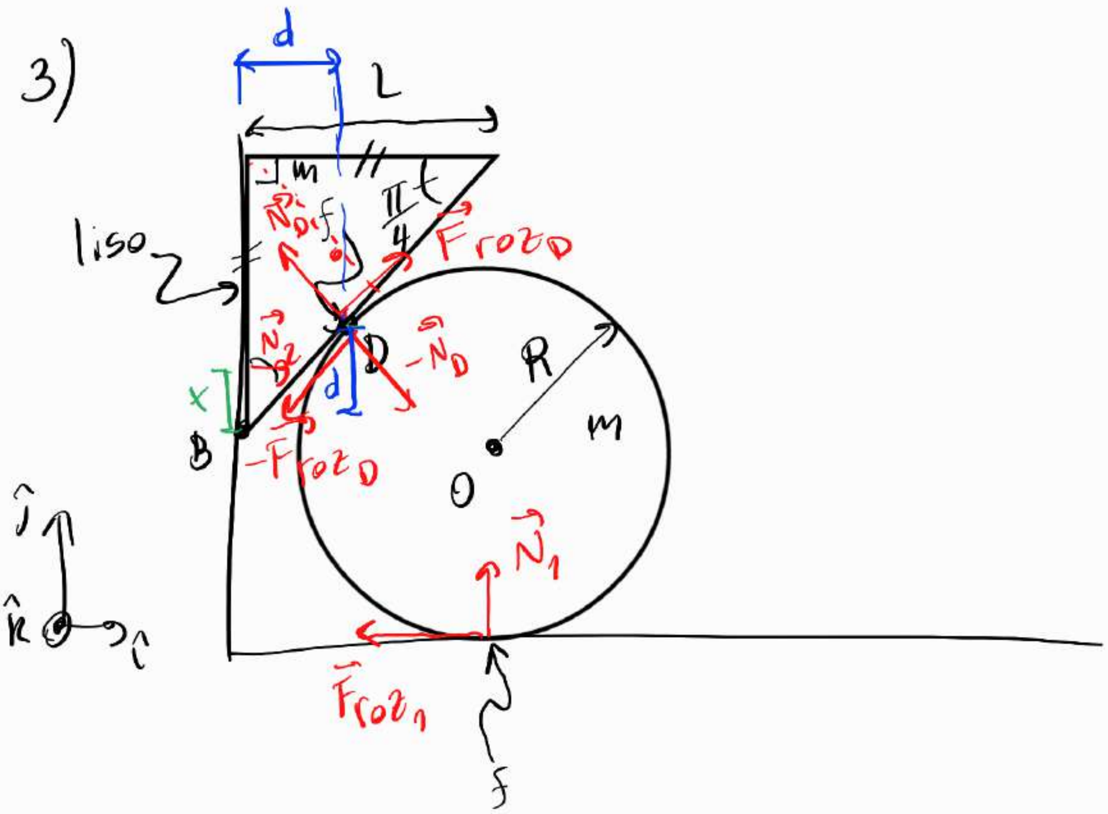
$$\rightarrow \quad P_\nu\Big|_{\theta=0} \approx \frac{(m_{\pi^+}^2 - m_\mu^2)E_{\pi^+}}{m_\pi^2 c}$$

$$\frac{1}{2} = \frac{P_\nu}{P_\nu\big|_{\theta=0}} = \frac{1 - 1 + \frac{m_\pi^2 c^4}{2E_\pi^2}}{1 - \left(1 - \frac{m_\pi^2 c^4}{2E_\pi^2}\right)\cos\theta}$$

$$\Rightarrow \quad 1 - \cos\theta + \frac{m_\pi^2 c^4}{2E_\pi^2}\cos\theta = \frac{m_\pi^2 c^4}{E_\pi^2} \rightarrow \underbrace{1 - \cos\theta}_{\frac{\theta^2}{2}} = \frac{m_\pi^2 c^4}{2E_\pi^2}(\overbrace{2 - \cos\theta}^{\sim 1})$$

$$\Rightarrow \quad \theta \approx \frac{m_\pi c^2}{E_\pi} \sim 7 \times 10^{-4} \text{ rad}$$

3)



1ra cardinal

Placa triangular

$$m\vec{g} + \vec{N}_2 + \vec{N}_D + \vec{F}_{roz,D} = 0$$

$$-mg\hat{j} + N_2\hat{i} + \frac{N_D}{\sqrt{2}}\hat{j} - \frac{N_D}{\sqrt{2}}\hat{i} + \frac{F_{roz_D}}{\sqrt{2}}\hat{i} + \frac{F_{roz_D}}{\sqrt{2}}\hat{j} = 0$$

$$\Rightarrow \boxed{\begin{array}{ll} \sqrt{2}\,mg = N_D + F_{roz_D} & (\text{I}) \\ \sqrt{2}\,N_2 = N_D - F_{roz_D} & (\text{II}) \end{array}}$$

Disco

$$m\vec{g} + \vec{N}_1 + \vec{F}_{roz_1} - \vec{N}_D - \vec{F}_{roz,D} = 0$$

$$-mg\hat{j} + N_1\hat{j} - F_{roz_1}\hat{i} - \frac{N_D}{\sqrt{2}}\hat{j} + \frac{N_D}{\sqrt{2}}\hat{i} - \frac{F_{roz_D}}{\sqrt{2}}\hat{i} - \frac{F_{roz_D}}{\sqrt{2}}\hat{j} = 0$$

$$\Rightarrow \boxed{\begin{array}{ll} \sqrt{2}\,mg = \sqrt{2}\,N_1 - N_D - F_{roz_D} & (\text{III}) \\ \sqrt{2}\,F_{roz_1} = N_D - F_{roz_D} & (\text{IV}) \end{array}}$$

2da cardinal

Placa triangular

$$\vec{\tau}_{placa,B} = 0 = \underbrace{\vec{\tau}_{N_2,B}}_{-xN_2\hat{k}} + \underbrace{\vec{\tau}_{N_D,B}}_{\sqrt{2}\,dN_D\hat{k}} + \cancel{\vec{\tau}_{F_{roz_D},B}}^{0} + \underbrace{\vec{\tau}_{m\vec{g},B}}_{-mg\frac{L}{3}\hat{k}} = \left(\sqrt{2}\,dN_D - mg\frac{L}{3} - xN_2\right)\hat{k}$$

$$\Rightarrow \boxed{\sqrt{2}\,dN_D - mg\frac{L}{3} - xN_2 = 0 \quad (\text{V})}$$

Disco

$$\vec{\tau}_{Disco,O} = \cancel{\vec{\tau}_{mg,O}}^{0} + \cancel{\vec{\tau}_{-N_D,O}}^{0} + \cancel{\vec{\tau}_{N_1,O}}^{0} + \vec{\tau}_{F_{roz_1},O} + \vec{\tau}_{-F_{roz_D},O} = R\left(F_{roz_D} - F_{roz_1}\right)\hat{k} = 0$$

$$\Rightarrow \boxed{F_{roz_1} = F_{roz_D} \quad (\text{VI})}$$

De (II) y (IV) es $F_{roz_1} = N_2 \geq 0 \Rightarrow 0 \leq F_{roz_1} \leq f N_1$

C1

De (IV) y (IV) es $\boxed{(\sqrt{2}+1) F_{roz_D} = N_D} \geq 0 \Rightarrow 0 \leq \underset{\frac{1}{\sqrt{2}+1} N_D}{\overset{\shortparallel}{F_{roz_D}}} \leq f N_D \Rightarrow \boxed{f \geq \frac{1}{\sqrt{2}+1}}$

$*$

De lo contrario hay deslizam. entre placa y disco

Sustituyendo $*$ en (I) es

$$\sqrt{2}\, mg = N_D \left(1 + \frac{1}{\sqrt{2}+1}\right) \Rightarrow N_D = \frac{(2+\sqrt{2})\, mg}{\sqrt{2}+2} \Rightarrow \left.\begin{array}{l} \boxed{N_D = mg} \\ \boxed{F_{roz_D} = \frac{mg}{\sqrt{2}+1}} \end{array}\right\} **$$

$***$

Sustituyendo $**$ en (IV) es $N_2 = \frac{mg}{\sqrt{2}}\left(1 - \frac{1}{\sqrt{2}+1}\right) \Rightarrow \boxed{N_2 = \frac{mg}{\sqrt{2}+1}}$

sustituyendo $**$ en (III) es $N_1 = mg\left(\underbrace{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}+2}}\right) \Rightarrow \boxed{N_1 = 2\, mg}$

$$\frac{2(\sqrt{2}+1) + \sqrt{2}+2 + \sqrt{2}}{\sqrt{2}(\sqrt{2}+2)} = 2$$

de (VI) y $**$ es $F_{roz_1} = \frac{mg}{\sqrt{2}+1} \leq f N_1 = 2 f mg$

$\Rightarrow f \geq \frac{1}{2(\sqrt{2}+1)}$ (C2)

pero de (C1) es

$f \geq \frac{1}{\sqrt{2}+1} > \frac{1}{2(\sqrt{2}+1)}$ por lo

que ya existía una restricción mayor.

si $f < \frac{1}{2(\sqrt{2}+1)}$ hay deslizamiento entre placa y disco y entre disco y piso.

si $\frac{1}{\sqrt{2}+1} < f < \frac{1}{2(\sqrt{2}+1)}$ hay desliz. sólo entre placa y disco.

sustituyendo $**$ y $***$ en (V) es

$$\sqrt{2}\, d\, mg - mg\frac{L}{3} - x\,\frac{mg}{\sqrt{2}+1} = 0 \Rightarrow x = (3\sqrt{2}\, d - L)\frac{(\sqrt{2}+1)}{3}$$

pero $0 \leq x \leq L \Rightarrow \underline{3\sqrt{2}\, d \geq L}$

para que no rote la placa horario

$(4+\sqrt{2})L \geq 3\sqrt{2}\, d$

pero $d \leq L$ y $(4+\sqrt{2}) \geq 3\sqrt{2}$

(nunca va a rotar antihorario)