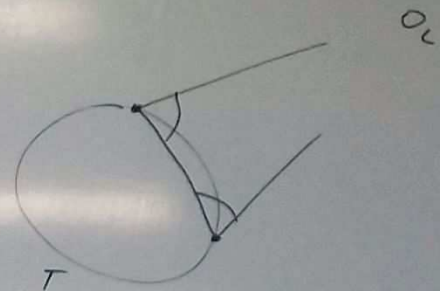
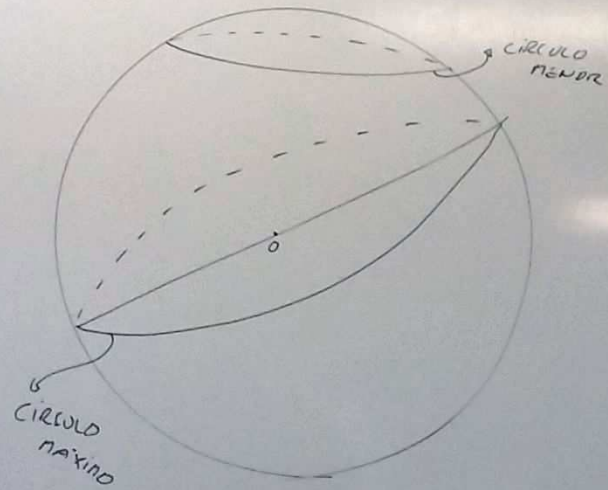
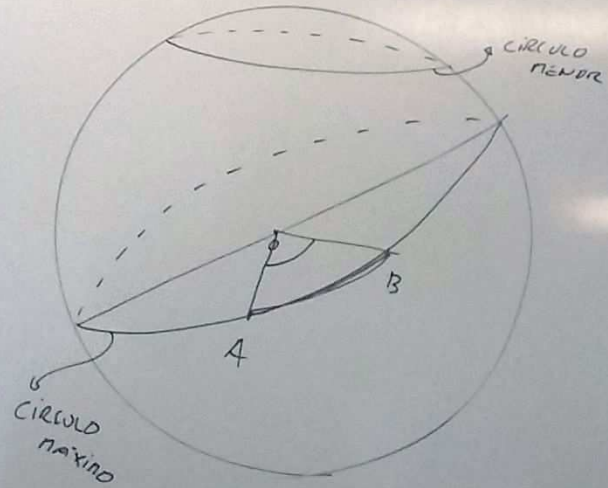


AFundamental2018clase01	2
AFundamental2018clase02	13
AFundamental2018clase03	34
AFundamental2018clase04	60
AFundamental2018clase05	86
AFundamental2018clase06	110
AFundamental2018clase07	128
AFundamental2018clase08	143
AFundamental2018clase09	168
AFundamental2018clase10	191
AFundamental2018clase11	216
AFundamental2018clase12	239
AFundamental2018clase13	262
AFundamental2018clase14	279
AFundamental2018clase15	305
AFundamental2018clase16	330
AFundamental2018clase17	353
AFundamental2018clase18	375
AFundamental2018clase19	401
AFundamental2018clase20	421
AFundamental2018clase21	443

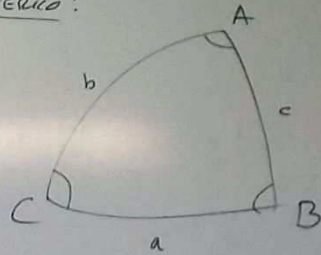
① TRIG. ESFÉRICA



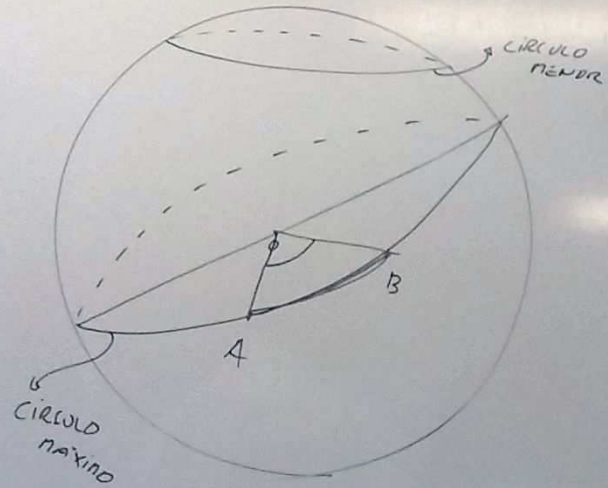
① TRIG. ESFÉRICA



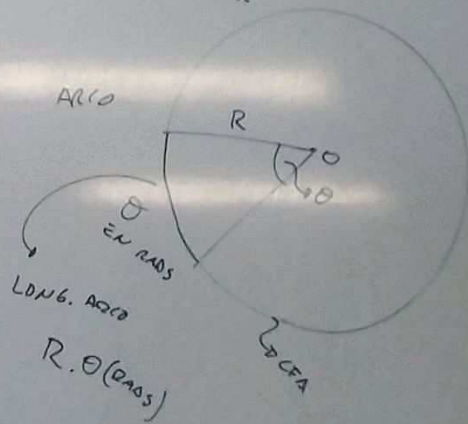
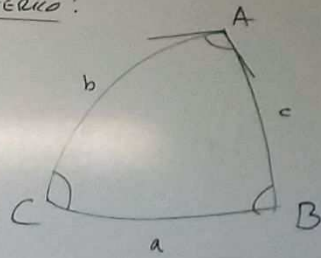
DESFERICO:



① TRIG. ESFÉRICA



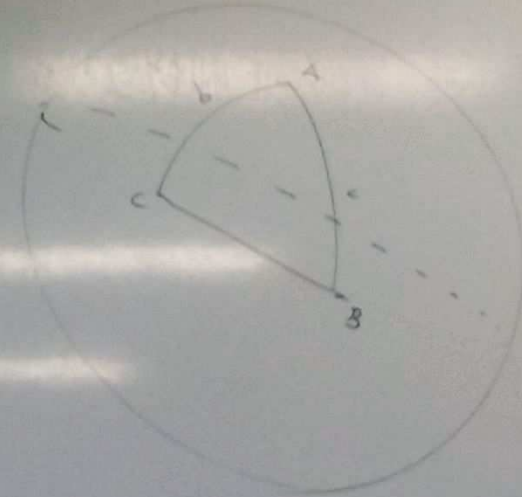
DESFÉRICO:



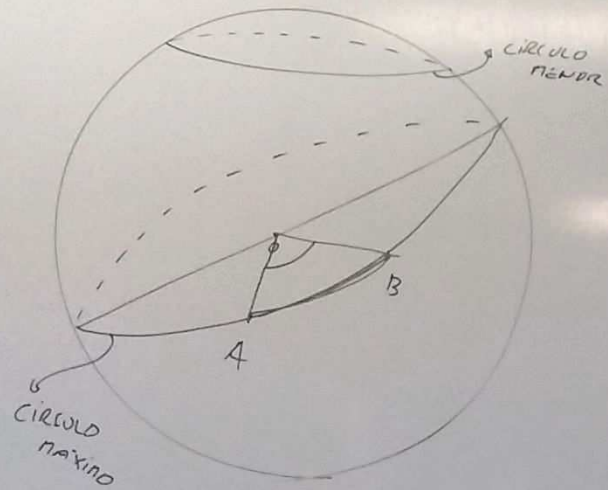
$$\pi \rightarrow 180^\circ$$

$$a, b, c \leq \pi$$

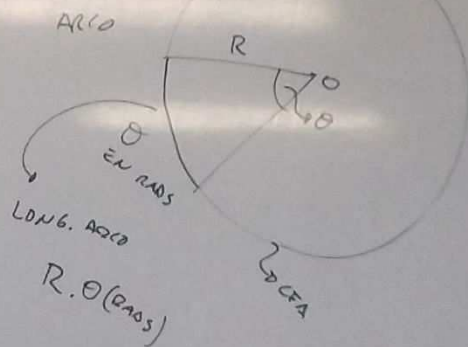
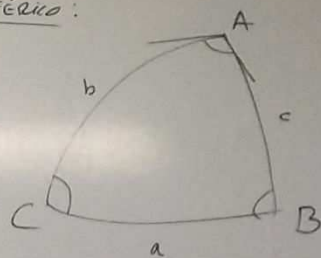
$$\pi < A + B + C < 3\pi$$



① TRIG. ESFÉRICA



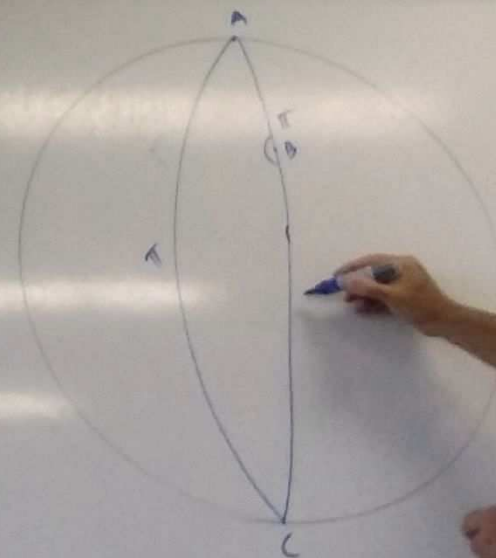
DESFÉRICO:



$\pi \rightarrow 180^\circ$

$a, b, c \leq \pi$

$\pi < A+B+C < 3\pi$



① TRIG. ESFERICA

ESFERA  
ÁREA  $4\pi \cdot R^2$

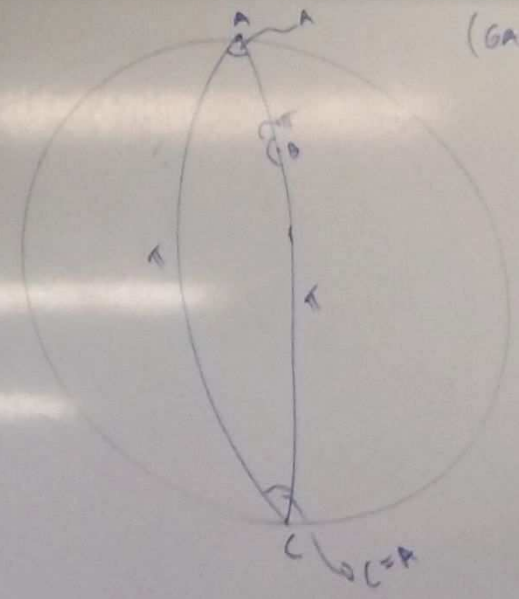
$A = 2\pi \rightarrow$  AREA "base"  $4\pi$

$\pi \rightarrow 180^\circ$

$a, b, c \leq \pi$

$\pi < A+B+C < 3\pi$

TRIANGULO BI-ANGULO  
(GASU)

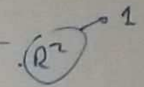


① TRIG. ESFÉRICATEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

ESFERA  
ÁREA  $4\pi \cdot R^2$

$A = 2\pi \rightarrow$  "ÁREA 'base'"  
 $4\pi$

$A \rightarrow 2A$   
↑  
ÁREA GAÏO





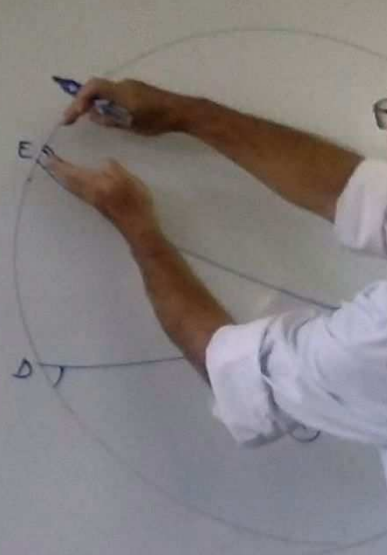
① TRIG. ESFÉRICA

TEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

ESFERA  
ÁREA  $4\pi \cdot R^2$

$A = 2\pi$  → ÁREA "caso"  $4\pi$

$A$  →  $2A$   
↑  
ÁREA GAUO



$\Delta$ ?

GAUO A :  $\textcircled{1} + \Delta$

GAUO B :



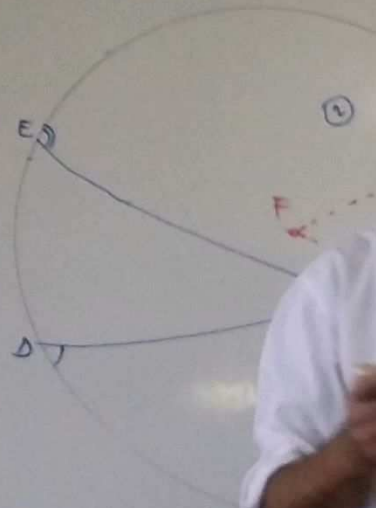
① TRIG. ESFÉRICA

TEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

ESFERA  
ÁREA  $4\pi \cdot R^2$

$A = 2\pi$  → ÁREA "cabo"  $4\pi$

$A$  →  $2A$   
↑  
ÁREA GAUDO



$\sum \Delta?$   
CASO A : ① +  $\Delta$  =  $2A$   
CASO B : ② +  $\Delta$  =  $2B$   
CASO C : ~~ÁREA~~ +  $\Delta$  =  $2C$

① TRIG. ESFÉRICA

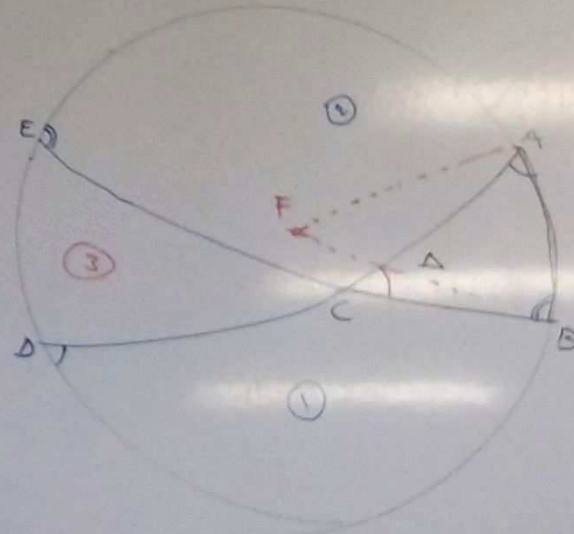
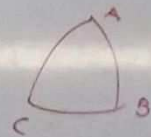
TEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

ÁREA  $\Delta = A + B + C - \pi$

ESFERA  
ÁREA  $4\pi \cdot R^2$

$A = 2\pi \rightarrow$  ÁREA "cabo"  $4\pi$

$A \rightarrow 2A$   
ÁREA GAJO



$\Delta?$   
CASO A : ① +  $\Delta = 2A$

CASO B : ② +  $\Delta = 2B$

CASO C :  $\overline{A'B'C'}$  +  $\Delta = 2C$

$① + ② + ③ + 3\Delta = 2(A+B+C)$

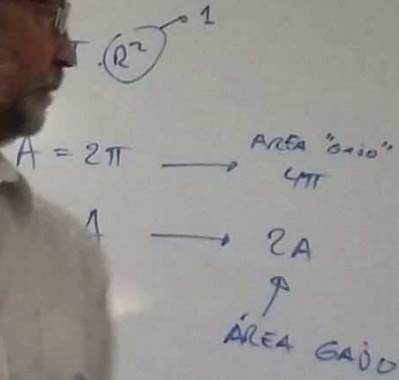
$① + ② + ③ + \Delta + 2\Delta = 2(A+B+C)$   
 $2\pi$

① TRIG. ESFÉRICA

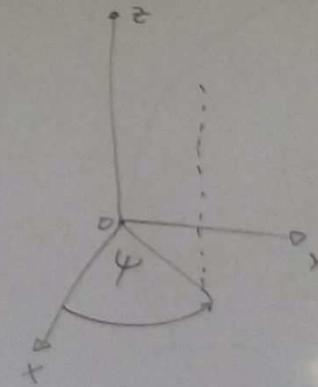
TEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

ÁREA  $\Delta = A + B + C$

EXCESO ESFÉRICO



COORD. ESFÉRICAS



① TRIG. ESFÉRICA

TEO GIRARD : ÁREA DE  $\Delta$  ESFÉRICO

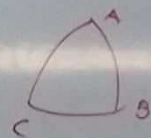
$$\text{ÁREA } \Delta = A + B + C - \pi$$

EXCESO ESFÉRICO

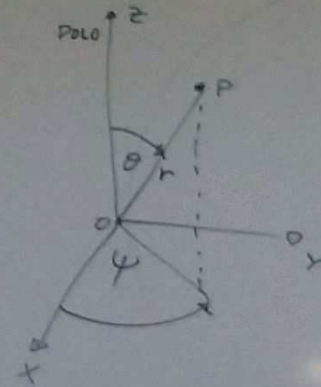
ESFERA  
ÁREA  $4\pi \cdot R^2$

$A = 2\pi \rightarrow$  ÁREA "base"  $4\pi$

$A \rightarrow 2A$   
ÁREA GAIRD



COORD. ESFÉRICAS



$0 \leq \psi < 360$   
ACIMUTAL

$0 \leq \theta < 180$   
POLAR

$(\psi, \theta)$

XY : PLANO FUNDAMENTAL

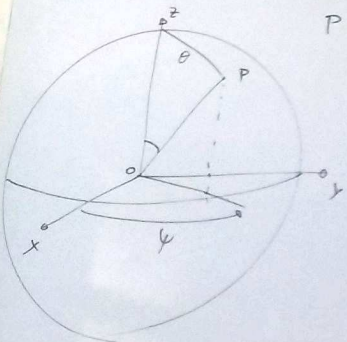
RECTANGULARES:

$$z = r \cdot \cos \theta$$

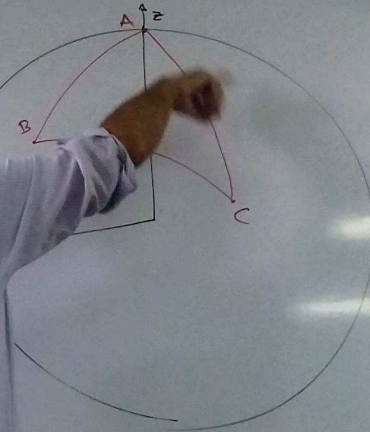
$$y = r \cdot \sin \theta \cdot \sin \psi$$

$$x = r \cdot \sin \theta \cdot \cos \psi$$

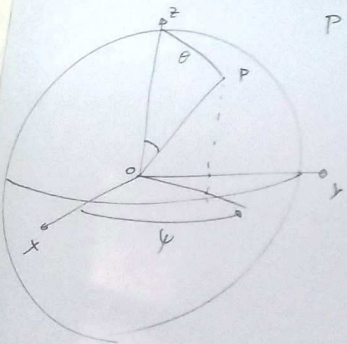
## DEDUCCION FÓRM. TRIG. ESFÉRICA

 $P(\psi, \theta)$ 

$$\begin{cases} x = r \cdot \sin \theta \cdot \cos \psi \\ y = r \cdot \sin \theta \cdot \sin \psi \\ z = r \cdot \cos \theta \end{cases}$$

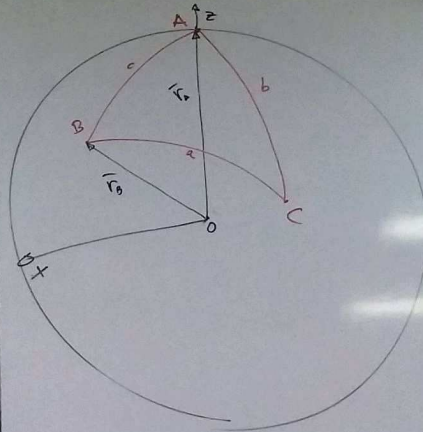


### DEDUCCION FÓRM. TRIG. ESFÉRICA



$P(\psi, \theta)$

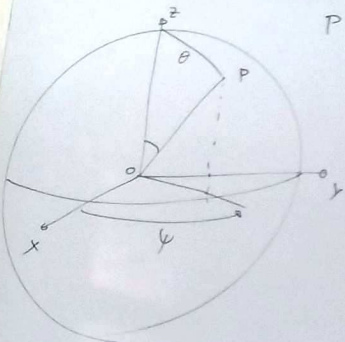
$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\vec{r}_A = (\psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

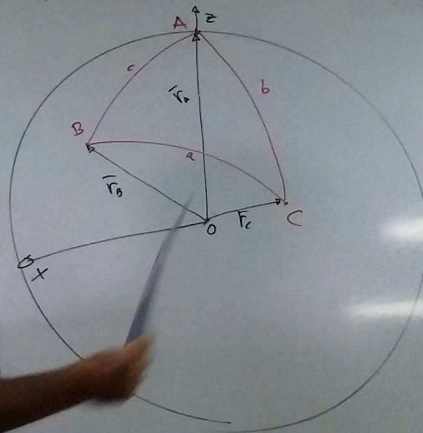
$$\vec{r}_B = (\psi = 0^\circ, \theta = \dots)$$

DEDUCCIÓ FÓRM. TRIG. ESFÉRICA



$P(\psi, \theta)$

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

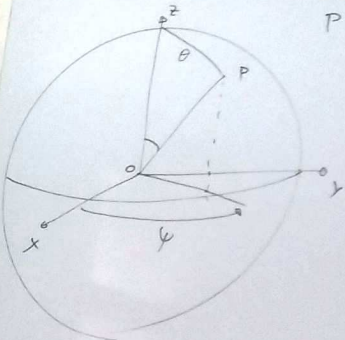


$$\vec{r}_A = (\sin \theta, \psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

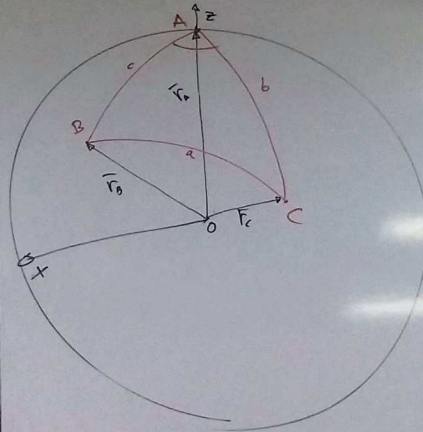
$$\vec{r}_B = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_C = (\psi =$$

## DEDUCCION FÓRM. TRIG. ESFÉRICA


 $P(\psi, \theta)$ 

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$



$$\vec{r}_a = (\psi=0, \theta=0) = (0, 0, 1)$$

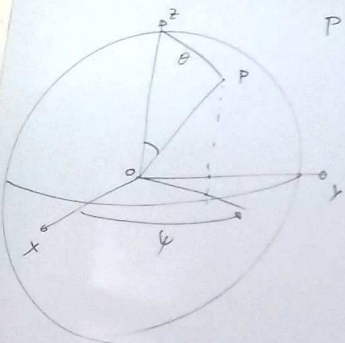
$$\vec{r}_b = (\psi=0, \theta=c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi=A, \theta=b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

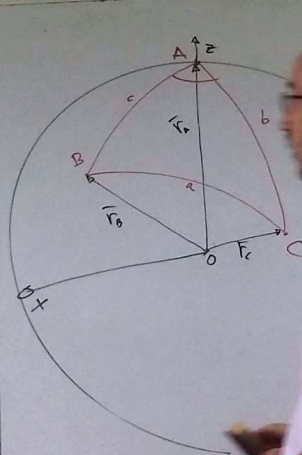


DEDUCCION FORM. TRIG. ESFERICA



$P(\psi, \theta)$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\vec{r}_a = (\psi=0^\circ, \theta=0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi=0^\circ, \theta=c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi=a, \theta=b) = (\sin b \cdot \cos a, \sin b \cdot \sin a, \cos b)$$

$$\vec{r}_b = \cos a$$

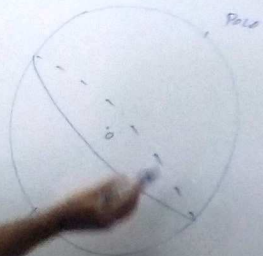
$$\sin c \cdot \sin b \cdot \cos a + 0 + \cos c \cdot \cos b = \cos a$$

$$= \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos a$$

F. COSENO

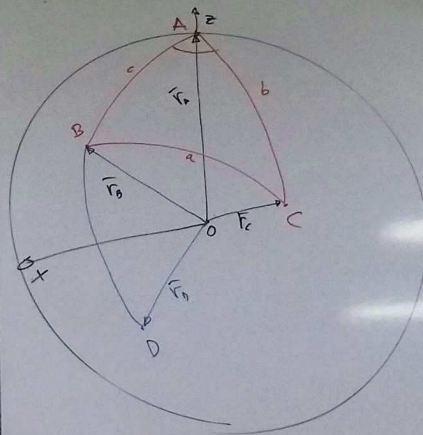
DEDUCCION FORM. TRIG. ESFERICA

POLO



$$\begin{cases} x = r \cdot \theta \cdot \cos \psi \\ y = r \cdot \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

$$\vec{r}_c \wedge \vec{r}_b = \sin a$$



$$\vec{r}_a = (\psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

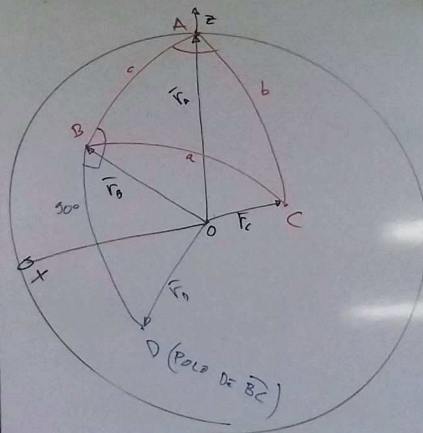
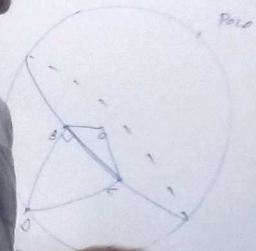
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

EXPRESIÓN FÓRM. TRIG. ESFÉRICA

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

$$\vec{r}_c \wedge \vec{r}_b = \sin a$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

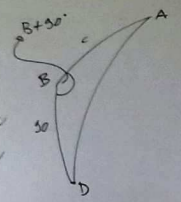
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DEDUCCION TRIG. ESFERICA

POLO

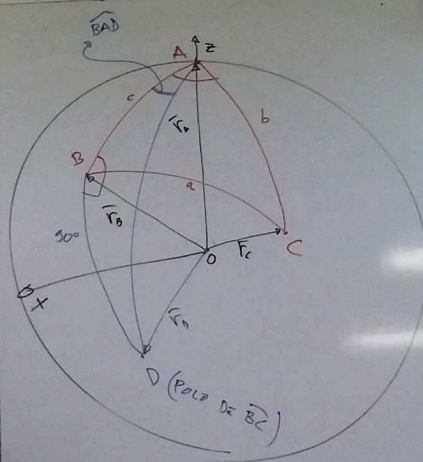
$$\begin{cases} x = r \sin \theta \cos \psi \\ y = r \sin \theta \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_d$$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} =$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

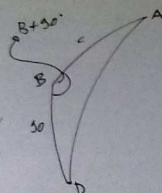
$$\cos a = \cos b \cdot \sin c + \sin b \cdot \cos c \cdot \cos A$$

F. COSENO

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA Z DE  $\vec{r}_0$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

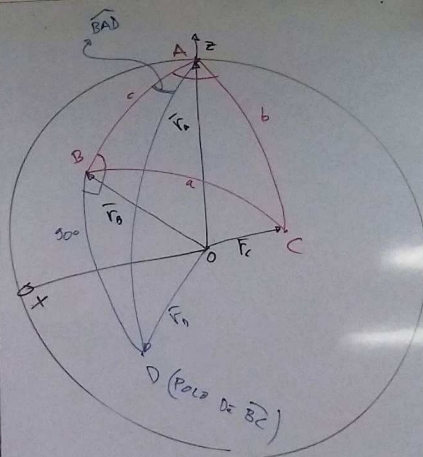


$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_a$$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90^\circ \cdot \cos c + \sin 90^\circ \cdot \sin c \cdot \cos(B+90^\circ)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

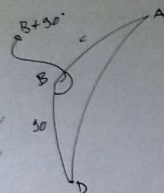
$$\cos a = \sin b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -\sin c \sin B$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



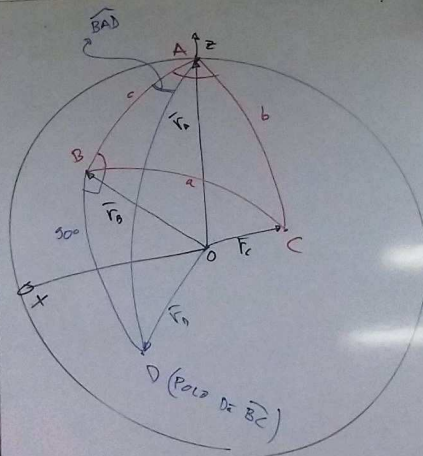
i	j	k
$\sin b \cdot \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\cos c$

$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_0$$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90^\circ \cdot \cos c + \sin 90^\circ \cdot \sin c \cdot \cos(B+90^\circ)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B \quad -\sin B$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

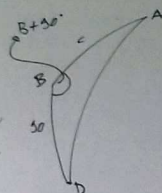
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

### DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -\sin \theta \sin \psi$

$$\begin{cases} x = r \sin \theta \cos \psi \\ y = r \sin \theta \sin \psi \\ z = r \cos \theta \end{cases}$$



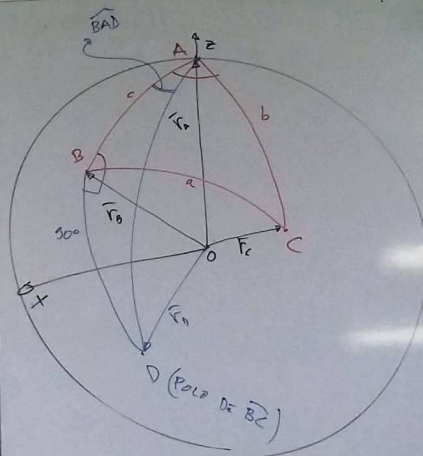
$$\begin{vmatrix} i & j & k \\ \sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta \\ \sin \psi & \cos \psi & 0 \end{vmatrix} =$$

$$\vec{r}_c \wedge \vec{r}_b = \sin \theta \vec{r}_0$$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90^\circ \cos c + \sin 90^\circ \sin c \cos (B+90^\circ)$$

$$\cos \widehat{AD} = -\sin c \sin B \quad -\sin B$$



$$-\sin \theta \sin \psi = \cos \widehat{AD} = \cos \theta \cos \psi$$

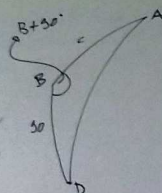
$\uparrow$   
 COORD. Z DE  
 $\vec{r}_c \wedge \vec{r}_b$

$$\cos \theta = \cos \psi \sin A$$

### DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -m.c.m.B$

$$\begin{cases} x = r \cdot \sin \theta \cdot \cos \psi \\ y = r \cdot \sin \theta \cdot \sin \psi \\ z = r \cdot \cos \theta \end{cases}$$



$$\begin{vmatrix} i & j & k \\ m.b.\cos A & m.b.m.A & \cos b \\ m.c & 0 & \cos c \end{vmatrix} =$$

( $m.c \cdot \cos b.m.c - m.b.m.A.m.c$ ,  
 $-m.b.m.A.m.c$ )

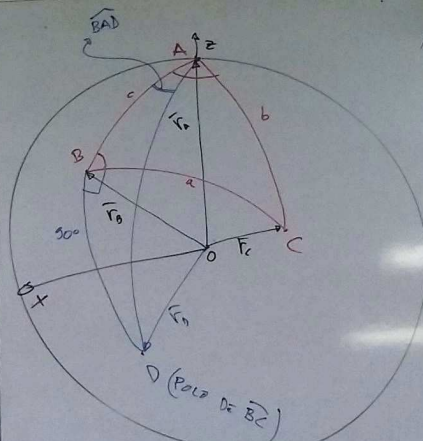
$$\vec{r}_c \wedge \vec{r}_b = m.a \cdot \vec{r}_0$$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90 \cdot \cos c + \sin 90 \cdot \cos c \cdot \cos (B+90)$$

$$\cos \widehat{AD} = -m.c \cdot m.B$$

-m.B



$$\pm m.b.m.A.m.c = \pm m.a \cdot \cos \theta \cdot m.B$$

$\uparrow$   
 $\cos \theta = z / r_0$   
 $\vec{r}_c \wedge \vec{r}_b$

$$\frac{m.a}{m.A} = \frac{m.b}{m.B} = \frac{m.c}{m.C}$$

T. SENO

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



### DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -\sin \theta \sin \phi$

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

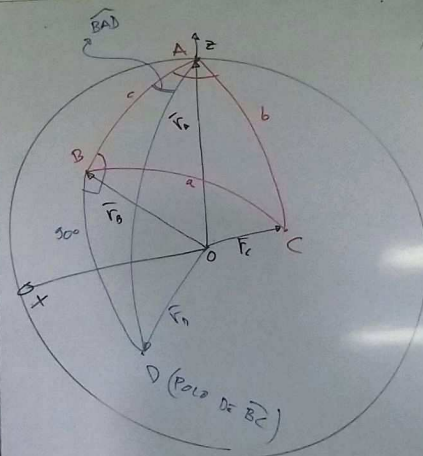
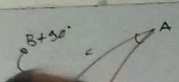
i	j	k
$\sin \theta \cos \phi$	$\sin \theta \sin \phi$	$\cos \theta$
$\sin \phi$	0	$-\cos \phi$

$$\vec{r}_c \wedge \vec{r}_b = \sin \theta \vec{r}_0$$

COORDENADA Z DE  $\vec{r}_0$

$$\sin \theta \sin \theta \cos \theta = \sin \theta \cos \theta$$

$$\sin \theta \sin \theta \cos \theta = \sin \theta \cos \theta$$



$$\sin \theta \sin \theta \cos \theta = \sin \theta \cos \theta$$

COORDENADA Z DE  $\vec{r}_c \wedge \vec{r}_b$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

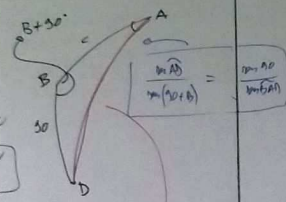
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO

### DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -\cos C \sin B$

$$\begin{cases} x = r \sin \theta \cos \psi \\ y = r \sin \theta \sin \psi \\ z = r \cos \theta \end{cases}$$



i	j	k
$\sin b \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\cos c$

$\vec{r}_c \wedge \vec{r}_b = \sin A \vec{r}_0$

$\psi = \widehat{BAD}$   
 $\theta = \widehat{AD}$

COORDENADA Z DE  $\vec{r}_0$

$\sin \widehat{AD} \cdot \sin \widehat{BAD} = \sin(B+90) \sin 90 = \cos B$

F. SENO     $\cos B$     1

$\sin b \sin A \cos c$      $\cos b \sin c - \sin b \cos A \cos c$

$-\sin b \sin A \cos c$

$\cos b \sin c - \sin b \cos A \cos c = \sin b \cos A$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

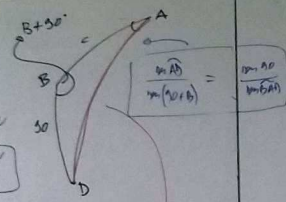
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO

DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE  $\vec{r}_0 = -\cos \theta$

$$\begin{cases} x = r \sin \theta \cdot \cos \phi \\ y = r \sin \theta \cdot \sin \phi \\ z = r \cos \theta \end{cases}$$



$$\frac{\sin \widehat{AB}}{\sin(\widehat{ADB})} = \frac{\sin \widehat{AD}}{\sin(\widehat{BAD})}$$

i	j	k
$\sin b \cdot \cos A$	$\sin b \cdot \sin A$	$\cos b$
$\sin c$	0	$\cos c$

$\vec{r}_c \wedge \vec{r}_b = \sin \theta \cdot \vec{r}_0$

COORDENADA Z DE  $\vec{r}_0$

$\sin \widehat{AD} \cdot \sin \widehat{BAD} = \sin(B+90) \cdot \sin 90 = \cos B$

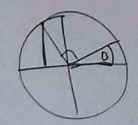
F. SENO

$\cos b \sin c - \sin b \cos c \cdot \cos A = \sin a \cdot \cos B$

F. ANALOGA

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO



$\cos c = \dots$

$\cos b = \cos A \cdot \cos c + \sin A \cdot \sin c \cdot \cos B$

$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

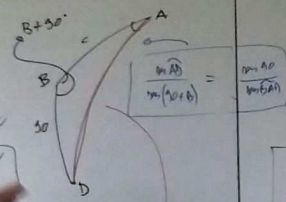
F. COSENO

DEDUCCION

G. ESFERICA

COORDENADA  $\geq$  DE  $\hat{A}$

$$\begin{cases} x = \rho \cdot \cos \psi \\ y = \rho \cdot \sin \psi \\ z = \rho \cdot \cos \theta \end{cases}$$



$$\frac{\sin \hat{A}}{\sin(\hat{B}+\hat{C})} = \frac{\sin \hat{A}D}{\sin \hat{B}A\hat{D}}$$

$\hat{A}$   
 $\hat{B}$   
 $\hat{C}$   
 $\hat{A}$   
 $\hat{B}$   
 $\hat{C}$

$$\hat{A}D \cdot \sin \hat{B}A\hat{D} = \sin(B+90) \cdot \sin 90 = \cos B$$

↑  
F. SEND

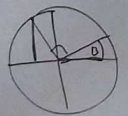
$$\cos b \sin c - \sin b \cos c \cos A = \sin a \cos B$$

F. ANALOGA

OPERANDO CON F. SEND Y F. COSEND:

F. 4 PARTES:

$$\cot b \cdot \sin a = \cos a \cos c + \sin c \cot B$$



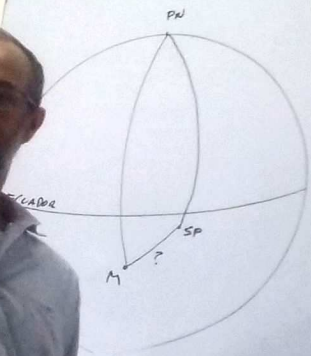
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SEND

$$\begin{aligned} \cos c &= \dots \\ \cos b &= \cos a \cos c + \sin a \sin c \cos B \end{aligned}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSEND



DIST. M-SP  
 $M (\lambda = -56^\circ, \phi = -35^\circ)$   
 SP  $(\lambda =$

$$\cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A = \sin a \cdot \cos B$$

F. ANÁLOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cos^2 b \cdot \sin a = \sin a \cdot \cos c + \sin c \cdot \cos^2 B$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

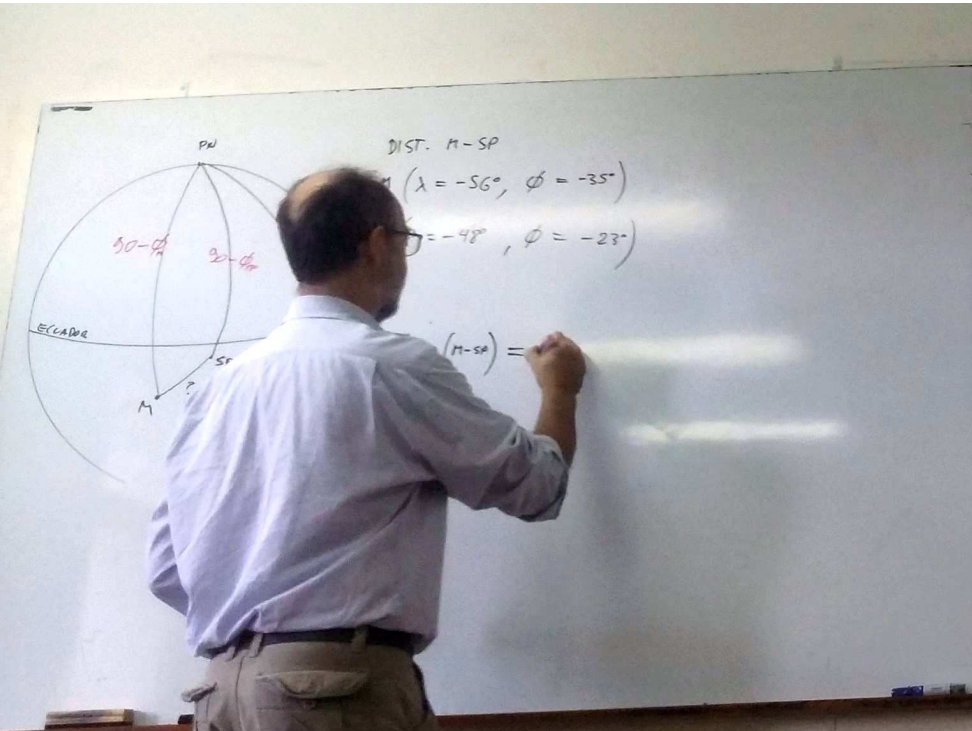
T. SENO



$$\cos b = \cos a \cdot \cos c + \sin a \cdot \sin c \cdot \cos B$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



DIST. M-SP

$$M (\lambda = -56^\circ, \phi = -35^\circ)$$

$$M (\lambda = -48^\circ, \phi = -23^\circ)$$

$$(M-SP) =$$

$$\cos mc - mb \cdot \cos A = ma \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENDO Y F. COSENO:  
 F. 4 PARTES:  
 $\cot b \cdot \sin a = \cos a \cdot \cos C + \sin C \cdot \cot B$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

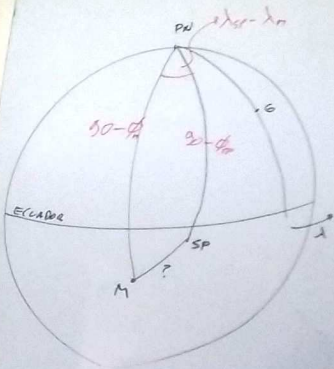
T. SENDO



$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



DIST. M-SP

$$M (\lambda = -56^\circ, \phi = -35^\circ)$$

$$SP (\lambda = -48^\circ, \phi = -23^\circ)$$

$$\cos(M-SP) = \sin \phi_m \cdot \sin \phi_{sp} + \cos \phi_m \cdot \cos \phi_{sp} \cdot \cos(\lambda_{sp} - \lambda_m)$$

$$\widehat{M-SP} = 33^\circ \cdot \frac{\pi}{180} \text{ (RAD)} \cdot R_T \rightarrow 6400 \text{ km}$$

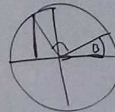
$$\cos m c - m b \cdot \cos A = m a \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \sin a = \cos a \cdot \cos C + \sin C \cdot \cot B$$



$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

$\text{DIST. M-SP}$   
 $M (\lambda = -35^\circ)$   
 $SP (\lambda = -23^\circ)$

$\text{CPE} \quad \text{CTO} \quad \text{CPE}$   
 $= \sin \phi_m \cdot \sin \phi_{sp} + \cos \phi_m \cdot \cos \phi_{sp} \cdot \cos (\lambda_{sp} - \lambda_m)$

$\text{DADO } \Delta \phi_m$   
 $(\Delta \lambda = 0) \Rightarrow \Delta$

$\phi_m \cdot \Delta \phi_m - \cos \phi_{sp} \cdot \sin \phi_m \cdot \cos (\lambda_{sp} - \lambda_m) \cdot \Delta \phi_m$

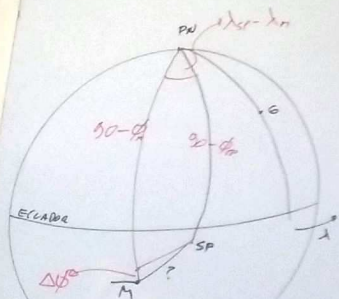
$\cos b \cdot \cos c - \cos a \cdot \cos B = \sin a \cdot \cos B$   
 F. ANALOGA

OPERANDO CON F. SEENO Y F. COSENO:  
 F. 4 PARTES:  
 $\cos b \cdot \sin a = \sin a \cdot \cos c + \sin c \cdot \cos B$

$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$   
 T. SEENO

$\cos c = \dots$   
 $\cos b = \cos a \cdot \cos c + \sin a \cdot \sin c \cdot \cos B$   
 $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$   
 F. COSENO





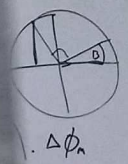
DIST. M-SP  
 M ( $\lambda = -56^\circ, \phi = -35^\circ$ )  
 SP ( $\lambda = -48^\circ, \phi = -23^\circ$ )

DADO  $\Delta\phi_M$   
 $(\Delta\lambda = 0) \Rightarrow \Delta(\pi-SP) ?$

$$\cos(\pi-SP) = \sin\phi_M \cdot \sin\phi_{SP} + \cos\phi_M \cdot \cos\phi_{SP}$$

$$\pi-SP = 33^\circ \cdot \frac{\pi}{180} \text{ (RAD)} \cdot \frac{R_T}{R_T} \rightarrow G'$$

$$-\sin(\pi-SP) \cdot \Delta(\pi-SP) = \sin\phi_M \cdot \cos\phi_{SP} \cdot \Delta\phi$$



$$\cos b \sin c - \sin b \cos c \cos A = \sin a \cos B$$

F. ANALOGA

OPERANDO CON F. SENDO Y F. COSENO :

F. 4 PARTES :

$$\cot b \cdot \sin a = \cos a \cdot \cos c + \sin c \cdot \cot B$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

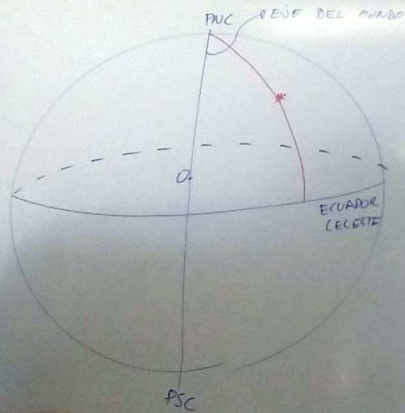
T. SENDO

$$\cos b = \cos a \cdot \cos c + \sin a \cdot \sin c \cdot \cos B$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

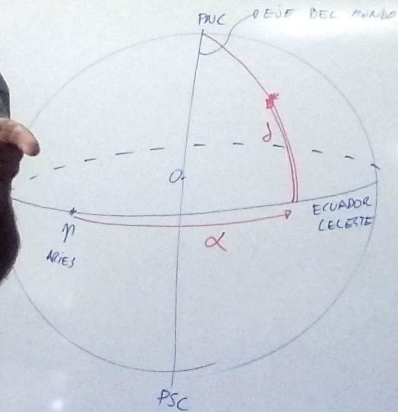
# SISTEMA DE COORD. ECUATORIALES



ESFÉRICAS ( $\alpha, \delta$ )  
RECTANGULARES ( $x, y, z$ )

$\alpha$ : ASCENSIÓN RECTA  
 $\delta$ : DECLINACIÓN

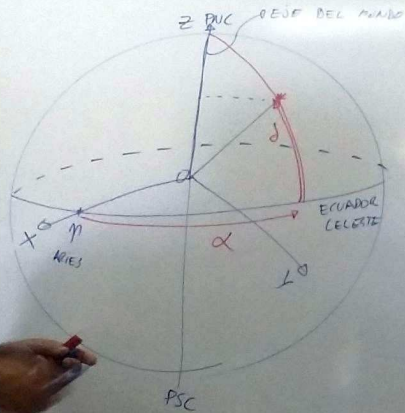
### SISTEMA DE COORD. ECUATORIALES



ESFERICAS  $(\alpha, \delta)$   
RECTANGULARES  $(x, y, z)$

$\alpha$ : ASCENSION RECTA  $(0^h, 24^h)$   
 $\delta$ : DECLINACION  $(-90^\circ, +90^\circ)$

SISTEMA DE COORD ECUATORIALES



ESFERICAS  $(\alpha, \delta)$   
 RECTANGULARES  $(X, Y, Z)$

$X =$

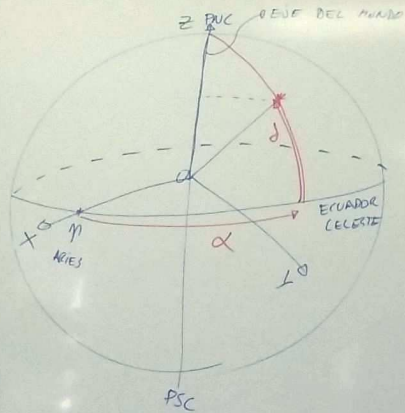
$Y =$

$Z = r \sin \delta$

$\alpha$ : ASCENSION RECTA  $(0^h, 24^h)$

$\delta$ : DECLINACION  $(-90^\circ, +90^\circ)$

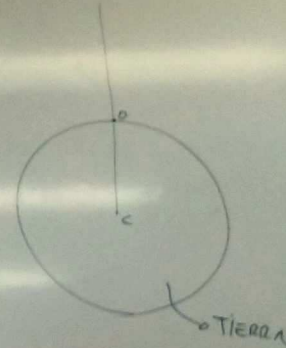
SISTEMA DE COORD ECUATORIALES



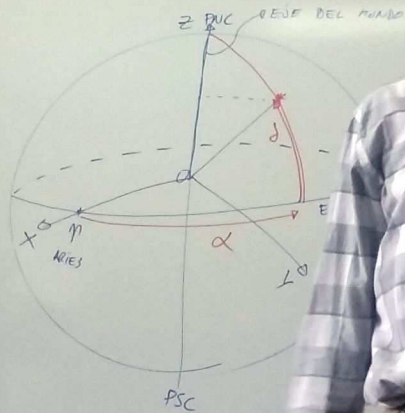
ESFERICAS  $(\alpha, \delta)$   
 RECTANGULARES  $(X, Y, Z)$

ASCENSION RECTA  $(0^h, 24^h)$   
 DECLINACION  $(-90^\circ, +90^\circ)$

$$\begin{cases} X = \cos \delta \cdot \cos \alpha \\ Y = \cos \delta \cdot \sin \alpha \\ Z = \sin \delta \end{cases}$$



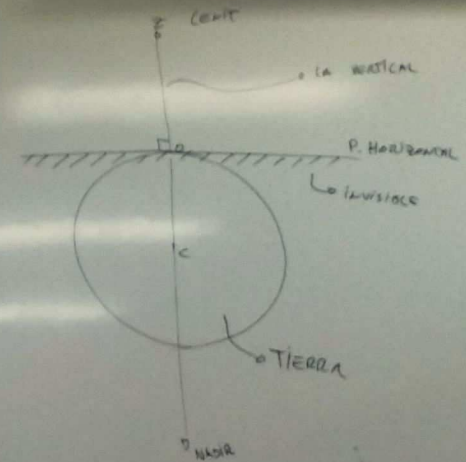
SISTEMA DE COORD. ECUATORIAL



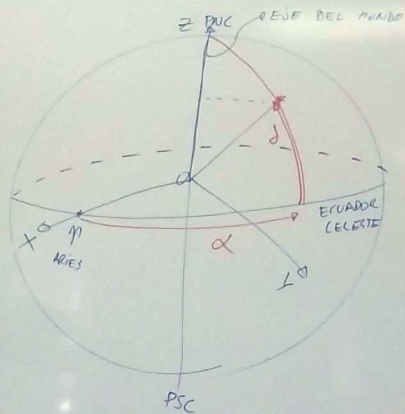
ESFERICAS  $(\alpha, \delta)$

RECTANGULARES  $(x, y, z)$

$\alpha$ : ASCENSION RECTA  $(0^h, 24^h)$   
 $\delta$ : DECLINACION  $(-90^\circ, +90^\circ)$

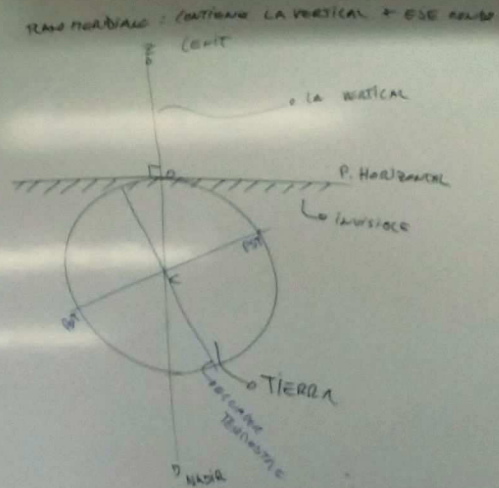


SISTEMA DE COORD. ECUATORIALES

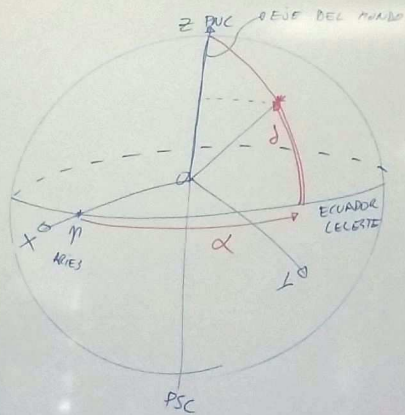


$$\begin{cases} X = \cos \delta \\ Y = \sin \delta \\ Z = \sin \delta \end{cases}$$

$\alpha$ : ASCENSION RECTA ( $0^h, 24^h$ )  
 $\delta$ : DECLINACION ( $-90^\circ, +90^\circ$ )



SISTEMA DE COORD. ECUATORIALES



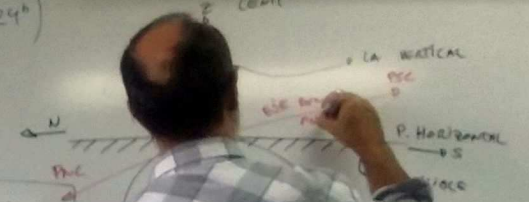
ESFERICAS ( $\alpha, \delta$ )  
 RECTANGULARES ( $x, y, z$ )

$$\begin{cases} X = \cos \delta \cdot \cos \alpha \\ Y = \cos \delta \cdot \sin \alpha \\ Z = \sin \delta \end{cases}$$

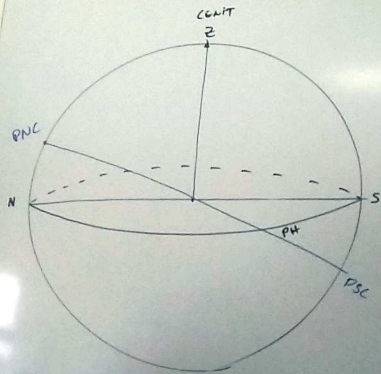
$\alpha$ : ASCENSION RECTA ( $0^h, 24^h$ )  
 $\delta$ : DECLINACION ( $-90^\circ, +90^\circ$ )

MERIDIANA: P. Horiz  $\cap$  P. Meridiano  
 LINEA N-S

TRANS-MERIDIANO: CONTIENE LA VERTICAL + ESE PLANO

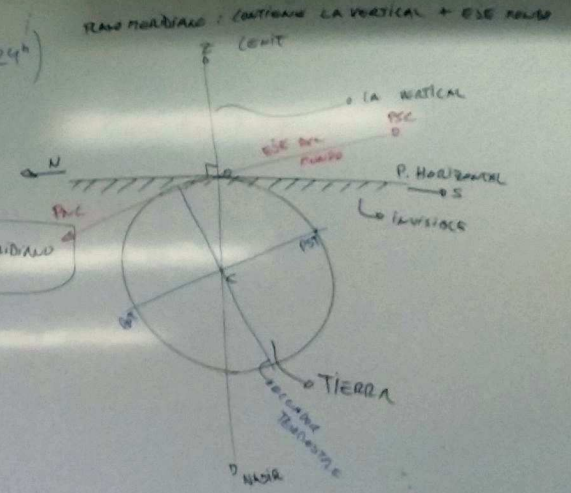




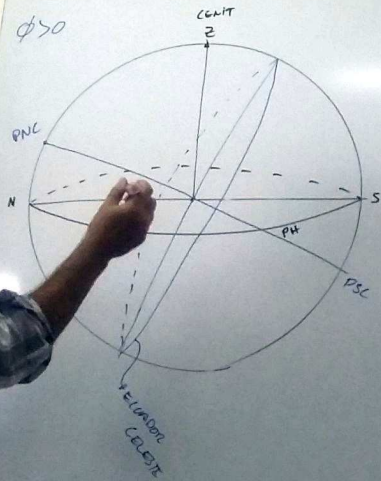


$\alpha$ : ASCENSION RECTA ( $0^h, 24^h$ )  
 $\delta$ : DECLINACION ( $-90^\circ, +90^\circ$ )

MERIDIANA: P. Horiz  $\cap$  P. Meridiano  
 LINEA N-S



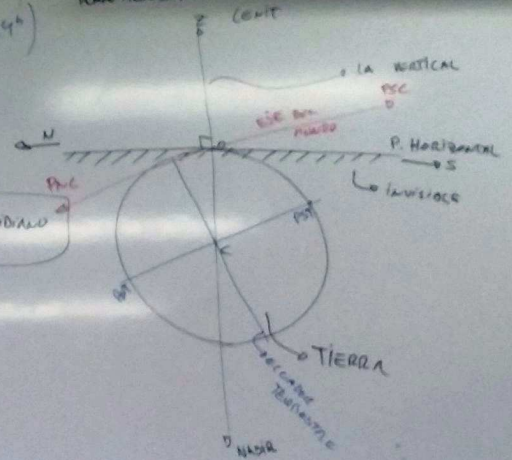
$\phi > 0$

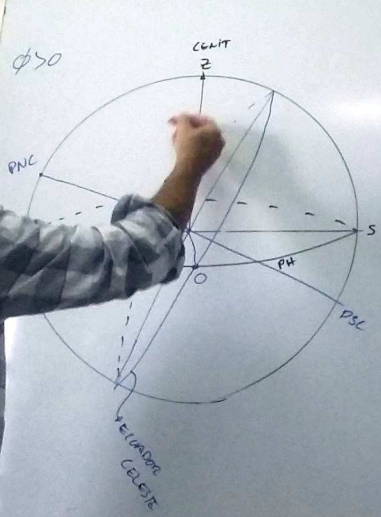


$\alpha$ : ASCENSION RECTA ( $0^h, 24^h$ )  
 $\delta$ : DECLINACION ( $-90^\circ, +90^\circ$ )

MERIDIANA: P. Horiz  $\cap$  P. Meridiano  
 LINEA N-S

TRANS MERIDIANO: CONTIENE LA VERTICAL + EJE MUNDO





20/3/2018 13:15m

$\delta_0 = 0^\circ$  (EQUADOR CELESTE)

$\alpha_0 = 0^h$

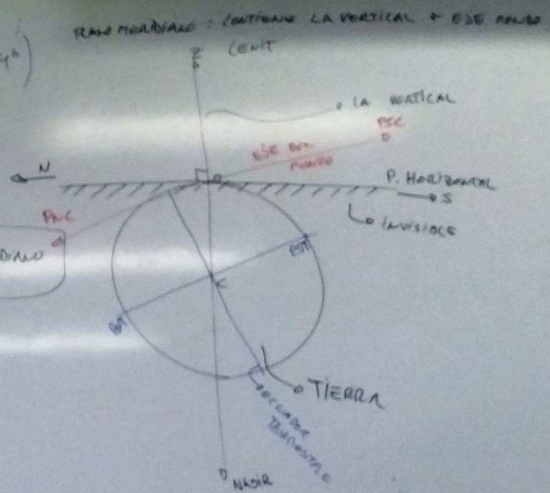
↓ PASÓ POR EL PUNTO ARIETIS

$\alpha$ : ASCENSION RECTA ( $0^h, 24^h$ )

$\delta$ : DECLINACION ( $-90^\circ, +90^\circ$ )

MERIDIANA: P. Horiz  $\perp$  P. Meridiano

LÍNEA N-S

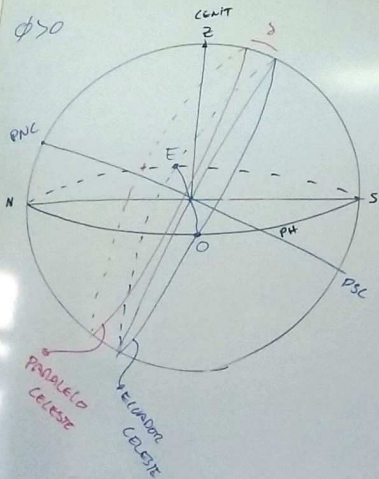


20/3/2018 13:15

$$\delta_0 = 0^\circ \text{ (EQUINOXIO)}$$

$$\alpha_0 = 0^h$$

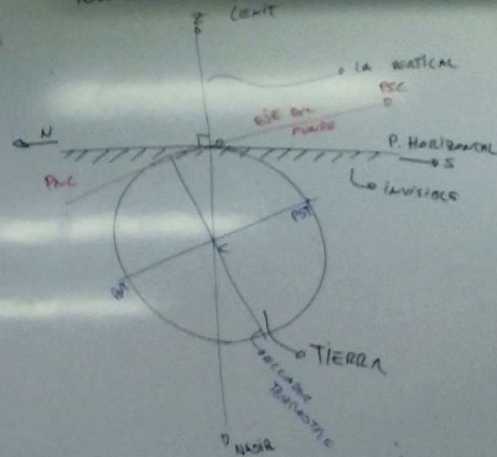
↓  
PASÓ POR EL PUNTO ARIES  
☿



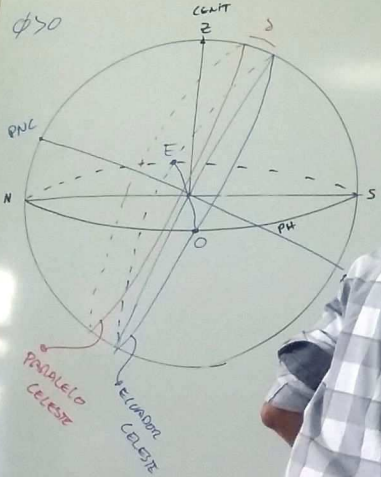
TEO. LATITUD

ALTURA DE PNC =  $\phi$

PLANO MERIDIANO = CONTIENE LA VERTICAL + EJE RADIAL



20/3/2018 13:15



$$\delta_0 = 0^\circ \text{ (EQUADOR CELESTE)}$$

$$\alpha_0 = 0^h$$

↓  
PLANO AN EL PLANO MERIDIANO

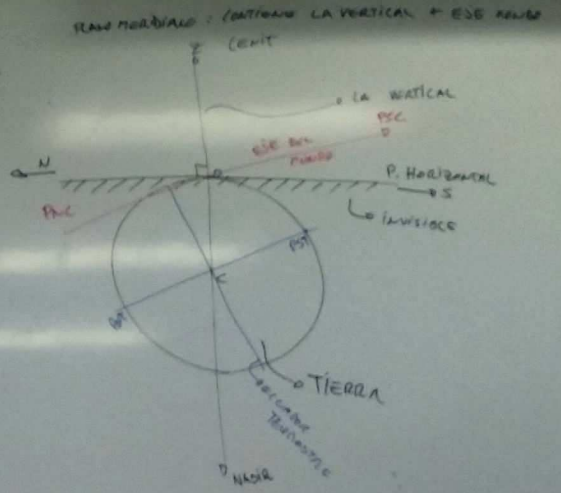
PLANOS HORIZONTALES

+ VERTICAL

A  
a  
(-30°, +30°)

TEO. LATITUD

ALTURA DE PNC =  $\phi$

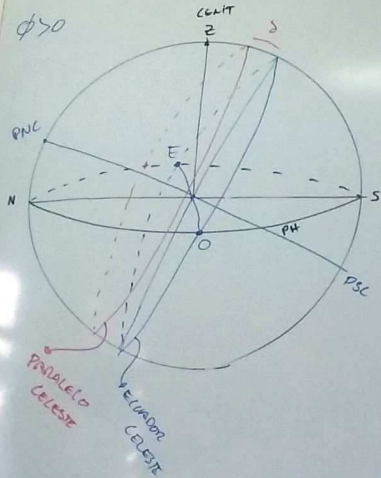


20/3/2018 13:15

$$\delta_0 = 0^\circ \text{ (Ecuador Celeste)}$$

$$\alpha_0 = 0^h$$

↓ PASÓ POR EL PUNTO ARIETES



**COORDENADAS HORIZONTALES**

P. Horiz + VERTICAL

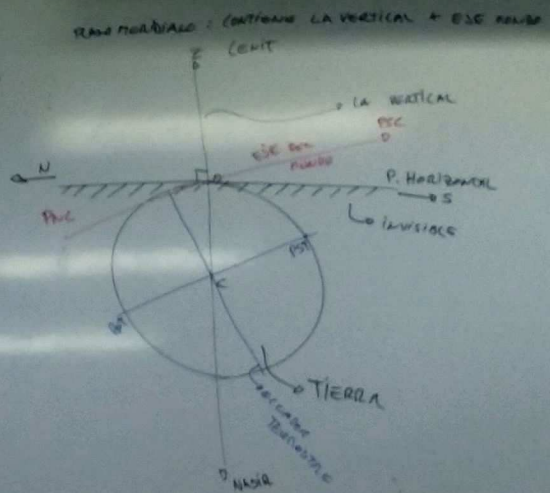
ACIUT :  $A$  (0°, 360°) SENTIDO NOSE

ALTURA :  $a$  (-90°, +90°)

↓ NADIR → CENIT

**TEO. LATITUD**

ALTURA DE PVC =  $\phi$

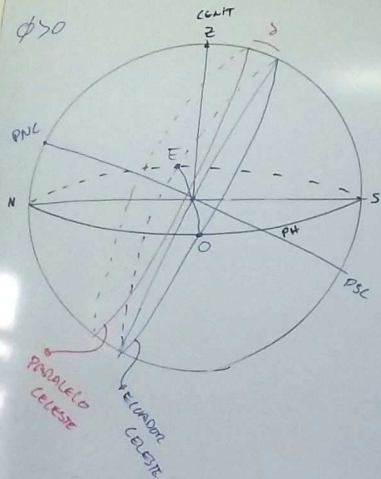


20/3/2018 13:15

$$\delta_0 = 0^\circ \quad (\text{ECUADOR CELESTE})$$

$$\alpha_0 = 0^h$$

↓ PASÓ POR EL PUNTO ARIES  
↑



COORDENADAS HORIZONTALES

P. HORIZ + VERTICAL

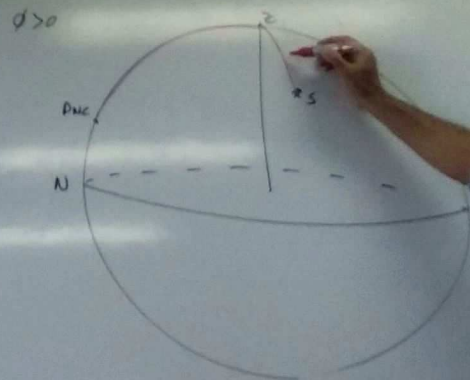
ACIUT :  $A$   $(0^\circ, 360^\circ)$  SENTIDO NOSE

ALTURA :  $a$   $(-90^\circ, +90^\circ)$   
 ↓ NADIR                      → CENIT

TEO. LATITUD

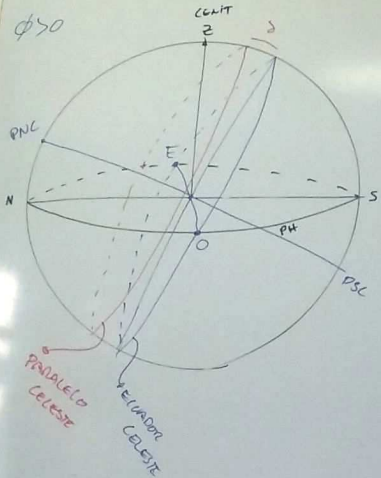
ALTURA DE PNC =  $\phi$

TRIÁNGULO DE POSICIÓN



20/3/2018 13:15

(EJERCICIO)



HORIZONTALES

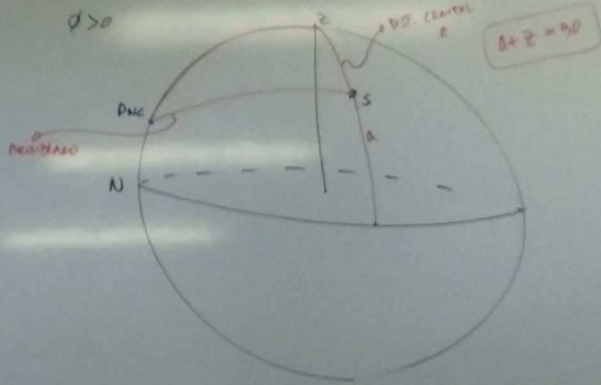
$0^\circ, 360^\circ$  SENTIDO NOSE

$90^\circ +$

TEO. LATITUD

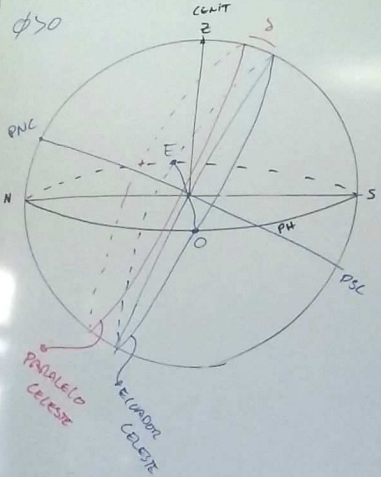
ALTURA DE PNC =  $\phi$

TRIANGULO DE POSICION





$\phi > 0$



20/3/2018 13:15

$$\left. \begin{aligned} \delta_0 &= 0^\circ \text{ (EQUADOR CELESTE)} \\ \alpha_0 &= 0^h \end{aligned} \right\} \text{ PASÓ POR EL PUNTO ARIETES}$$

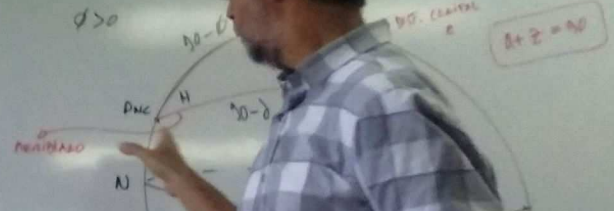
COORDENADAS HORIZONTALES

- P. HORIZ + VERTICAL
- ACIUT :  $A$   $(0^\circ, 360^\circ)$  SENTIDO NOSE
  - ALTURA :  $a$   $(-90^\circ, +90^\circ)$ 
    - $\downarrow$  NAJIE
    - $\rightarrow$  GRUIT
  - DI. ST. CENTRAL  $z$   $(0^\circ, 180^\circ)$

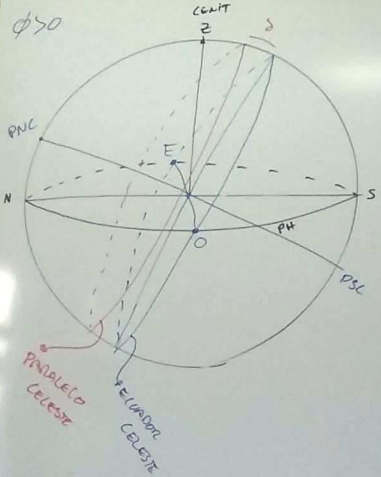
TEO. LATITUD

ALTURA DE PNC =  $\phi$

TRIANGULO POSICION



20/3/2018 13:10



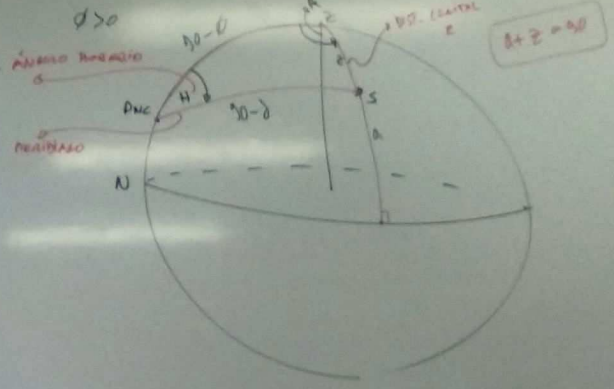
POSICION  
CELESTRE  
MEDIAS  
ARCS  
 $\eta$   
HORIZONTALES  
360° SENTIDO  
NOSE  
0° + 90°  
ZENIT

TEO. LATITUD

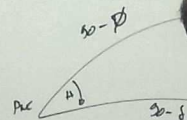
ALTURA DE PNC =  $\phi$

(0h, 24h)

TRIANGULO DE POSICION



Δ. Posición



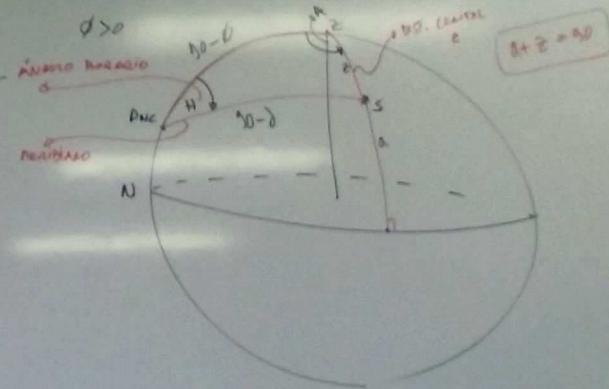
T. coseno :

TEO. LATITUD

ALTURA DE PNC =  $\phi$

$(0^h, 24^h)$

TRIÁNGULO DE POSICIÓN

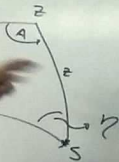


Δ. Posición

T. coseno :

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

PNC

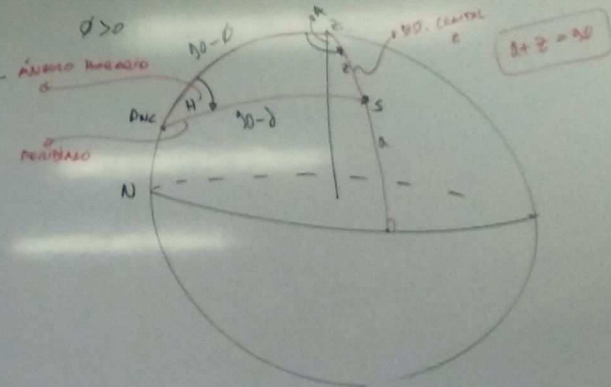


TEO. LATITUD

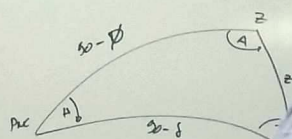
ALTURA DE PNC =  $\phi$

(0<sup>h</sup>.24<sup>h</sup>)

TRIÁNGULO DE POSICIÓN



Δ. Posición



$$\cos Z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

$$= \frac{\sin H}{\sin Z}$$

$$\cos \phi \cdot \cos \delta \cdot \cos H$$

$$- \tan \phi \cdot \tan \delta$$

S, P

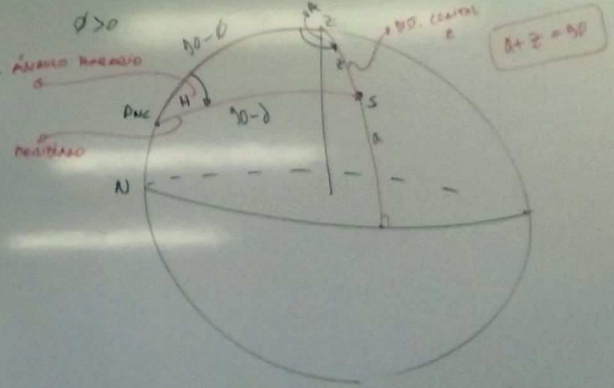
TEO. LATITUD

$$\text{ALTURA DE PNC} = \phi$$

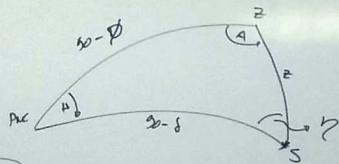
(0<sup>h</sup>.24<sup>h</sup>)

si  $\delta = 0$

TRIÁNGULO DE POSICIÓN



Δ. Posición



CASO 1)

$H(\tau) \equiv$  TIEMPO SIDÉREO LOCAL

T. COSENO :

$$\cos Z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

$$\frac{\sin A}{\cos \delta} = \frac{\sin H}{\sin Z}$$

SALIDA Y PUESTA

$$Z = 50^\circ$$

$$0 = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

$$\Rightarrow \cos H = -\frac{\tan \phi \cdot \tan \delta}{\sin \phi}$$

TEO. LATITUD

ALTURA DE PNC = 0

(0h, 24h)

si  $\delta = 0$

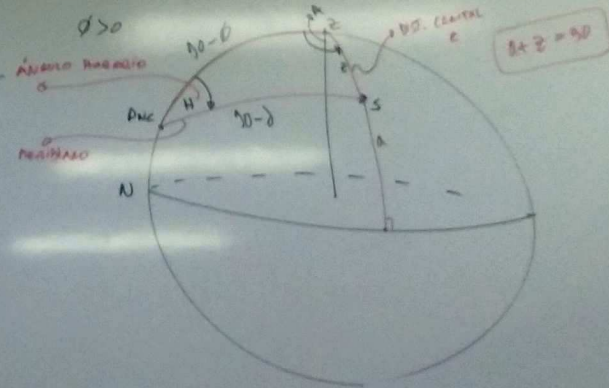
$$\cos H_{sup} = 0$$

$$\Rightarrow H = 90^\circ$$

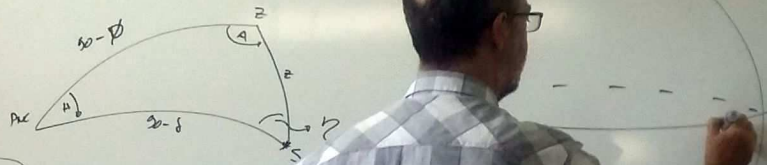
6h

15° → 1h (360° → 24h)

TRIÁNGULO DE POSICIÓN



Δ. Posición



CASO  $\eta$

$H(\eta) \equiv$  TIEMPO SIDÉREO L

TEO. LATITUD

ALTURA DE PNC =  $\phi$

( $0^h, 24^h$ )

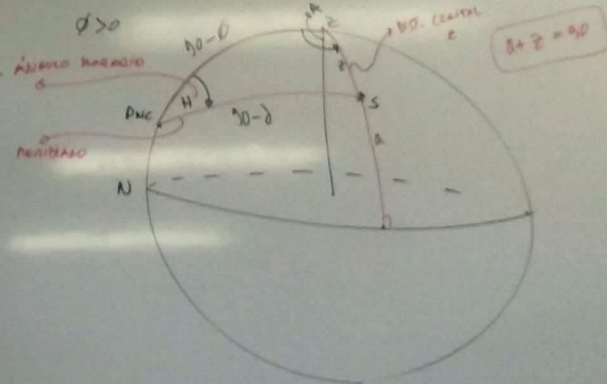
si  $\delta = 0$

$\hookrightarrow H_{sur} = 0$

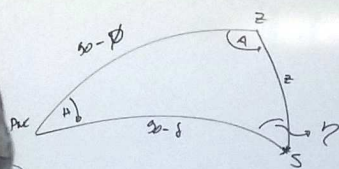
$\Rightarrow H = 90^\circ$   
6h

$15^\circ \rightarrow 1^h$  ( $360^\circ \rightarrow 24^h$ )

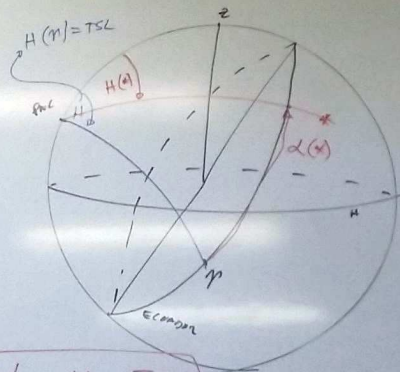
TRIÁNGULO DE POSICIÓN



Δ. Posición



TIEMPO SIDÉREO LOCAL



$$\alpha_* + H_* = TSL$$

TEO. LATITUD

ALTURA DE PNC = 0

(0h, 24h)

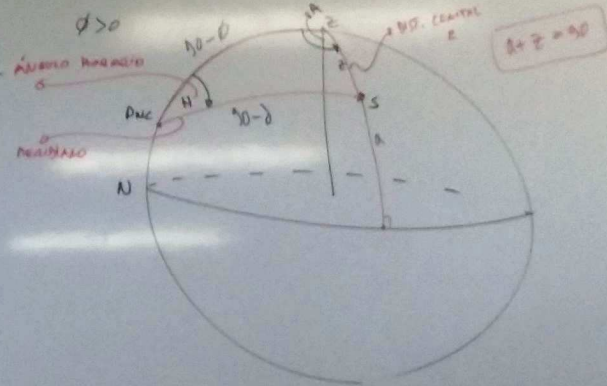
si  $\delta = 0$

$\Rightarrow H_{sur} = 0$

$\Rightarrow H = 90^\circ$   
6h

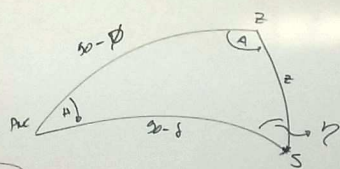
15° → 1h (360° → 24h)

TRIÁNGULO DE POSICIÓN



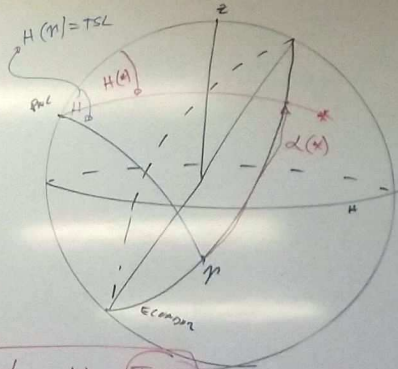


Δ. Posición



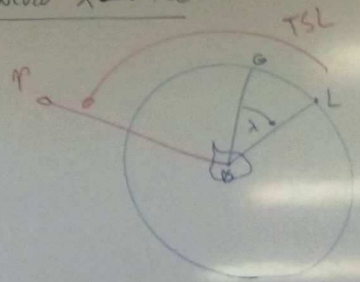
CASO  $\mu$

$H(\mu) \equiv$  TIEMPO SIDÉREO LOCAL

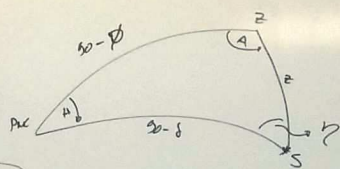


$\alpha_* + H_* = TSL$   
 RELOJ

VINCULO  $\lambda \leftrightarrow TSL$

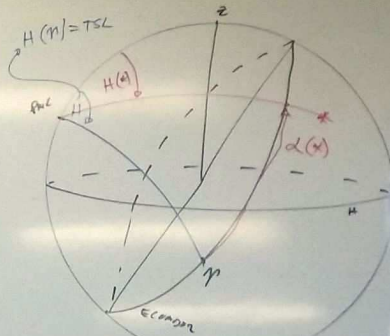


Δ. Posición



CASO  $\eta$

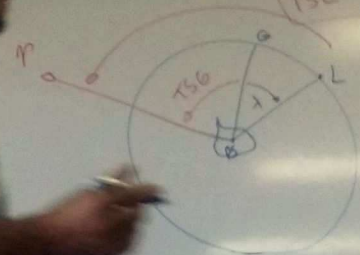
$H(\eta) \equiv$  TIEMPO SIDÉREO LOCAL



$\alpha_* + H_* = TSL$

RELOJ

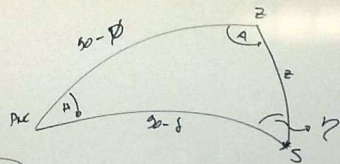
ECUADOR  $\lambda \leftrightarrow TSL$



$TSL = TSG + \lambda$

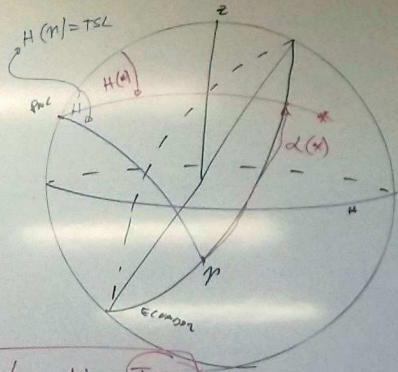
RELOJ "universal"

Δ. Posición



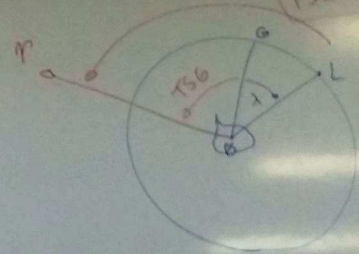
CASO  $\eta$

$H(\eta) \equiv$  TIEMPO SIDÉREO LOCAL



$\alpha_{*} + H_{*} = TSL$   
RELOJ

VINCULO  $\lambda \leftrightarrow TSL$



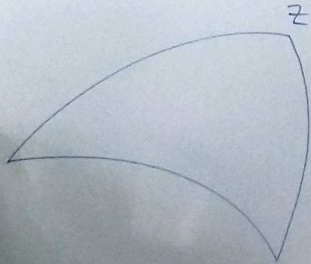
$TSL = TSG + \lambda$

Reloj "universal"

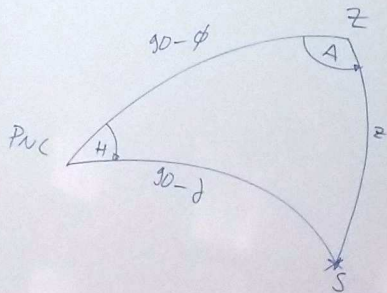
FÓRMULA

DADA FECHA  
HORA }  $\rightarrow$  TSG

COORDENADAS ECUATORIALES TERRESTRES (GEOGRAFICAS)  $(\lambda, \phi)$   
" " CELESTES  $(\alpha, \delta)$   
" HORIZONTALES  $(A, a)$



[ COORDENADAS ECUATORIALES TERRESTRES (GEOGRAFICAS)  $(\lambda, \phi)$   
 " " CELESTES  $(\alpha, \delta)$   
 " HORIZONTALES  $(A, a)$  ]



[ DADO  $t$  y ESTRELLA  $(\alpha, \delta)$   
 OBSERVANDO  $(A, a)$   
 HALLAR  $(\lambda, \phi)$  ]

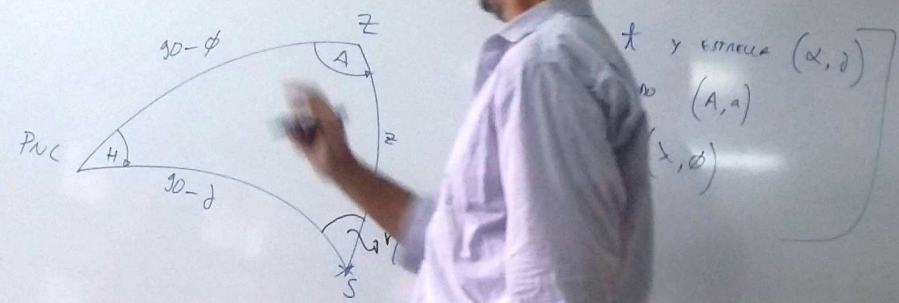
$$TSL = \alpha + H$$

$$TSL = \alpha_T + H_T$$

F. SERVO:

$$\frac{y-A}{d} = \frac{y-H}{d}$$

COORDENADAS ECUATORIALES TERRESTRES (GEOGRAFICAS)  $(\lambda, \phi)$   
 " " " " (CELESTIALES)  $(\alpha, \delta)$   
 " " " " HORIZONTALES  $(A, a)$



$$TSL = \alpha + H$$

$$TSL = \alpha_p + H_p$$

F. seno:

$$\frac{\sin A}{\sin \delta} = \frac{\sin H}{\sin \phi} = \frac{\sin \lambda}{\sin \phi}$$

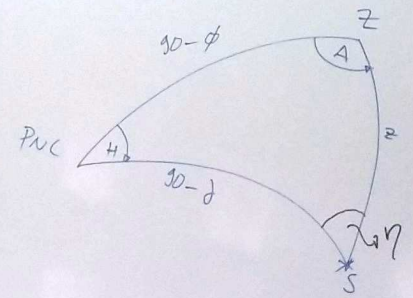
F. coseno:

$$\cos \phi = \sin \delta \cdot \cos z + \cos \delta \cdot \sin z \cdot \cos \lambda$$

$$\cos z = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos H$$

$$\sin \delta =$$

[ COORDENADAS ECUATORIALES TERRESTRES (GEOGRÁFICAS)  $(\lambda, \phi)$   
 " " CELESTES  $(\alpha, \delta)$   
 " " HORIZONTALES  $(A, a)$  ]



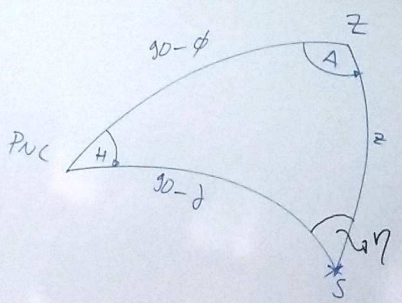
[ DADO  $t$  y estrella  $(\alpha, \delta)$   
 OBSERVANDO  $(A, a)$   
 HALLAR  $(\lambda, \phi)$  ]

$TSL = \alpha + H$   
 $TSL = \alpha_T + H_T$   
 -0

F. SERO:  $\frac{\sin A}{\sin \delta} = \frac{\sin H}{\sin z} = \frac{\sin \eta}{\sin \phi}$   
 $= \sin \delta \cdot \cos z + \cos \delta \cdot \sin z \cdot \cos \eta$   
 $= \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos H$   
 $\cos z \cdot \sin \phi + \sin z \cdot \cos \phi \cdot \cos A$

$\cos \phi = \pm \sqrt{1 - \sin^2 \delta}$

COORDENADAS ECUATORIALES TERRESTRES (GEOGRAFICAS)  $(\lambda, \phi)$   
 " " CELESTES  $(\alpha, \delta)$   
 " " HORIZONTALES  $(A, a)$



DADO  $t$  y estrella  $(\alpha, \delta)$   
 OBSERVANDO  $(A, a)$   
 HALLAR  $(\lambda, \phi)$

$$TSL = \alpha + H$$

$$TSL = \alpha_T + H_T$$

F. seno:

$$\frac{\sin A}{\sin \delta} = \frac{\sin H}{\sin \phi} = \frac{\sin \eta}{\sin \phi}$$

F. coseno:

$$\cos \phi = \sin \delta \cdot \cos z + \cos \delta \cdot \sin z \cdot \cos \eta$$

$$\cos z = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos H$$

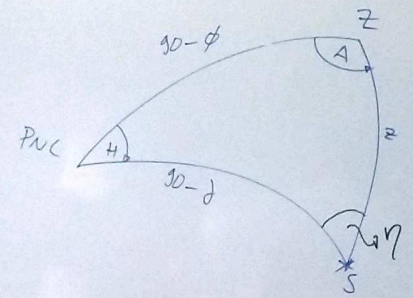
$$\sin \delta = \cos z \cdot \sin \phi + \sin z \cdot \cos \phi \cdot \cos A$$

$\rightarrow x, y \Rightarrow \phi$

$$\cos \phi = + \sqrt{1 - \sin^2 \phi}$$



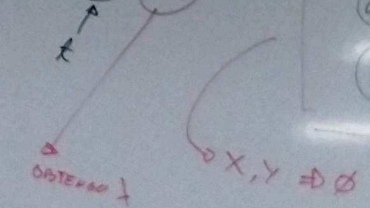
COORDENADAS ECUATORIALES TERRESTRES (GEOGRAFICAS)  $(\lambda, \phi)$   
 " " CELESTES  $(\alpha, \delta)$   
 " " HORIZONTALES  $(A, a)$



DADO  $t$  y estrella  $(\alpha, \delta)$   
 OBSERVANDO  $(A, a)$   
 HALLAR  $(\lambda, \phi)$

$TSL = \lambda + H$   
 $TSL = \lambda_T + H_T$

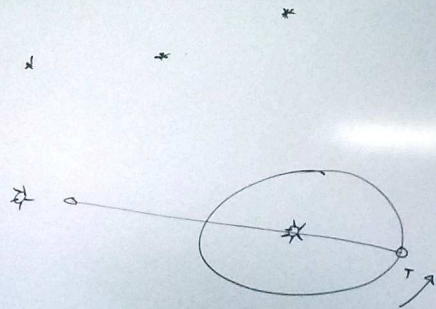
$TSL = TSG + \lambda$



F. seno:  $\frac{\sin A}{\sin \delta} = \frac{\sin H}{\sin \phi} = \frac{\sin \eta}{\sin \phi}$

F. coseno:  
 $\cos \phi = \sin \delta \cdot \cos z + \cos \delta \cdot \sin z \cdot \cos \eta$   
 $\cos z = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos H$   
 $\sin \delta = \cos z \cdot \sin \phi + \sin z \cdot \cos \phi \cdot \cos A$

$\cos \phi = + \sqrt{1 - \sin^2 \phi}$



Estimular Púas

F. SERVO:

$$\frac{r_A}{\cos \delta} = \frac{r_H}{\cos \epsilon} = \frac{r_H \eta}{\cos \phi}$$

F. COSMO:

$$r \sin \phi = r \cos \delta \cdot \cos \epsilon + r \sin \delta \cdot \cos \epsilon \cdot \cos \eta$$

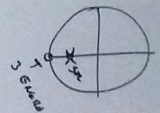
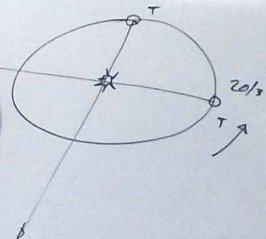
$$\cos \delta = r \cos \phi \cdot \cos \epsilon + r \sin \phi \cdot \cos \epsilon \cdot \cos \eta$$

$$r \cos \delta = r \cos \phi \cdot \cos \epsilon + r \sin \phi \cdot \cos \epsilon \cdot \cos \eta$$

APP  
SINUS  
CLOCK

$\phi, \delta, \epsilon, \eta$

$$\cos \phi = \sqrt{1 - \sin^2 \phi}$$



F. seno:

$$\frac{\sin A}{\sin a} = \frac{\sin H}{\sin z} = \frac{\sin \varphi}{\sin \phi}$$

F. coseno:

$$\cos \phi = \cos \delta \cdot \cos z + \sin \delta \cdot \sin z \cdot \cos \varphi$$

$$\cos z = \cos \delta \cdot \sin \phi + \sin \delta \cdot \cos \phi \cdot \cos H$$

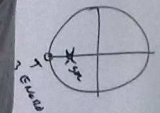
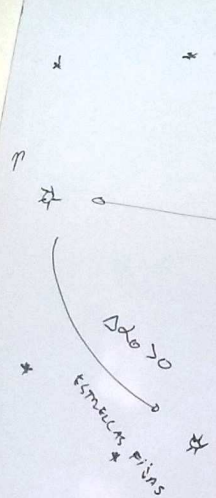
$$\sin \delta = \sin z \cdot \sin \phi + \cos z \cdot \cos \phi \cdot \cos A$$

APP  
Spherical  
triangle

$\phi, \delta, \gamma \Rightarrow \phi$

$$\cos \phi = \sqrt{1 - \sin^2 \phi}$$

LA ECLIPTICA  $\equiv E$



F. seno:

$$\frac{\sin A}{\sin D} = \frac{\sin H}{\sin \phi} = \frac{\sin \eta}{\sin \phi}$$

F. coseno:

$$\begin{aligned} \cos \phi &= \cos \delta \cdot \cos z + \sin \delta \cdot \sin z \cdot \cos \eta \\ \cos z &= \cos \delta \cdot \sin \phi + \sin \delta \cdot \cos \phi \cdot \cos H \\ \sin \delta &= \sin z \cdot \sin \phi + \cos z \cdot \cos \phi \cdot \cos A \end{aligned}$$

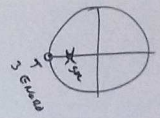
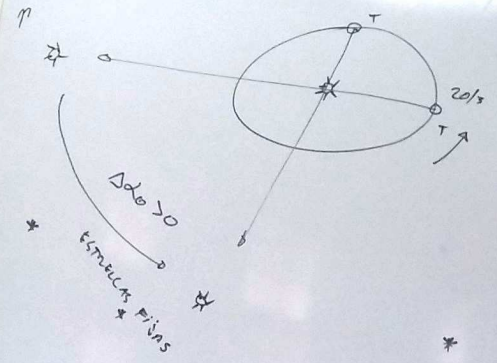
App Spherical trigonometry

$\delta, \eta, \phi, \eta, \delta, \phi$

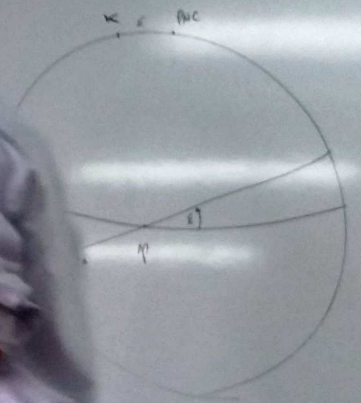
$$\cos \phi = \sqrt{1 - \sin^2 \delta}$$

LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORBITAL TERRESTRE

COORD. ECLIPTICAS



$\Sigma =$   
 OBLICUO



APP  
 SINGULAR  
 CLASE

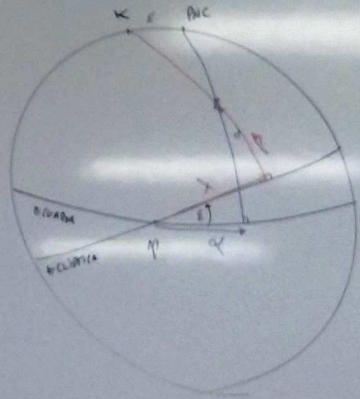
LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRESTRE

COORD. ECLIPTICAS

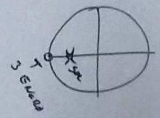
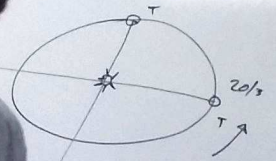
$\epsilon = 23^{\circ} 27'$

OBLICUIDAD

- $\lambda =$  LONGITUD ECLIPTICA  $(0^{\circ}, 360^{\circ})$
- $\beta =$  LATITUD ECLIPTICA  $(-90^{\circ}, +90^{\circ})$



APP  
 SYSTEM  
 CLOCK



COORD. ECUAL. CELESTES

$(x, y, z)$

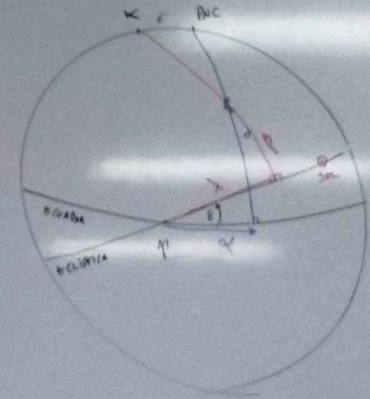
LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRESTRE

COORD. ECLIPTICAS

RECTANGULARES  $(x, y, z)$   
 ESPHERICAS  $(\lambda, \beta)$

$\epsilon = 23^\circ 27'$   
 OBLICUIDAD

$\lambda =$  LONGITUD ECLIPTICA  
 $(0^\circ, 360^\circ)$   
 $\beta =$  LATITUD ECLIPTICA  
 $(-90^\circ, +90^\circ)$



$\beta_0 = 0^\circ$

APP  
 SIDEREAL  
 CLOCK

LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA

COORD. ECLIPTICAS

RECTANGULARES  $(x, y, z)$

ESFERICAS  $(\lambda, \beta)$

OBLICUIDAD  $\epsilon = 23^\circ 27'$

$\lambda =$  LONGITUD ECLIPTICA  $(0^\circ, 360^\circ)$

$\beta =$  LATITUD ECLIPTICA  $(-90^\circ, +90^\circ)$

$\beta_0 = 0^\circ$

APR SISTEMAS CLASE

CELESTES  $(x, y, z)$

(2.1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$

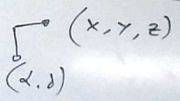
$X =$

$Y =$

$Z =$



T. CELESTES



LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRAZA

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} x = \cos \delta \cdot \cos \alpha \\ y = \cos \delta \cdot \sin \alpha \\ z = \sin \delta \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

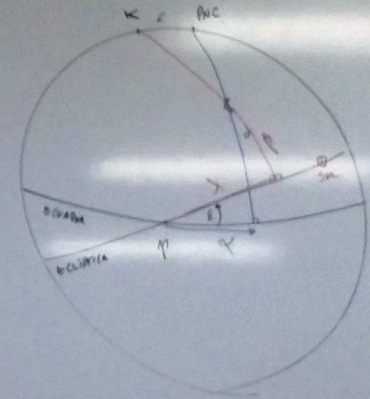
$$\begin{cases} \xi = x + 0 + 0 \\ \eta = 0 + y \cos \epsilon + z \sin \epsilon \\ \zeta = 0 - y \sin \epsilon + z \cos \epsilon \end{cases}$$

COORD. ECLIPTICAS

RECTANGULARES  $(x, y, z)$   
 ESPHERICAS  $(\lambda, \beta)$

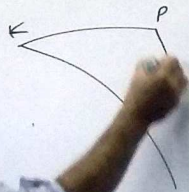
$\epsilon = 23^\circ 27'$   
 OBLICUIDAD

$\lambda =$  LONGITUD ECLIPTICA  $(0^\circ, 360^\circ)$   
 $\beta =$  LATITUD ECLIPTICA  $(-90^\circ, +90^\circ)$



$\beta_0 = 0^\circ$

$$\begin{cases} \xi = \cos \beta \cdot \cos \lambda \\ \eta = \cos \beta \cdot \sin \lambda \\ \zeta = \sin \beta \end{cases}$$



LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA ORIGINAL TERRESTRE

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$\xi = X + 0 + 0$$

$$\eta = 0 + Y \cos \epsilon + Z \sin \epsilon$$

$$\zeta = 0 - Y \sin \epsilon + Z \cos \epsilon$$

$$\cos \beta \cos \lambda = \cos \delta \cos \alpha$$

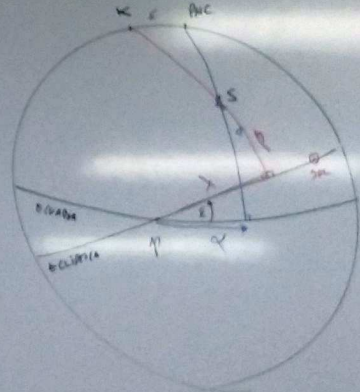
$$\sin \beta = -\cos \delta \sin \alpha \cos \epsilon + \sin \delta \cos \epsilon$$

COORD. ECLIPTICAS

RECTANGULARES  $(\xi, \eta, \zeta)$   
ESFERICAS  $(\lambda, \beta)$

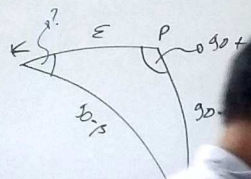
$\epsilon = 23^\circ 27'$   
OBLICUIDAD

$\lambda =$  LONGITUD ECLIPTICA  $(0^\circ, 360^\circ)$   
 $\beta =$  LATITUD ECLIPTICA  $(-90^\circ, +90^\circ)$



$\beta_0 = 0^\circ$

$$\begin{cases} \xi = \cos \beta \cdot \cos \lambda \\ \eta = \cos \beta \cdot \sin \lambda \\ \zeta = \sin \beta \end{cases}$$



LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRESTRE

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$\xi = x + 0 + 0$$

$$\cos \beta \cos \lambda = \cos \delta \cos \alpha$$

$$\eta = 0 + y \cos \epsilon + z \sin \epsilon$$

$$\zeta = 0 - y \sin \epsilon + z \cos \epsilon$$

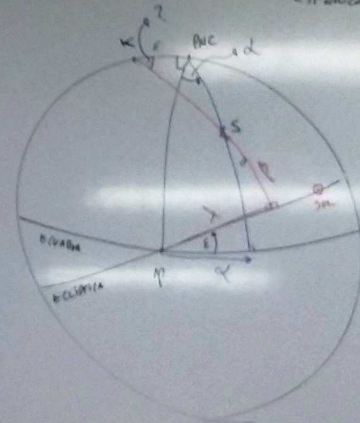
$$\sin \beta = -\cos \delta \sin \alpha \sin \epsilon + \sin \delta \cos \epsilon$$

COORD. ECLIPTICAS

RECTANGULAS  $(x, y, z)$   
 ESFERICAS  $(\lambda, \beta)$

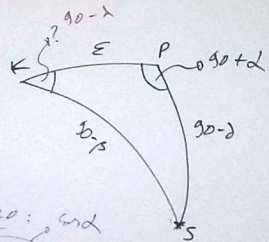
$\epsilon = 23^\circ 27'$   
 OBLICUIDAD

$\lambda$  = LONGITUD ECLIPTICA  
 ( $0^\circ, 360^\circ$ )  
 $\beta$  = LATITUD ECLIPTICA  
 ( $-90^\circ, +90^\circ$ )



$\beta_0 = 0^\circ$

$$\begin{cases} \xi = \cos \beta \cdot \cos \lambda \\ \eta = \cos \beta \cdot \sin \lambda \\ \zeta = \sin \beta \end{cases}$$



F. SELD:  $\cos \delta$

$$\frac{\sin(90 + \delta)}{\sin(90 - \lambda)} = \frac{\sin(90 - \beta)}{\sin(90 - \delta)}$$

$\cos \delta$

$\cos \delta$

$$\Rightarrow \cos \delta \cdot \cos \lambda = \cos \beta \quad (1)$$

LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRESTRE

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$\xi = X + 0 + 0$$

$$\cos \delta \cos \lambda = \cos \delta \cdot \cos \lambda \quad (1)$$

$$\eta = 0 + Y \cos \epsilon + Z \sin \epsilon$$

$$\zeta = 0 - Y \sin \epsilon + Z \cos \epsilon$$

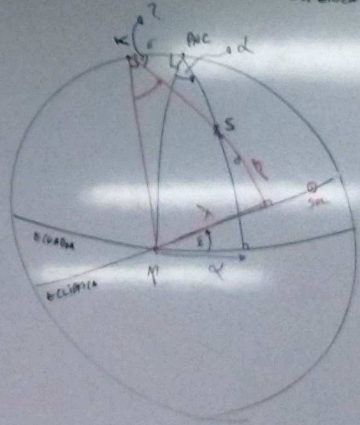
$$\sin \beta = -\cos \delta \cdot \sin \lambda \cdot \sin \epsilon + \sin \delta \cdot \cos \epsilon$$

COORD. ECLIPTICAS

RECTANGULAS  $(\xi, \eta, \zeta)$   
ESPHERICAS  $(\lambda, \beta)$

$\epsilon = 23^\circ 27'$   
OBLICUIDAD

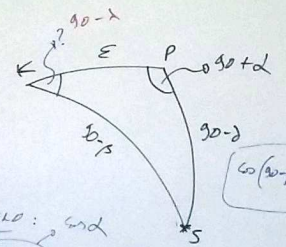
$\lambda =$  LONGITUD ECLIPTICA  
( $0^\circ, 360^\circ$ )  
 $\beta =$  LATITUD ECLIPTICA  
( $-90^\circ, +90^\circ$ )



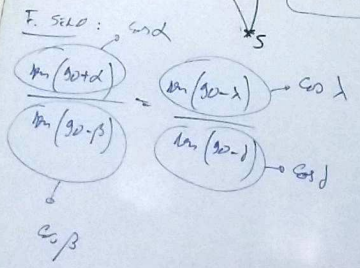
$\beta_0 = 0^\circ$

$$\begin{cases} \xi = \cos \beta \cdot \cos \lambda \\ \eta = \cos \beta \cdot \sin \lambda \\ \zeta = \sin \beta \end{cases}$$

LA ECLIPTICA  $\equiv$  ESFERA CELESTE  $\cap$  PLANO DE LA ECLIPTICA  
PLANO ORIGINAL TERRA-SOL



$$\cos(90-\rho) = \cos(90-d) \cdot \cos \epsilon + \sin(90-d) \cdot \sin \epsilon \cdot \cos(90+d) \quad (2)$$



$$\Rightarrow \cos d \cdot \cos \lambda = \cos \lambda \cdot \cos \beta \quad (1)$$

$$\xi = X + 0 + 0 \rightarrow \cos \rho \cos \lambda = \cos d \cdot \cos \lambda \quad (1)$$

$$\eta = 0 + y \cos \epsilon + z \sin \epsilon$$

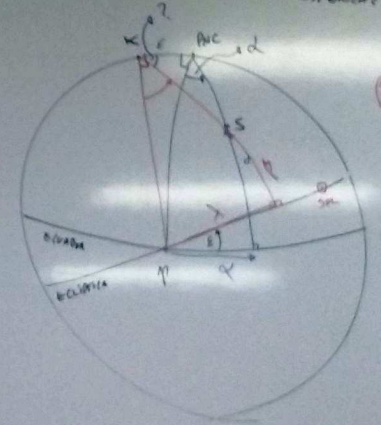
$$\zeta = 0 - y \sin \epsilon + z \cos \epsilon \rightarrow \sin \rho = -\cos d \cdot \sin \epsilon + \sin d \cdot \cos \epsilon \quad (2)$$

$\epsilon = 23^\circ 27'$   
 OBLICUIDAD

$\lambda =$  LONGITUD ECLIPTICA  
 ( $0^\circ, 360^\circ$ )  
 $\beta =$  LATITUD ECLIPTICA  
 ( $-90^\circ, +90^\circ$ )

COORD. ECLIPTICAS

- RECTANGULAS ( $\xi, \eta, \zeta$ )
- ESFERICAS ( $\lambda, \beta$ )

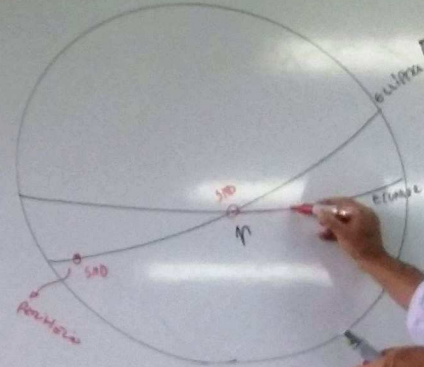


$$\begin{cases} \xi = \cos \beta \cdot \cos \lambda \\ \eta = \cos \beta \cdot \sin \lambda \\ \zeta = \sin \beta \end{cases}$$

$\beta_0 = 0^\circ$

TIEMPO → SIDÉREO :  $H(\pi)$   
 SOLAR → APARENTE :  $S_{\text{V}} \text{ VISIBL}$   
 MEDID :  $S_{\text{L}} \text{ MEDID}$

Soc  
 Med  
 Barrio }  $\frac{\Delta \lambda_{\text{sol}}}{\Delta t} = c \tau$



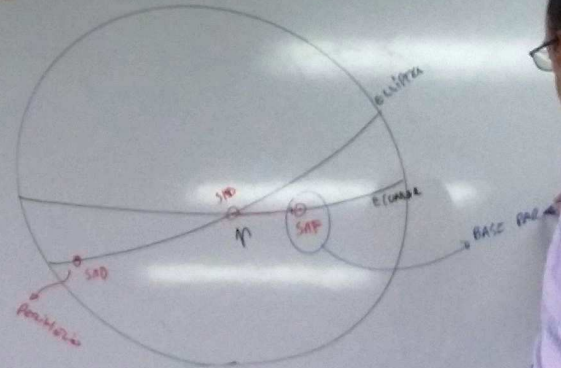
TIEMPO → SIDÉREO :  $H(\pi)$   
 SOLAR → APARENTE :  $S_{\text{V}} \text{ VISIBL}$   
 MEDIO :  $S_{\text{M}} \text{ MEDIO}$

$$\left. \begin{array}{l} \text{Sol} \\ \text{Medio} \\ \text{Barrido} \end{array} \right\} \frac{\Delta \lambda_{\text{sol}}}{\Delta t} = \text{cte}$$

$$\left[ \begin{array}{l} \text{Sol} \\ \text{Medio} \\ \text{Futuro} \end{array} \right]$$

"Sol ficticio"

$$\frac{\Delta \lambda_{\text{SF}}}{\Delta t} = \text{cte}$$



TIEMPO → SIDÉREO :  $H(\pi)$   
 SOLAR → APARENTE :  $S_{A}$  VISIBLE  
 MEDIO :  $S_{M}$  SOL MEDIO

$$T. \text{ SIDÉREO} = H(\pi)$$

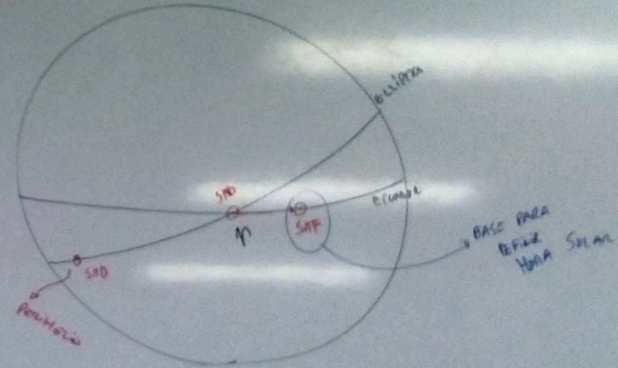
$$T. \text{ SOLAR APARENTE} = H_0 + 12^h$$

$$T. \text{ SOLAR MEDIO} = H_{SF} + 12^h$$

↑  
FICTICIO

SOL MEDIO QUINILIO }  $\frac{\Delta \lambda_{SMQ}}{\Delta t} = \text{cte}$

SOL MEDIO FICTICIO }  
 "SOL MEDIO"  
 $\frac{\Delta \lambda_{SF}}{\Delta t} = \text{cte}$





TIEMPO SIDÉREO :  $H(\pi)$

SOLAR  
 APARENTE :  $S_{\alpha}$  VISIBL  
 MEDIO :  $S_{\alpha}$  MEDIO

$$T. S_{\alpha} AP - T. S_{\alpha} MEDIO = H_{\theta} - H_{\varphi} = TSL - \alpha_{\theta} - (TSL - \alpha_{\varphi})$$

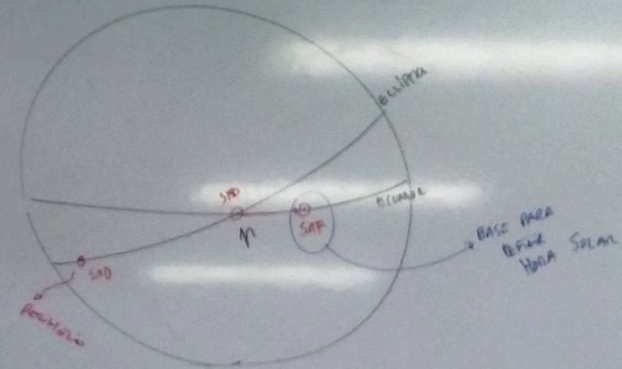
$$TSL = H + \alpha$$

$$T. SIDÉREO = H(\pi)$$

$$T. SOLAR APARENTE = H_{\theta}$$

$$T. SOLAR MEDIO = H_{\varphi}$$

↑  
FICTIVO



TIEMPO SIDÉREO :  $H(\pi)$   
 SOLAR  
 - APARENTE : SOL VISUAL  
 - MEDIO : SOL MEDIO

$T. \text{SIDÉREO} = H(\pi)$

$T. \text{SOLAR APARENTE} = H_0 + 12^h$

$T. \text{SOLAR MEDIO} = H_{SF} + 12^h$   
 ↑  
 FICTICIO

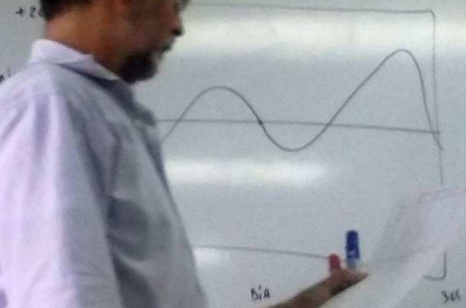
$$T. \text{Sol Ap} - T. \text{Sol Medio} = H_0 - H_{SF} =$$

$$= TSL - \alpha_0 - (TSL - \alpha_{SF})$$

$$= \alpha_{SF} - \alpha_0$$

$T. \text{Sol Ap} - T. \text{Sol Medio} = \alpha_{SF} - \alpha_0$   
 ECLIPCIÓN DEL TIEMPO

$TSL = H + \alpha$



TIEMPO SIDÉREO :  $H(\pi)$

SOLAR  
 - APARENTE : SOL VISIBLE  
 - MEDIO : SOL MEDIO

$T. \text{SIDÉREO} = H(\pi)$

$T. \text{SOLAR APARENTE} = H_0 + 12^h$

$T. \text{SOLAR MEDIO} = H_{SF} + 12^h$   
FICTICIO

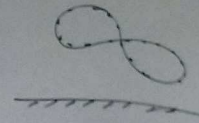
$T. \text{Sol. Ap} - T. \text{Sol. Medio} = H_0 - H_{SF} =$

$= TSL - (TSL - \alpha_{sf})$   
 $= \alpha_{sf}$

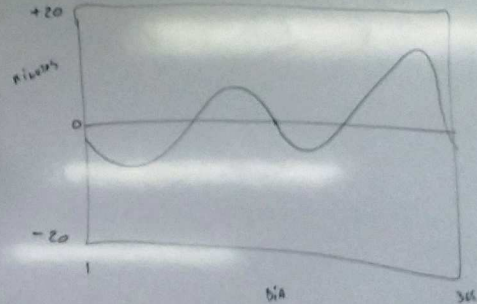
$T. \text{Sol. Ap} - T. \text{Sol. Medio} = \alpha_{sf}$

ECLIPCIÓN DEL

$TSL = H + \alpha$



ANALEMA



Tiempo  
 ↓  
 S

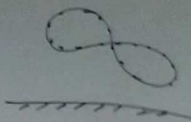
T. Sin AP - T. Sin Medio =  $H_0 - H_{sf} =$   
 $= TSL - d_0 - (TSL - d_{sf})$   
 $= d_{sf} - d_0$

$T. Sin. AP - T. Sin. Medio = d_{sf} - d_0$   
 ECLACION DEL TIEMPO

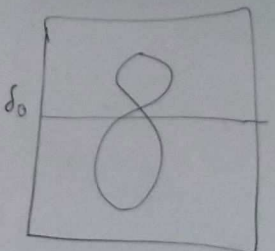
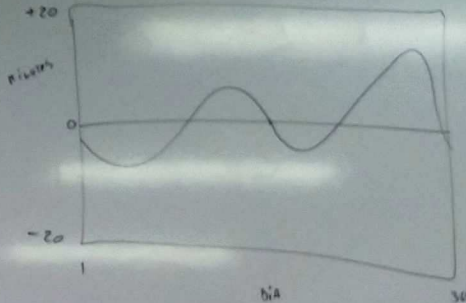
T. Sin AP  
 T. Sin Medio  
 T. Sin

TSL =  $H + d$

$d_{sf} = 0$



ALABAMA

TIEMPO SIDÉREO :  $H(\pi)$

SOLAR  
 APARENTE : SOL VISIBLE  
 MEDIO : SOL MEDIO

$T. \text{ SIDÉREO} = H(\pi)$

$T. \text{ SOLAR APARENTE} = H_0 +$

$T. \text{ SOLAR MEDIO} = H_{SF} + \text{FICTICIO}$

$T. \text{ SOL MEDIO} = H_0 - H_{SF} =$

$= TSL - \alpha_0 - (TSL - \alpha_{SF})$

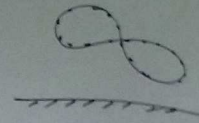
$= \alpha_{SF} - \alpha_0$

$\alpha_{SF} - \alpha_0$

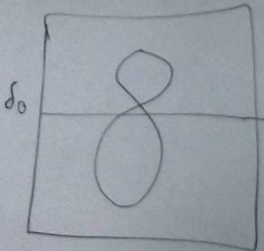
varianza

$TSL = H + \alpha$

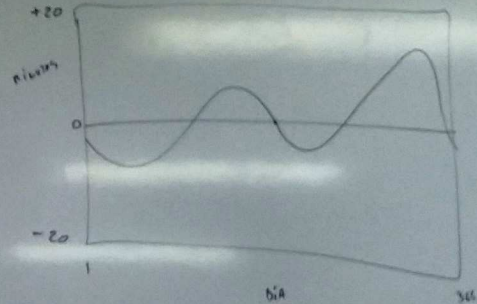
$\alpha_{SF} = 0$



ANALEMA



$\alpha_{SF} - \alpha_0$



$\eta$   
T. SIDÉREO → DÍA SIDÉREO : 2 CULMINACIONES DE  $\eta$  = 24 HORAS SIDÉREAS

S. FICTICIO  
T. SOLAR MEDIO → DÍA SOLAR MEDIO : 2 CULN. DEL S. FICTICIO = 24 HORAS

T. SOLAR APARENTE → DÍA SOLAR VERDADERO : 2 CULN. DE  $\odot$

= 365.2

PARCIAL :  
20 ABRIL

VER WEB CURSO:  
- ESP. CELESTE INTER.  
- MOV. SOL

$\varphi$   
**T. SIDÉREO** → DÍA SIDÉREO : 2 CULMINACIONES DE  $\varphi$   $\approx$  24 HORAS SIDÉREAS

**MEDIO** → DÍA SOLAR MEDIO : 2 CULM. DEL S. FICTICIO = 24 HORAS

**APARENTE** → DÍA SOLAR VERDADERO : 2 CULM. DE  $\odot$

**1 AÑO** = 365.25 DÍAS SOLARES (MEDIO)  
 365.25 + 1 DÍAS SIDÉREOS

$$\text{DÍA SIDÉREO} = \frac{365.25}{366.25} \text{ DÍAS SOLARES}$$

$$= 0.997269... = (1,002737)^{-1}$$

**1 DÍA SIDÉREO = 23<sup>h</sup> 56<sup>m</sup> 4<sup>s</sup> DE T. SOLAR MEDIO**

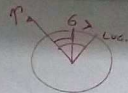
**PALCIAL :**  
 20 ABRIL

VER WEB CURSO:  
 - ESP. CELESTE INTER.  
 - NOV. SOL

T. Solm Medio Greenwich =

$$T_{Sol L} = T_{SG} + \lambda$$

↑  
R.A.O.S



$$1 \text{ año} = 365.25 \text{ DÍAS SOLARES (MEDIOS)}$$

$$365.25 + 1 \text{ DÍAS SIDEREOS}$$

$$\text{DÍA SIDEREAL} = \frac{365.25}{366.25} \text{ DÍAS SOLARES}$$

$$= 0.997269 \dots = (1.002737)^{-1}$$

$$1 \text{ DÍA SIDEREAL} = 23^h 56^m 4^s \text{ DE T. SOLM MEDIO}$$

PARCIAL:

20 Abril

VER WEB CURSO:

- ESP. CELESTE INTER.
- NOV. SOL



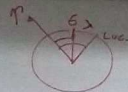
$$T. \text{ Solar Medio Greenwich} = H_{SP} (\text{GREENWICH}) + 12^h = T. \text{ UNIVERSAL (TU)}$$

$$HLU = TU - 3^h$$

E  
S  
T  
A  
L  
A  
D  
A  
Y  
A

$$T_{SioL} = T_{SG} + \lambda$$

RAOJ



$$1 \text{ AÑO} = 365.25 \text{ DÍAS SOLARES (MEDIOS)}$$

$$365.25 + 1 \text{ DÍAS SIDEREOS}$$

$$\text{DÍA SIDEREAL} = \frac{365.25}{366.25} \text{ DÍAS SOLARES}$$

$$= 0.997269 \dots = (1.002737)^{-1}$$

$$1 \text{ DÍA SIDEREAL} = 23^h 56^m 4^s \text{ DE T. SOLAR MEDIO}$$

PARCIAL:  
20 abril

VER WEB CURSO:  
- ESP. CELESTE INTER  
- MOV. SOL

$$T. \text{ Solar Medio GREENWICH} = H_{SP}(\text{GREENWICH}) + 12^h = T. \text{ UNIVERSAL (TU)}$$

$$HLU = \text{TU} - 3^h$$

E  
G  
A  
L  
R  
U  
S  
D  
A  
Y  
Δ

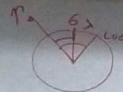
$$T_{SM} = T_S$$

Greenwich  
Sideral

$$T_{Sol Ap (Mon)} - \lambda$$

$$T_{Sol L} = T_{SG} + \lambda$$

raos



$$1 \text{ año} = 365.25 \text{ DÍAS SOLARES (medios)}$$

$$365.25 + 1 \text{ DÍAS SIDEREOS}$$

$$\text{DÍA SIDEREAL} = \frac{365.25}{366.25} \text{ DÍAS SOLARES}$$

$$= 0.997269... = (1.002737)^{-1}$$

$$1 \text{ DÍA SIDEREAL} = 23^h 56^m 4^s \text{ DE T. SOLAR MEDIO}$$

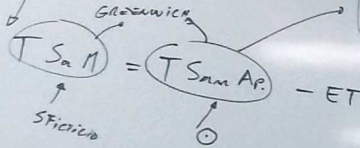
PARCIAL:  
20 abril

VER WEB CURSO:  
- ESP. CELESTE INTER  
- NOV. SOL

$$T. \text{ Solar Medio Greenwich} = H_{SP}(\text{Greenwich}) + 12^h = T. \text{ Universal (TU)}$$

$$HLU = TU - 3^h$$

EQUAL  
SUBDAY  
Δ



$$T_{SAp}(G) = T_{Sol Ap}(Mon) - \lambda_{Mon}$$

RELOJ SOLAR

$$\Rightarrow HLU = T_{Sol Ap}(Mon) - \lambda_{Mon} - ET - 3^h = T_{SAp}(Mon) + 3^h + 44^m - ET - 3^h$$

$$HLU = T$$

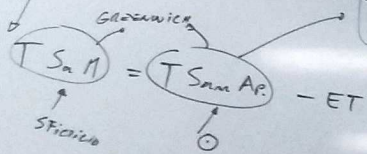
PARCIAL:  
20 Abril

- VER WEB CURSO:
- ESP. CELESTE INTER
  - NOV. SOL

$$T. \text{ Solar Medio Greenwich} = H_{SM}(\text{Greenwich}) + 12^h = T. \text{ Universal (TU)}$$

$$HLU = TU - 3^h$$

ER  
GAL  
SOL  
DAY



$$T_{SAp}(G) = T_{SAp}(Mon) - \lambda_{Mon}$$

$$\Rightarrow HLU = T_{SAp}(Mon) - \lambda_{Mon} - ET - 3^h = T_{SAp}(Mon) + 3^h + 44^m - ET - 3^h$$

$$HLU = T_{SAp}(Mon) + 44^m - ET$$

RELOJ RELOJ SOLAR

PARCIAL:  
20 Abril

$$T_{Sol} \text{ Greenwich} = f(\dots)$$

WEB CURSO:  
CELESTE INTER  
SOL

T.  $E = H_{SP}(\text{GREENWICH}) + 12^h = T. \text{ UNIVERSAL (TU)}$

$(TU) - 3^h$

$T_{SAM AP.} = T_{SAM AP.} - ET$

Greenwich  
M  
Ficilia

$T_{SAP(G)} = T_{SAP(Mon)} - \lambda_{MON}$

$\cancel{\lambda_{MON}} - ET - 3^h = T_{SAP(Mon)} + 3^h + 44^m - ET - 3^h$

$(3^h 44^m)$

$HLU = T_{SAP(Mon)} + 44^m - ET$

RELOJ RELOJ SAM

$T_{SID \text{ GREENWICH}} = f(TU)$

INSTANTE EXPANSION



PARCIAL:  
20 Abril

VER WEB CURSO:  
- ESP. CELESTIC INTER.  
- NOV. SOL

T. Solar Medio Greenwich =  $H_{SM}$  (GREENWICH)

T. UNIVERSAL (TU)

$$HLU = TU - 3^h$$

ESTAD  
AL  
S  
U  
D  
Y  
A

$$T_{SM} = \dots$$

GREENWICH

SEFICIA

RELOJ SOLAR

$$\Rightarrow HLU = T_{SM} Ap(Mon) - ET - (3^h 44^m)$$

$$T_{SM} Ap(Mon) \rightarrow MON$$

$$HLU = T_{SM} Ap(Mon) + 44^m - ET$$

RELOJ

RELOJ SOLAR

$$F(Año, mes, día, TU)$$

JD

PARCIAL:  
20 abril

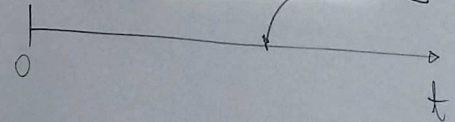
VER WEB CURSO:  
- ESP. CELESTE INTER.  
- NOV. SOL

$$T_{SIO} Greenwich = f(TU)$$

INSTANTE  
EXPOSICION

1 Enero 12 TU  
-4713

JD = 0



FECHA JULIANA  
JD

T. Solar Medio Greenwich =  $H_{SM}(\text{Greenwich}) + 12^h = T. \text{ Universal (TU)}$

$HLU = TU - 3^h$

E  
S  
T  
A  
D  
I  
A  
L  
D  
A  
Y

$T_{SM} = T_{SM}(\text{Greenwich}) - ET$

$T_{SAp}(G) = T_{SAp}(\text{Mon}) - \text{Mon}$

RELOJ SOLAR

$\Rightarrow HLU = T_{SAp}(\text{Mon}) - \text{Mon} - ET - 3^h = T_{SAp}(\text{Mon}) + 3^h + 44^m - ET - 3^h$

$HLU = T_{SAp}(\text{Mon}) + 44^m - ET$

RELOJ RELOJ SOLAR

$F(\text{Ano}, \text{Mes}, \text{Dia}, \text{TU}) \rightarrow JD$

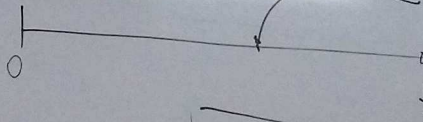
PARCIAL: 20 Abril

VER WEB CURSO:  
- ESP. CELESTE INTER.  
- NOV. SOL

$T_{SID} \text{ Greenwich} = f(TU)$

1 Enero 12 TU - 4713

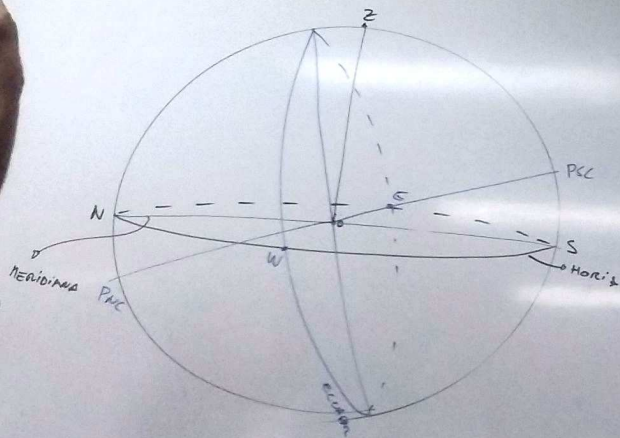
JD = 0



2000.0 JD = 2451545.0

FECHA JULIANA  
JD

$$a(\text{PAC}) = 0$$



PARCIAL :

20 abril

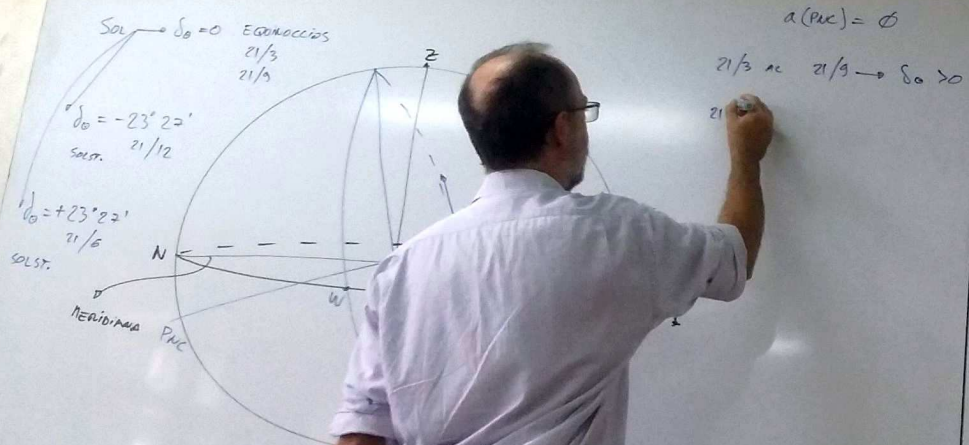
VÉR WEB CURSO:

- ESP. CELESTE INTER
- NOV. SOL

FECHA JULIANA

JD



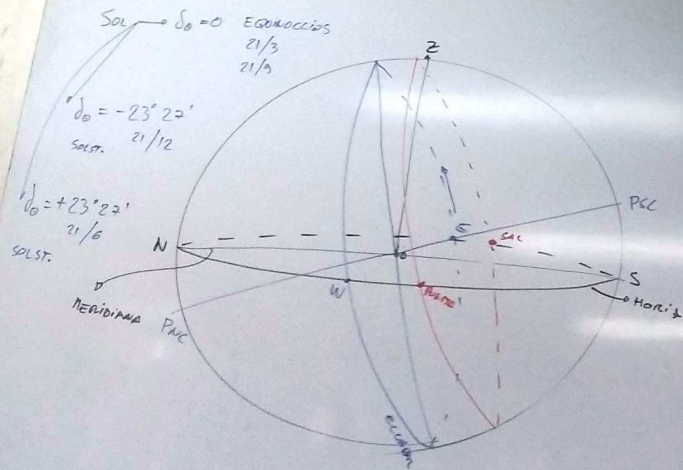


$a(PNC) = 0$   
 21/3 ac 21/9  $\rightarrow \delta_0 > 0$   
 21

PARCIAL:  
 20 abril

VER WEB CURSO:  
 - ESP. CELESTE INTER  
 - NOV. SOL

FECHA JULIANA  
 JD



$$a(\text{PAC}) = \emptyset$$

$$21/3 \text{ al } 21/9 \rightarrow \delta_0 > 0$$

$$21/9 \text{ al } 21/3 \rightarrow \delta_0 < 0$$

$\delta_0 \rightarrow$  VARIACIÓN EN DURACIÓN DEL DÍA  
↓  
VARIACIÓN ANUAL DE LA INSOLACIÓN

PARCIAL:

20 abril

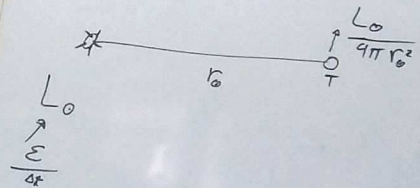
VER WEB CURSO:

- ESP. CELESTE INTER.
- NOV. SOL

FECHA JULIANA

JD

CÁLCULO DE INSOLACIÓN



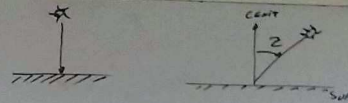
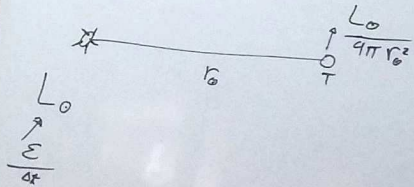
$J_0 \rightarrow$  VARIACIÓN EN DURACIÓN DEL DÍA  
 $\downarrow$   
 VARIACIÓN ANUAL DE LA INSOLACIÓN

PARCIAL:  
 20 Abril

VER WEB CURSO:  
 - ESP. CELESTE INTER.  
 - NOV. SOL

FECHA JULIANA  
 JD

CÁLCULO DE INSOLACIÓN



$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \Delta t \cdot \cos z$$

↑  
ENERGÍA TOTAL RECIBIDA

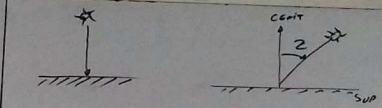
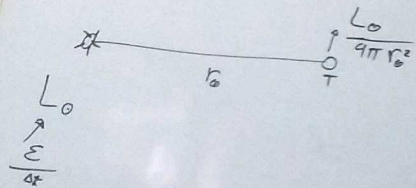
$$\Delta Q = \frac{L_0}{4\pi r_0^2} \int_{t_{sun}}^{t_{atmos}} \cos z(\ast) \cdot dt$$

PARCIAL:  
20 abril

VER WEB CURSO:  
- ESP. CELESTE INTER.  
- NOV. SOL

FECHA JULIANA  
JD

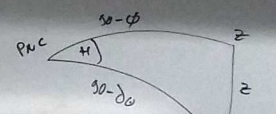
CÁLCULO DE INSOLACIÓN



↑  
ENERGÍA TOTAL RECIBIDA

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \Delta t \cdot \cos z$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \int_{x_{sun}}^{x_{horizon}} \cos z(*) \cdot dt$$



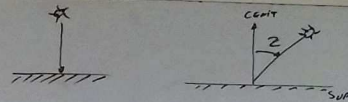
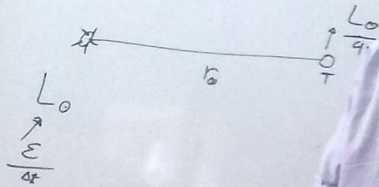
$$\cos z = \underbrace{\cos \delta_0}_{CTE} \underbrace{\cos \phi}_{CTE} + \underbrace{\cos \delta_0}_{CTE} \underbrace{\sin \phi}_{CTE} \cdot \cos H$$

$$\Delta H = \frac{2\pi}{D_p}$$

CÁLCULO DE INSOLACIÓN

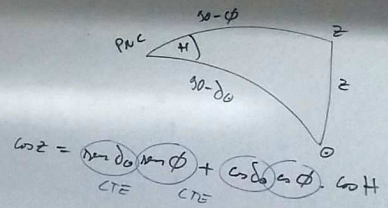
$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{\text{Día}}{2\pi} \int_{\theta=0}^{\theta=\delta_0} \cos \theta \, d\theta$$

$H_{\text{sol}} \uparrow$



$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \Delta t \cdot \cos z$$

$\uparrow$   
ENERGÍA TOTAL RECIBIDA



$$\Delta Q = \frac{L_0}{4\pi r_0^2} \int_{t_{\text{trans}}}^{t_{\text{trans}} + \Delta t} \cos z(\ast) \cdot dt$$

$$\Delta H = \frac{2\pi}{24 \text{ h}} \cdot \Delta t$$

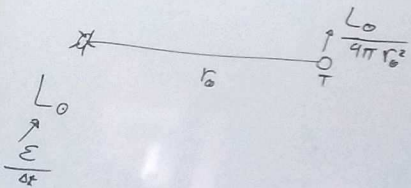
$\text{Día}$   
 $\downarrow$   
 $24 \text{ h}$

$$= \frac{L_0}{4\pi r_0^2} \int_{t_{\text{tra}}}^{t_{\text{tra}} + \Delta t} (\sin \delta_0 \sin \phi + \cos \delta_0 \cos \phi \cdot \cos H) dt$$

$\rightarrow$   $\frac{dH \cdot \text{Día}}{2\pi}$

CÁLCULO DE INSOLACIÓN

$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left( \cos \delta_0 \sin \phi + \sin \delta_0 \cos \phi \cdot \cos H \right) dH$$



$$\cos \delta_0 \sin \phi \cdot (H_p - H_{sun})$$

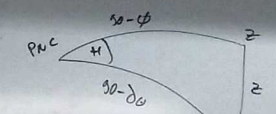
$2 H_p$

$$\cos \delta_0 \cos \phi \cdot \tan H \Big|_{H_{sun}}^{H_p} = \cos \delta_0 \cos \phi \left( \tan H_p - \tan H_{sun} \right)$$

$2 \tan H_p$



$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \dots$$



$$\cos z = \tan \delta_0 \tan \phi + \cos \delta_0 \cos \phi \cdot \cos H$$

$\text{CTE} \quad \text{CTE}$

$$\Delta H = \frac{2\pi}{D_{ra}} \cdot \Delta t$$

$D_{ra} \rightarrow 24 \text{ h}$

CÁLCULO DE INSOLACIÓN

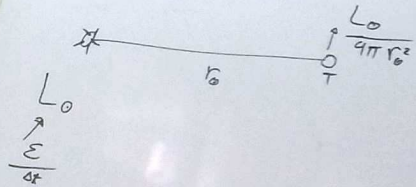
$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left( \cos \delta_0 \sin \phi + \sin \delta_0 \cos \phi \cdot \cos H \right) dH$$

$H_{solar}$

$$\cos \delta_0 \sin \phi \cdot (H_p - H_{solar})$$

$$\cos \delta_0 \cos \phi \cdot \tan H \Big|_{H_{solar}}^{H_p} = \cos \delta_0 \cos \phi (m H_p - m H_{solar})$$

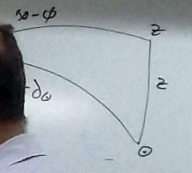
$$2 m H_p$$



$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left[ \sin \delta_0 \sin \phi \cdot 2 H_p + \cos \delta_0 \cos \phi \cdot 2 m H_p \right]$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \cdot 2 \left( \sin \delta_0 \sin \phi \cdot H_p + \cos \delta_0 \cos \phi \cdot m H_p \right)$$

$$H_p(\delta)$$

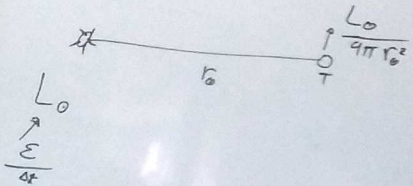


$$\frac{1}{2} \phi$$



CÁLCULO DE INSOLACIÓN

$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left( \cos \delta_0 \sin \phi + \sin \delta_0 \cos \phi \cdot \cos H \right) dH$$



$$\cos \delta_0 \sin \phi \cdot (H_p - H_{sun})$$

$2 H_p$

$$\cos \delta_0 \cos \phi \cdot \tan H \Big|_{H_{sun}}^{H_p} = \cos \delta_0 \cos \phi (H_p - H_{sun})$$

$2 \tan H_p$

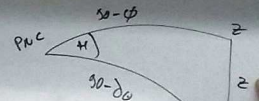
$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left[ \sin \delta_0 \sin \phi \cdot 2 H_p + \cos \delta_0 \cos \phi \cdot 2 \tan H_p \right]$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \cdot 2 \left( \sin \delta_0 \sin \phi \cdot H_p + \cos \delta_0 \cos \phi \cdot \tan H_p \right)$$

$$\Delta Q(\delta_0, \phi, r_0)$$

+23° 27'      -35°  
-23° 27'

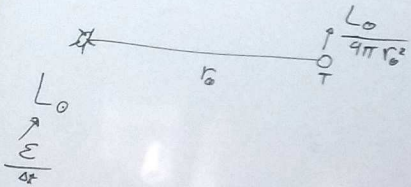
$$H_p(\delta_0, \phi)$$



$$\cos H_p = -\tan \delta_0 \tan \phi$$

CÁLCULO DE INSOLACIÓN

$$\Rightarrow \Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left( \cos \delta_0 \sin \phi + \sin \delta_0 \cos \phi \cdot \cos H \right) dH$$



$$\cos \delta_0 \sin \phi \cdot (H_p - H_{sac})$$

$$2 H_p$$

$$\cos \delta_0 \cos \phi \cdot \tan H \Big|_{H_{sac}}^{H_p} = \cos \delta_0 \cos \phi (H_p - H_{sac})$$

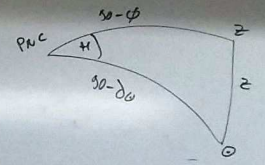
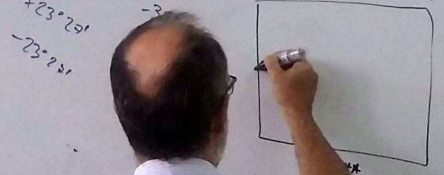
$$2 H_p$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left[ \sin \delta_0 \sin \phi \cdot 2 H_p + \cos \delta_0 \cos \phi \cdot 2 H_p \right]$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \cdot 2 \left( \sin \delta_0 \sin \phi \cdot H_p + \cos \delta_0 \cos \phi \cdot H_p \right)$$

$$\Delta Q(\delta_0, \phi, r_0)$$

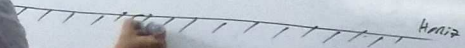
$$H_p(\delta_0, \phi)$$



$$\cos H_p = -\tan \delta_0 \tan \phi$$

$$\cos \delta_0 = \sin \delta_0 \sin \phi + \cos \delta_0 \cos \phi \cdot \cos H$$

CREPÚSCULOS



$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left[ \sin \delta_0 \cos \phi \cdot 2H_p + \cos \delta_0 \sin \phi \cdot 2 \cdot \sin H_p \right]$$

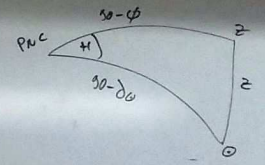
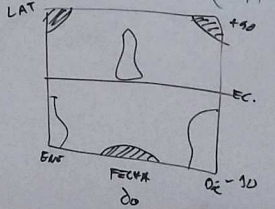
$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \cdot 2 \left( \sin \delta_0 \cos \phi \cdot H_p + \cos \delta_0 \sin \phi \cdot \sin H_p \right)$$

$$\Delta Q(\delta_0, \phi, r_0)$$

+23° 27'

-23° 27'

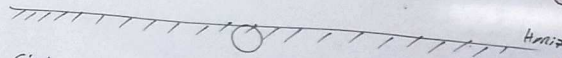
-35°



$$\cos H_p = -\frac{1}{2} \delta_0 \frac{1}{2} \phi$$

$$\cos \delta_0 = \sin \phi + \sin \delta_0 \cdot \cos H$$

CREPÚSCULOS



CIVIL

NAÚTICO

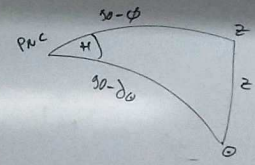
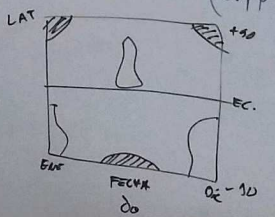
ASTRONÓMICO

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \left[ \sin \delta_0 \sin \phi \cdot 2H_p + \cos \delta_0 \cos \phi \cdot 2 \cdot m \cdot H_p \right]$$

$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{ia}}{2\pi} \cdot 2 \left( \sin \delta_0 \sin \phi \cdot H_p + \cos \delta_0 \cos \phi \cdot m \cdot H_p \right)$$

$$\Delta Q(\delta_0, \phi, r_0)$$

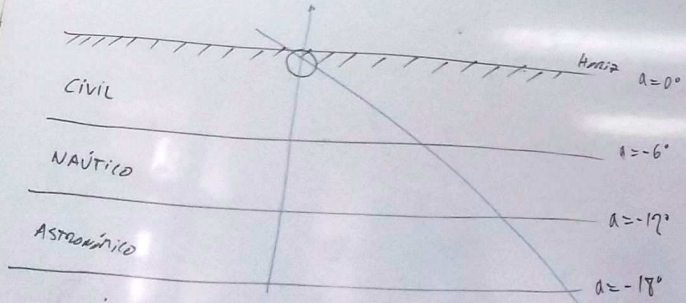
+23° 27'  
-23° 27'



$$\cos H_p = -\tan \delta_0 \tan \phi$$

$$\cos \delta_0 = \tan \phi \tan \delta_0 + \cos \phi \cdot \cos H$$

CREPÚSCULOS



$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{\text{dia}}}{2\pi} \left[ \sin \delta_0 \cos \phi \cdot 2H_p + \cos \delta_0 \sin \phi \cdot 2 \cdot \sin H_p \right]$$

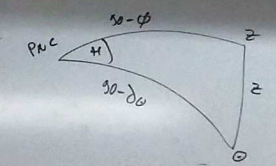
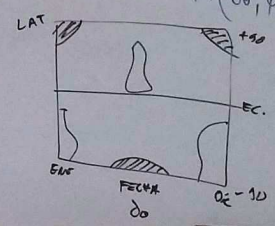
$$\Delta Q = \frac{L_0}{4\pi r_0^2} \cdot \frac{D_{\text{dia}}}{2\pi} \cdot 2 \left( \sin \delta_0 \cos \phi \cdot H_p + \cos \delta_0 \sin \phi \cdot \sin H_p \right)$$

$$\Delta Q(\delta_0, \phi, r_0)$$

+23° 29'

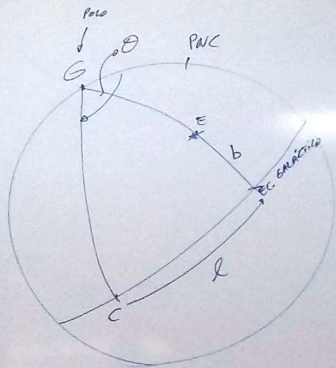
-23° 29'

-35°



$$\cos H_p = -\tan \delta_0 \tan \phi$$

$$\cos \delta_0 = \frac{\cos H_p}{\sin \phi}$$



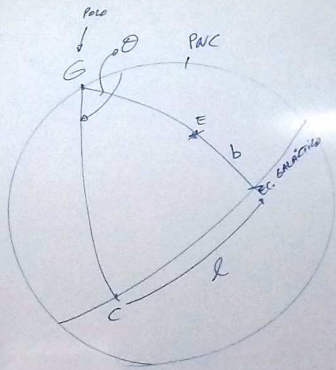
$$\alpha_G = 12^h 45^m$$

$$\delta_G = +27^\circ.4$$

$$\Theta = 123^\circ$$

$$\text{LAT. GALÁCTICA: } b \quad -90^\circ \leq b \leq +90^\circ$$

$$\text{LONG. GALÁCTICA: } l \quad 0 < l < 360^\circ$$



$$\alpha_G = 12^h 45^m$$

$$\delta_G = +27^\circ.4$$

$$\theta = 123^\circ$$

LAT. GALÁCTICA:  $b \quad -90^\circ \leq b \leq +90^\circ$

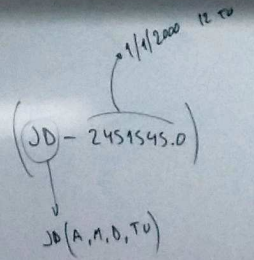
LONG. GALÁCTICA:  $l \quad 0 \leq l \leq 360^\circ$

T. SIDÉREO GREENWICH

TSG

$$GMST = 18^h.657375 + 24^h.06570982 \left( \text{JD} - 2451545.0 \right)$$

$$TSC_{LOCAL} = TSG + \lambda$$



REFRACCIÓN ATMOSFÉRICA

T. SIDÉREO GREENWICH

TSG

$$GMST = 18^h.697375 + 24^h.06570982 \left( \text{JD} - 2451545.0 \right)$$

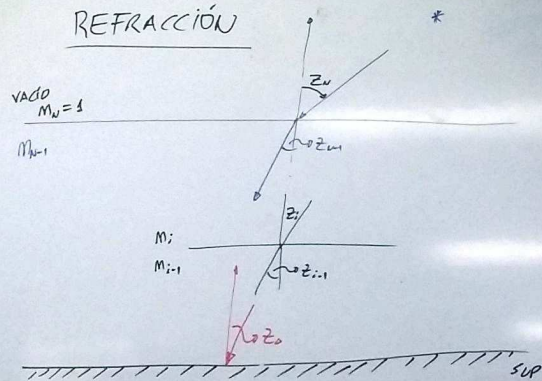
$$TSC = TSG + \lambda$$

LOCAL

 $\text{JD}(A, M, D, Y)$ 
 $\frac{1}{4} / 2000 \text{ 12 TU}$



REFRACCIÓN



SUCC

$$M_u \sin z_u = M_{i-1} \sin(z_{i-1})$$

$$M_i \sin z_i = M_{i-1} \sin z_{i-1}$$

T. SIDÉREO GREENWICH

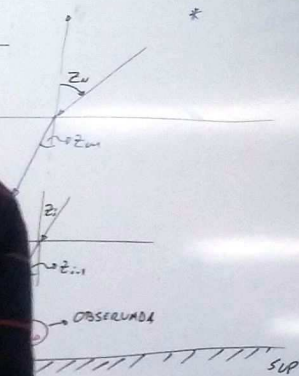
TSG

$$GMST = 18^h.697375 + 24^h.06570982 \left( \text{JD} - 2451545.0 \right) + 1/12000 \text{ 12 TU}$$

$$TSC = TSG + \lambda$$

JD(A, M, D, TU)

CIÓN



SUPL

$$M_u \cdot z_u = M_{i-1} \cdot z_{i-1}$$


---


$$M_i \cdot z_i = M_{i-1} \cdot z_{i-1}$$


---


$$M_i \cdot z_i = M_o \cdot z_o$$

$$M_u \cdot z_u = M_o \cdot z_o \rightarrow \text{OBSERVADO}$$

z  
"TOPOCÉNTRICO"

T. SIDÉREO GREENWICH

TSG

$$GMST = 18^h.657375 + 24^h.06570982 \left( \text{JD} - 2451545.0 \right)$$

1/1/2000 12 TU

JD(A, M, D, TU)

$$TSC = TSG + \lambda$$

LOCAL

# REFRACCIÓN

VACIO  
 $M_N = 1$

$M_{N-1}$

$M_i$

$M_{i-1}$

OBSERVADA

SUP

SINEL

$$M_N \sin z_N = M_{N-1} \sin(z_{N-1})$$

$$M_i \sin z_i = M_{i-1} \sin z_{i-1}$$

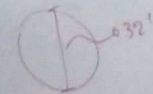
$$M_1 \sin z_1 = M_0 \sin z_0$$

$$M_N \sin z_N = M_0 \sin z_0 \rightarrow \text{OBSERVADO}$$

VACIO  
1

"TOPOCÉNTRICO"

REF. HORIZONTAL : 34'



Horiz

SE  
+34' = R

# REFRACCIÓN

"REFRACCIÓN"

$$R = z - z_0 > 0$$

↑  
REDUCCIÓN

①:  $n_a z = n_0 n_a z_0$

$(R + z_0)$

SUCEL

$$n_0 n_a z_n = n_{a-1} n_a (z_{n-1})$$

$$n_i n_a z_i = n_{i-1} n_a z_{i-1}$$

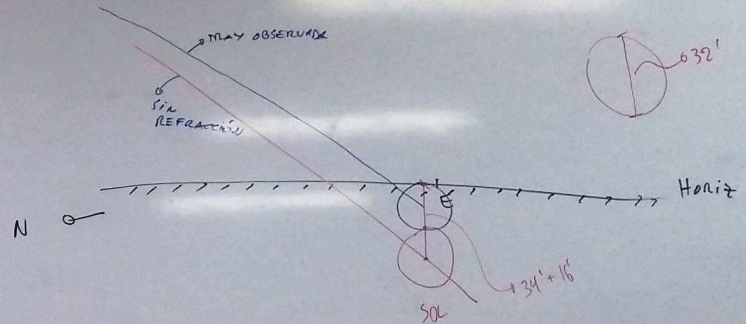
$$n_1 n_a z_1 = n_0 n_a z_0$$

$$n_a n_a z_a = n_0 n_a z_0 \rightarrow \text{OBSERVADO}$$

① VACIO

"TOPOCÉNTRICO"

REF. HORIZONTAL: 34'



# REFRACCIÓN

"REFRACCIÓN"

$$R = z - z_0 > 0$$

↑  
REDUCCIÓN

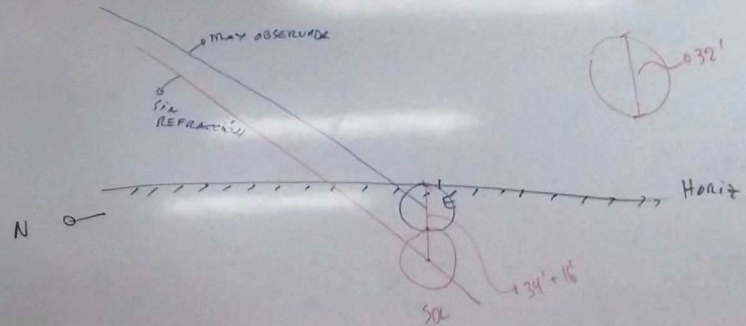
①:  $n_1 z = n_0 n_2 z_0$

$$n_1 (R + z_0) = n_1 R \cdot \cos z_0 + n_2 z_0$$

$\sim R$

$$\rightarrow n_0 n_2 z_0 \approx R \cdot \cos z_0 + n_2 z_0$$

REF. HORIZONTAL : 34'



# REFRACCIÓN

"REFRACCIÓN"

$$= z - z_0 > 0$$

$M_0 \sin z_0$

$$= \sin R \cdot \cos z_0 + \cos R \cdot \sin z_0$$

$\approx R$

$$z_0 \approx R \cdot \cos z_0 + \sin z_0$$

$$R \cdot \cos z_0 = (M_0 - 1) \cdot \sin z_0$$

$$\Rightarrow R = (M_0 - 1) \cdot \frac{1}{\cos z_0}$$

RADIAVES

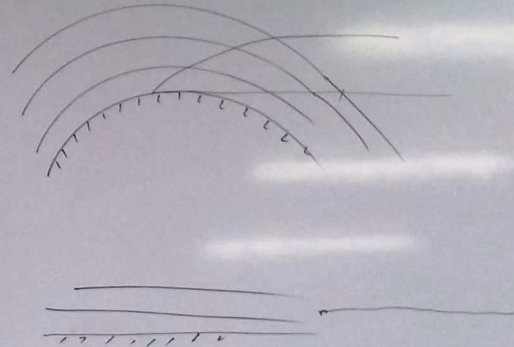
$$R = 206265 \cdot \overset{\text{ATM STD } 1.0002922}{(M_0 - 1)} \cdot \frac{1}{\cos z_0}$$

(")

$$\Rightarrow R^{(")} = (K) \cdot \frac{1}{\cos z_0}$$

$\cdot 60'' \cdot 4$   
CTE REFRACCIÓN

REF. HORIZONTAL : 34'



# REFRACCIÓN

"REFRACCIÓN"

$$R = z - z_0 > 0$$

↑  
REDUCCIÓN

①:  $n_n z = n_0 n_n z_0$

$$n_n (R + z_0) = n_0 n_n z_0$$

~ R

$$n_0 n_n z_0 \approx R \cdot \cos z_0$$

$$R \cdot \cos z_0 = (n_0 - 1) \cdot n_n z_0$$

$$\Rightarrow R = (n_0 - 1) \cdot \frac{1}{k} z_0$$

↑  
DIÁMETROS

$$206265 \cdot (n_0 - 1) \cdot \frac{1}{k} z_0$$

ATM STD  
1.0002522

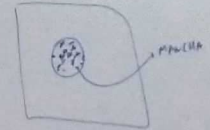
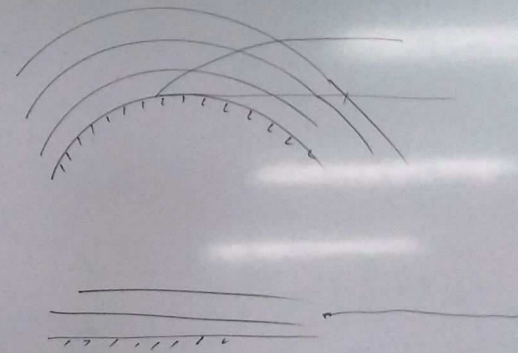
$$R^{(") = (K) \cdot \frac{1}{k} z_0$$

VALE PARA

$$z < 75^\circ$$

· 60".4  
CTE REFRACCIÓN

REF. HORIZONTAL : 34'



# REFRACCIÓN

"Refracción"

$$R = z - z_0 > 0$$

↑  
REDUCCIÓN

①:  $n(z) = M_0 n(z_0)$

$$n(R+z_0) = n(R) \cdot \cos z_0 + \cos R \cdot n(z_0)$$

$\sim R$

$$M_0 n(z_0) \approx R \cdot \cos z_0 + n(z_0)$$

$$R \cdot \cos z_0 = (M_0 - 1) \cdot n(z_0)$$

$$\Rightarrow R = (M_0 - 1) \cdot \frac{1}{\cos z_0}$$

↑  
RADIANTES

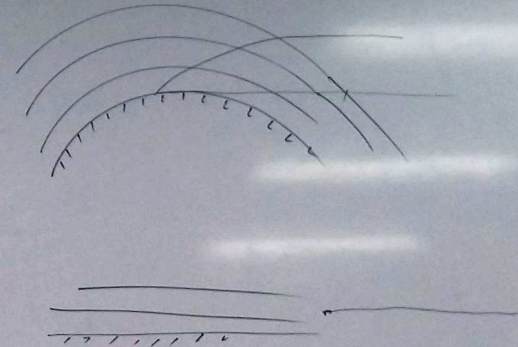
$$R = 206265 \cdot (M_0 - 1) \cdot \frac{1}{\cos z_0}$$

ATM STD  
1.0002922

$$\Rightarrow R^{(1)} = K \cdot \frac{1}{\cos z_0}$$

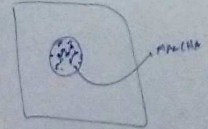
VALE PARA  
 $z < 75^\circ$

· 60"·4  
CTE REFRACCIÓN



REF. HORIZONTAL : 34'

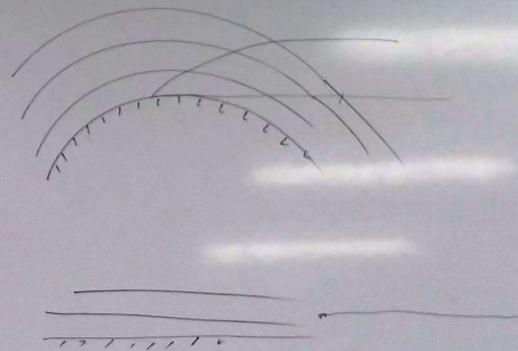
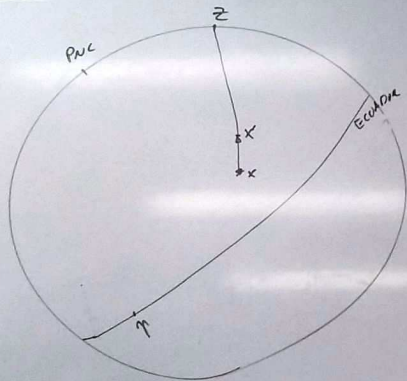
SEEING





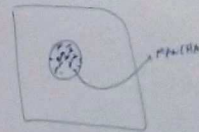
EFECTO EN  $\alpha$  y  $\delta$

$A = 0$   
 $= R$   
 $\Delta\alpha, \Delta\delta?$   
 $X = R$



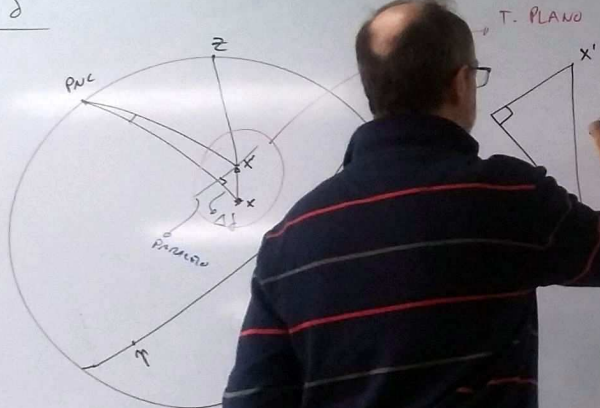
REF. HORIZONTAL : 34'

SEE 106



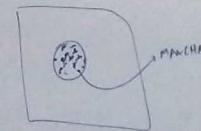
EFFECTO EN  $\alpha$  y  $\delta$

$$\begin{cases} \Delta A = 0 \\ \Delta a = R \\ \Delta \alpha, \Delta \delta? \\ X'X = R \end{cases}$$

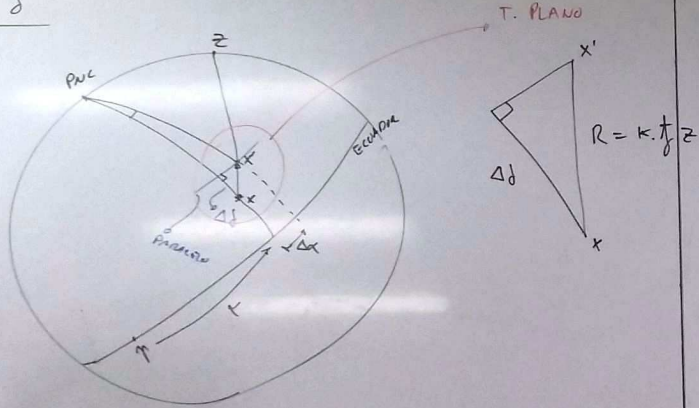


REF. HORIZONTAL : 34'

SEEING

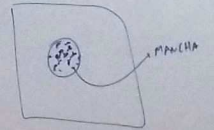


EFFECTO EN  $\alpha$  y  $\delta$



REF. HORIZONTAL : 34'

SEEING



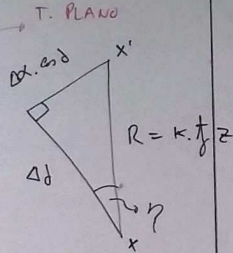
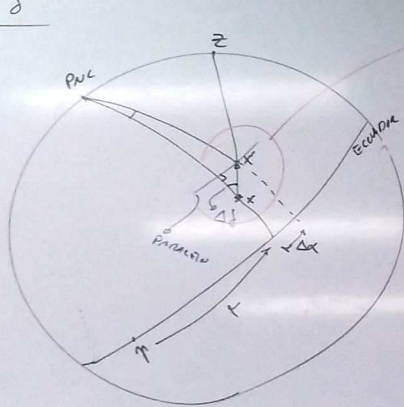
TOPO

EFFECTO EN  $\alpha$  y  $\delta$

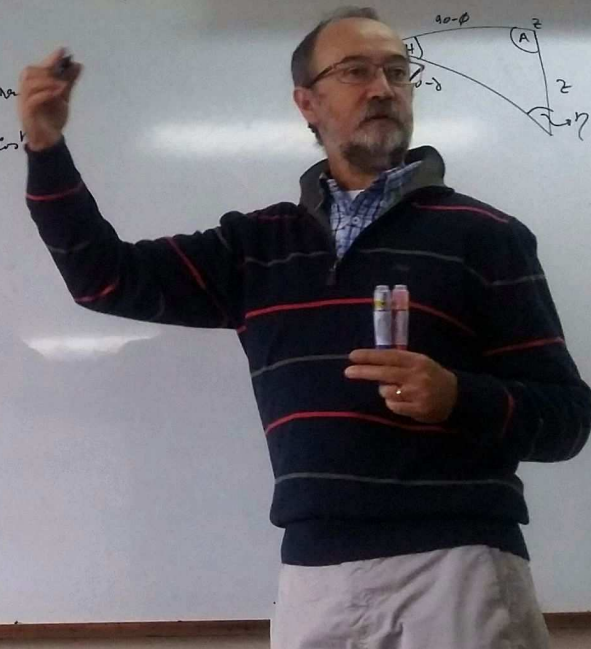
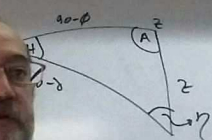
$$\begin{cases} \Delta A = 0 \\ \Delta a = R \\ \Delta \alpha, \Delta \delta? \end{cases}$$

$$X'X = R$$

$$\Delta = \text{OBS} - \text{TOPO}$$



$$\begin{cases} \Delta \alpha \cdot \cos \delta = R \cdot \sin \delta \\ \Delta \delta = R \cdot \cos \delta \end{cases}$$



EFFECTO EN  $\alpha, \delta$

$$\begin{cases} \Delta \alpha = 0 \\ \Delta \delta = R \end{cases}$$

$d(\Delta \alpha, \Delta \delta)?$

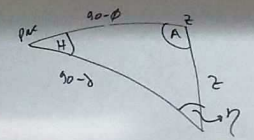
$$X'X = R$$

$$\Delta = \text{OBS} - \text{TOPO}$$

$$\Delta \alpha \cdot \cos \delta = \dots \sin H$$



$$\begin{cases} \Delta \alpha \cdot \cos \delta = R \cdot \frac{\sin H}{\cos \phi} \\ \Delta \delta = R \cdot \cos \delta \end{cases}$$



$$\frac{\sin z}{\cos \phi} = \frac{\sin H}{\cos \delta}$$

$$\begin{aligned} \cos(90-\phi) &= \cos(90-\delta) \cdot \cos z + \sin(90-\delta) \sin z \cdot \cos H \\ \sin \phi &= \sin \delta \cdot \cos z + \cos \delta \sin z \cdot \cos H \end{aligned}$$

EFFECTO EU

$$\begin{cases} \Delta \alpha = 0 \\ \Delta \delta = R \\ \text{¿} \Delta \alpha, \Delta \delta? \end{cases}$$

$$X'X = R$$

$$\Delta = \text{OBS}$$

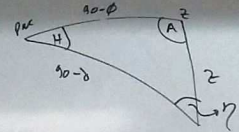
$$\Delta \alpha \cdot \cos \delta = K \frac{f}{z} \cdot \frac{\cos \phi \sin H}{\sin z} = K \cdot \frac{\cos \phi \sin H}{\cos z}$$

$$\Rightarrow \Delta \alpha \cong K \cdot \frac{\cos \phi \sin H}{\cos \delta \cdot \cos z}$$

$$\frac{f}{z}$$

$$\begin{cases} \Delta \alpha \cdot \cos \delta = R \cdot \sin \eta \\ \Delta \delta = R \cdot \cos \eta \end{cases}$$

$$\cos \phi \cdot \frac{\sin H}{\sin z}$$



$$\frac{\sin z}{\cos \phi} = \frac{\sin H}{\sin z}$$

$$\begin{aligned} \cos(90 - \phi) &= \cos(90 - \delta) \cdot \cos z + \sin(90 - \delta) \sin z \cdot \cos \eta \\ \sin \phi &= \sin \delta \cdot \cos z + \cos \delta \sin z \cdot \cos \eta \end{aligned}$$

EFFECTO EN  $\alpha, \delta$

$$\begin{cases} \Delta A = 0 \\ \Delta a = R \\ \text{¿} \Delta \alpha, \Delta \delta? \end{cases}$$

$$X'X = R$$

$$\Delta = \text{OBS} - \text{TOPO}$$

$$\Delta \alpha \cdot \cos \delta = K \cdot f_{\text{g}} \cdot z \cdot \frac{\cos \phi \sin H}{\sin z} = K \cdot \frac{\cos \phi \sin H}{\cos z}$$

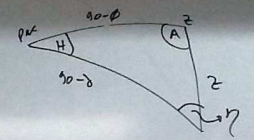
$$\Rightarrow \Delta \alpha \cong K \cdot \frac{\cos \phi \sin H}{\cos \delta \cdot \cos z}$$

$$\Delta \delta = K \cdot f_{\text{g}} \cdot z \cdot \frac{(\sin \phi - \sin \delta \cos z)}{\cos \delta \sin z} = K \cdot \frac{(\sin \phi - \sin \delta \cos z)}{\cos \delta \cdot \cos z}$$

$\downarrow$   
 $z \text{ ó } z_0$

$$\begin{cases} \Delta \alpha \cdot \cos \delta = R \cdot \sin \eta \\ \Delta \delta = R \cdot \cos \eta \end{cases}$$

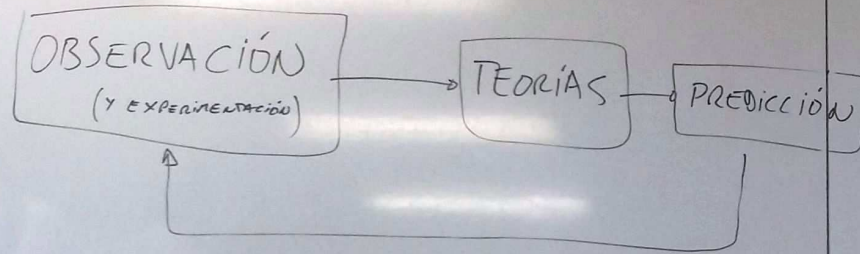
$\swarrow$   $\cos \phi \cdot \frac{\sin H}{\sin z}$



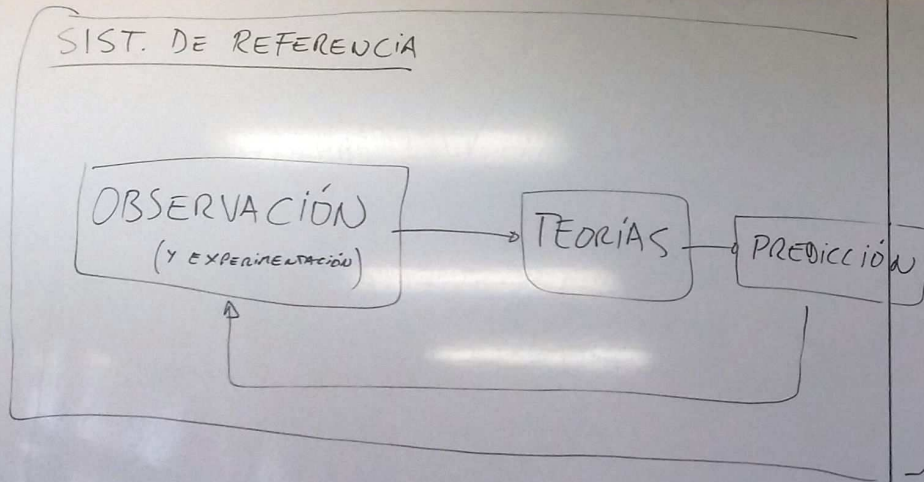
$$\frac{\sin z}{\cos \phi} = \frac{\sin H}{\sin z}$$

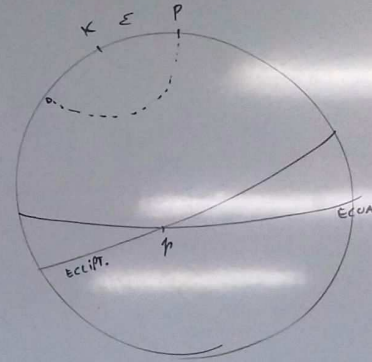
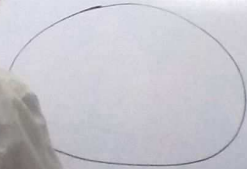
$$\begin{aligned} \cos(90-\phi) &= \cos(90-d) \cdot \cos z + \sin(90-d) \sin z \cdot \cos \eta \\ \sin \phi &= \sin \delta \cdot \cos z + \cos \delta \sin z \cdot \cos \eta \end{aligned}$$

SIST. DE REFERENCIA



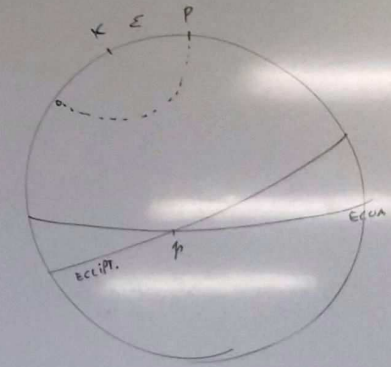
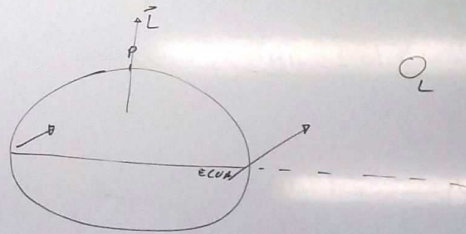




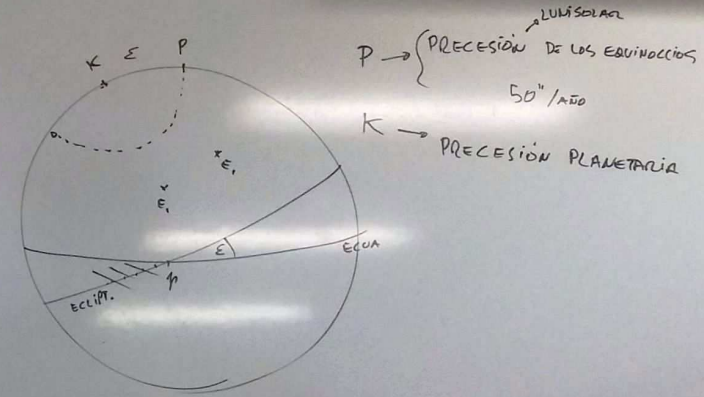
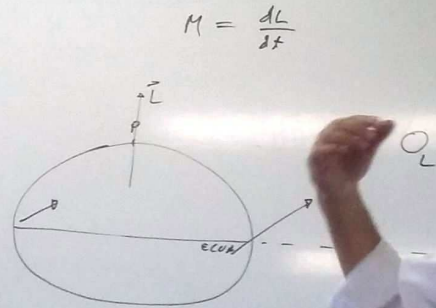


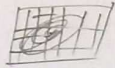
PRECESIÓN DE LOS EQUINOCCIOS  
50" / AÑO

$$M = \frac{dL}{dt}$$



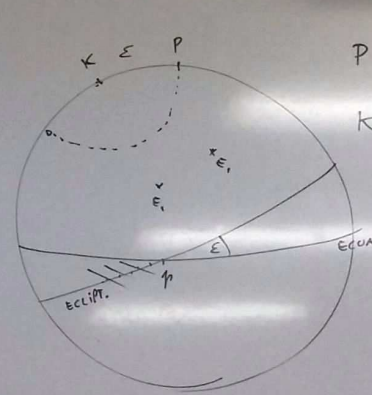
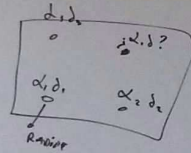
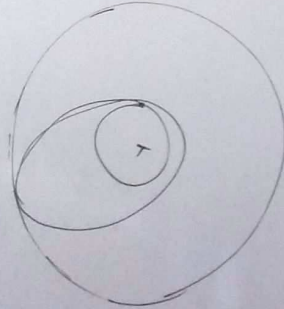
PRECESIÓN DE LOS EQUINOCCIOS  
50" / AÑO





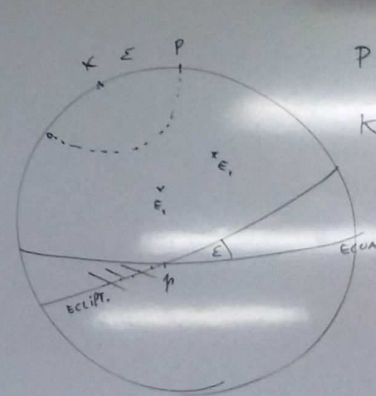
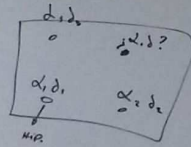
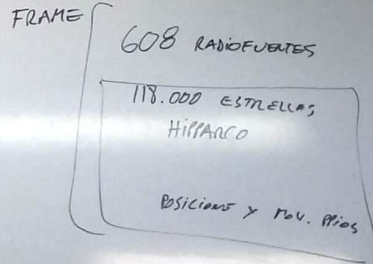
608 RADIOFUENTES

118.000 ESTRELLAS  
HIPPARCO

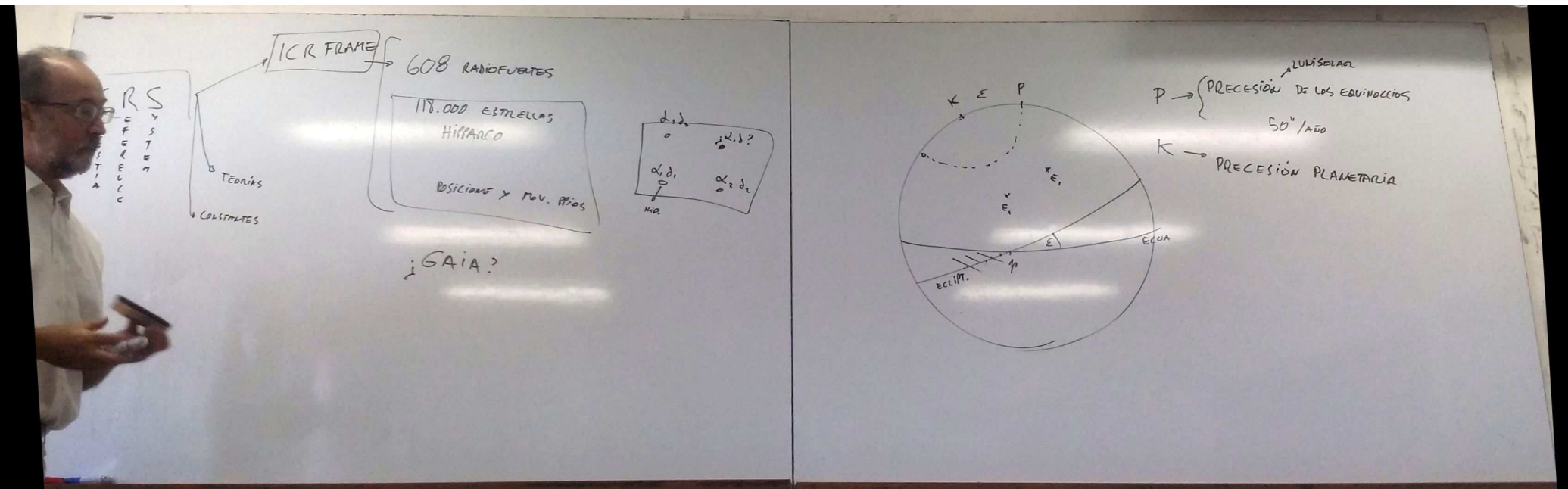


$P \rightarrow$  PRECESIÓN DE LOS EQUINOCCIOS  
 $50''/\text{AÑO}$   
 $K \rightarrow$  PRECESIÓN PLANETARIA

ICRS



LUNISOLAR  
 $P \rightarrow$  PRECESIÓN DE LOS EQUINOCCIOS  
 50" / AÑO  
 $K \rightarrow$  PRECESIÓN PLANETARIA



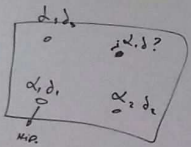
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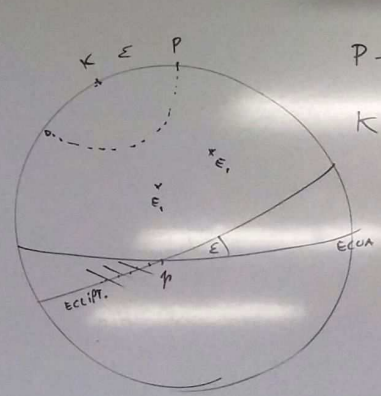
608 RADIOFUENTES

118.000 ESTRELLAS  
HIPPARCO  
POSICIONES y Mov. Prios

TECNICAS  
CONSTANTES



¿GAIA?



LUNISOLAR  
P → PRECESION DE LOS EQUINOCCIOS  
50" / AÑO  
K → PRECESION PLANETARIA

ICRS  
 D E F Y  
 T L E S T  
 E S T E N  
 N. A C I O N  
 A L E

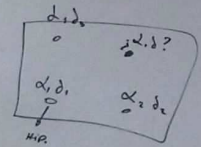
ICR FRAME

8 RADIOFUENTES

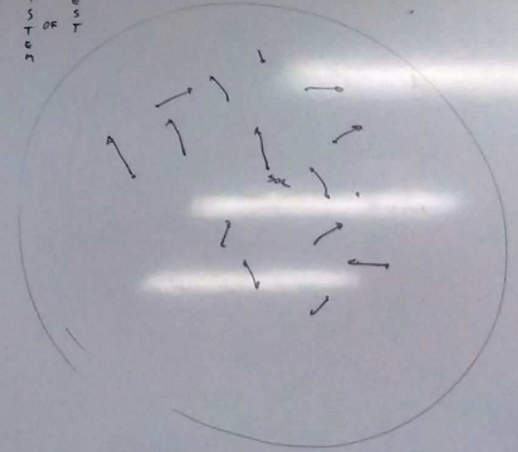
100.000 ESTRELLAS  
 HIPPARCO

POSICIONES Y MOV. PROPIOS

GAIA?



LSR  
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 SUN





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608 RADIOFUENTES

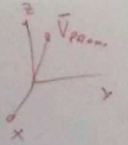
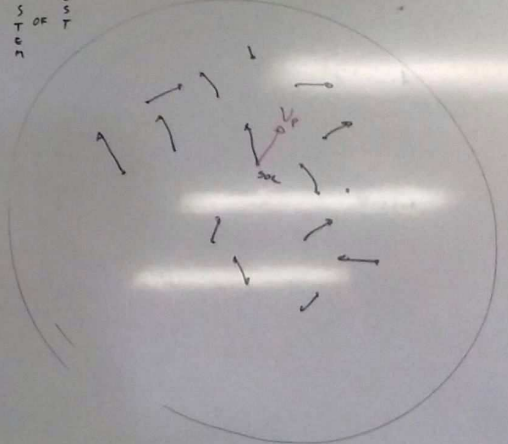
118.000 ESTRELLAS  
 HIPPARCO  
 POSICIONES Y M.

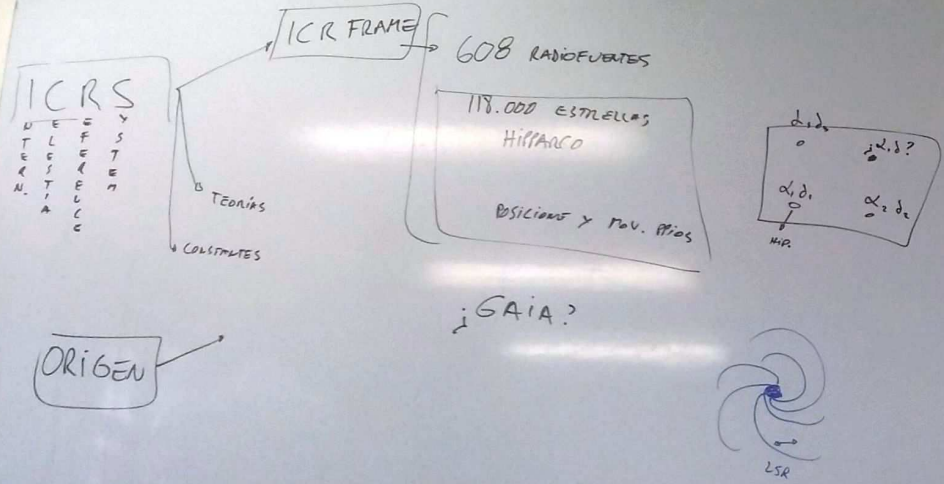
TEORIAS  
 CONSTANTES

$\alpha, \delta$ ?  
 $\alpha, \delta, d$

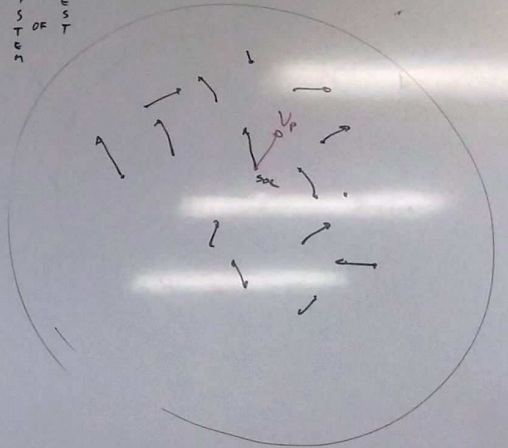
¡GAIA?

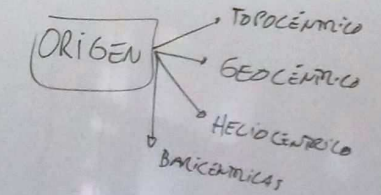
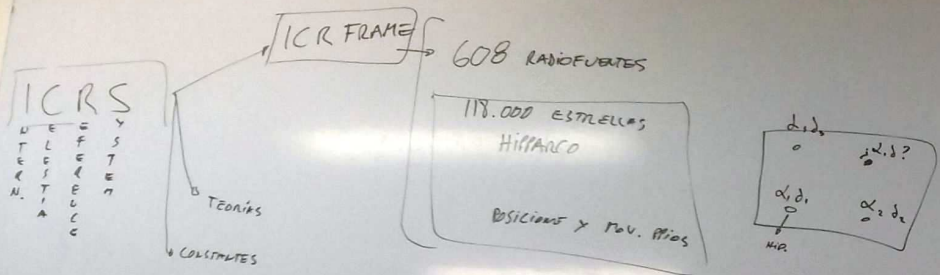
LSR  
 LOCAL  
 SYSTEM  
 OF  
 STARS



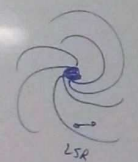


LSR  
LOCAL  
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SUN

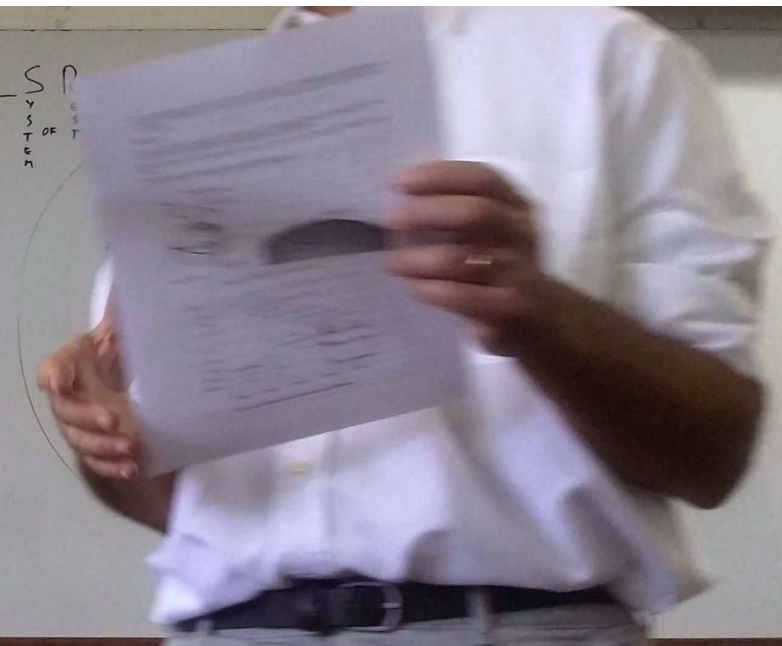


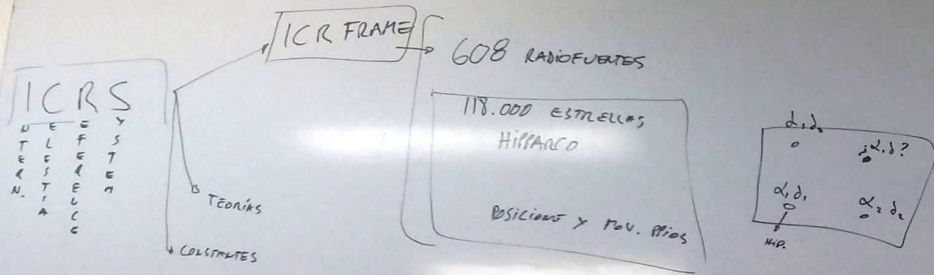


¿GAIA?

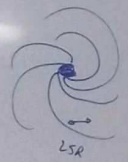
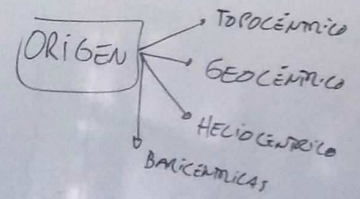


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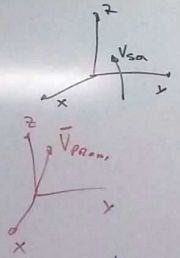
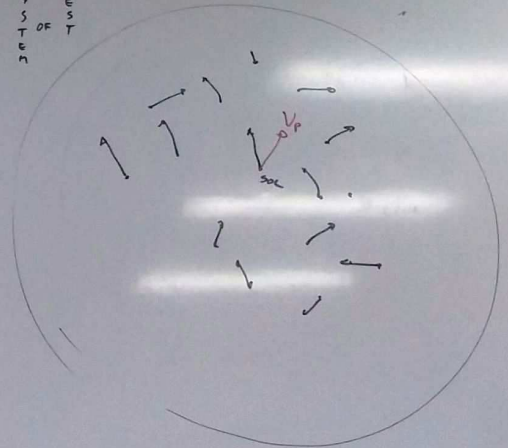




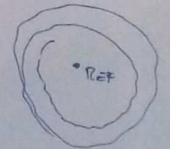
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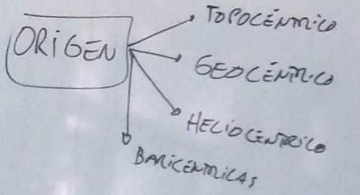
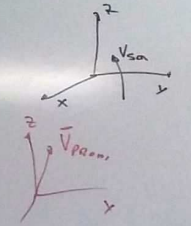
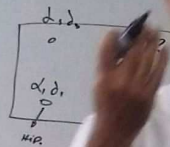
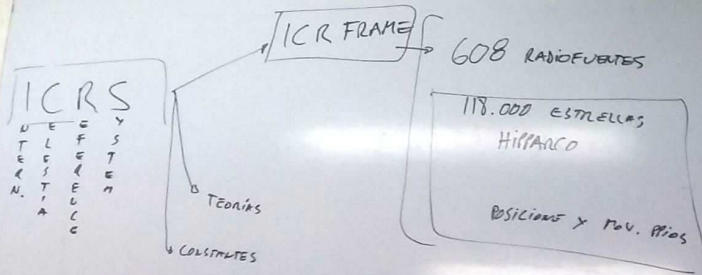


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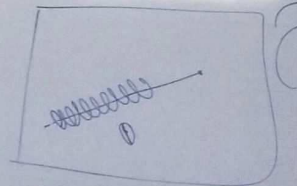


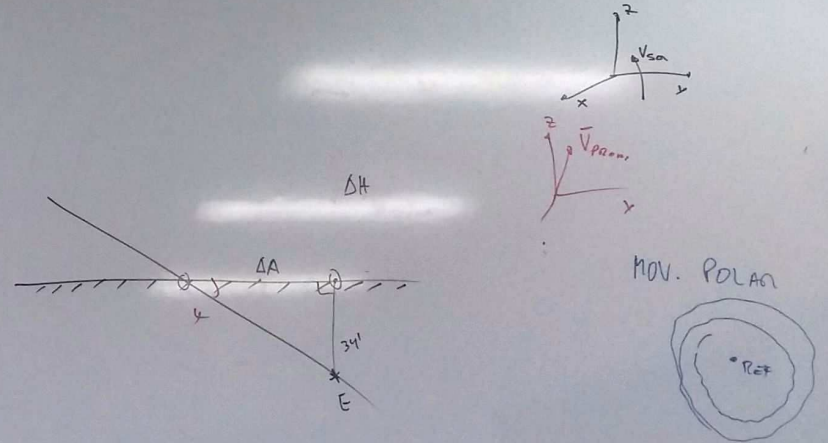
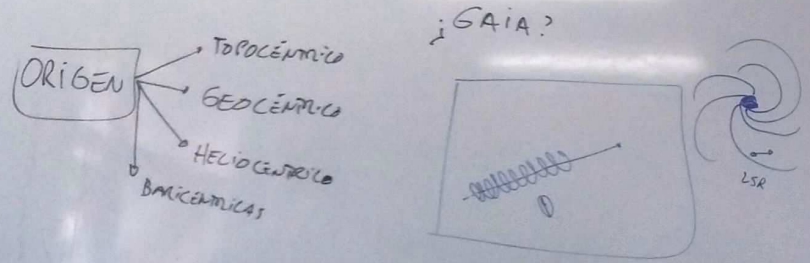
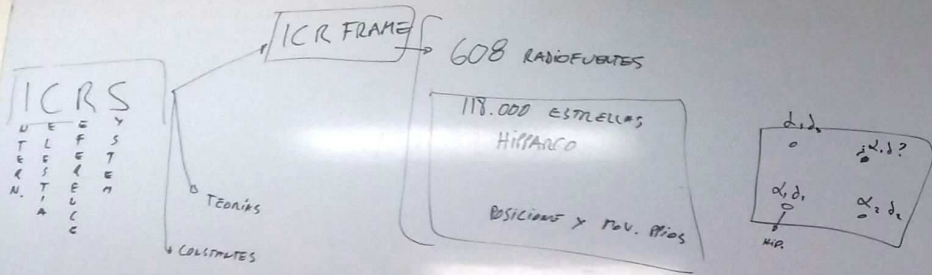
MOV. POLAR





¿GAIA?



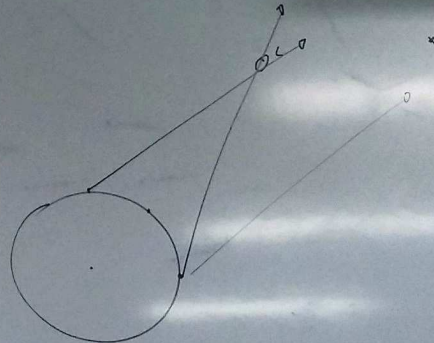


RELACION TOPOCÉNTRICAS - GECÉNTRICAS

PARALAJE DIURNA

ABERRACION DIURNA

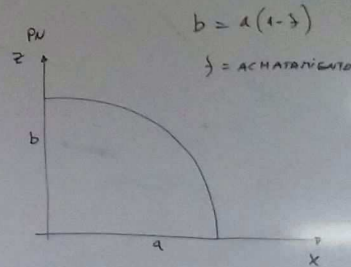
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RELACION TOPOCÉNTRICAS - GEOFÉNICAS

PARALAJE DIURNA

ABERRACION DIURNA

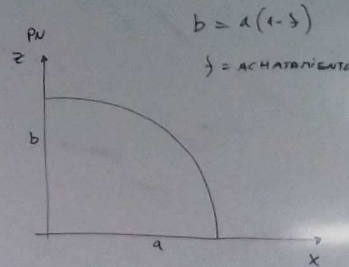


GEOIDE = SUP. EQUIPOT.



RELACIÓN TOPOCÉNTRICA Y GEOFICÉNTRICAS

PARA  
ABERRACIÓN

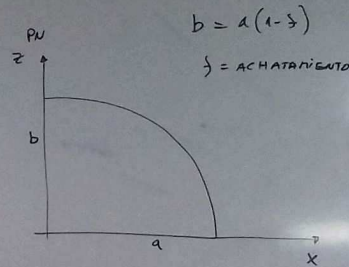
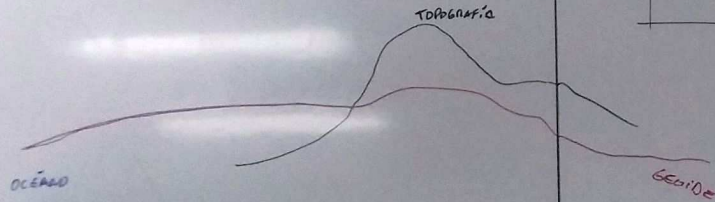


DIFERENCIA DE METROS {  
 "GEOIDE" = SUP. EQUIPOTENCIAL  
 "ESFEROIDE ESTÁNDAR" = ELIPSOIDE DE REVOLUCIÓN

TRANSFORMACIONES TOPOCÉNTRICAS - GEOCÉNTRICAS

RAJÉ DIURNA

RAJÉ DIURNA



DIFERENCIA DE METROS

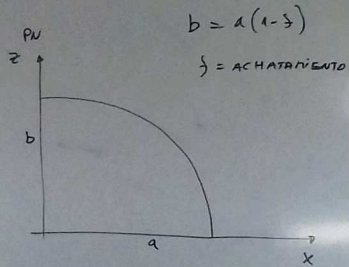
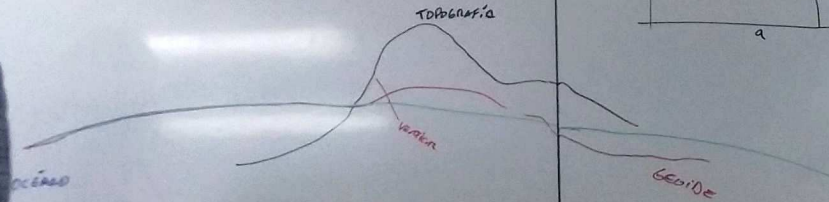
"GEOIDE" = SDP. EQUIPOTENCIAL

"ESFEROIDE ESTÁNDAR" = ELIPSOIDE DE REVOLUCIÓN

RELACIÓN TROPICAS - GEOCÉNTRICAS

PARALAJE DIURNO

ABERRACIÓN



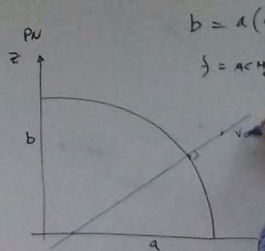
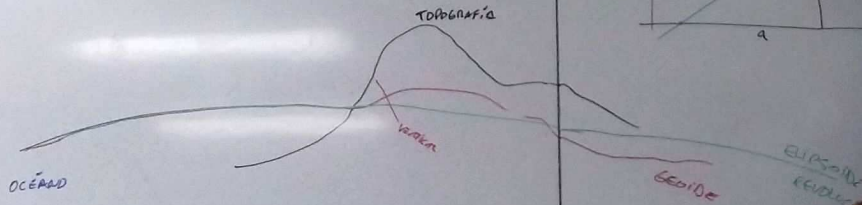
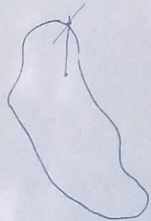
DIFERENCIA DE METROS {  
 "GEOIDE" = SUP. EQUIPOTENCIAL  
 "ESFEROIDE ESTÁNDAR" = ELIPSOIDE DE REVOLUCIÓN

GRAVEDAD LOCAL  $\perp$  GEOIDE

# RELACION TOPOCÉNTRICAS - GEODÉNTRICAS

PARALAJE DIURNA

ABERRACION DIURNA



$$b = a(1 - \dots)$$

$$\dots = \text{ACH}$$

DIFERENCIA DE METROS

"GEODE" = SUP. EQUIPOTENCIAL

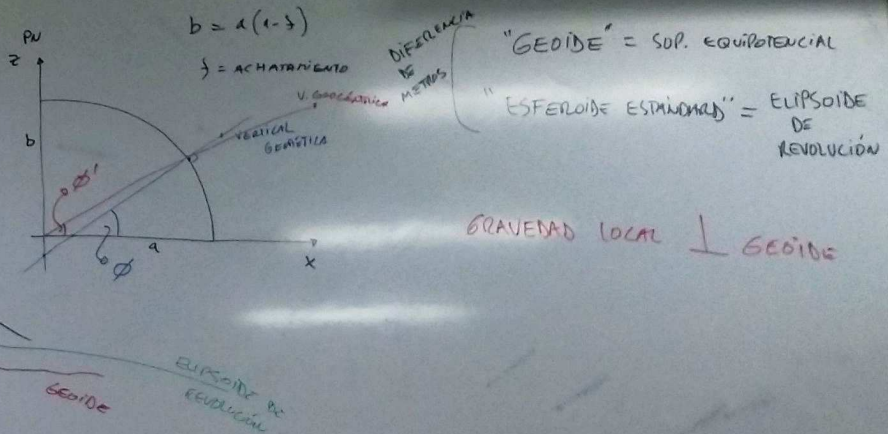
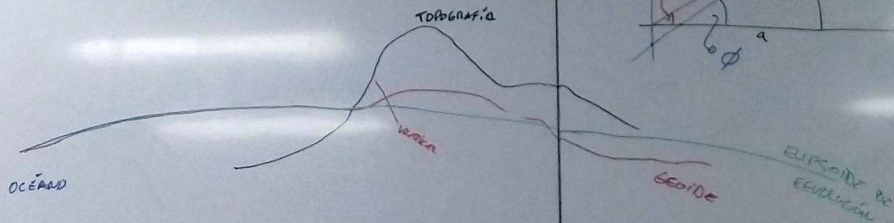
"ESFEROIDE ESTÁNDAR" = ELIPSOIDE DE REVOLUCIÓN

TRAVERSA LOCAL  $\perp$  GEODE

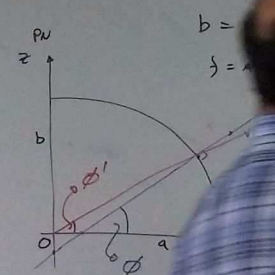
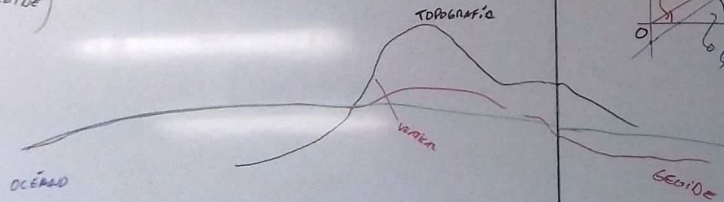
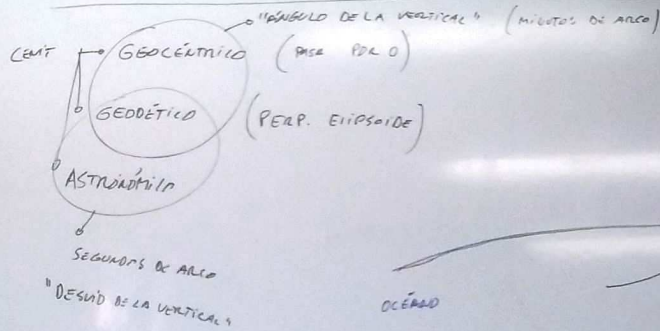
# RELACION TOPOCÉNTRICAS - GEODÉNTRICAS

PARALAJE DIURNA

ABERRACION DIURNA



# RELACION TOPOCÉNTRICAS - GEODÉNTRICAS

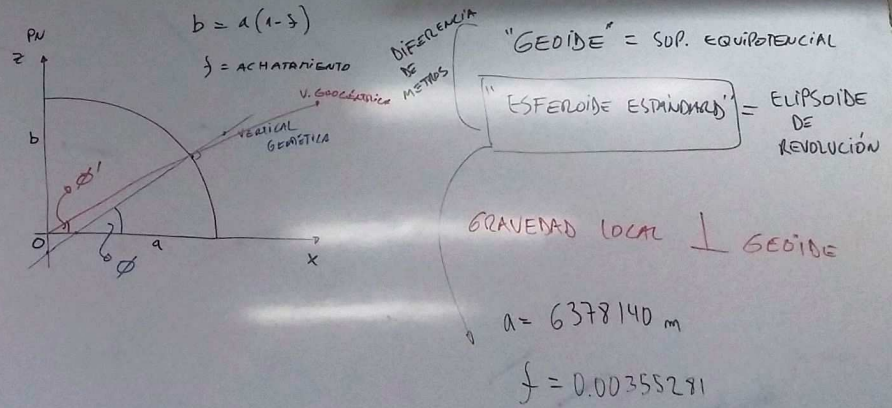
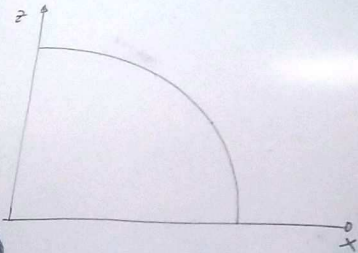


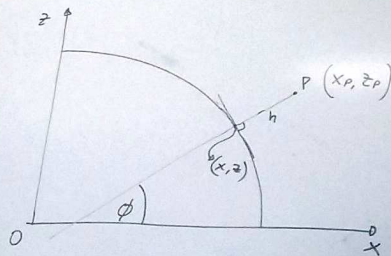
DIFERENCIA DE METROS

"GEOIDE" = SDP. EQUIPOTENCIAL

"ESFEROIDE ESTANDAR" = ELIPSOIDE DE REVOLUCIÓN

GRAVEDAD LOCAL  $\perp$  GEOIDE



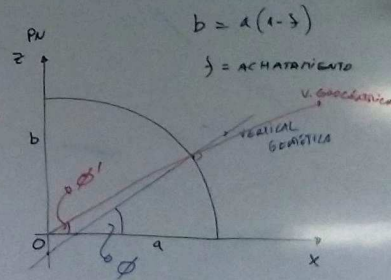


$$x_p = x + h \cdot \cos \phi$$

$$z_p = z + h \cdot \sin \phi$$

EIPSE

$h =$  ALTURA SOBRE EL MAR



$$b = a(1-f)$$

$f =$  ACHATAMIENTO

DIFERENCIA  
DE  
METROS

"GEOIDE" = SUP. EQUIPOTENCIAL

"ESFEROIDE ESTIMADO" = ELIPSOIDE DE REVOLUCIÓN

GRAVEDAD LOCAL  $\perp$  GEOIDE

$$a = 6378140 \text{ m}$$

$$f = 0.00355291$$



$$b = a(1-f)$$

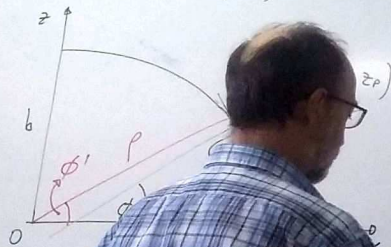
$$x_p = x + h \cos \phi = \rho \cos \phi'$$

$$z_p = z + h \sin \phi = \rho \sin \phi'$$

↑  
EIPSE

BUSCAMOS  $(h, \phi) \longleftrightarrow (\rho, \phi')$

$$\frac{x^2}{a^2} + \frac{z^2}{\underbrace{b^2}_{a^2(1-f)^2}} = 1$$



$h = \text{altura}$

"GEOIDE" = SUP. EQUIPOTENCIAL

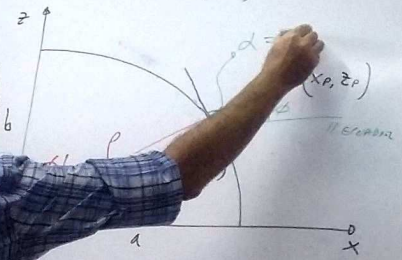
"ESFEROIDE ESTANDAR" = ELIPSOIDE DE REVOLUCION

GRAVEDAD LOCAL  $\perp$  GEOIDE

$$a = 6378140 \text{ m}$$

$$f = 0.00355291$$

$$b = a(1-f)$$



$$x_p = x + h \cdot \cos \phi = \rho \cdot \cos \phi'$$

$$z_p = z + h \cdot \sin \phi = \rho \cdot \sin \phi'$$

EIPSE

BUSCAMOS  $(h, \phi) \longleftrightarrow (\rho, \phi')$

$$\frac{x^2}{a^2} + \frac{z^2}{\underbrace{b^2}_{a^2(1-f)^2}} = 1$$

$$\frac{z}{a^2} dx + \frac{z}{a^2(1-f)^2} dz = 0 \Rightarrow \frac{dx}{dz} = -\frac{z}{x(1-f)^2}$$

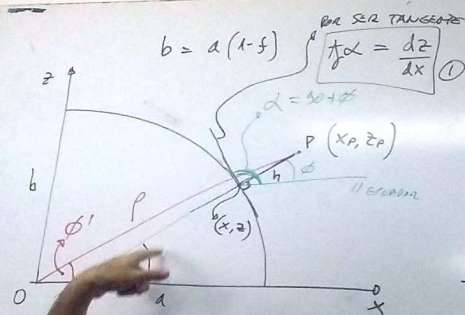
"GEOIDE" = SUP. EQUIPOTENCIAL

"ESFEROIDE ESTIMADO" = ELIPSOIDE DE REVOLUCIÓN

GRAVEDAD LOCAL  $\perp$  GEOIDE

$$a = 6378140 \text{ m}$$

$$f = 0.00355291$$



$b = a(1-f)$

PARA SER TANGENTE  
 $f \alpha = \frac{dz}{dx}$  ①

$x_p = x + h \cdot \cos \phi = \rho \cdot \cos \phi'$

$z_p = z + h \cdot \sin \phi = \rho \cdot \sin \phi'$

EURSE

BUSCAMOS  $(h, \phi) \leftrightarrow (\rho, \phi')$

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$$

$b^2 = a^2(1-f)^2$

$$\frac{2x dx}{a^2} + \frac{2z dz}{a^2(1-f)^2} = 0 \Rightarrow \frac{dx}{dz} = -\frac{z}{x(1-f)^2}$$

DE ①:  $f \alpha = f(\phi + \phi') = -\frac{1}{f \phi} = \frac{dz}{dx}$

$\Rightarrow -\frac{1}{f \phi} = -\frac{z}{x(1-f)^2}$

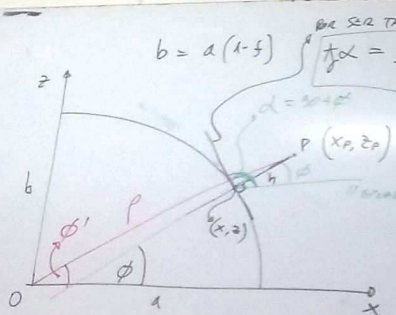
"GEOIDE" = SUP. EQUIPOTENCIAL

"ESFEROIDE ESTIMADO" = ELIPSOIDE DE REVOLUCIÓN

GRAVEDAD LOCAL  $\perp$  GEOIDE

$a = 6378140 \text{ m}$

$f = 0.00355291$



$$b = a(1-f)$$

DE SER TANGENTE

$$\frac{dz}{dx} = \frac{dz}{dx} \quad (1)$$

$$x_p = x + h \cdot \cos \phi = \rho \cdot \cos \phi'$$

$$z_p = z + h \cdot \sin \phi = \rho \cdot \sin \phi'$$

EIPSE

BUSCAMOS  $(h, \phi) \leftrightarrow (\rho, \phi')$

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$$

$$\frac{2x dx}{a^2} + \frac{2z dz}{a^2(1-f)^2} = 0$$

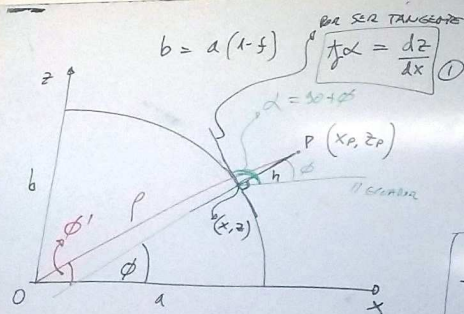
$$\text{DE (1): } \frac{dz}{dx} = \frac{dz}{dx} (1+f) = -\frac{1}{\frac{dz}{dx}} = \frac{dz}{dx}$$

$$(2) \quad \frac{dz}{dx} = f \frac{z}{x(1-f)^2}$$

$$\frac{dz}{z} = f \frac{dx}{x(1-f)^2}$$

$$\frac{z^2}{a^2} + \frac{z^2(1-f)^2 \frac{dz}{dx}}{a^2} = 1$$

h = ALTURA SOBRE EL MAR



$b = a(1-f)$

DE SER TANGENTE  
 $f \alpha = \frac{dz}{dx}$  ①

$x_p = x + h \cdot \cos \phi = p \cdot \cos \phi'$

$z_p = z + h \cdot \sin \phi = p \cdot \sin \phi'$

EUPSE

BUSCAMOS  $(h, \phi) \leftrightarrow (p, \phi')$

$$\frac{x^2}{a^2} + \frac{z^2}{a^2(1-f)^2} = 1$$

$$\frac{z \cdot x \cdot dx}{a^2} + \frac{z \cdot z \cdot dz}{a^2(1-f)^2} = 0 \Rightarrow \frac{dx}{dz} = -\frac{z}{x(1-f)^2}$$

h = ALTURA SOBRE EL MAR

DE ①:  $f \alpha = f(90 + \phi) = -\frac{1}{f \phi} = \frac{dz}{dx}$

②  $f + f \phi = f \frac{z}{x(1-f)^2}$

$z = x(1-f)^2 \cdot f \phi$  ③

$$\frac{x^2}{a^2} + \frac{x^2(1-f)^2 f^2 \phi^2}{a^2} = 1$$

④  $x^2 \left( 1 + (1-f)^2 f^2 \phi^2 \right) = a^2$

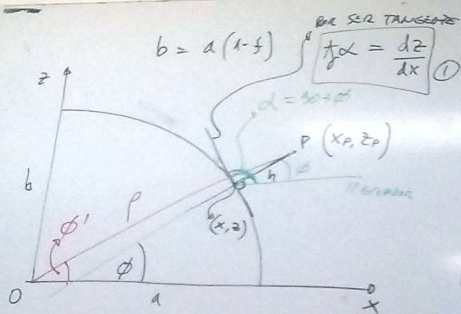
$$x^2 = \frac{a^2 \cdot \cos^2 \phi}{\cos^2 \phi + (1-f)^2 \sin^2 \phi}$$

$$x = a \cdot \left[ \cos^2 \phi + (1-f)^2 \sin^2 \phi \right]^{-1/2} \cdot \cos \phi$$

z = ...

$x = a \cdot C \cdot \cos \phi$

z =



$b = a(1-f)$   
 DE SER TANGENTE  
 $f \alpha = \frac{dz}{dx}$  ①

$x_p = x + h \cdot \cos \phi = p \cdot \cos \phi'$   
 $z_p = z + h \cdot \sin \phi = p \cdot \sin \phi'$

EIPSE

$a \cdot C \cdot \cos \phi + h \cdot \cos \phi = p \cdot \cos \phi'$   
 $a \cdot S \cdot \sin \phi + h \cdot \sin \phi = p \cdot \sin \phi'$

$a \cdot \cos \phi \cdot (C + h/a) = p \cdot \cos \phi'$   
 $a \cdot \sin \phi \cdot (S + h/a) = p \cdot \sin \phi'$

FUNCIÓN DE  $\phi$

$h =$  ALTURA SOBRE EL MAR

DE ①:  $f \alpha = f \phi (90 + \phi) = -\frac{1}{f \phi} = \frac{dz}{dx}$

②  $f \phi + f \phi = f \frac{z}{x(1-f)^2}$

$z = x(1-f)^2 \cdot f \phi$  ③

$\frac{x^2}{a^2} + \frac{x^2(1-f)^2 f^2 \phi}{a^2} = 1$

$x^2 (1 + (1-f)^2 f^2 \phi) = a^2$

$x^2 = \frac{a^2 \cdot \cos^2 \phi}{\cos^2 \phi + (1-f)^2 \sin^2 \phi}$

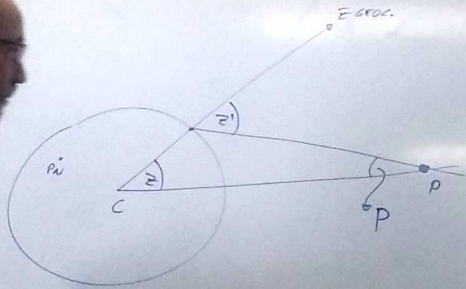
$x = a \cdot [\cos^2 \phi + (1-f)^2 \sin^2 \phi]^{-1/2} \cdot \cos \phi$

$z = \dots$

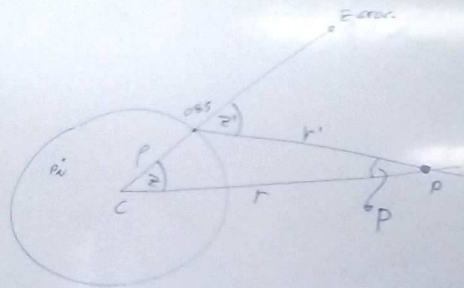
$x = a \cdot C \cdot \cos \phi$   
 $z = a \cdot S \cdot \sin \phi$

$C \cdot (1-f)^2$

PARALASE GEOCÉNTRICA (ó DIURNA)



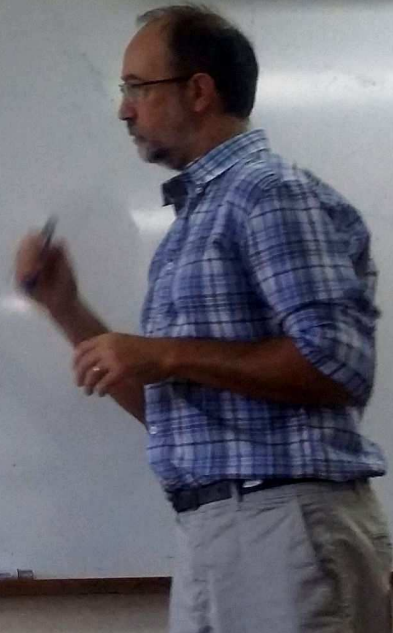
PARALAJE GEOCÉNTRICA (ó SURTA)



$$\frac{\sin P}{p} = \frac{\sin z}{r'} = \frac{\sin(180-z')}{r} \Rightarrow \sin P = \frac{p}{r} \cdot \sin z'$$

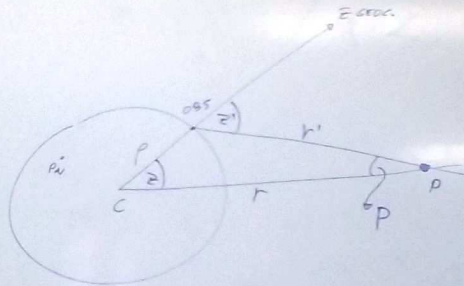
$$z' = z + p$$

$$\Rightarrow z = z' - p$$





PARALAJE GEOCÉNTRICA (ó DIURNA)



$$\frac{\sin P}{p} = \frac{\sin z}{r'} = \frac{\sin(180 - z')}{r}$$

$$z' = z + p$$

$$\Rightarrow \sin P = \frac{p}{r} \cdot \sin z'$$

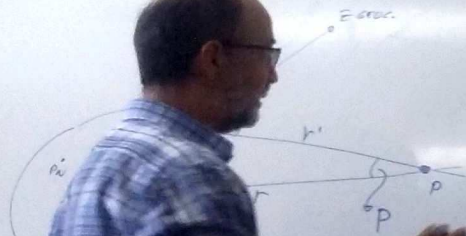
$$\Rightarrow z = z' - p$$

P. HORIZONTAL ES

$z' = 90^\circ$

$$\Rightarrow \sin P_{\text{max}} = \frac{p}{r}$$

PARALAJE GEOCÉNTRICA (ó DIURNA)



$$\frac{\sin P}{\rho} = \frac{\sin z}{r'} = \frac{\sin(180 - z')}{r}$$

$$z' = z + p$$

$$\Rightarrow \sin P = \frac{\rho}{r} \cdot \sin z'$$

$$\Rightarrow z = z' - p$$

P. HORIZONTAL ES P TAL QUE

$$p = a \quad \text{y} \quad z' = 90^\circ$$

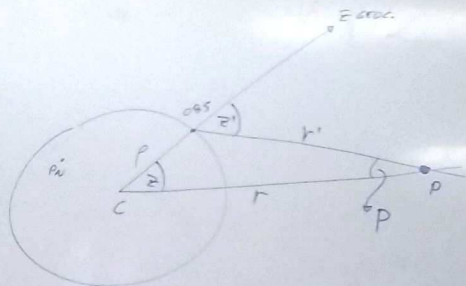
$$\Rightarrow \sin P_{\text{hor}} = \frac{1}{r} \cdot 1$$

$\rightarrow$  LUNA  $P_{\text{hor}} \approx 57'$   
 $\rightarrow$  SOL

$$\sin P_{\text{sol}} \approx \frac{6400 \text{ km}}{150.000.000 \text{ km}}$$

$$8,8$$

PARALAJE GEOCÉNTRICO (ó DIURNO)



$$\frac{\sin P}{p} = \frac{\sin z}{r'} = \frac{\sin(180 - z')}{r}$$

$$\Rightarrow \sin P = \frac{p}{r} \cdot \sin z'$$

$$z' = z + p$$

$$\Rightarrow z = z' - p$$

$$\Delta \alpha, \Delta \delta$$

ANÁLOGO A REFRACCIÓN

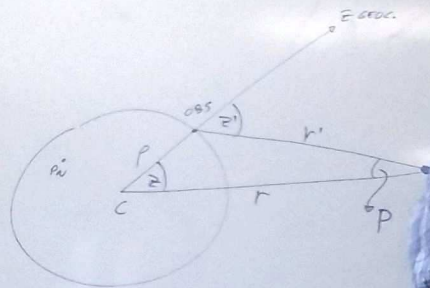
(DESCONCIAMOS  $\Delta \alpha$ )

FÓRMULA RIGOROSA



$$\Delta z = k \cdot \frac{1}{r} z$$

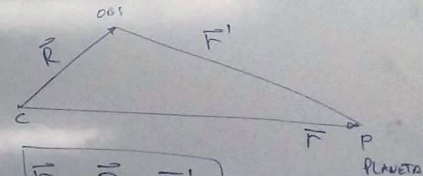
PARALAJE GEOCÉNTRICA (ó DIURNA)



DADO  $\vec{r} = (\alpha, \delta, r)$

$$\begin{cases} x = \cos \delta \cdot \cos \alpha \cdot r \\ y = \cos \delta \cdot \sin \alpha \cdot r \\ z = r \cdot \sin \delta \end{cases}$$

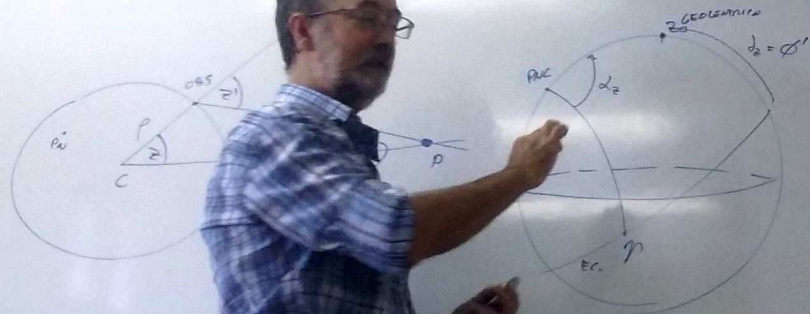
FÓRMULA RIORDANA



$$\vec{F} = \vec{R} + \vec{R}'$$

PARALAJE GEOCÉNTRICO (ó DIÓTRIA)

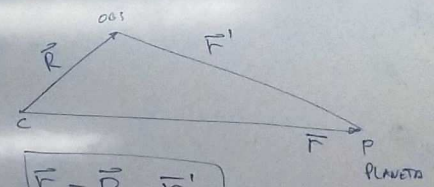
COORD. ECUI. CELESTES DEL OBS.



DADO  $\vec{r} (\alpha, \delta, r)$

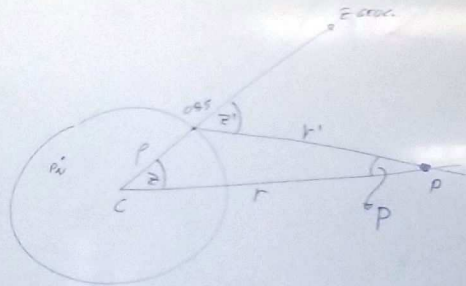
$$\begin{cases} x = \cos \delta \cdot \sin \alpha \cdot r \\ y = \cos \delta \cdot \cos \alpha \cdot r \\ z = \sin \delta \cdot r \end{cases}$$

FÓRMULA RICORDOSA

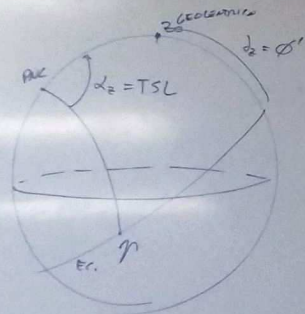


$$\vec{F} = \vec{R} + \vec{F}'$$

PARALAJE GEOCÉNTRICA (ó DIURNA)



(COORD. ECUI. CELESTES DEL OBS.)



DADO  $\vec{R} (\alpha, \delta, r)$

$$\begin{cases} x = \cos \delta \cdot \sin \alpha \cdot r \\ y = \cos \delta \cdot \cos \alpha \cdot r \\ z = \sin \delta \cdot r \end{cases} \quad \text{PLA}$$

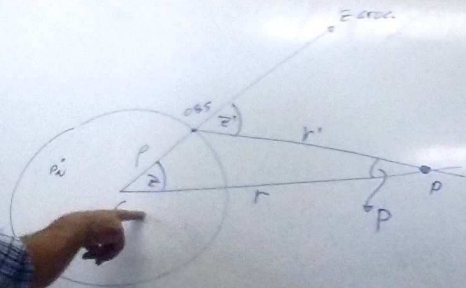
$$\vec{R} = \begin{cases} x = P \cdot \cos TSL \cdot \cos \phi' \\ y = P \cdot \sin TSL \cdot \cos \phi' \\ z = P \cdot \sin \phi' \end{cases} \quad \text{OBS}$$

$$\Rightarrow \vec{R}' = \vec{R} - \vec{R} \Rightarrow x'$$

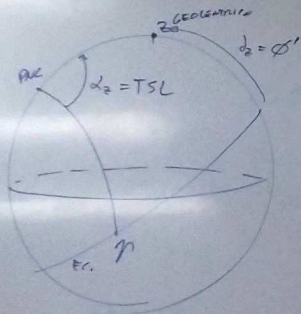
FÓRMULA RECURRENTE



PARALAJE GEOLÉNTRICA (ó DIURNA)



(COORD. ECUI. CELESTES DEL OBS.)



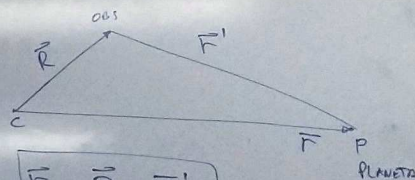
DADO  $\bar{r} (\alpha, \delta, r)$

$$\begin{cases} x = \cos \delta \cdot \sin \alpha \cdot r \\ y = \cos \delta \cdot \cos \alpha \cdot r \\ z = \sin \delta \cdot r \end{cases} \quad \text{PLA}$$

$$\bar{R} = \begin{cases} x = P \cdot \cos TSL \cdot \cos \phi' \\ y = P \cdot \sin TSL \cdot \cos \phi' \\ z = P \cdot \sin \phi' \end{cases} \quad \text{OBS}$$

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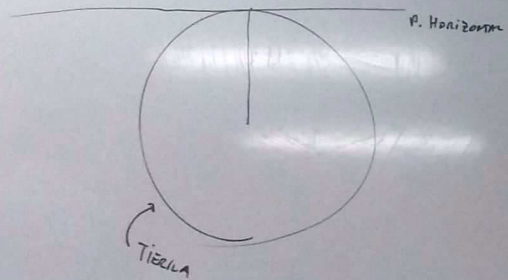
FÓRMULA RIGUROSA



$$\bar{F} = \bar{R} + \bar{r}'$$

$$\Rightarrow \bar{r}' = \bar{F} - \bar{R} \Rightarrow \begin{cases} x' = r' \cdot \cos \delta' \cdot \cos \alpha' \\ y' = r' \cdot \cos \delta' \cdot \sin \alpha' \\ z' = r' \cdot \sin \delta' \end{cases} \Rightarrow \alpha', \delta'$$

# DEPRESIÓN DEL HORIZONTE

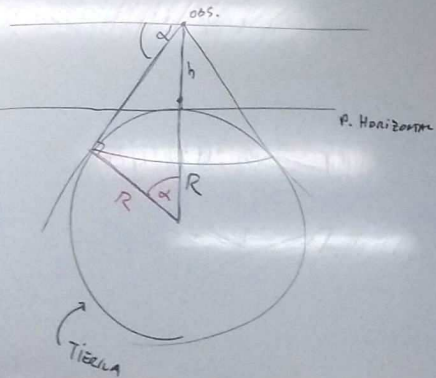




## DEPRESIÓN DEL HORIZONTE

$\alpha$  = Depresión

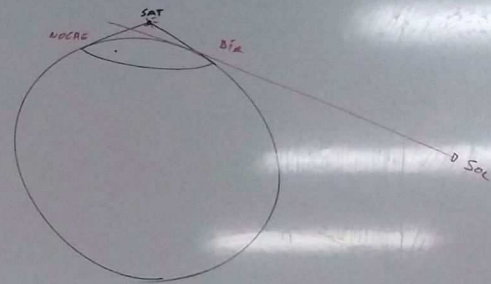
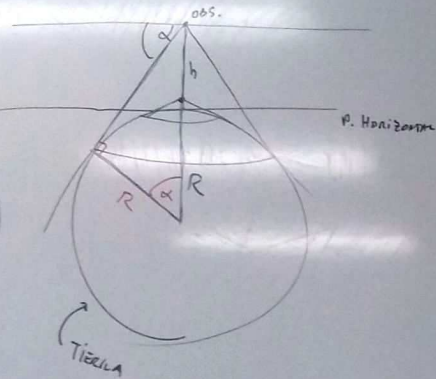
$$\cos \alpha = \frac{R}{R+h} = \frac{1}{1+b/R}$$



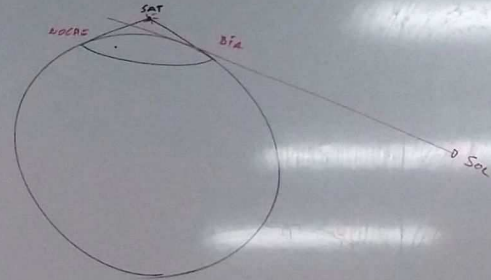
# DEPRESIÓN DEL HORIZONTE

$\alpha$  = Depresión

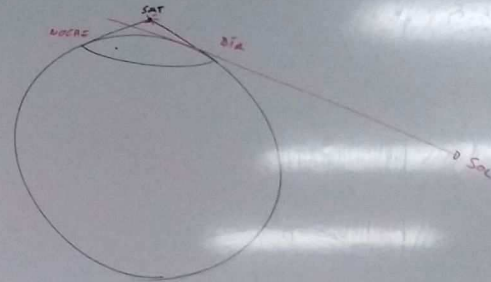
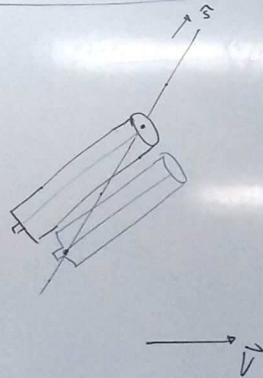
$$\cos \alpha = \frac{R}{R+h} = \frac{1}{1+h/R}$$



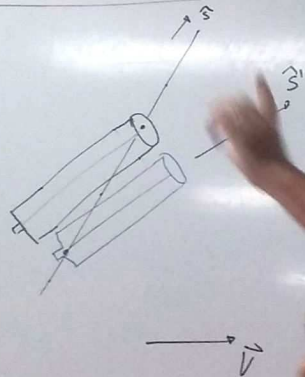
# ABERRACIÓN DE LA LUZ



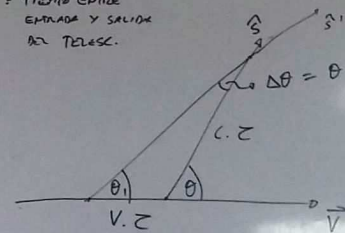
# ABERRACIÓN DE LA LUZ



# ABERRACIÓN DE LA LUZ



$Z$  = TIEMPO ENTRE ENTRADA Y SALIDA DEL TELESC.



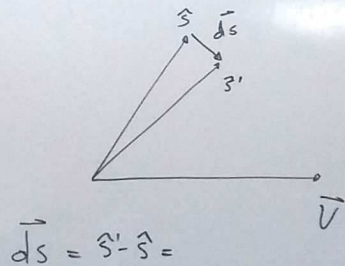
$$\Rightarrow \frac{\Delta \theta}{V \cdot Z} = \frac{\sin \theta_1}{c \cdot Z}$$

$$\Rightarrow \Delta \theta \approx \left(\frac{V}{c}\right) \cdot \sin \theta_1 \ll \theta$$

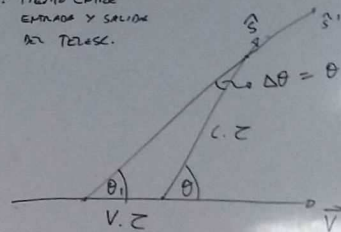
$$\Delta \theta (\text{rads}) \approx \frac{V}{c} \cdot \sin \theta_1 \approx \frac{V}{c} \cdot \sin \theta$$

$$V \ll c$$

# ABERRACIÓN DE LA LUZ



$z$  = TIEMPO ENTRE ENTRADA Y SALIDA DEL TELESC.



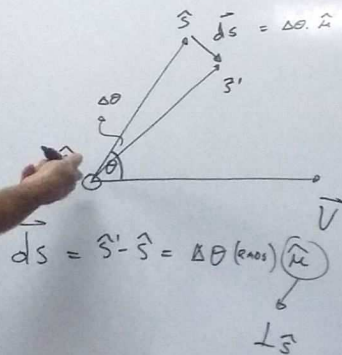
$V \ll c$

$$\Rightarrow \frac{\sin \Delta \theta}{V \cdot z} = \frac{\sin \theta_1}{c \cdot z}$$

$$\Rightarrow \sin \Delta \theta = \left( \frac{V}{c} \right) \cdot \sin \theta_1$$

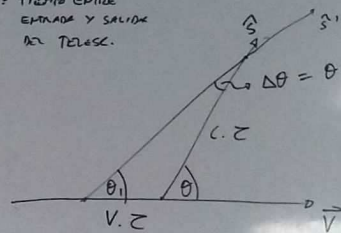
$$\Delta \theta (\text{RADS}) \approx \frac{V}{c} \cdot \sin \theta_1 \approx \frac{V}{c} \cdot \sin \theta$$

# ABERRACIÓN DE LA LUZ



$$\hat{r} = \frac{\vec{V} \wedge \hat{s}}{V \cdot \sin\theta}$$

$z$  = TIEMPO ENTRE ENTRADA Y SALIDA DEL TELESC.



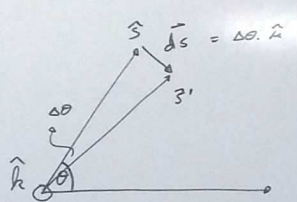
$$\Rightarrow \frac{\sin\Delta\theta}{V \cdot z} = \frac{\sin\theta'}{c \cdot z}$$

$$\Rightarrow \sin\Delta\theta = \left(\frac{V}{c}\right) \cdot \sin\theta'$$

$$\Delta\theta \text{ (rads)} \approx \frac{V}{c} \cdot \sin\theta' \approx \frac{V}{c} \cdot \sin\theta$$

$$V \ll c$$

# ABERRACIÓN DE LA LUZ



$$\hat{h} = \frac{\vec{V} \wedge \hat{s}}{V \cdot \sin \theta}$$

$$\hat{h} = \hat{s} \wedge \hat{V}$$

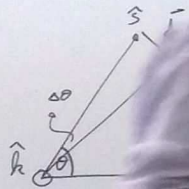
$$\vec{ds} = \hat{s}' - \hat{s} = \Delta \theta (\hat{h}) = \frac{V}{c} \cdot \hat{s} \wedge \left( \frac{\vec{V} \wedge \hat{s}}{V \sin \theta} \right) = \frac{1}{c} \cdot \hat{s} \wedge (\vec{V} \wedge \hat{s})$$

$$\frac{\sin \theta_1}{c \cdot z} \ll \ll$$

$$\theta_1 \approx \frac{V}{c} \cdot \sin \theta$$



# ABERRACIÓN LUZ

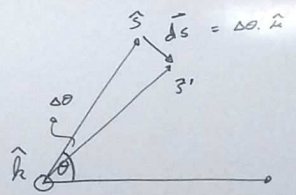


$$\vec{ds} = \hat{s}' - \hat{s}$$

$$\hat{s}' \approx \hat{s} + \frac{\vec{v} \wedge \hat{s}}{c} = \frac{1}{c} \cdot \hat{s} \wedge (\vec{v} \wedge \hat{s})$$

$$\vec{ds} = \frac{1}{c} \cdot \left[ (\hat{s} \cdot \hat{s}) \cdot \vec{v} - (\hat{s} \cdot \vec{v}) \cdot \hat{s} \right] = \frac{1}{c} \left[ \vec{v} - (\hat{s} \cdot \vec{v}) \cdot \hat{s} \right]$$

# ABERRACIÓN DE LA LUZ



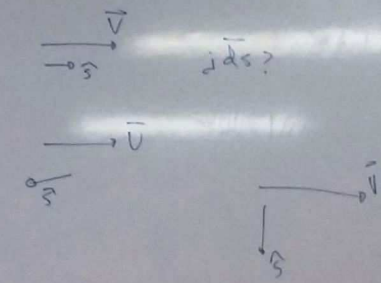
$$\hat{h} = \frac{\vec{V} \wedge \hat{s}}{V \sin \theta}$$

$$\hat{h} = \hat{s} \wedge \hat{h}$$

$$\vec{d}s = \hat{s}' - \hat{s} = \Delta \theta (\text{rad}) \hat{h} = \frac{V}{c} \cdot \hat{s} \wedge \left( \frac{\vec{V} \wedge \hat{s}}{V \sin \theta} \right) = \frac{1}{c} \cdot \hat{s} \wedge (\vec{V} \wedge \hat{s})$$

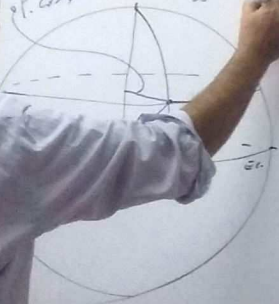
$$\vec{d}s = \frac{1}{c} \cdot \left[ (\hat{s} \cdot \hat{s}) \cdot \vec{V} - (\hat{s} \cdot \vec{V}) \cdot \hat{s} \right] = \frac{1}{c} \left[ \vec{V} - (\hat{s} \cdot \vec{V}) \cdot \hat{s} \right]$$

↓
90. max
90. min



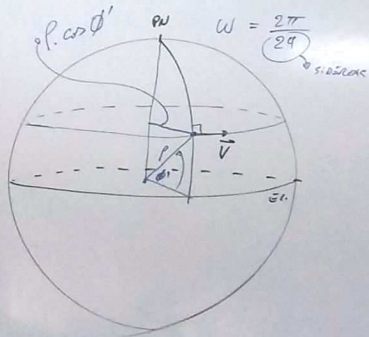
# ABERRACIÓN DIURNA

$\rho \cdot \cos \delta'$      $\rho \omega$      $\omega = 2\pi$



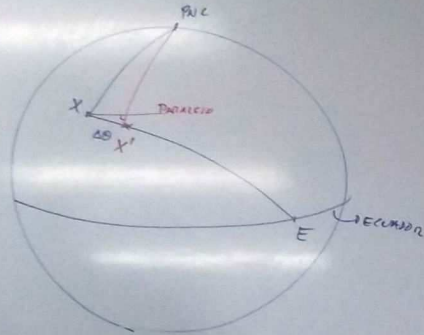
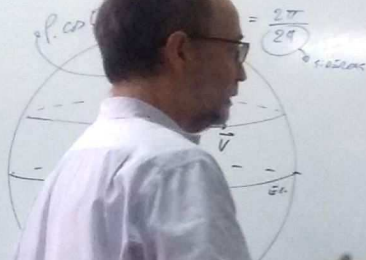
## ABERRACIÓN DIURNA

$$\vec{V} = P \cdot \sin \phi' \cdot \omega \cdot \hat{E}$$

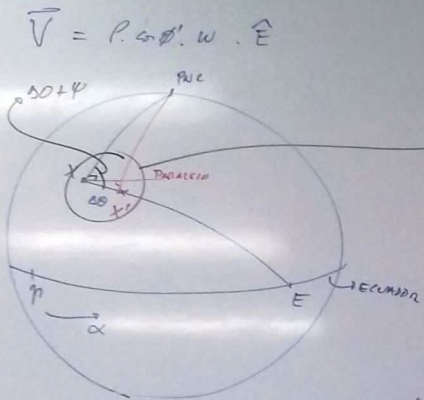
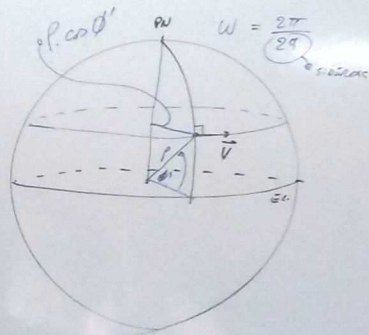


ABERRACION DIURNA

$$\vec{V} = P \cdot \sin \delta \cdot \omega \cdot \hat{E}$$

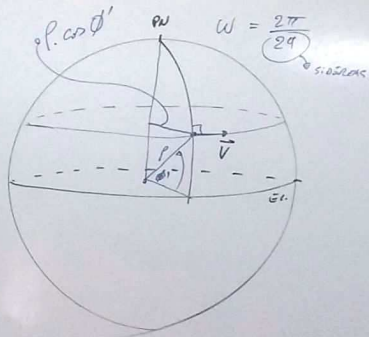


ABERRACION DIURNA

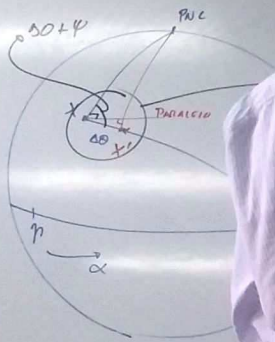


$$\Delta \theta = \frac{V}{c} \cdot \sin \theta$$

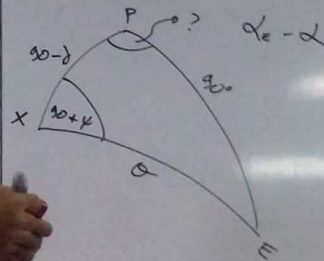
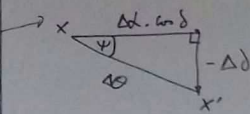
ABERRACIÓN DIURNA



$$\vec{V} = R \cdot \sin \delta' \cdot \omega \cdot \hat{E}$$



Δ PLANO

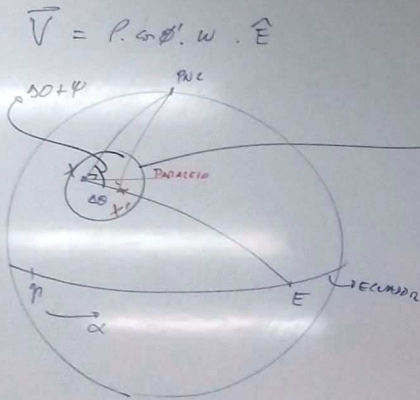
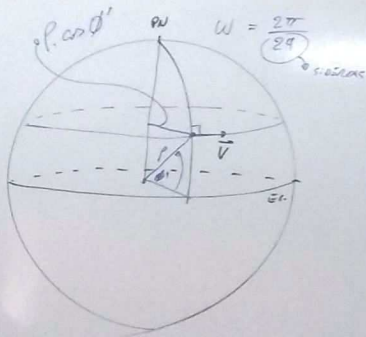


$$\Delta \theta = \frac{V}{c} \cdot \sin \theta \cdot \hat{X} \hat{E}$$

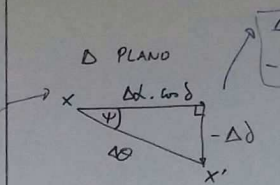
$$TSL = H + \alpha = H_e + \alpha_e$$

$$\alpha_e - \alpha = H - H_e$$

ABERRACIÓN DIURNA



$$\vec{V} = P \cdot \sin \delta \cdot \omega \cdot \hat{E}$$



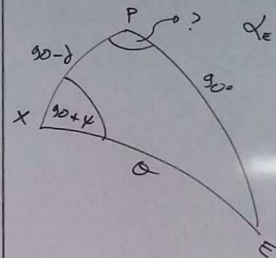
$$\Delta \alpha \cdot \cos \delta = \Delta \theta \cdot \cos \psi$$

$$-\Delta \delta = \Delta \theta \cdot \sin \psi$$

$$\Delta \alpha \cdot \cos \delta = \frac{v}{c} \cdot \sin \theta$$

$$TSL = H + \alpha = H_E + \alpha_E$$

$$\Rightarrow \alpha_E - \alpha = H - H_E =$$



$$\alpha_E - \alpha = H + 90^\circ$$



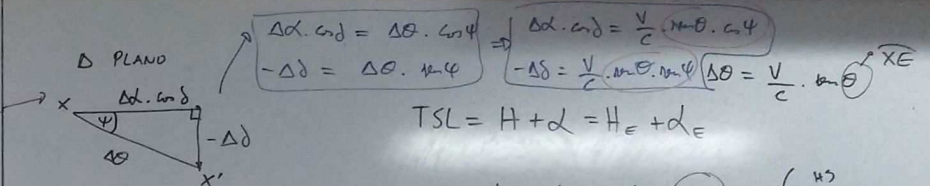
ABERRACION DIURNA

$$\vec{V} = P \cdot \sin \theta \cdot \omega \cdot \hat{E}$$

$$\frac{\cos \psi}{\sin(90 - \psi)}$$

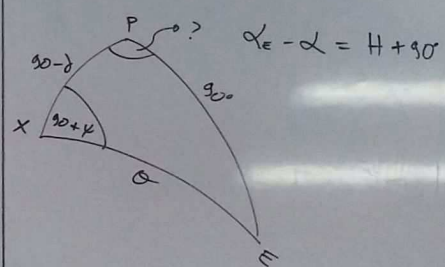
$$\Rightarrow \cos \psi \cdot \omega \theta = \cos H$$

$$= \frac{V}{\omega \theta} \cdot \frac{H}{\cos \psi}$$



$$TSL = H + \alpha = H_E + \alpha_E$$

$$\Rightarrow \alpha_E - \alpha = H - H_E = -6^{\text{h}} - 50^{\circ}$$



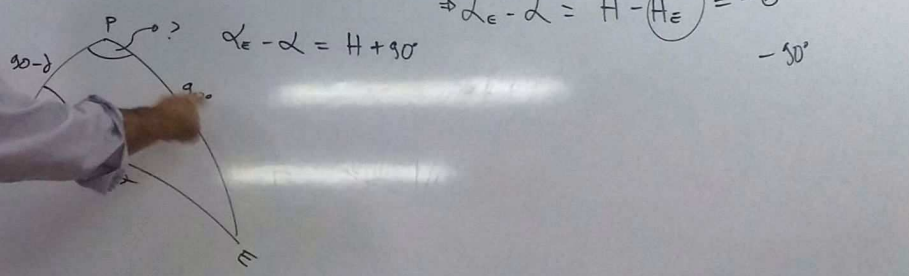
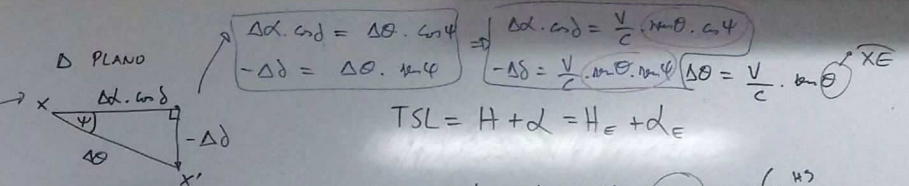
ABERRACION DIURNA

$$\vec{V} = P \cdot \cos \phi' \cdot \omega \cdot \hat{E}$$

$$\frac{\cos \psi}{\cos 90^\circ} = \frac{\cos(H + 90^\circ)}{\cos \theta} \Rightarrow \boxed{\cos \psi \cdot \cos \theta = \cos H}$$

$$\Delta \alpha = \frac{P \cdot \omega \cdot \cos \phi'}{c} \cdot \frac{\cos H}{\cos \delta} = \frac{P \omega}{c}$$

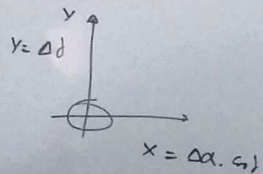
PROBAR  $\Rightarrow \Delta \delta = \frac{P \omega}{c} \cdot \cos \phi' \cdot \sin \delta \cdot \sin \theta$



ABERRACION DIURNA

$$\vec{V} = P \cdot \sin \phi' \cdot \omega \cdot \hat{E}$$

$$\frac{\cos(90 + \psi)}{\cos 90'} = \frac{\sin(H + 90)}{\sin \theta} \Rightarrow \cos \psi \cdot \sin \theta = \cos H$$



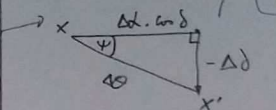
$$\Delta \alpha = \frac{P \cdot \omega \cdot \cos \phi'}{c} \cdot \frac{\cos H}{\cos \delta} = \frac{P \omega}{c} \cdot \cos \phi' \cdot \frac{\cos H}{\cos \delta}$$

PROBAR

$$\Delta \delta = \frac{P \omega}{c} \cdot \cos \phi' \cdot \sin \delta \cdot \sin H$$

ECUACION

\Delta PLANO



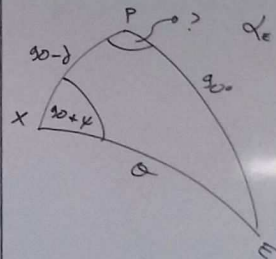
$$\Delta \alpha \cdot \cos \delta = \Delta \delta \cdot \cos \psi \Rightarrow \Delta \alpha \cdot \cos \delta = \frac{V}{c} \cdot \sin \theta \cdot \cos \psi$$

$$-\Delta \delta = \Delta \alpha \cdot \sin \psi \Rightarrow -\Delta \delta = \frac{V}{c} \cdot \sin \theta \cdot \sin \psi \quad \Delta \alpha = \frac{V}{c} \cdot \sin \theta$$

$$TSL = H + \alpha = H_e + \alpha_e$$

$$\alpha_e - \alpha = H - H_e = -6''$$

$$-50'$$



$$\alpha_e - \alpha = H + 90'$$

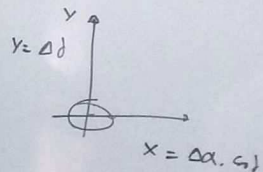
ABERRACION DIURNA

$$\vec{V} = P \cdot \cos \phi' \cdot \omega \cdot \hat{E}$$

↘  $\cos \psi$

$$\frac{\cos(90 + \psi)}{\cos 90'} = \frac{\cos(H + 90)}{\cos \theta} \Rightarrow \cos \psi \cdot \cos \theta = \cos H$$

1



PROBAR

$$\Delta \alpha = \left( \frac{V}{c} \right) \cdot \frac{\cos H}{\cos \delta} = \frac{P \cdot \omega \cdot \cos \phi'}{c} \cdot \frac{\cos H}{\cos \delta}$$

$$\Delta \delta = \frac{P \cdot \omega}{c} \cdot \cos \phi' \cdot \sin \delta \cdot \sin H$$

ERENDIN

$$\begin{aligned} \Delta \alpha \cdot \cos \delta &= \Delta \theta \cdot \cos \psi \\ -\Delta \delta &= \Delta \theta \cdot \sin \psi \end{aligned} \Rightarrow \begin{aligned} \Delta \alpha \cdot \cos \delta &= \frac{V}{c} \cdot \cos \theta \cdot \cos \psi \\ -\Delta \delta &= \frac{V}{c} \cdot \sin \theta \cdot \sin \psi \end{aligned} \quad \Delta \theta = \frac{V}{c} \cdot \sin \theta \cdot \hat{X}_E$$

$$x = \Delta \alpha \cdot \cos \delta = \frac{P \cdot \omega}{c} \cdot \cos \phi' \cdot \cos H$$

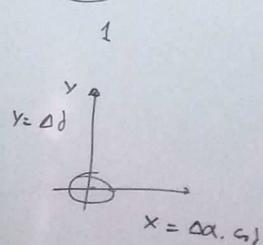
$$TSL = H + \alpha = H_E + \alpha_E$$

$$\Rightarrow \alpha_E - \alpha = H - H_E = -6^{\text{h}} - 50'$$

ABERRACION DIURNA

$$\vec{V} = P \cdot \sin \phi' \cdot \omega \cdot \hat{E}$$

$$\frac{\cos \psi}{\cos(90 + \psi)} = \frac{\cos(H + 90)}{\cos \theta} \Rightarrow \cos \psi \cdot \cos \theta = \cos H$$



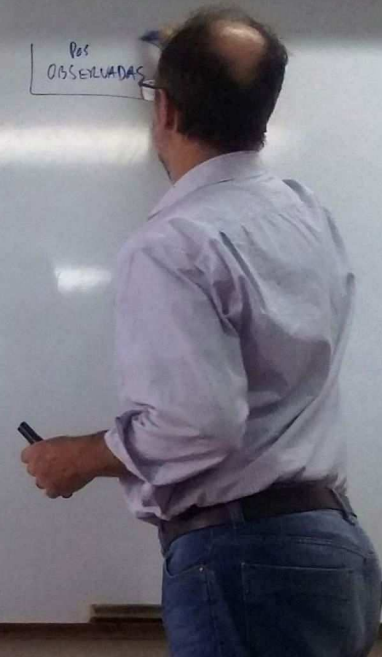
PROBAR

$$\Delta \alpha = \frac{V}{c} \cdot \frac{\cos H}{\cos \delta} = \frac{P \cdot \omega \cdot \cos \phi'}{c} \cdot \frac{\cos H}{\cos \delta}$$

$$\Delta \delta = \frac{P \cdot \omega}{c} \cdot \cos \phi' \cdot \sin \delta \cdot \sin H$$

$$\Rightarrow \begin{aligned} \Delta \alpha &\cong 0.0213 \cdot \cos \phi' \cdot \cos H / \cos \delta \\ \Delta \delta &\cong 0.320 \cdot \cos \phi' \cdot \sin \delta \cdot \sin H \end{aligned}$$

Pos  
OBSERVADAS



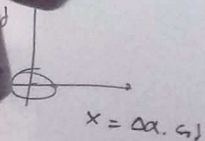
ABERRACION DIURNA

$$\vec{V} = P \cdot \sin \phi' \cdot \omega \cdot \hat{E}$$

$$\frac{\cos \psi}{\cos(\psi + \phi)} = \frac{\sin(H + 90)}{\sin \theta} \Rightarrow \cos \psi \cdot \sin \theta = \cos H$$

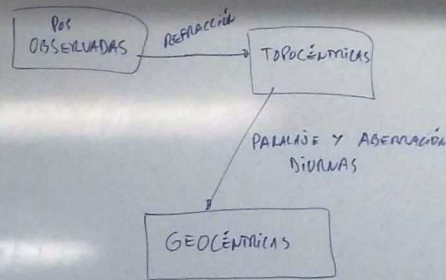
$$\Delta \alpha = \frac{V}{c} \cdot \frac{\cos H}{\cos \delta} = \frac{P \cdot \omega \cdot \cos \phi'}{c} \cdot \frac{\cos H}{\cos \delta}$$

PROBAR  $\rightarrow \Delta \delta = \frac{P \omega}{c} \cdot \cos \phi' \cdot \sin \delta \cdot \sin H$



$$\Rightarrow \Delta \alpha \cong 0.0213 \cdot \cos \phi \cdot \cos H / \cos \delta$$

$$\Delta \delta \cong 0.320 \cdot \cos \phi \cdot \sin \delta \cdot \sin H$$



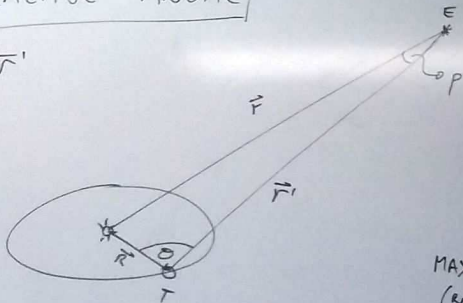
"COORD. ABARENTES"

CORRECCIONES MOVIMIENTO POLAR

# PARALAJE ANUAL

1838 G1 del cisne

$$F = \vec{R} + \vec{r}'$$



$$\frac{\sin p}{R} = \frac{\sin \theta}{r}$$

$$\Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta$$

$$p \text{ (rads)} = \frac{R}{r} \cdot \sin \theta$$

$$\text{MAX } p \text{ (rads)} = \pi = \frac{R}{r} \cdot \sin 90^\circ$$

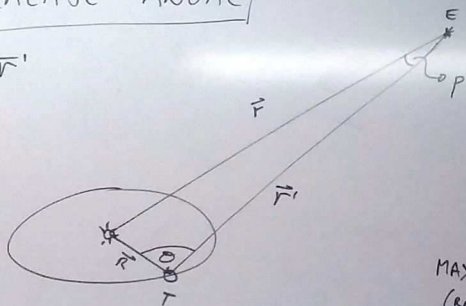
PAR-SEC

$$\pi = 1'' = \frac{1}{206265}$$

PARALAJE ANUAL

1838 G1 del CISE

$F = \vec{R} + \vec{V}'$



$\sin p = \frac{\sin \theta}{R}$

$\Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta$

$p \text{ (RAD)} = \frac{R}{r} \cdot \sin \theta$

$\text{MAX } p \text{ (RAD)} = \pi = \frac{R}{r} \cdot \sin 90^\circ$

PAR-SEC

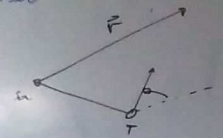
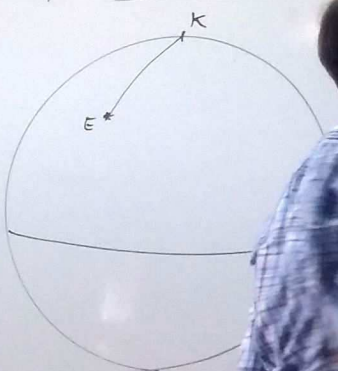
$\pi = 1'' = \frac{1}{206265} \text{ RAD} = \frac{R}{r} = \frac{1 \text{ ua}}{r} \Rightarrow r = 206265 \text{ ua} = 1 \text{ pc}$

1 pc =



PARALAJE ANUAL

1838 G1 del círculo



$$p(\text{rads}) = \frac{R}{r} \cdot \sin \theta$$

PAR-SEC

$$\pi = 1'' = \frac{1}{206265} = \frac{R}{r} = \frac{1 \mu a}{r} \Rightarrow r = 206265 \mu a = 1 \text{ PC}$$

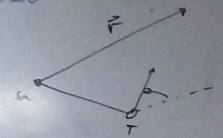
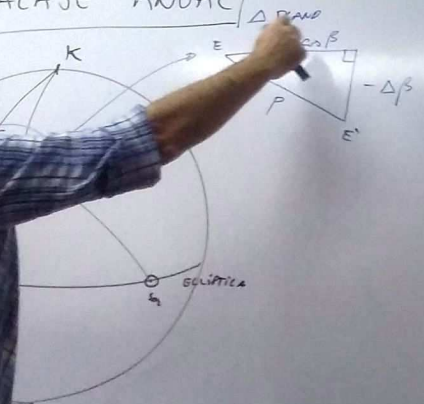
$1 \mu a = 150 \times 10^6 \text{ km}$

$$1 \text{ PC} = 3,26 \text{ AL}$$

$$1 \text{ AL} = 300.000 \frac{\text{km}}{\text{seg}} \times 60 \times 60 \times 24 \times 365,25$$

PARALAJE ANUAL

1838 G1 de CIGUE



$$p(\text{rads}) = \frac{R}{r} \cdot \sin \theta$$

PAR-SEC

$$\pi = 1'' = \frac{1}{206265} \text{ rads} = \frac{R}{r} = \frac{1 \text{ ua}}{r} \Rightarrow r = 206265 \text{ ua} = 1 \text{ PC}$$

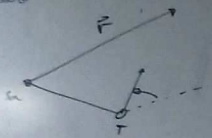
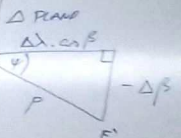
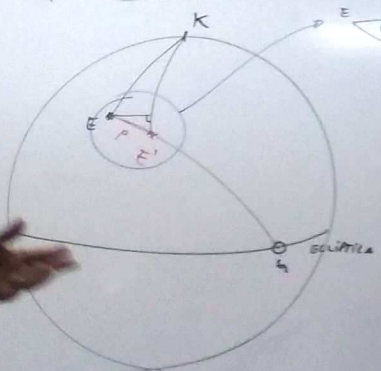
$$1 \text{ ua} = 150 \times 10^6 \text{ km}$$

$$1 \text{ PC} = 3,26 \text{ AL}$$

$$1 \text{ AL} = 300.000 \frac{\text{km}}{\text{seg}} \times 60 \cdot 60 \cdot 24 \cdot 365,25$$

PARALAJE ANUAL

1838 GI de CIGUIS



$$\Rightarrow \Delta \lambda \cdot \cos \beta = p \cos \psi$$

$$-\Delta \beta = p \sin \psi$$

$$\Rightarrow \begin{cases} \Delta \lambda \cdot \cos \beta = \frac{R}{r} \cdot \sin \theta \cdot \cos \psi \\ -\Delta \beta = \frac{R}{r} \cdot \sin \theta \cdot \sin \psi \end{cases}$$

$$p(\text{rads}) = \frac{R}{r} \cdot \sin \theta$$

PAR-SEC

$$\pi = 1'' = \frac{1}{206265} \text{ rads}$$

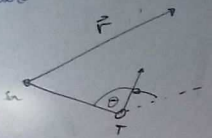
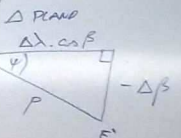
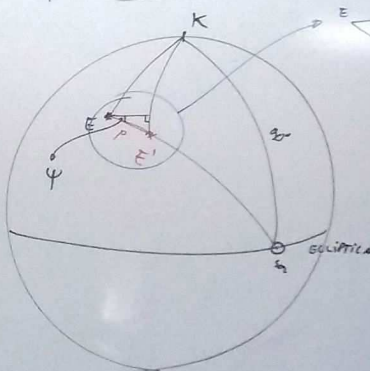
$$\frac{R}{r} = \frac{1 \mu a}{r} \Rightarrow r = 206265 \mu a = 1 \text{ pc}$$

$$1 \text{ pc} = 3,26 \text{ AL}$$

$$1 \text{ AL} = 300.000 \frac{\text{km}}{\text{seg}} \times 60 \cdot 60 \cdot 24 \cdot 365,25$$

PARALAJE ANUAL

1838 G1 de CIGUE

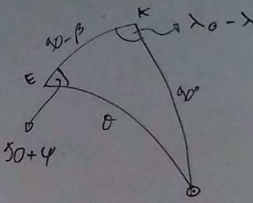


$$p(\text{eas}) = \frac{R}{r} \cdot \sin \theta$$

$$\Rightarrow \Delta \lambda \cdot \cos \beta = p \cdot \cos \psi$$

$$-\Delta \beta = p \cdot \sin \psi$$

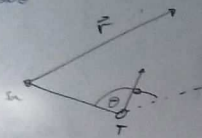
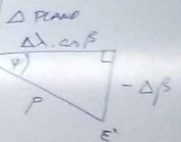
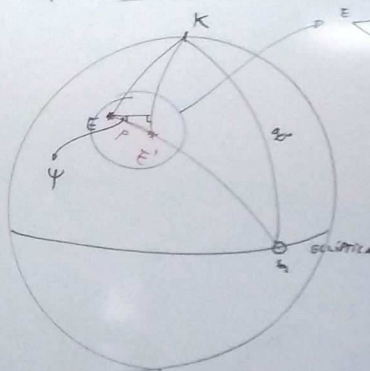
$$\Rightarrow \begin{cases} \Delta \lambda \cdot \cos \beta = \frac{R}{r} \cdot \sin \theta \cdot \cos \psi \\ -\Delta \beta = \frac{R}{r} \cdot \sin \theta \cdot \sin \psi \end{cases}$$



$$\frac{\sin(\lambda_0 - \lambda)}{\sin \theta} = \frac{\sin(90 - \beta)}{\sin(90 + \psi)}$$

PARALAJE ANUAL

1838 GI de CIGUS

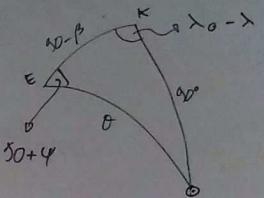


$$p(\text{eas}) = \frac{R}{r} \cdot \sin \theta$$

$$\Rightarrow \Delta \lambda \cdot \cos \beta = p \cos \psi$$

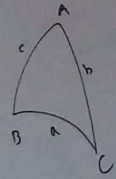
$$-\Delta \beta = p \cdot \sin \psi$$

$$\Rightarrow \begin{cases} \Delta \lambda \cdot \cos \beta = \frac{R}{r} \cdot \sin \theta \cdot \cos \psi & (1) \\ -\Delta \beta = \frac{R}{r} \cdot \sin \theta \cdot \sin \psi \end{cases}$$



$$\frac{\sin(\lambda_0 - \lambda)}{\sin \theta}$$

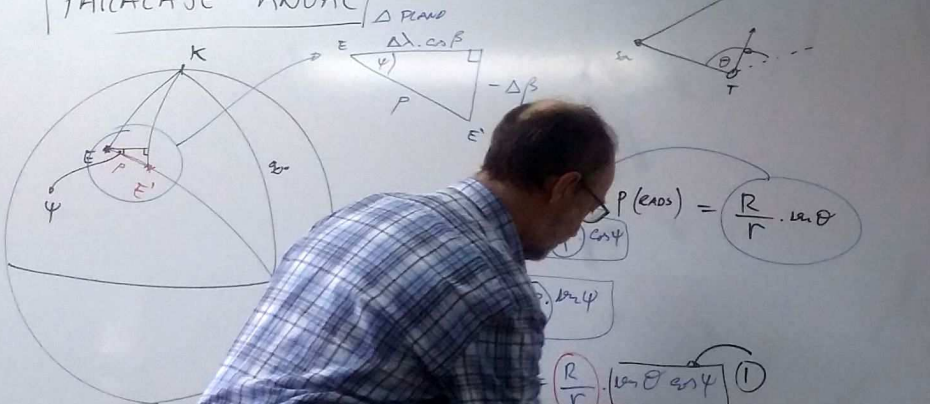
$$\Rightarrow \sin(\lambda_0 - \lambda) = \sin \theta \cdot \cos \psi \quad (1)$$



$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

PARALAJE ANUAL

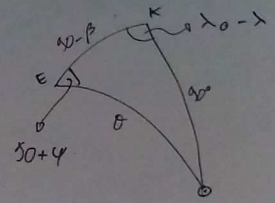
1838 G1 de CIGUE



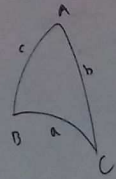
$$p(\text{rad}) = \frac{R}{r} \cdot \sin \theta$$

$$\frac{R}{r} \cdot \sin \theta \sin \psi \quad (1)$$

$$\frac{R}{r} \cdot \sin \theta \cdot \sin \psi$$



$$\frac{\sin(\lambda_0 - \lambda)}{\sin \theta} = \frac{\sin(90 + \psi)}{\sin 90} \Rightarrow \sin(\lambda_0 - \lambda) = \sin \theta \cdot \cos \psi \quad (1)$$



$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

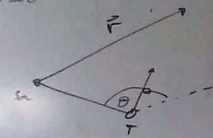
$$\sin \theta \cdot \cos(90 + \psi) = 0 - 1 \cdot \cos(90 - \beta) \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \cdot (-\sin \psi) = -\sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \sin \psi = \sin \beta \cdot \cos(\lambda_0 - \lambda) \quad (2)$$

**PARALAJE ANUAL**

1838 G1 del círculo



$$\Delta \lambda \cdot \cos \beta = \pi \cdot \sin(\lambda_0 - \lambda)$$

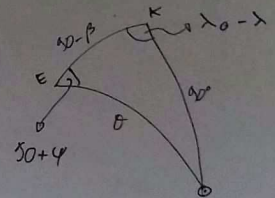
$$\Delta \beta = -\pi \cdot \sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$\lambda_0$  (\*)

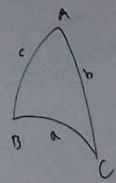
$$\Rightarrow \Delta \lambda \cdot \cos \beta = p \cdot \cos \psi \quad p(\text{en AU}) = \frac{R}{r} \cdot \sin \theta$$

$$-\Delta \beta = p \cdot \sin \psi$$

$$\Rightarrow \begin{cases} \Delta \lambda \cdot \cos \beta = \frac{R}{r} \cdot \sin \theta \cdot \cos \psi & (1) \\ -\Delta \beta = \frac{R}{r} \cdot \sin \theta \cdot \sin \psi & (2) \end{cases}$$



F. ANALOGA



$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

$$\sin \theta \cdot \cos(90+\psi) = 0 - 1 \cdot \cos(90-\beta) \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \cdot (-\sin \psi) = -\sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \sin \psi = \sin \beta \cdot \cos(\lambda_0 - \lambda) \quad (2)$$

F. SENO

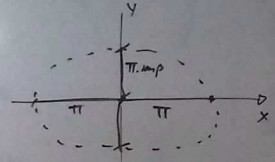
$$\frac{\sin(\lambda_0 - \lambda)}{\sin \theta} = \frac{\sin(90+\psi)}{\sin 90} \Rightarrow \sin(\lambda_0 - \lambda) = \sin \theta \cdot \cos \psi \quad (1)$$

PARALAJE ANUAL

1838 G1 de CIGUE

ELIPSE PARALÁCTICA

$$\frac{x^2}{\pi^2} + \frac{y^2}{\pi^2 \mu \beta} = 1$$

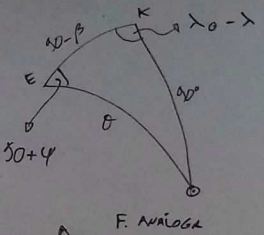


ESTRELLA con  $\beta = \pm 90$

$$\Delta \lambda \cdot \cos \beta = \pi \cdot \mu$$

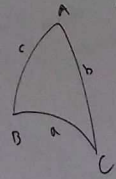
$$y \cdot \Delta \beta = -\pi \cdot \mu \beta$$

$\lambda_0(\ast)$



F. SEND

$$\frac{\mu (\lambda_0 - \lambda)}{\mu \theta} = \frac{\mu \cos(\beta + \psi)}{\mu \sin 90^\circ} \Rightarrow \mu (\lambda_0 - \lambda) = \mu \theta \cdot \cos \psi \quad (1)$$



F. ANALOGA

$$\mu a \cos B = \cos b \mu c - \mu b \cos c \cos A$$

$$\mu \theta \cdot \cos(\beta + \psi) = 0 - 1 \cdot \cos(90 - \beta) \cdot \cos(\lambda_0 - \lambda)$$

$$\mu \theta \cdot (-\mu \cos \psi) = -\mu \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\mu \theta \mu \cos \psi = \mu \beta \cdot \cos(\lambda_0 - \lambda) \quad (2)$$



**PARALAJE ANUAL**

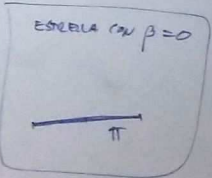
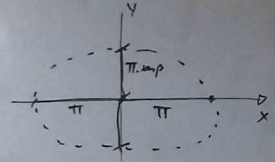
1838 G1 DA CIGUE

ELIPSE PARALÁCTICA

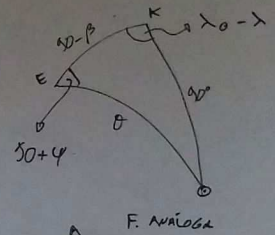
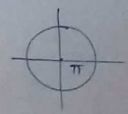
$$\Delta \lambda \cdot \cos \beta = \pi \cdot \sin(\lambda_0 - \lambda)$$

$$y = -\pi \cdot \sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\frac{x^2}{\pi^2} + \frac{y^2}{\pi^2 \sin^2 \beta} = 1$$

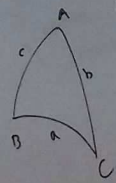


ESTRELLA con  $\beta = \pm 90$



F. SEMO

$$\frac{\sin(\lambda_0 - \lambda)}{\sin \theta} = \frac{\sin(90 + \psi)}{\sin 90} \Rightarrow \sin(\lambda_0 - \lambda) = \sin \theta \cdot \cos \psi \quad (1)$$



$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

$$\sin \theta \cdot \cos(90 + \psi) = 0 - 1 \cdot \cos(90 - \beta) \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \cdot (-\sin \psi) = -\sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\sin \theta \sin \psi = \sin \beta \cdot \cos(\lambda_0 - \lambda) \quad (2)$$

PARALAJE ANUAL

1838

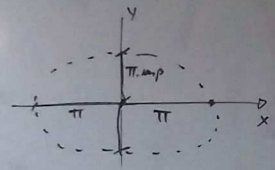
$$\Delta \lambda \cdot \cos \beta = \pi \cdot \sin(\lambda_0 - \lambda)$$

$$\Delta \beta = -\pi \cdot \sin \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\frac{x^2}{\pi^2 l^2} +$$

$\lambda_0 (*)$

PARALÁCTICA

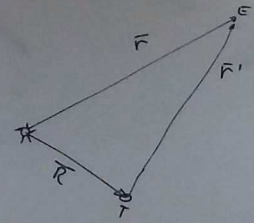


para  $\sin \beta = \pm 90$



FORMULACIÓN VECTORIAL

$$\vec{r} = \vec{R} + \vec{r}'$$



**PARALAJE ANUAL**

1838 G1 de CIGUE

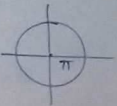
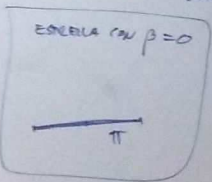
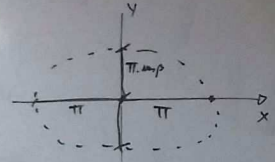
ELIPSE PARALÁCTICA

$$\Delta \lambda \cos \beta = \pi \cdot \mu \cdot \sin(\lambda_0 - \lambda)$$

$$\Delta \beta = -\pi \cdot \mu \cdot \cos(\lambda_0 - \lambda)$$

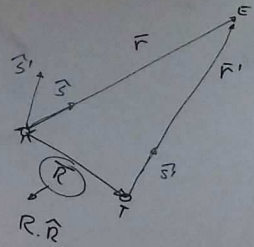
$\lambda_0 (*)$

$$\frac{x^2}{\pi^2 \mu^2} + \frac{y^2}{\pi^2 \mu^2 \cos^2 \beta} = 1$$



FORMULACIÓN VECTORIAL

$$\vec{r} = \vec{R} + \vec{r}'$$



$$r' \cdot \hat{S}' = r \cdot \hat{S} - R \cdot \hat{R}$$

$$r' \cdot \hat{S} \hat{S}' = r \cdot \hat{S} \hat{S}' - R \cdot \hat{S} \hat{R}$$

$$r' \cdot \hat{S} \hat{S}' = -R \cdot \hat{S} \hat{R}$$

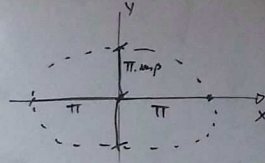
$\vec{d}S =$

**PARALAJE ANUAL**

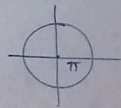
1838 G1 del CIGUE

ELIPSE PARALÁCTICA

$$\frac{x^2}{\pi^2} + \frac{y^2}{\pi^2 \sin^2 \beta} = 1$$



ESTRELLA con  $\beta = \pm 90$



ESTRELLA con  $\beta = 0$



$$\Delta \lambda \cos \beta = \pi \sin(\lambda_0 - \lambda)$$

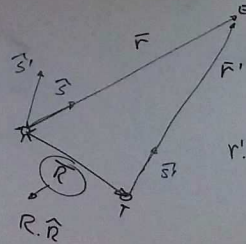
$$y \Delta \beta = -\pi \sin \beta \cos(\lambda_0 - \lambda)$$

$\lambda_0 (*)$

FORMULACIÓN VECTORIAL

$$\overline{ds} = \hat{s}' - \hat{s}$$

$$\vec{r} = \vec{R} + \vec{r}'$$



$$r' \hat{s}' = r \hat{s} - R \hat{n}$$

$$r' \hat{s} \hat{s}' = r \hat{s} \hat{s} - R \hat{s} \hat{n}$$

$$r' \hat{s} \hat{n} (\hat{s} \hat{s}') = -R \hat{s} \hat{n} (\hat{s} \hat{n})$$

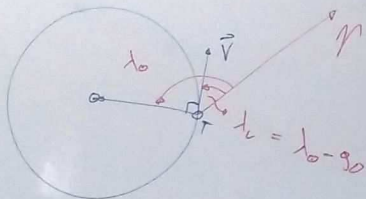
$$r' \left[ (\hat{s} \cdot \hat{s}') \hat{s} - (\hat{s} \cdot \hat{s}) \hat{s}' \right] = -R \left[ (\hat{s} \cdot \hat{n}) \hat{s} - (\hat{s} \cdot \hat{s}) \hat{n} \right]$$

$$r' (\hat{s} - \hat{s}') = -R \left[ (\hat{s} \cdot \hat{n}) \hat{s} - \hat{n} \right]$$

$$\overline{ds} = \left( \frac{R}{r'} \right) \left[ (\hat{s} \cdot \hat{n}) \hat{s} - \hat{n} \right]$$

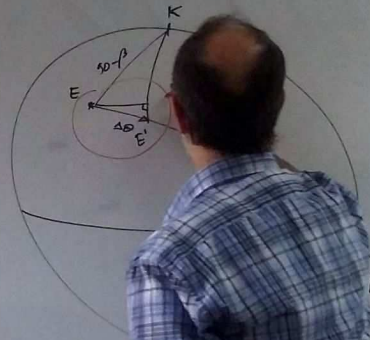
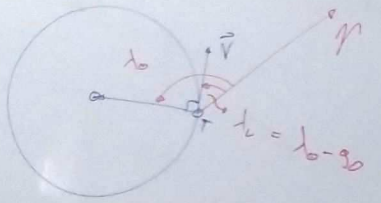
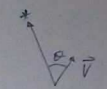
## ABERRACIÓN ANUAL

$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$



ABERRACIÓN ANUAL

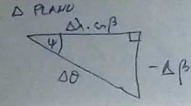
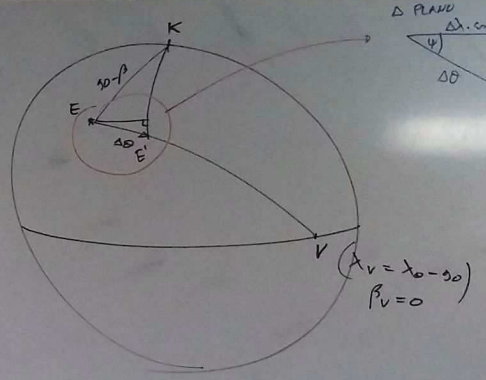
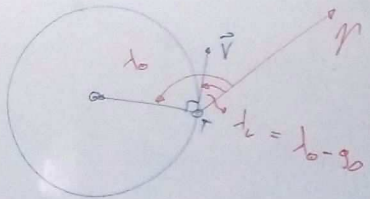
$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$



$$\begin{aligned} \lambda &= \lambda_0 - \beta_0 \\ \beta_V &= 0 \end{aligned}$$

ABERRACIÓN ANUAL

$$\Delta\theta = \frac{V}{c} \sin\theta$$



$$\Delta\lambda \cdot \cos\beta = (\Delta\theta) \cdot \cos\psi$$

$$-\Delta\beta = \frac{\Delta\theta}{\cos\psi} \cdot \cos\psi$$

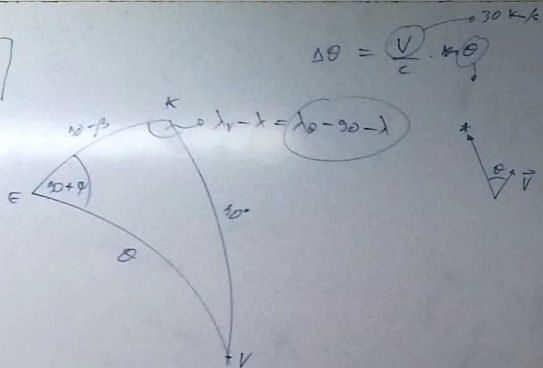
$$\Rightarrow \Delta\lambda \cdot \cos\psi = \frac{V \sin\psi}{c} \cdot \cos\psi$$

$$-\Delta\beta = \frac{V}{c} \sin\psi$$

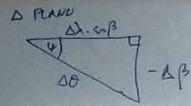
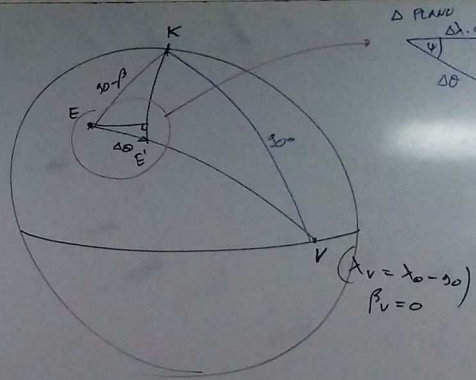
$$\frac{V}{c} = \frac{30}{300000}$$

$$1 \text{ rad} = 206264''$$

ABERRACIÓN ANUAL



$$\Delta\theta = \frac{V}{c} \cdot \sin\theta$$



$$\Delta\lambda \cdot \cos\beta = \Delta\theta \cdot \cos\psi$$

$$-\Delta\beta = \Delta\theta \cdot \sin\psi$$

$$\frac{V}{c} \cdot \sin\theta$$

$$\Rightarrow \Delta\lambda \cdot \cos\beta = \frac{V}{c} \cdot \sin\theta \cdot \cos\psi$$

$$-\Delta\beta = \frac{V}{c} \cdot \sin\theta \cdot \sin\psi$$

$$\frac{V}{c} = \frac{30}{300.000} = 1 \times 10^{-4} \text{ RAD}$$

$$1 \text{ RAD} = 206265''$$

$$10^{-4} \text{ RAD} = 20.6''$$

$$20.4'' = K$$

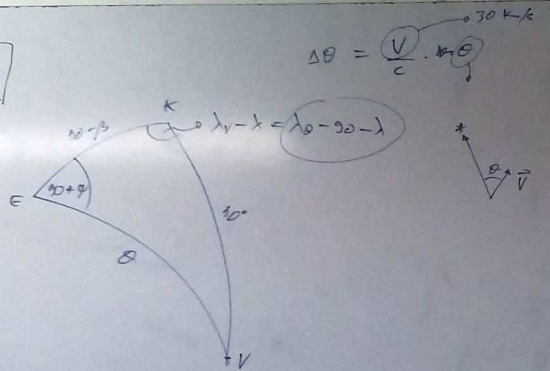
cte de aberración



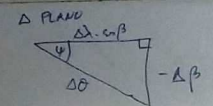
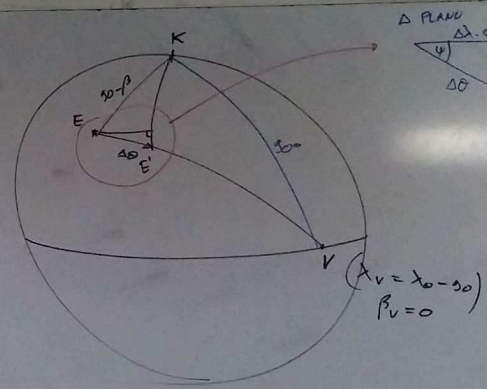
ABERRACIÓN ANUAL

$$\Rightarrow \begin{cases} \Delta\lambda \cdot \cos\beta = -K \cdot \cos(\lambda_0 - \lambda) \\ \Delta\beta = -K \cdot \sin\beta \cdot \sin(\lambda_0 - \lambda) \end{cases}$$

$$\frac{x^2}{K^2} + y^2 = 1$$



$$\Delta\theta = \frac{V}{c} \cdot \sin\theta$$



$$\begin{aligned} \Delta\lambda \cdot \cos\beta &= \Delta\theta \cdot \cos\psi \\ -\Delta\beta &= \Delta\theta \cdot \sin\psi \\ &= \frac{V}{c} \cdot \sin\theta \end{aligned}$$

$$\Rightarrow \begin{cases} \Delta\lambda \cdot \cos\beta = \frac{V}{c} \cdot \sin\theta \cdot \cos\psi \\ -\Delta\beta = \frac{V}{c} \cdot \sin\theta \cdot \sin\psi \end{cases}$$

$$\frac{V}{c} = \frac{30}{300000} = 1 \cdot 10^{-4} \text{ RAD}$$

$$\begin{aligned} 1 \text{ RAD} &= 206265'' \\ 10^{-4} \text{ RAD} &= 20.6'' \end{aligned}$$

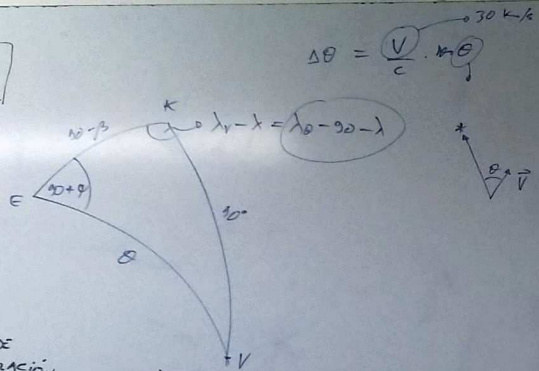
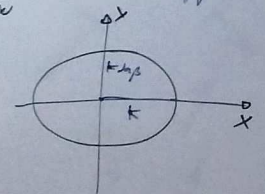
$$20.4'' = K \text{ CTE ABERRACIÓN}$$

ABERRACIÓN ANUAL

$$\Rightarrow \begin{cases} \Delta\lambda \cdot \cos\beta = -K \cdot \cos(\lambda_0 - \lambda) \\ \Delta\beta = -K \cdot \sin\beta \cdot \sin(\lambda_0 - \lambda) \end{cases}$$

$$\frac{X^2}{K^2} + \frac{Y^2}{K^2 \sin^2\beta} = 1$$

ELIPSE DE ABERRACIÓN



$$\Delta\theta = \frac{V}{c} \cdot \sin\theta$$

$$\begin{aligned} \Delta\lambda \cdot \cos\beta &= \Delta\theta \cdot \cos\psi \\ -\Delta\beta &= \Delta\theta \cdot \sin\psi \\ &= \frac{V}{c} \cdot \sin\theta \end{aligned}$$

$$\Rightarrow \begin{cases} \Delta\lambda \cdot \cos\beta = \frac{V}{c} \cdot \sin\theta \cdot \cos\psi \\ -\Delta\beta = \frac{V}{c} \cdot \sin\theta \cdot \sin\psi \end{cases}$$

$$\frac{V}{c} = \frac{30}{300.000} = 1 \cdot 10^{-4} \text{ RAD}$$

$$\begin{aligned} 1 \text{ RAD} &= 206265'' \\ 10^{-4} \text{ RAD} &= 20.6'' \end{aligned}$$

$$20.4'' = K \text{ CTE DE ABERRACIÓN}$$

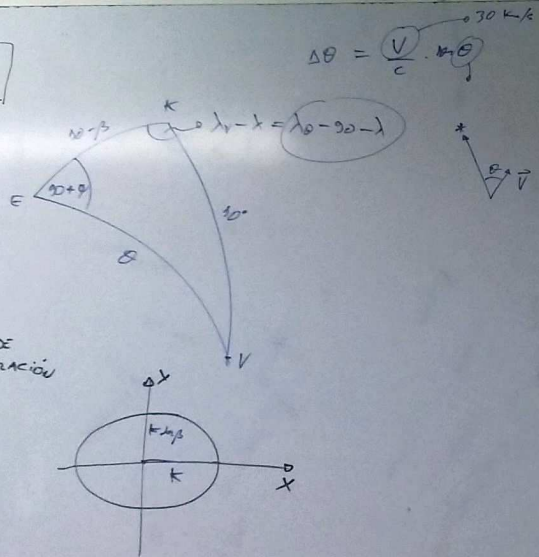
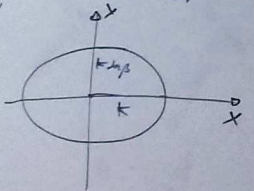
ABERRACIÓN ANUAL

$$\Delta\lambda \cdot \cos\beta = -K \cdot \cos(\lambda_0 - \lambda)$$

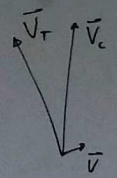
$$\Delta\beta = -K \cdot \sin\beta \cdot \sin(\lambda_0 - \lambda)$$

$$\frac{x^2}{K^2} + \frac{y^2}{K^2 \sin^2\beta} = 1$$

ELIPSE DE ABERRACIÓN



$$\Delta\theta = \frac{V}{c} \cdot \sin\theta$$



$$\Delta\lambda \cdot \cos\beta = \Delta\theta \cdot \cos\psi$$

$$-\Delta\beta = \Delta\theta \cdot \sin\psi$$

$$\frac{V}{c} \cdot \sin\theta$$

$$\Delta\lambda \cdot \cos\beta = \frac{V}{c} \cdot \sin\theta \cdot \cos\psi$$

$$-\Delta\beta = \frac{V}{c} \cdot \sin\theta \cdot \sin\psi$$

$$\frac{V}{c} = \frac{30}{300000} = 1 \cdot 10^{-4} \text{ RAD}$$

$$1 \text{ RAD} = 206265''$$

$$10^{-4} \text{ RAD} = 20.6''$$

$$20.4'' = K$$

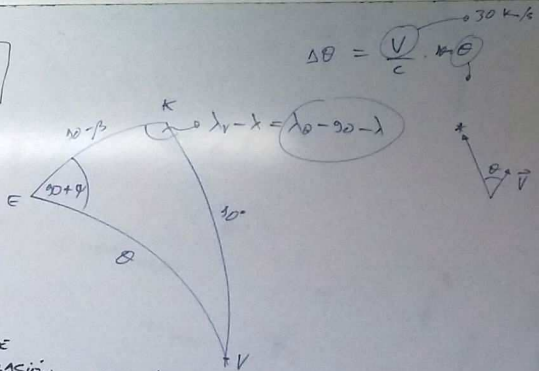
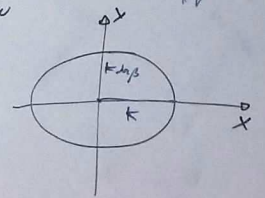
cte de aberración

ABERRACIÓN ANUAL

$$\Rightarrow \begin{cases} \Delta \lambda \cos \beta = -K \cos(\lambda_0 - \lambda) \\ \Delta \beta = -K \sin \beta \sin(\lambda_0 - \lambda) \end{cases}$$

$$\frac{x^2}{K^2} + \frac{y^2}{K^2 \sin^2 \beta} = 1$$

ELIPSE DE ABERRACIÓN



$$\Delta \theta = \frac{V}{c} \cdot K \sin \beta$$

$\Delta \lambda \cos \beta$

ABERRACIÓN ANUAL

$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$

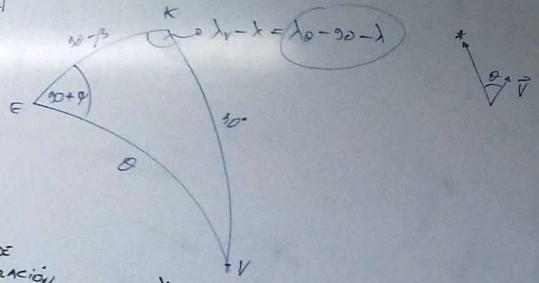
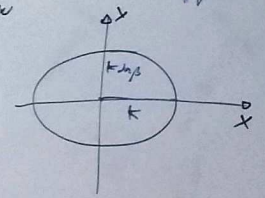
30 km/s

$$\Delta\lambda \cdot \cos\beta = -K \cdot \cos(\lambda_0 - \lambda)$$

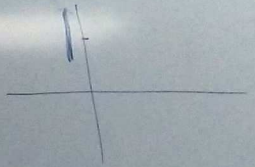
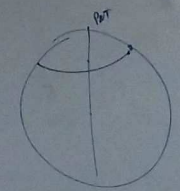
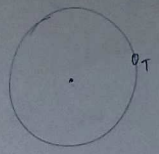
$$\Delta\beta = -K \cdot \sin\beta \cdot \sin(\lambda_0 - \lambda)$$

$$\frac{x^2}{K^2} + \frac{y^2}{K^2 \sin^2\beta} = 1$$

ELIPSE DE ABERRACIÓN



$$y = \frac{p}{r} (\sin\delta \sin\phi' \cos H - \cos\delta \sin\phi')$$

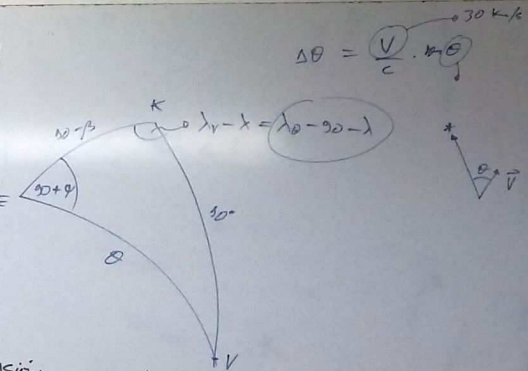
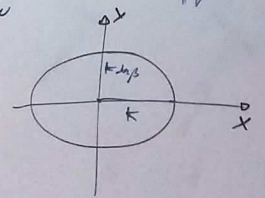


ABERRACIÓN ANUAL

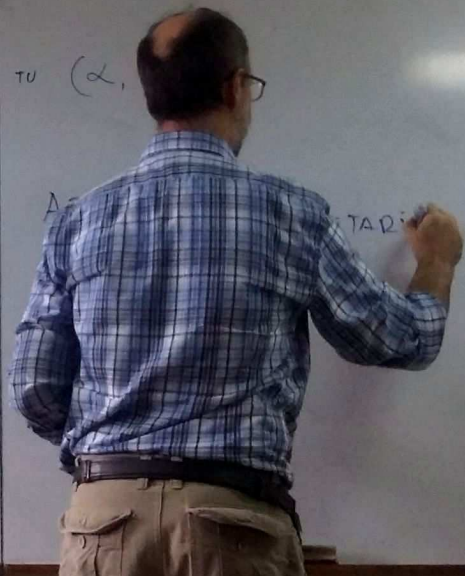
$$\Rightarrow \begin{cases} \Delta\lambda \cdot \cos\beta = -K \cdot \cos(\lambda_0 - \lambda) \\ \Delta\beta = -K \cdot \sin\beta \cdot \sin(\lambda_0 - \lambda) \end{cases}$$

$$\frac{x^2}{K^2} + \frac{y^2}{K^2 \sin^2\beta} = 1$$

ELIPSE DE ABERRACIÓN



$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$



TU (alpha,

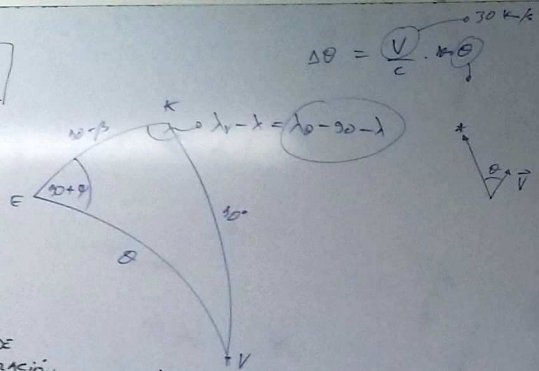
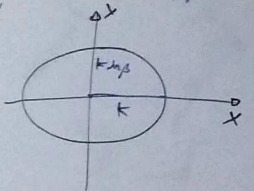
A = ... TARI

**ABERRACIÓN ANUAL**

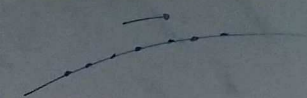
$$\Rightarrow \begin{cases} \Delta \lambda \cdot \cos \beta = -K \cdot \cos(\lambda_0 - \lambda) \\ \Delta \beta = -K \cdot \sin \beta \cdot \sin(\lambda_0 - \lambda) \end{cases}$$

$$\frac{x^2}{K^2} + \frac{y^2}{K^2 \sin^2 \beta} = 1$$

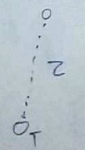
ELIPSE DE ABERRACIÓN



$$\Delta \theta = \frac{V}{c} \cdot \sin \theta$$



TU  $(\alpha, \delta) \rightarrow$  DIST OBJETO - TIERRA  $\Rightarrow D \hat{z}$



ABERRACIÓN PLANETARIA = CORRECCIÓN POR TIEMPO-LUZ

$$\begin{aligned} \alpha_{obs} &= \alpha - z \cdot \frac{d\alpha}{dt} \\ \delta_{obs} &= \delta - z \cdot \frac{d\delta}{dt} \end{aligned}$$

TOPO - GEO - HELIO

MOV. POLAR

} ⇒ MOV. OBSERVADOR

PRECESION Y NUTACION

→ MOV. SIST. REFERENCIA

MOV. PROPIO  
(estrellas)

MOV. ORBITAL  
(S. Sol)

} ⇒ MOV. OBJETO

PARA  $h=0$ :

ÁNGULO DE LA VERTICAL

$$N = \phi' - \phi = -11' 32''.7 \cdot \sin(2\phi) + \dots$$

$$p = a \left( 0.998327 + 0.0016764 \cdot \cos(2\phi) + \dots \right)$$



TOPO - GEO - HELIO

MOV. POLAR

⇒ MOV. OBSERVADOR

PRECESIÓN Y NUTACIÓN

⇒ MOV. SIST. REFERENCIA

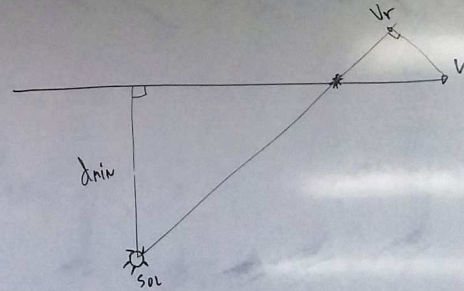
MOV. PROPIO  
(esmarlas)

⇒ MOV. OBJETO

MOV. ORBITAL  
(S. Sol)

TIEMPO

5 MOV. PROPIO



TOPO - GEO - HELIO

MOV. POLAR

⇒ MOV. OBSERVADOR

PRECESIÓN Y NUTACIÓN

⇒ MOV. SIST. REFERENCIA

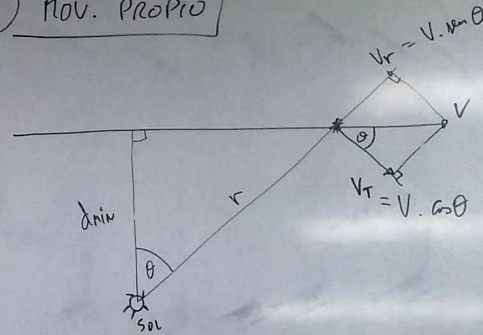
MOV. PROPIO  
(esmales)

MOV. ORBITAL  
(s. sol)

⇒ MOV. OBJETO

TIEMPO

5 MOV. PROPIO



$$V_r = V \cdot \sin \theta = \frac{dr}{dt}$$

$$V_t = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

TOPO - GEO - HELIO

MOV. POLAR

⇒ MOV. OBSERVADOR

PRECESION Y NUTACION

⇒ MOV. SIST. REFERENCIA

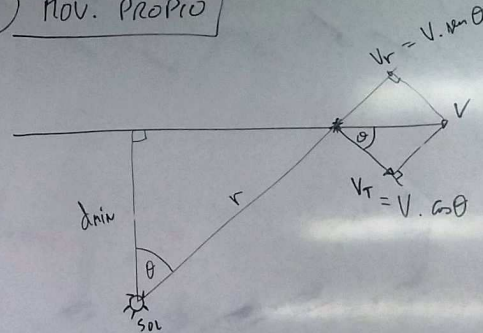
MOV. PROPIO  
(esmales)

⇒ MOV. OBJETO

MOV. ORBITAL  
(s. solar)

TIEMPO

5 MOV. PROPIO

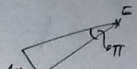


$$V_r = V \cdot \sin \theta = \frac{dr}{dt}$$

$$V_t = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = V_t \cdot \frac{1}{r} = \pi (\text{rads}) \cdot V_t \rightarrow \mu \text{ "/>$$

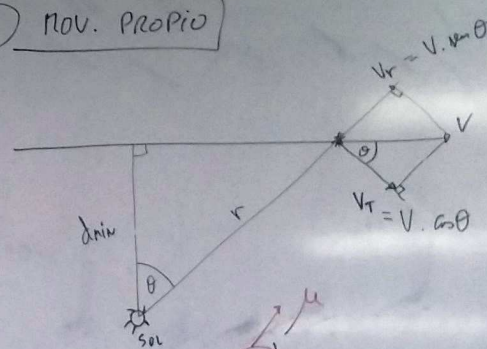
$$V_T (u_A/A_{10}) = V_T \times \frac{150 \times 10^6 \text{ km}}{r}$$



$$\sin \pi = \frac{1 u_A}{r}$$

$$\pi (\text{rads}) = \frac{1 u_A}{r (\text{rads})}$$

5 NOV. PROPIO



$$V_r = V \cdot \sin \theta = \frac{dr}{dt}$$

$$V_T = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = V_T \cdot \frac{1}{r} = \pi (\text{rads}) \cdot V_T$$

$$\Rightarrow \frac{d\theta}{dt} = \pi^{(11)} \cdot V_T (u_A/A_{10})$$

$$V_T (u/año) = V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}$$

Km/Sec

4.74



$$\sin \pi = \frac{u}{r}$$

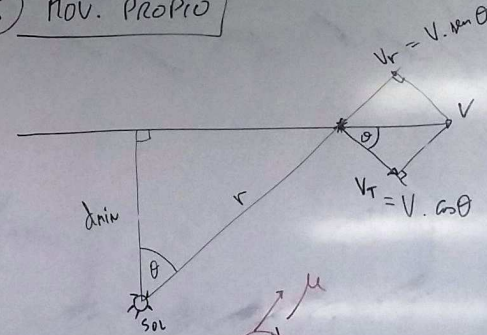
$$\pi (\text{rads}) = \frac{u}{r(\text{mas})}$$

$$\mu = 4.74 \cdot \frac{u}{r} \cdot V_T$$

u/año

Km/Sec

5) Nov. Propio



$$V_r = V \cdot \sin \theta = \frac{dr}{dt}$$

$$V_t = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = V_t \cdot \frac{1}{r} = \pi (\text{rads}) \cdot V_T$$

$$\Rightarrow \frac{d\theta}{dt} = \pi (\text{''}) \cdot V_T (u/año)$$

$$V_T (UA/AU) = V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}$$

Km/Sec

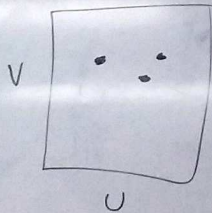
4.74

$$\mu = \pi \frac{v}{r}$$

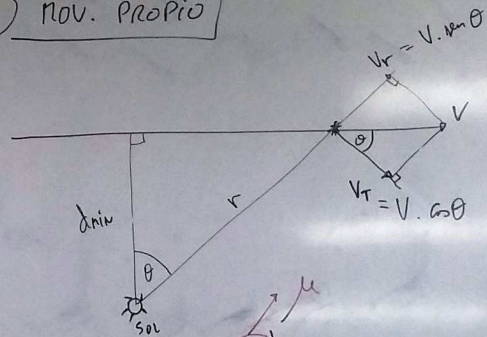
$$\pi (\text{rads}) = \frac{v}{r (\text{rads})}$$

$$\mu = 4.74 \cdot \pi \cdot V_T$$

UA/AU



5 NOV. PROPIO



$$V_r = v \cdot \sin \theta = \frac{dr}{dt}$$

$$V_t = v \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

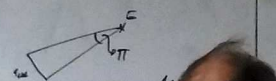
$$\frac{d\theta}{dt} = V_t \cdot \frac{1}{r} = \pi (\text{rads}) \cdot V_T$$

UA/AU

$$\Rightarrow \frac{d\theta}{dt} = \pi (\text{rads}) \cdot V_T (UA/AU)$$

$$V_T \left( \frac{M}{M_{\odot}} \right) = \frac{V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}}{\text{km/sec}}$$

4.74

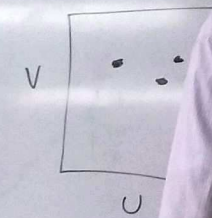
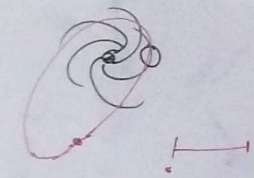


$$1 \text{ yr} = \frac{1 \text{ yr}}{1 \text{ yr}}$$

$$\Pi \text{ (in yr)} = \frac{1 \text{ yr}}{1 \text{ yr}}$$

$$\mu = 4.74 \cdot \Pi \cdot V_T$$

km/sec



EFFECT

$$V_T (u_a / A_{70}) = V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}$$

Km/sec

4.74

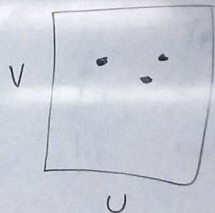
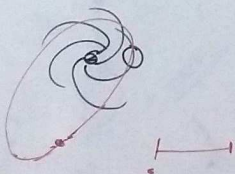
$$10 \rightarrow \pi = \frac{1 u_a}{r}$$

$$\pi (\text{mas}) = \frac{1 u_a}{r (\text{mas})}$$

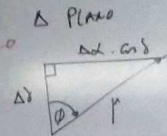
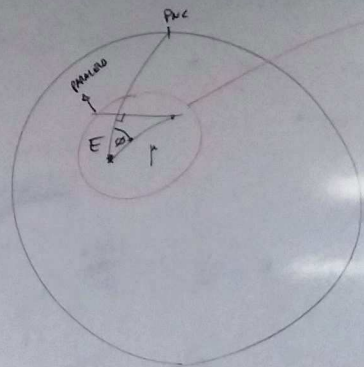
$$M = 4.74 \cdot \pi \cdot V_T$$

10 / A<sub>70</sub>

Km/sec



EFFECTO EN  $\alpha$  Y  $\delta$



$$\left\{ \begin{aligned} \Delta \alpha \cdot \cos \delta &= \mu \cdot \sin \phi \\ \Delta \delta &= \mu \cdot \cos \phi \end{aligned} \right.$$



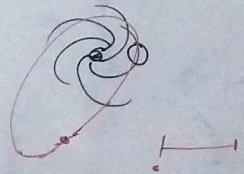
$$V_T (\text{ua/año}) = V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}$$

Km/sec

4.74

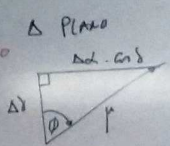
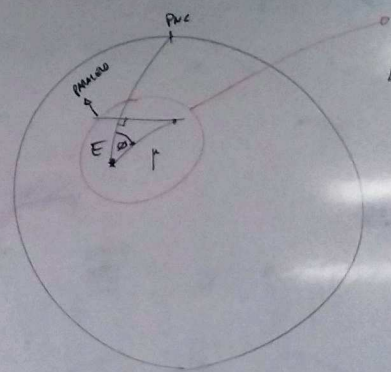
$$\mu = 4.74 \cdot \frac{1}{a} \cdot V_T$$

1/año      Km/sec



CATÁLOGO:  
2000.0  
 $\alpha, \delta, \mu_\alpha, \mu_\delta$

EFFECTO EN  $\alpha, \delta$



$$\begin{cases} \Delta \alpha \cdot \cos \delta = \mu \cdot \sin \phi \\ \Delta \delta = \mu \cdot \cos \phi \end{cases}$$

$$\Delta \alpha = \frac{\mu \cdot \sin \phi}{\cos \delta} \cdot \frac{1}{15}$$

$$\Delta \delta = \mu \cdot \cos \phi$$

$$\Rightarrow \begin{cases} \mu_\alpha = \frac{1}{15} \cdot \mu \cdot \sin \phi / \cos \delta \\ \mu_\delta = \mu \cdot \cos \phi \end{cases}$$

$$V_T (\mu\text{a}/\text{año}) = V_T \times \frac{150 \times 10^6 \text{ km}}{365.25 \times 24 \times 60 \times 60}$$

Km/Sec

4.74

CATÁLOGO:

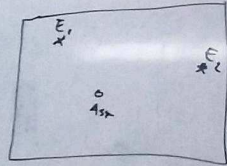
2000.0

$\alpha, \delta, \mu_\alpha, \mu_\delta$

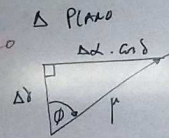
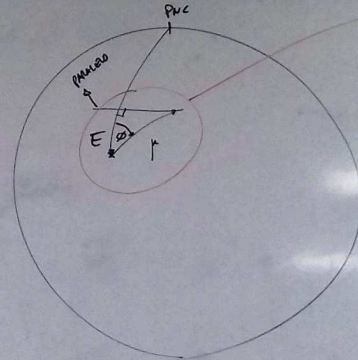
$$\mu = 4.74 \cdot \frac{V_T}{d}$$

Km/Sec

$\mu/\text{año}$



EFFECTO EN  $\alpha, \delta$



$$\begin{cases} \Delta \alpha \cdot \cos \delta = \mu \cdot \sin \phi \\ \Delta \delta = \mu \cdot \cos \phi \end{cases}$$

$$\Delta \alpha = \frac{\mu \cdot \sin \phi}{\cos \delta} \cdot \frac{1}{15}$$

$$\Delta \delta = \mu \cdot \cos \phi$$

$$\begin{aligned} \Rightarrow \mu_\alpha &= \frac{1}{15} \cdot \mu \cdot \sin \phi / \cos \delta \\ \mu_\delta &= \mu \cdot \cos \phi \end{aligned}$$

$$V_T (\mu\text{a}/\text{Año}) = V_T \times \frac{150 \cdot 10^6 \text{ km}}{365.25 \times 24 \times 60 \cdot 60}$$

Km/Sec

4.74

CATÁLOGO :

2000.0

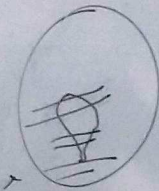
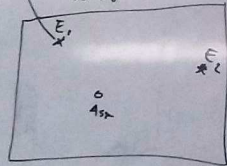
$\alpha, \delta, \mu_\alpha, \mu_\delta$

$$\mu = 4.74 \cdot \frac{V_T}{d}$$

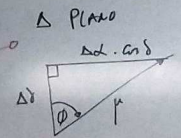
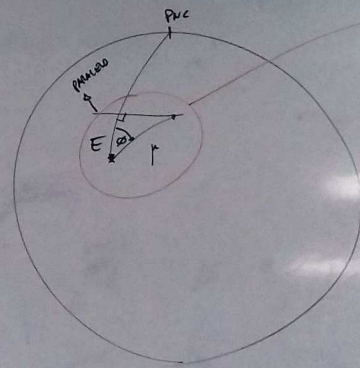
$\mu/\text{Año}$

$\alpha_{2000} + \mu_\alpha \cdot 18$

2018



EFFECTO EN  $\alpha, \delta$



$$\begin{cases} \Delta \delta \cdot \cos \delta = \mu \cdot \sin \phi \\ \Delta \delta = \mu \cdot \cos \phi \end{cases}$$

$$\Delta \alpha = \frac{\mu \cdot \sin \phi}{\cos \delta} \cdot \frac{1}{15}$$

$$\Delta \delta = \mu \cdot \cos \phi$$

$$\begin{aligned} \mu_\alpha &= \frac{1}{15} \cdot \mu \cdot \sin \phi / \cos \delta \\ \mu_\delta &= \mu \cdot \cos \phi \end{aligned}$$

$$V_T (M/AU) = V_T \times \frac{150 \cdot 10^6 \text{ km}}{365.25 \times 24 \times 60 \cdot 60}$$

Km/sec

4.74

$$\vec{V}_* - \vec{V}_\odot = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_\odot - \vec{V}_{LSR})$$

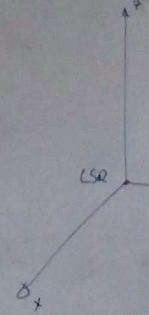
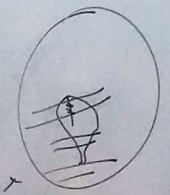
μ peculiar      μ parallático

$$M = 4.74 \cdot \frac{d}{\text{pc}} \cdot V_T$$

M/AU      Km/sec



$$V_{LSR} \approx 220 \text{ km/sec}$$



$$V_T (U^*/A^*) = V_T \times \frac{150 \times 10^6 \text{ km}}{355,25 \text{ km/sec}}$$

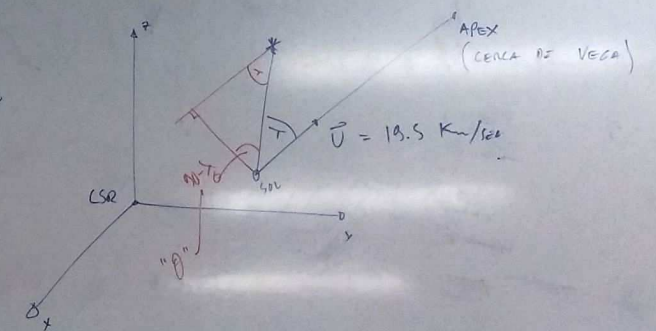
$\uparrow$   
Km/sec

$$\vec{V}_* - \vec{V}_\odot = \underbrace{(\vec{V}_* - \vec{V}_{LSR})}_{\mu \text{peculiar}} - \underbrace{(\vec{V}_\odot - \vec{V}_{LSR})}_{\mu \text{paraláctico}}$$

$$V_T = V$$

$$\mu = 4.74 \frac{V}{d}$$

$\downarrow$   
N/AU



$$V_T \left( \frac{U_A}{A_{50}} \right) = \underset{\substack{\uparrow \\ \text{Km/sec}}}{V_T} \times \left( \frac{150 \cdot 10^6 \text{ Km}}{365.25 \times 24 \times 60 \cdot 60} \right)^{-1}$$

4.74

$$\vec{V}_* - \vec{V}_\odot = \underbrace{(\vec{V}_* - \vec{V}_{LSR})}_{\mu \text{ peculiar}} - \underbrace{(\vec{V}_\odot - \vec{V}_{LSR})}_{\mu \text{ paraláctico}}$$

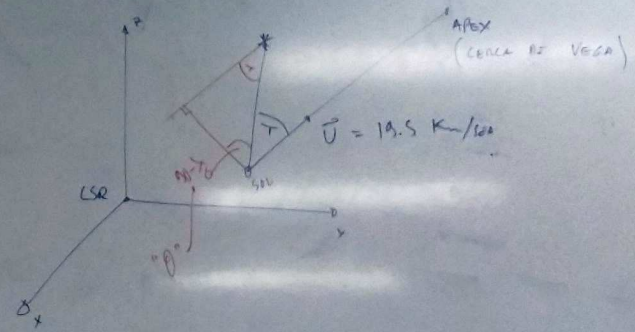
$$\mu = [4.74]^{-1} \cdot \frac{V_T}{d}$$

Km/sec

$\frac{U_A}{A_{50}}$

$$V_T = V \cdot \sin \theta$$

$$V_T \text{ (paraláctica)} = U \cdot \sin \lambda$$



$$V_T (U^*/A^*) = V_T \times \left( \frac{150 \times 10^6 \text{ km}}{365 \times 24 \times 60 \times 60} \right)^{-1}$$

Km/sec

$$\vec{V}_* - \vec{V}_\odot = \underbrace{(\vec{V}_* - \vec{V}_{LSR})}_{\mu \text{ peculiar}} - \underbrace{(\vec{V}_\odot - \vec{V}_{LSR})}_{\mu \text{ paraláctico}}$$

$$\mu = [4.74]^{-1}$$

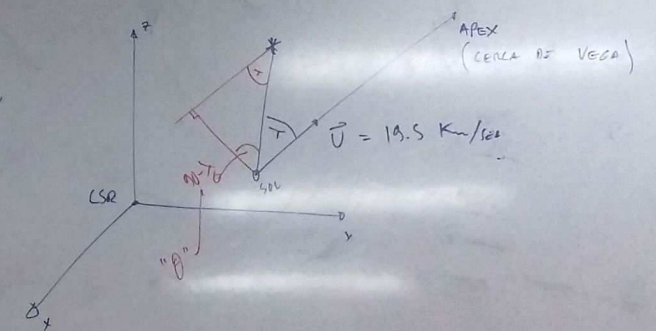
Km/sec

$$V_T = V \cdot \mu$$

$$V_T \text{ (paraláctico)} = U \cdot \mu \sin \lambda$$

$$\mu = \frac{U \cdot \mu \sin \lambda}{4.74}$$

H



$$V_T (U^*/A^*) = V_T \times \left( \frac{150 \times 10^6 \text{ km}}{355,25 \times 24 \times 60,40} \right)^{-1}$$

Km/sec

4,74

$$\vec{V}_* - \vec{V}_\odot = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_\odot - \vec{V}_{LSR})$$

μ peculiar      μ paraláctico

$$\mu = [4,74]^{-1} \cdot \pi \cdot V_T$$

Km/sec

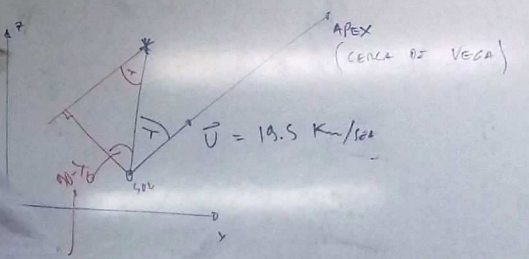
μ/A^\*

$$V_T = V \cdot \sin \lambda$$

$$V_T (\text{paraláctico}) = U \cdot \sin \lambda$$

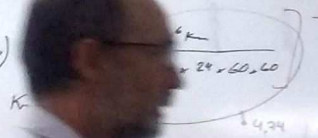
$$\mu_{\text{paraláctico}} = \frac{\pi \cdot U \cdot \sin \lambda}{4,74}$$

H





$$V_T (U^*/A70)$$



$$\vec{V}_* - \vec{V}_\odot = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_\odot - \vec{V}_{LSR})$$

μ<sub>paraláxica</sub>                      μ<sub>paraláxico</sub>

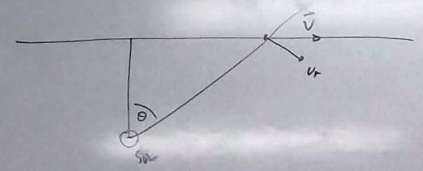
$$V_T = V \cdot \sin \theta$$

$$V_T (\text{paraláxica}) = U \cdot \sin \lambda$$

$$\mu_{\text{paraláxica}} = \frac{\pi \cdot U \cdot \sin \lambda}{4,74}$$

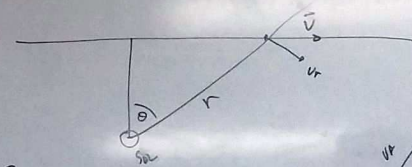
H

ACELERACIÓN DE PERSPECTIVA



$$\frac{d\mu}{dt}$$

ACELERACIÓN DE PERSPECTIVA



$$V_r = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

$$- \underbrace{V \cdot \sin \theta}_{V_r} \cdot \frac{d\theta}{dt} = \underbrace{\frac{dr}{dt}}_{V_r} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2} \Rightarrow -2V_r \frac{d\theta}{dt} = r \frac{d^2\theta}{dt^2}$$

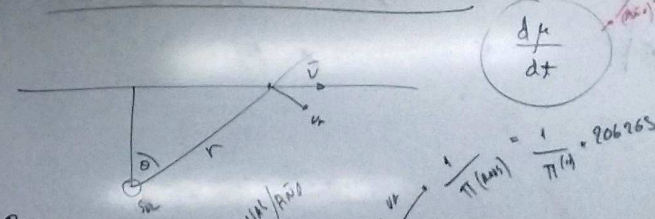
$\frac{1}{\pi} \text{ (ans)}$

$\frac{1}{(2\pi)^2}$

$$\frac{d^2\theta}{dt^2} = -\frac{2}{r} \cdot V_r \cdot \frac{d\theta}{dt} = -2 \cdot \frac{\pi^{(r)}}{206265} \cdot \frac{d\theta}{dt} \cdot V_r (U_r/A_{r0})$$

$V_r (km/sec) / 4.74$

ACELERACIÓN DE PERSPECTIVA



$$V_r = V \cdot \cos\theta = r \cdot \frac{d\theta}{dt}$$

$$-V \cdot \sin\theta \cdot \frac{d\theta}{dt} = \frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow -2V_r \frac{d\theta}{dt} = r \frac{d^2\theta}{dt^2}$$

$\frac{1}{\pi^{(r)}} = \frac{1}{\pi^{(r)}} \cdot 206265$

$$\frac{d^2\theta}{dt^2} = -\frac{2}{r} \cdot V_r \cdot \frac{d\theta}{dt}$$

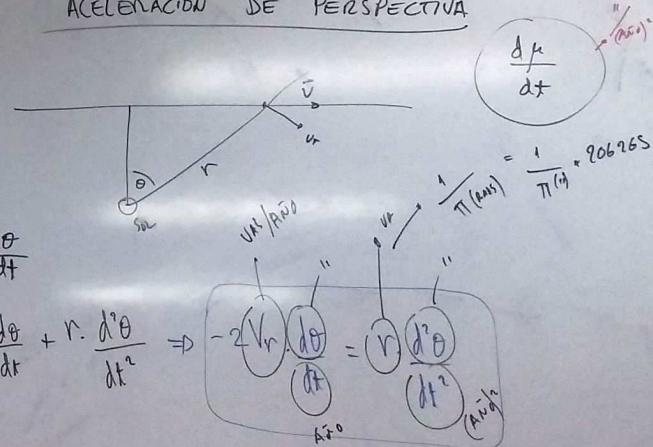
$\frac{d\theta}{dt} = \frac{\pi(\text{rad})}{206265}$

$$\frac{d\theta}{dt} = \frac{V_r (u/año)}{V_r (km/sec) / 4.74}$$

$$\frac{d\mu}{dt} = -\frac{2}{4.74 \cdot 206265} \cdot \pi \cdot \mu \cdot V_r = \frac{d\mu}{dt}$$

AC. DE PERSPECTIVA

ACELERACIÓN DE PERSPECTIVA



$$V_r = V \cdot \cos\theta = r \cdot \frac{d\theta}{dt}$$

$$-V_r \cdot \frac{d\theta}{dt} = \frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2} \Rightarrow -2V_r \frac{d\theta}{dt} = r \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{2}{r} \cdot V_r \cdot \frac{d\theta}{dt} = -2 \cdot \frac{\pi^{(r)}}{206265} \cdot \frac{d\theta}{dt} \cdot V_r \left( \frac{V_r}{a_{\odot}} \right)$$

$$\frac{d\mu}{dt} = \frac{-2}{4.74 \cdot 206265} \cdot \pi \cdot \mu \cdot V_r = -2,05 \times 10^{-6} \cdot \pi \cdot \mu \cdot V_r = \frac{d\mu}{dt}$$

AC. DE PERSPECTIVA

GRAVITACIONAL

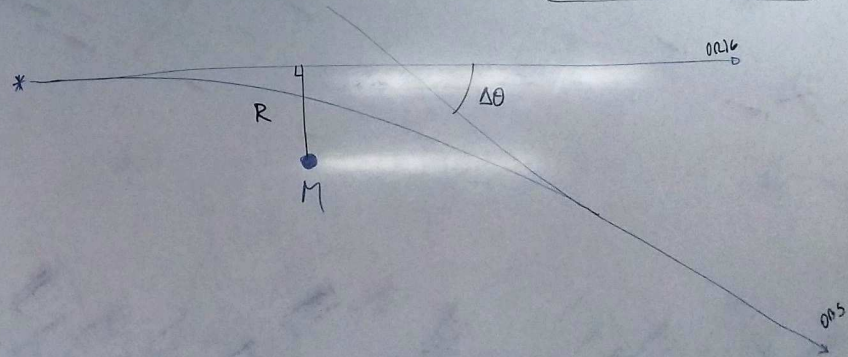
$$\frac{d^2\theta}{dt^2} = -\frac{2}{r} \cdot V_r \cdot \frac{d\theta}{dt} = -2 \cdot \frac{\pi^{(r)}}{206265} \cdot \frac{d\theta}{dt} \cdot V_r \left( \frac{V_r}{a_{\text{año}}} \right)$$

$$\frac{d\mu}{dt} = \frac{-2}{4.74 \cdot 206265} \cdot \pi \cdot \mu \cdot V_r = -2,05 \cdot 10^{-6} \cdot \pi \cdot \mu \cdot V_r = \frac{d\mu}{dt}$$

AC. DE PERSPECTIVA

DESVIÓ GRAVITACIONAL

$$\Delta\theta \cong \frac{GM}{R \cdot c^2} \cdot 4$$



PRECESIÓN Y NUTACIÓN

PRECESIÓN

- LUNISOLAR (MOV. PNC)
- + PLANETARIA (MOV. K)

NUTACIÓN

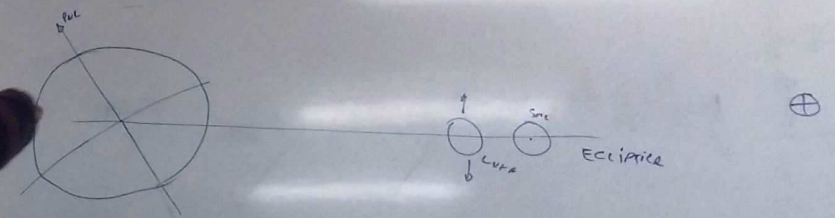
- LUNISOLAR (OSCILACIÓN PNC PEQUEÑA)

# PRECESIÓN Y NUTACIÓN

LUNISOLAR (MOV. PNC)  
+  
PLANETARIA (MOV. K)

LUNISOLAR  
(OSCILACIÓN PNC  
PEQUEÑA)

ABULMIGLTD EQUATORIAL → ROTACION





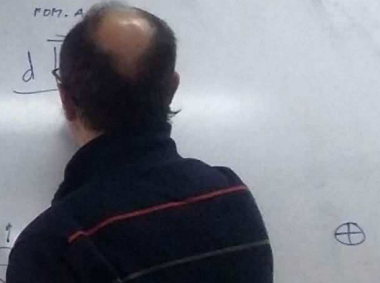
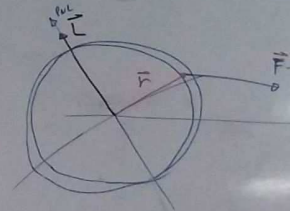
PRECESIÓN Y NUTACIÓN

LUNISOLAR (MOV. PNC)  
+  
PLANETARIA (MOV. K)

LUNISOLAR  
(OSCILACIÓN PNC)  
PEQUEÑA

ABOLVIMIENTO ECUATORIAL → ROTACION

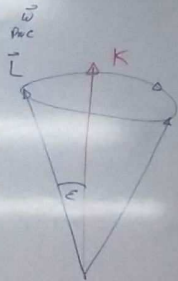
$$\vec{H} = \vec{v} \wedge \vec{F}$$



PRECESIÓN Y NUTACIÓN

LUNISOLAR (MOV. PNC)  
+  
PLANETARIA (MOV. K)

LUNISOLAR  
(OSCILACIÓN PNC  
PEQUEÑA)

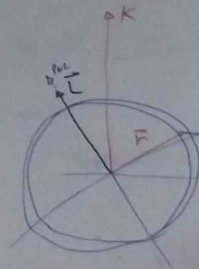


ABOLVIMIENTO EQUATORIAL → ROTACIÓN

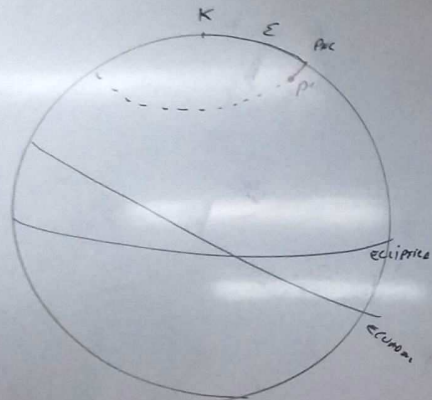
FORM. ANGULAR

$$\frac{d\vec{L}}{dt}$$

$$\vec{\pi} = \vec{v} \wedge \vec{F}$$



GEOSOLAR

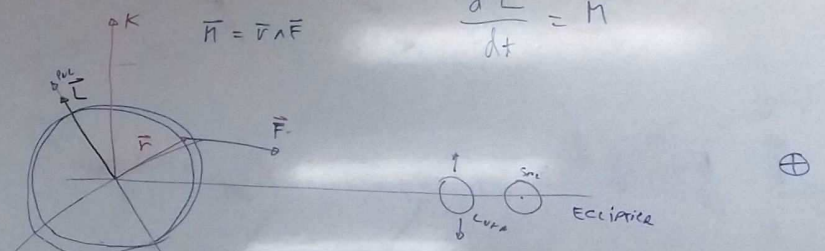


ABOLVIMIENTO ECATORIAL - ROTACION

MOM. ANGULAR

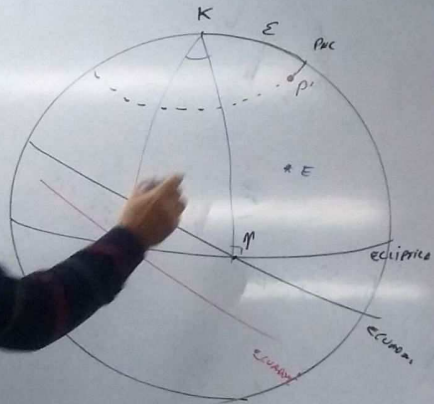
$$\frac{d\vec{L}}{dt} = \vec{M}$$

$$\vec{M} = \vec{v} \wedge \vec{F}$$



PREC. LUNAR

$$\vec{p}' \cdot \vec{q}' = 1$$



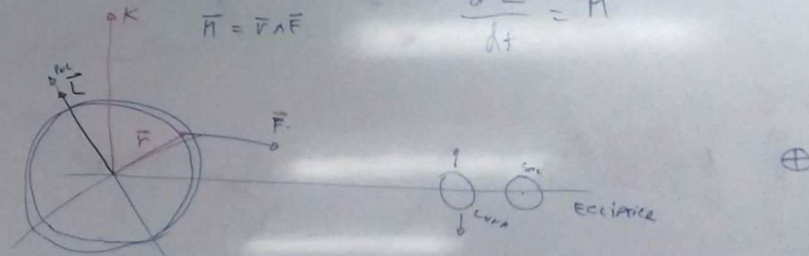
$$\begin{aligned} d\alpha \\ d\delta \\ d\lambda \\ d\beta = 0 \end{aligned}$$

ABOLIMIENTO ECUATORIAL → ROTACION

$$\vec{H} = \vec{v} \wedge \vec{F}$$

POSI. AUMENTAR

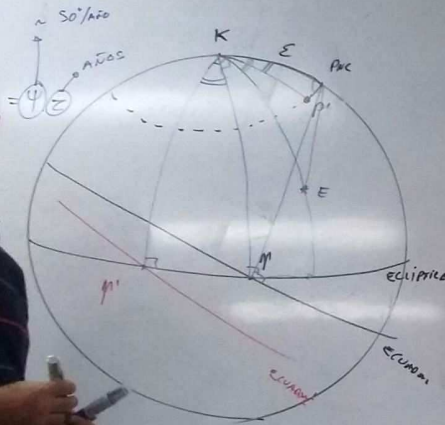
$$\frac{d\vec{L}}{dt} = \vec{M}$$



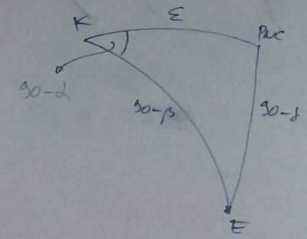


PREC. LUNISOLAR

$$\widehat{p'p} = \widehat{p'k} = \widehat{p'e}$$

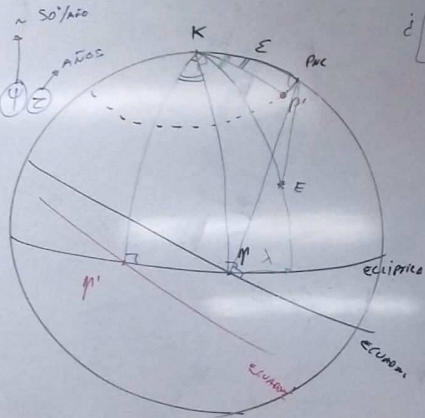


$$\begin{aligned} d\alpha & \\ d\delta & \\ d\lambda &= \psi \cdot z \\ d\beta &= 0 \end{aligned}$$



PREC. LUNISOLAR

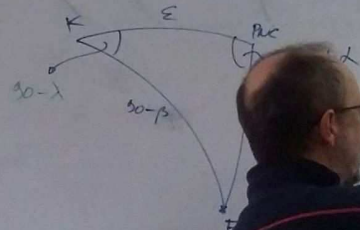
$$\widehat{P'K} = \widehat{P'K} = \widehat{P'K} = \Delta\lambda = \psi \oplus \epsilon$$



$$\frac{d\lambda}{d\delta} ?$$

$$d\lambda = \psi \cdot \epsilon$$

$$d\beta = 0$$



$$\cos(90-\delta) = \cos \epsilon \cdot \cos(90-\beta) + \sin \epsilon \cdot \sin(90-\beta) \cos(90-\lambda)$$

$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot \cos \beta \cdot \sin \lambda$$

$$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$$

PREC. LUNISOLAR

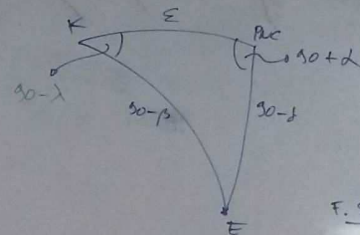
$$\widehat{P'K} = \widehat{P'K} = \widehat{P'K} = \Delta\lambda = \psi \oplus \epsilon$$

$\sim 50^\circ / \text{año}$   
 $\sim 15^\circ / \text{año}$

$$\frac{d\lambda}{d\delta} ?$$

$$d\lambda = \psi \cdot z$$

$$d\beta = 0$$



F. COSINE

$$\cos(90-\delta) = \cos \epsilon \cdot \cos(90-\beta) + \sin \epsilon \cdot \sin(90-\beta) \cdot \cos(90-\lambda)$$

$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot \cos \beta \cdot \sin \lambda$$

$$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$$

F. SECO

$$\frac{\sin(90+\delta)}{\sin(90-\beta)} = \frac{\sin(90-\lambda)}{\sin(90-\delta)} \Rightarrow \frac{\cos \delta}{\cos \beta} = \frac{\cos \lambda}{\cos \delta}$$

$$\Rightarrow \cos \beta \cdot \cos \lambda = \cos \delta \cdot \cos \delta$$



PREC. LUNISOLAR

$$\widehat{P'K} = \widehat{P'K} = \widehat{P'K} = \Delta\lambda = \psi \quad \begin{matrix} \sim 50^\circ/\text{año} \\ \sim 15^\circ/\text{año} \end{matrix}$$

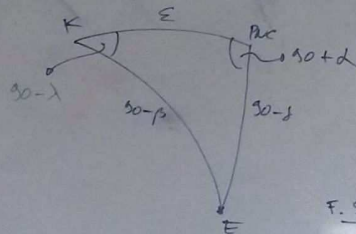
$$\cancel{\cos \delta} \cdot d\delta = \cancel{\sin \epsilon} \cdot d\lambda \cdot \cancel{\cos \delta}$$

$$\Rightarrow d\delta = \sin \epsilon \cdot \cos \delta \cdot d\lambda \quad \psi \cdot z$$

$$\frac{d\delta}{d\lambda} = ?$$

$$d\lambda = \psi \cdot z$$

$$d\beta = 0$$



F. COSEC

$$\cos(90 - \delta) = \cos \epsilon \cdot \cos(90 - \beta) + \sin \epsilon \cdot \sin(90 - \beta) \cdot \cos(90 - \lambda)$$

$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot \cos \beta \cdot \sin \lambda$$

$$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$$

F. SEFO

$$\frac{\sin(90 + \delta)}{\sin(90 - \beta)} = \frac{\sin(90 - \lambda)}{\sin(90 - \delta)} \Rightarrow \frac{\cos \delta}{\cos \beta} = \frac{\cos \lambda}{\cos \delta}$$

$$\Rightarrow \cos \beta \cdot \cos \lambda = \cos \delta \cdot \cos \delta$$

PREC. LUNISOLAR

$\widehat{P'K} = \widehat{P'K} = \widehat{P'K} =$

$d\delta = \sin \delta \cdot d\alpha$

$\Rightarrow d\delta = \sin \delta \cdot \psi \cdot z$

PROBAB

$\Rightarrow d\alpha = \frac{d\delta}{\sin \delta} = \frac{d\delta}{\sin \delta} \cdot \psi \cdot z$

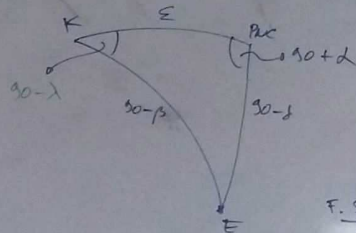
$\frac{d\delta}{d\alpha} ?$

$d\lambda = \psi \cdot z$

$d\beta = 0$

$\psi = 50'' \cdot 3878 + 0'' \cdot 0045 \cdot T$

$T = \frac{(t - 2000)}{100}$



F. CASE

$\cos(90-d) = \cos \epsilon \cdot \cos(90-\beta) + \sin \epsilon \cdot \sin(90-\beta) \cdot \cos(90-\lambda)$

$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot \cos \beta \cdot \sin \lambda$

$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$

F. SEFD

$\frac{\sin(90+d)}{\sin(90-\beta)} = \frac{\sin(90-\lambda)}{\sin(90-d)} \Rightarrow \frac{\cos d}{\cos \beta} = \frac{\cos \lambda}{\cos \delta}$

$\Rightarrow \cos \beta \cdot \cos \lambda = \cos \delta \cdot \cos d$

PREC. LUNAR

$$\widehat{P'K} = \widehat{P'K} = \widehat{P'K} = \Delta\lambda = \psi \cdot \epsilon$$

$\sim 50''/a_0$   
 $\sim a_0 \cos$

~~$\cos \delta \cdot d\delta = \sin \epsilon \cdot d\lambda \cdot \sin \delta \cdot \cos \delta$~~

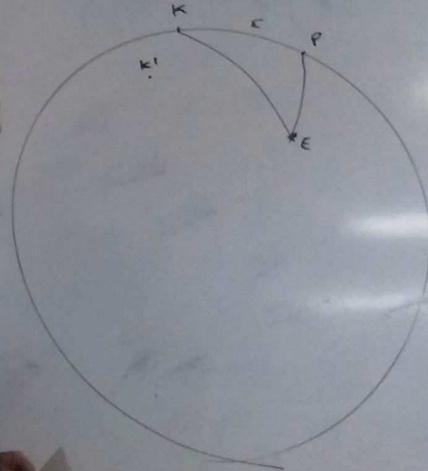
$$\Rightarrow d\delta = \sin \epsilon \cdot \cos \delta \cdot d\lambda \cdot \psi \cdot \epsilon$$

PROBAN

$$\Rightarrow d\alpha = \left( \cos \epsilon + \sin \epsilon \cdot \sin \delta \cdot \frac{1}{\cos \delta} \right) \cdot \psi \cdot \epsilon$$

$$\psi = 50'' \cdot 38778 +$$

$$T = \left( \frac{\int_{\delta}^{\delta_0} \dots}{\dots} \right)$$



PREC. LUISOLAR

$\hat{n}'\hat{n} = \hat{n}'\hat{k}\hat{n} = \hat{p}'\hat{k}\hat{p} = \Delta\lambda = \Psi \cdot z$

*So'afio*  
*afios*

$i \frac{d\lambda}{d\delta} ?$   
 $d\lambda = \Psi \cdot z$   
 $d\beta = 0$

~~$\cos \delta \cdot d\delta = \sin \epsilon \cdot d\lambda \cdot \cos \delta$~~

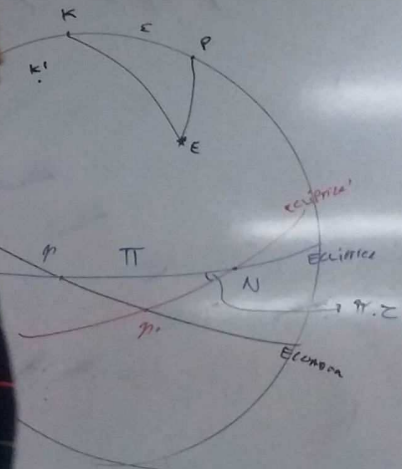
$\Rightarrow d\delta = \sin \epsilon \cdot \cos \delta \cdot d\lambda \cdot \Psi \cdot z$

PROBAR

$\Rightarrow d\alpha = \left( \cos \epsilon + \sin \epsilon \cdot \sin \delta \cdot \frac{1}{\tan \delta} \right) \cdot \Psi \cdot z$

$\Psi = 50'' \cdot 3878 + 0'' \cdot 0045$

$T = \left( \frac{1}{100} - 2000 \right)$



$\Pi, \pi \Rightarrow$  TEORÍA PLANETARIA

$\Pi = 174'' \cdot 8364 + 0'' \cdot 9137 \cdot T$

$\pi = 0'' \cdot 4200 - 0'' \cdot 0004 \cdot T$

PREC. PLANETARIA

$$d\alpha = -\dot{\lambda} \cdot c \rightarrow \text{O.E.}$$

$$d\delta = 0$$

$$d\lambda$$

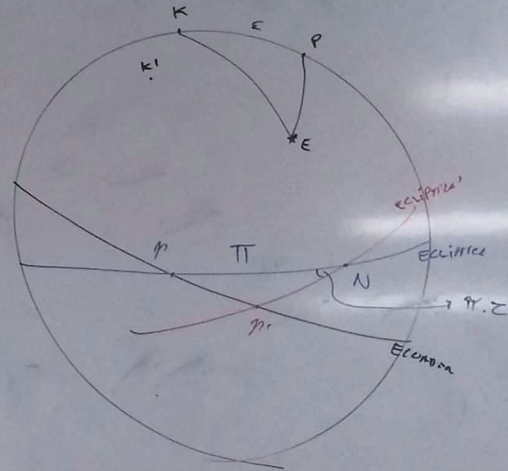
$$i \frac{dd}{ds} ?$$

$$d\lambda = \psi \cdot z$$

$$d\beta = 0$$

$$\psi = 50'' \cdot 3878 + 0'' \cdot 0045 \cdot T$$

$$T = \frac{\int_{\text{inst. en años}}^{t - 2000}}{100}$$



$\pi, \pi \Rightarrow$  TEORÍA PLANETARIO

$$\pi = 174'' \cdot 8764 + 0'' \cdot 9137 \cdot T$$

$$\pi = 0'' \cdot 4700 - 0'' \cdot 0007 \cdot T$$

PREC. PLANETARIA

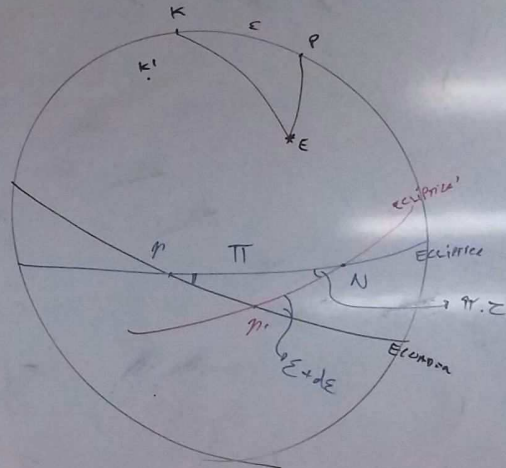
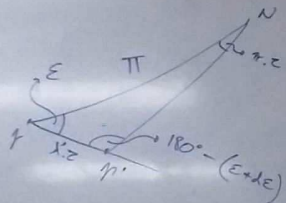
$d\alpha = -\dot{\lambda} \cdot C$  — COEF. DE PRECESIÓN PLANETARIA

$d\delta = 0$

$d\lambda =$

$d\beta =$

$\lambda', \pi, \pi'?$



$\pi, \pi' \Rightarrow$  TEORÍA PLANETARIA

$\pi = 174^{\circ}.8764 + 0^{\circ}.9137 \cdot T$

$\pi' = 0^{\circ}.4700 - 0^{\circ}.0007 \cdot T$

PREC. PLANETARIA

$d\alpha = -\lambda' \epsilon$  (DEF. OF PRECESSION PLANETARIA)

$d\delta = 0$

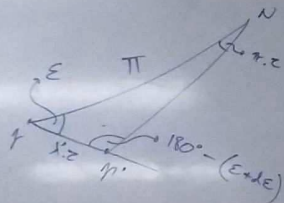
$d\lambda =$

$d\beta =$

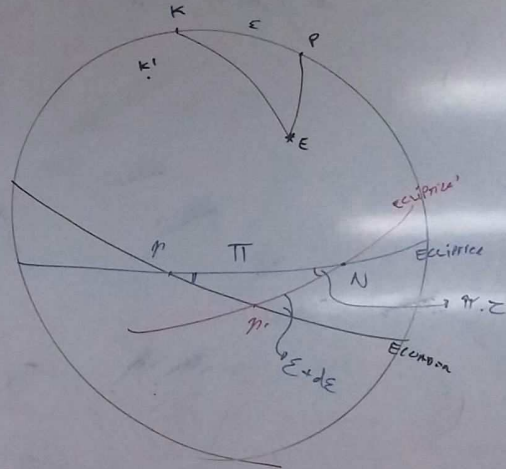
F. SERRA:

$$\frac{\sin(180 - (\epsilon + d\epsilon))}{\sin \pi} = \frac{\sin(\lambda \epsilon)}{\sin \lambda' \epsilon}$$

$\lambda', \pi, \pi'?$



$$\frac{\sin(\epsilon + d\epsilon)}{\sin \pi} = \frac{\pi \cdot \frac{1}{\epsilon}}{\lambda' \cdot \frac{1}{\epsilon}} \Rightarrow \lambda' = \frac{\pi}{\lambda \epsilon}$$



$\pi, \pi' \Rightarrow$  TEORÍA PLANETARIA

$\pi = 174^{\circ}.8764 + 0^{\circ}.9137 \cdot T$

$\pi' = 0^{\circ}.4700 - 0^{\circ}.0007 \cdot T$

PREC. PLANETARIA

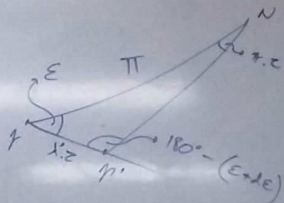
$d\lambda = -\lambda' \cdot \zeta$  (COEF. DE PRECESIÓN PLANETARIA)

$d\delta = 0$

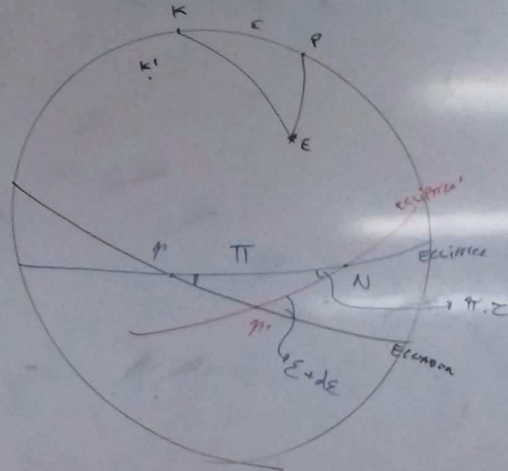
¿ $\lambda'$ ,  $\pi$ ,  $\pi'$ ?

F. SENA:

$$\frac{\tan(180 - (\epsilon + d\epsilon))}{\tan \pi} = \frac{\tan(\pi \cdot \zeta)}{\tan \lambda \cdot \zeta}$$



$$\frac{\tan(\epsilon + d\epsilon)}{\tan \pi} = \frac{\pi \cdot \zeta}{\lambda \cdot \zeta} \Rightarrow \lambda' = \pi \cdot \frac{\tan \pi}{\tan(\epsilon + d\epsilon)} \Rightarrow \lambda' = \pi \frac{\tan \pi}{\tan \epsilon}$$



$\pi$ ,  $\pi' \Rightarrow$  TEORÍA PLANETARIO

$\pi = 174^{\circ}.3364 + 0^{\circ}.9137 \cdot T$

$\pi' = 0^{\circ}.4200 - 0^{\circ}.0004 \cdot T$

SE PUEDE PROBAR QUE

$d\epsilon = \pi \cdot \zeta \cdot \cos \pi$



PREC. PLANETARIA + P. LUNISOLAR = PRECESION GENERAL

$$\Delta\alpha_{\text{TOTAL}} = \Delta\alpha_{\text{LS}} + \Delta\alpha_{\text{PLAN}} =$$

PREC. PLANETARIA + P. LUNISOLAR = PRECESION GENERAL

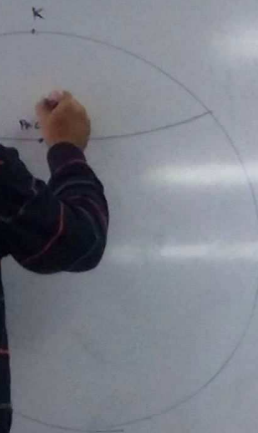
$$\Delta \alpha_{\text{total}} = \Delta \alpha_{\text{LS}} + \Delta \alpha_{\text{PLAN}} = (\cos \varepsilon + m \varepsilon \cdot \sin \delta \cdot \frac{1}{\cos \delta}) \cdot \psi \cdot z - \lambda' \cdot z$$

$$\Delta \delta_{\text{total}} = \Delta \delta_{\text{LS}} + \Delta \delta_{\text{PLAN}} = m \sin \varepsilon \cdot \cos \delta \cdot \psi \cdot z$$

"0"

$$\alpha, \delta, \varepsilon, \psi, \lambda', z \rightarrow \alpha', \delta'$$

LIBRACION : oscilación de PNC  $\approx 20''$



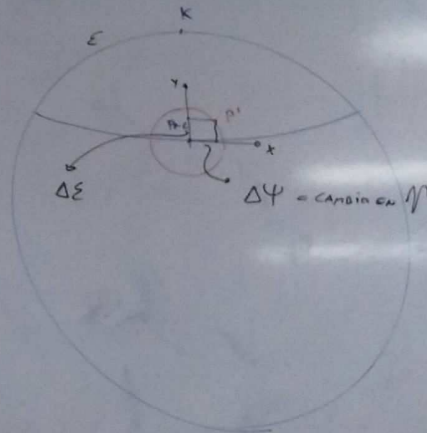
PRECESION GENERAL + P. LUNISOLAR = PRECESION GENERAL

$$\Delta \alpha + \Delta \alpha_{PL} + \Delta \alpha_{PL} = (\cos \epsilon + m \sin \epsilon \cos \delta) \cdot \psi \cdot z - \lambda' \cdot z$$

$$\Delta \delta + \Delta \delta_{PL} = m \sin \epsilon \cdot \sin \delta \cdot \psi \cdot z$$

$\alpha, \delta, \epsilon, \psi, \lambda', z \rightarrow \alpha', \delta'$

NUTACION : oscilación de  $\epsilon$   $\approx 20''$



$P' = \text{POLO INSTANTANEO}$

$P = \text{POLO MEDIO}$

$\Delta \psi = \text{cambio en } \psi$

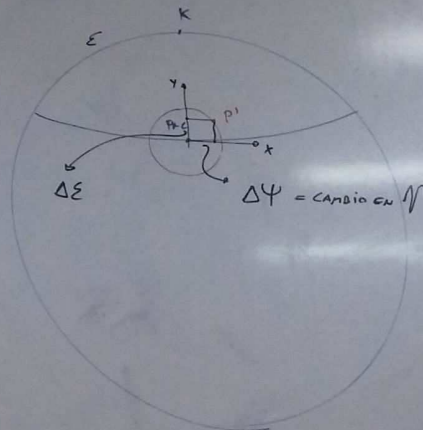
PREC. PLANETARIA + P. LUNISOLAR = PRECESION GENERAL

$$\Delta \alpha_{total} = \Delta \alpha_{ES} + \Delta \alpha_{PLN} = (G \epsilon + m \epsilon \cdot \cos \delta) \cdot \psi \cdot z - \lambda' \cdot z$$

$$\Delta \delta_{total} = \Delta \delta_{ES} + \Delta \delta_{PLN} = m \epsilon \cdot \cos \delta \cdot \psi \cdot z$$

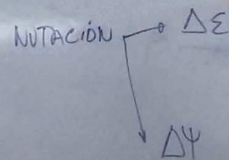
$$\alpha, \delta, \epsilon, \psi, \lambda', z \rightarrow \alpha', \delta'$$

NUTACION : oscilación de PNC  $\sim 20''$



P' = POLO INSTANTÁNEO

P = POLO MEDIO



ψ INSTANTÁNEO

ψ MEDIO DEFINIDO P

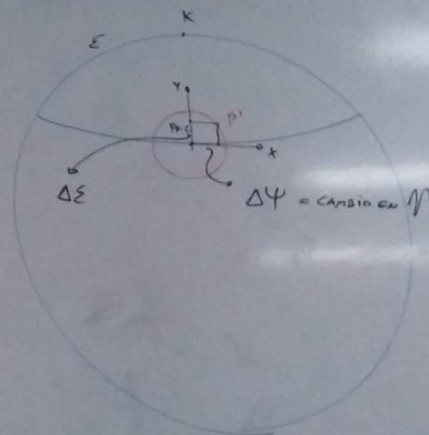
TEORÍA + P. LUNISOLAR = PRECESIÓN GENERAL

$$\Delta \alpha_{lis} + \Delta \alpha_{polar} = (\cos \epsilon + m \epsilon \cdot \sin \epsilon) \cdot \psi \cdot z - \lambda' \cdot z$$

$$+ \Delta \delta_{polar} = m \sin \epsilon \cdot \cos \alpha \cdot \psi \cdot z$$

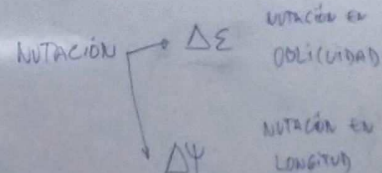
$\psi, \epsilon, \psi, \lambda', z \rightarrow \alpha', \delta'$

NUTACIÓN : oscilación de PNC  $\sim 20''$



P' = Polo Instantáneo

P = Polo Medio



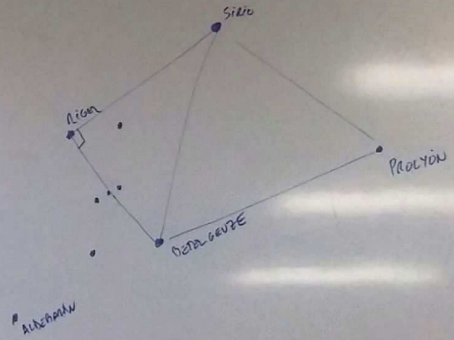
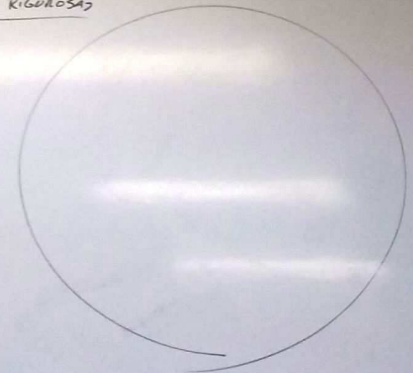
$\psi$  INSTANTÁNEO (P. INSTANTÁNEO)  $\Rightarrow$  TSV

$\psi$  MEDIO DEFINIDO POR PNC  $\Rightarrow$  TSM

PRECESIÓN: FORM. RIGUROSAS

Polo inicial  $P_0$

Polo Final  $P$



- HOY: ENTREGA
- 21: NO CLASE
- 23: RECUPERACION PRÁCTICO
- 10:00
- 24: P. ABIERTAS

PARCIAL:  
28/5  
10:00

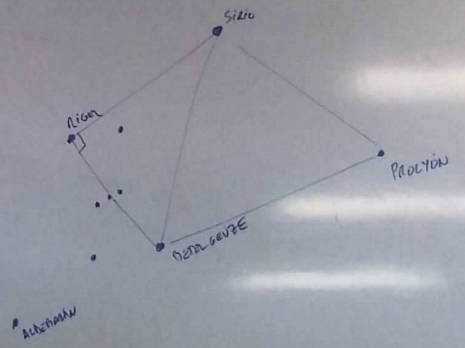
PRECESIÓN: FORM. RIGOROSAS

Polo inicial  $P_0$   
 Polo Final  $P$   
 $P$  RESPECTO A  $P_0$



TEORÍA PRECESIÓN GRAL.

$$\theta_A(t), \zeta_A(t), z_A(t)$$



H07: ENTREGA  
 21: NO CLASE  
 23: RECUPERACIÓN  
 PLÁSTICO  
 10:00  
 24: P. ABIERTAS

PARCIAL:  
 28/5  
 10:00

RESIÓN: Foan. Rigurosas

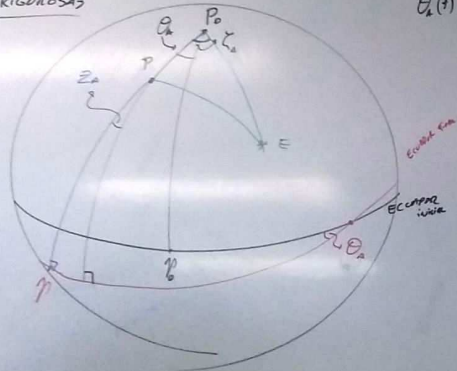
P<sub>0</sub>

P

P<sub>0</sub>

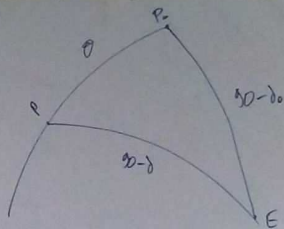
$(\alpha_0, \delta_0)$

en t



TEORÍA PROCESOS GRAL.

$$\theta_s(t), \zeta_s(t), \eta_s(t)$$



HOY: ENTREGA

21: NO CLASE

23: RECUPERACIÓN  
PRÁCTICO

10:00

24: P. ABIERTOS

PARCIAL:

28/5

10:00



PRECESIÓN: Form. Rigurosas

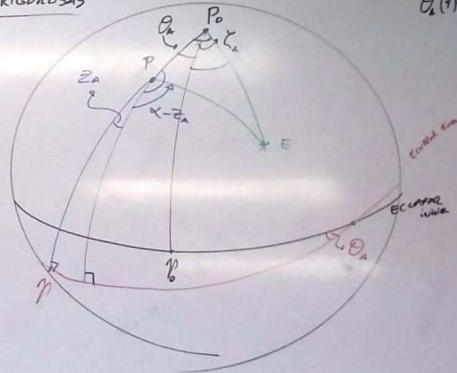
Polo inicial  $P_0$

Polo Final  $P$

$P$  respecto a  $P_0$

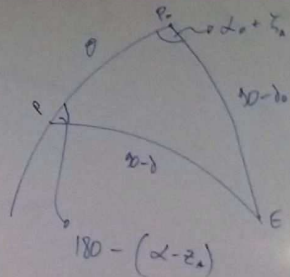
COORD. EN INST  $t_0$   $(\alpha_0, \delta_0)$

$\Rightarrow (\alpha, \delta)$  en  $t$



Teoría Precesión GRAL:

$$\theta_s(t), \zeta_s(t), z_s(t)$$



$$(\alpha_0, \delta_0) \leftrightarrow (\alpha, \delta)$$

R

PRECESIÓN: FÓRM. RIGUROSAS

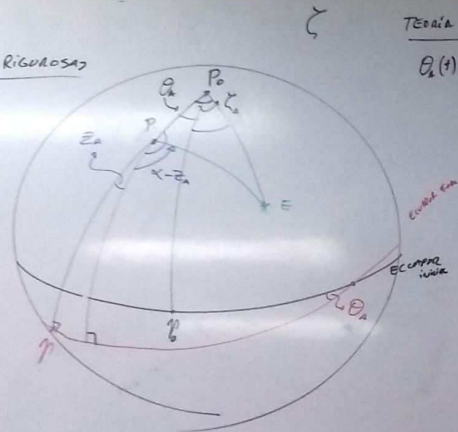
POLO INICIAL  $P_0$

POLO FINAL  $P$

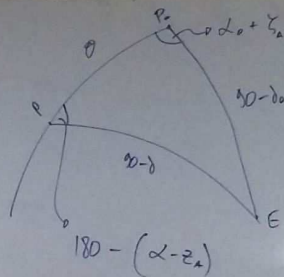
$P$  RESPECTO A  $P_0$

COORD. EN INST  $t_0$   $(\alpha_0, \delta_0)$

$\Rightarrow (\alpha, \delta)$  en  $t$



TEORÍA PRECESIÓN GRAL:  
 $\theta_A(t), \zeta_A(t), z_A(t)$

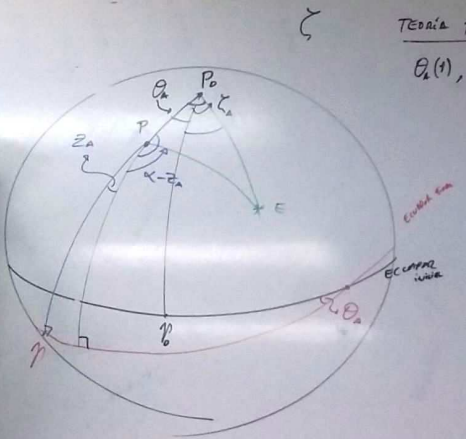


$$\begin{matrix} (\alpha_0, \delta_0) & \longleftrightarrow & (\alpha, \delta) \\ & \uparrow & \\ & \theta, \zeta, z & \end{matrix}$$

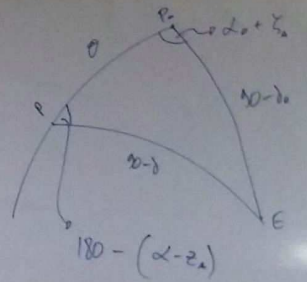
$$P(t_0, t) = R_z(-z_A) \cdot R_y(-\theta_A) \cdot R_z(-z_A)$$

↑  
MATRIZ PRECESIÓN

POS. MEDIA STANDARD: 2000.0  
 ↓ PRECESION GERAL + MOV. PROPRIO  
 POS. MEDIA DE LA FECHA (2011)



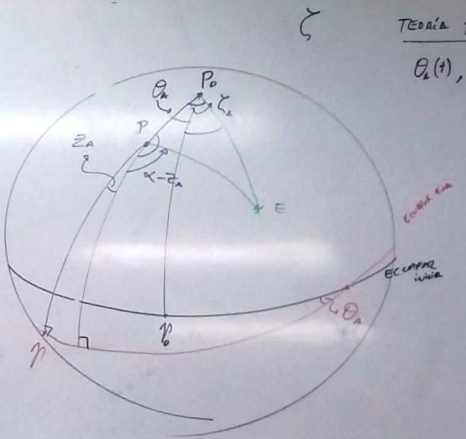
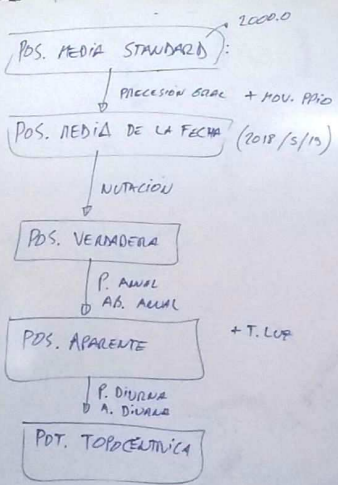
TEORIA PRECESION GERAL:  
 $\theta_a(t), \zeta_a(t), z_a(t)$



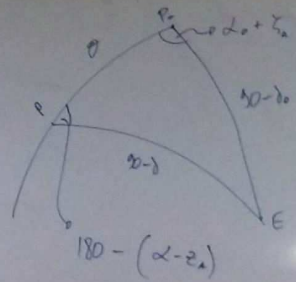
$(\alpha_0, \delta_0) \leftarrow (\alpha, \delta)$   
 $\uparrow$   
 $\theta, \zeta, z$

$$P(t_0, t) = R_z(-z_a) \cdot R_y(-\theta_a) \cdot R_x(-\zeta_a)$$

↑  
 MATRIZ PRECESION



TEORIA PRECESION GRAL:  
 $\theta_n(t), \zeta_n(t), \gamma_n(t)$

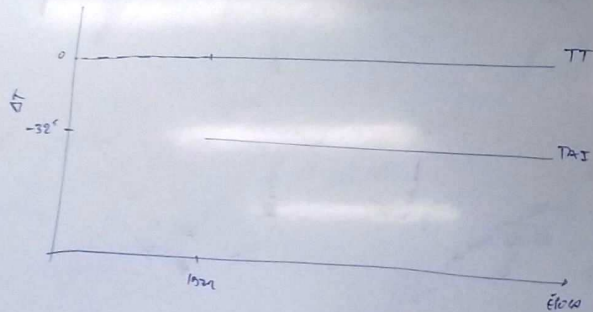


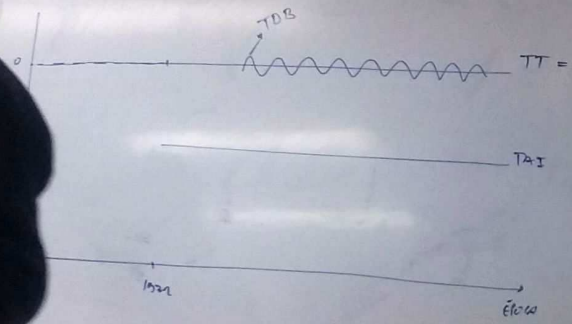
$$\begin{matrix} (\alpha_0, \delta_0) & \longleftrightarrow & (\alpha, \delta) \\ & \uparrow & \\ & \theta, \zeta, \gamma & \end{matrix}$$

$$P(t_0, t) = R_z(-\zeta_n) \cdot R_y(-\theta_n) \cdot R_z(-\gamma_n)$$

↑  
MATAIZ PRECESION

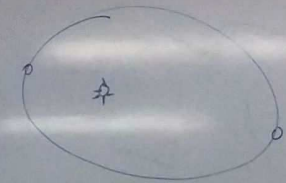
TIEMPO





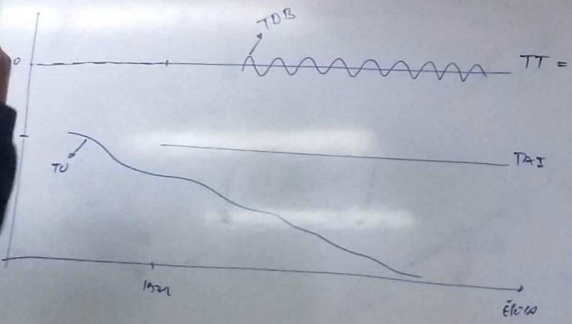
$$TDB - TT \sim 0.00166$$

$$TT = TAI + 32^s.184$$



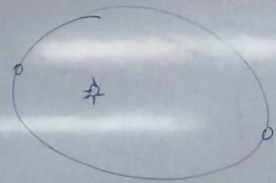
TIEMPO

TUC EASL



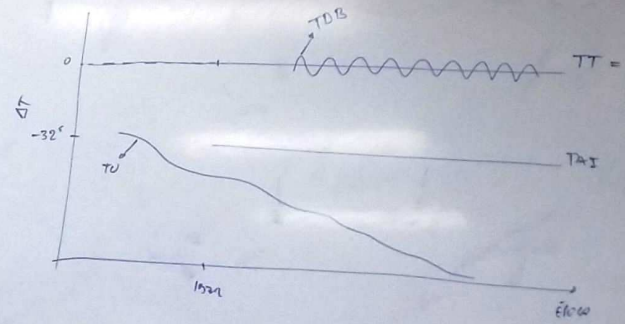
$$TDB - TT \sim 0.00166$$

$$TT = TAI + 32^s.184$$



**TIEMPO**

TUC : Tiempo Universal  
COORDINADO



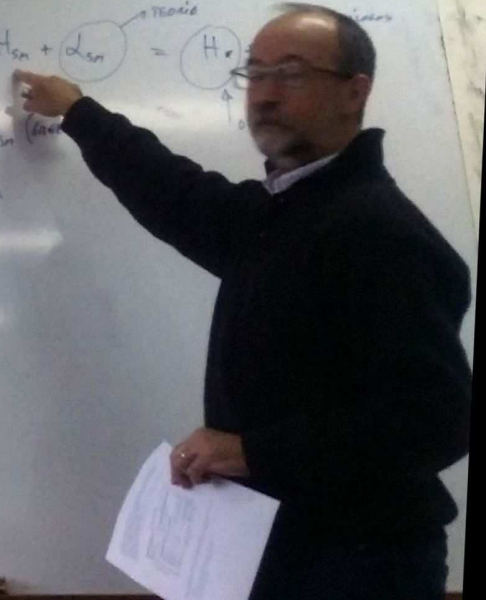
$TDB - TT \sim 0.000166$

$TT = TAI + 32.184$

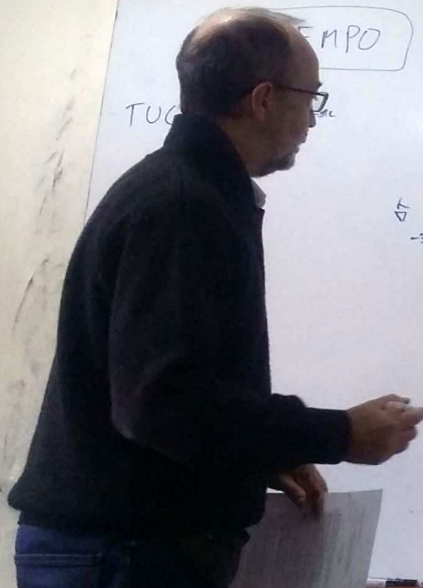
$TSL = H_{sm} + \Delta_{sm} = H_0$

$TU = 12^h + H_{sm}$

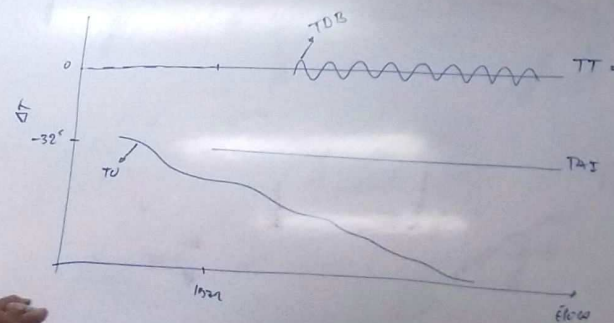
$T_{local} = TU + \lambda$







TEMPO



$$TDB - TT \sim 0.00166$$

$$TSL = H_{sm} + \Delta_{sm} = H_o + \Delta_o$$

↑ TEGAR
↑ COTA INCLINADA

$$TU = 12^\circ + H_{sm} \text{ (SOLARCO)}$$

$$T_{Lorel} = TU + (\times)$$

$$H_{sm} \rightarrow UT0 + CORP. MOV. POLAR \rightarrow UT1$$

**TIEMPO**

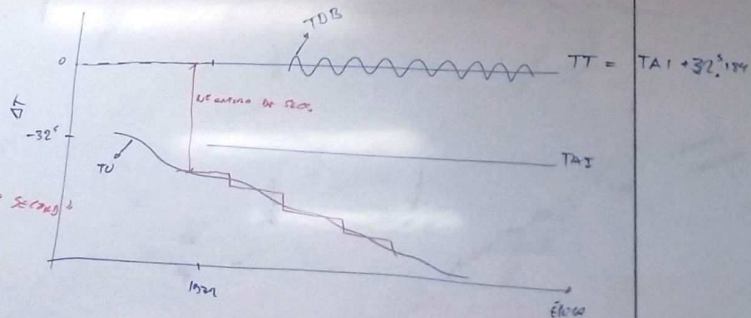
TUC : Tiempo Universal  
COORDINADO

23:59:59

23:59:60

24:00:00

LEAP SECOND



$TDB - TT \sim 0.00166$

$TSL = H_{sm} + \Delta_{sm} = H_o + \Delta_o$

*(Annotations: "TDB" above  $\Delta_{sm}$ , "obs." below  $H_o$ , "calculated" above  $\Delta_o$ )*

$TU = 12^h + H_{sm} \text{ (Greenwich)}$

$T_{local} = TU + (\times)$

$H_{sm} \rightarrow UT0 + \text{CORR. M.V. POLAR} \rightarrow UT1$

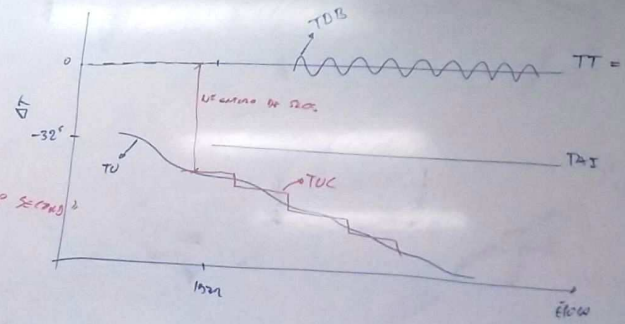
**TIEMPO**

TUC : Tiempo Universal  
COORDINADO

23:59:59

23:59:60 ← LEAP SECOND

24:00:00



$TDB - TT \sim 0.0016s$

$TT = TA1 + 32.184s$

$TSL = H_{sm} + \Delta_{sm} = H_o + \Delta_o$

*(Annotations: H<sub>sm</sub> and Δ<sub>sm</sub> are circled. Arrows point to 'TDB' and 'obs.')*

$TU = 12^h + H_{sm}$  (Greenwich)

$T_{local} = TU + (\text{X})$

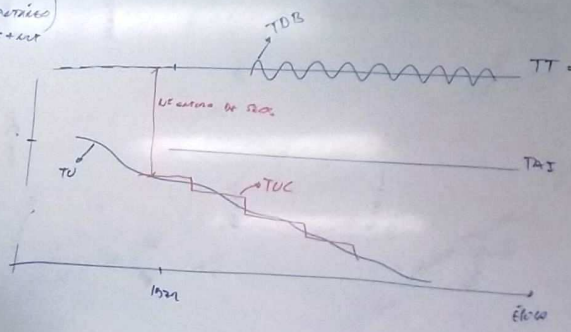
$H_{sm} \rightarrow UT0 + \text{CORR. MOV. POLAR} \rightarrow UT1$

$\frac{1}{24 \times 60 \times 60}$

$\Delta T = TT - UT$

**TIEMPO**

TS →  $\mu$  VARIACIONES (INSTANTANEAS)  $\mu_{oc} + \mu_{ut}$   
 $\mu$   $\mu_{medio}$  (PRECESION)



$TT - TAI \sim 0.00166$

$TT = TAI + 32.184$

$TSL = H_{sm} + \Delta_{sm} = H_o + \Delta_o$

*(Annotations: H<sub>sm</sub> is circled and labeled "teoría", Δ<sub>sm</sub> is circled and labeled "observación", H<sub>o</sub> is circled and labeled "teoría", Δ<sub>o</sub> is circled and labeled "observación")*

$TU = 12^h + H_{sm}$  (observación)

$T_{local} = TU + (\Delta)$

$H_u \rightarrow UT0 + \text{CORR. INV. POLAR} \rightarrow UT1$

$\frac{1}{24 \times 60 \times 60}$

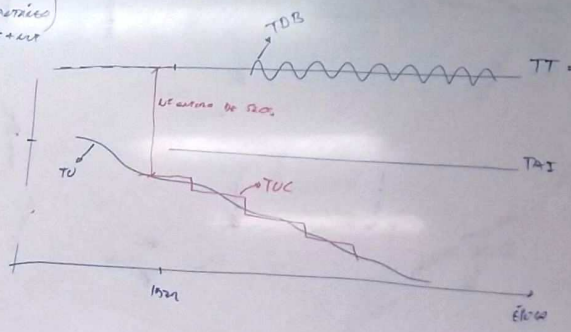
$\Delta T = TT - UT1$

**TIEMPO**

TS →  $n$  Verdadero (Instruico) TS aparente  $\rho_{oc} + \mu_{oc}$   
 ↓  
 $n$  Medio (Pneum)

$n_v - n_n =$  Ecuacion de los Equinoccios

$= \Delta \Psi \cdot \cos \epsilon$   
 ↑  
 obliquidad



$TDB - TT \sim 0.00166$

$TT = TAI + 32^s.184$

$TSL = H_{sm} + \alpha_{sm} = H_o + \alpha_o$   
 (Annotations:  $\alpha_{sm}$  is 'Fuerza',  $\alpha_o$  is 'Correccion',  $H_o$  is 'obs.')

$TU = 12^h + H_{sm}$  (observacion)

$T_{local} = TU + (\lambda)$

$H_{sm} \rightarrow UT0 + \text{CORR. MOV. POLAR} \rightarrow UT1$

$\frac{1}{24 \times 60 \times 60}$

$\Delta T = TT - UT1$

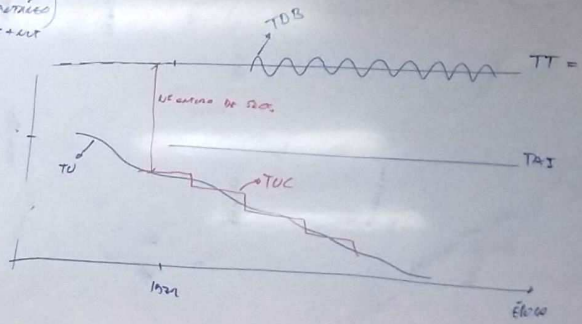
**TIEMPO**

TS →  $\mu$  Verdadero (Instantáneo)  
 TS aparente  $\mu_{app}$   
 $\mu_{app} - \mu = \Delta\mu$  (Percusión)

$\mu_V - \mu_n =$  Ecuación de los Equinoccios

$= \Delta\psi \cdot \cos \epsilon$   
 ↑  
 oblicuidad

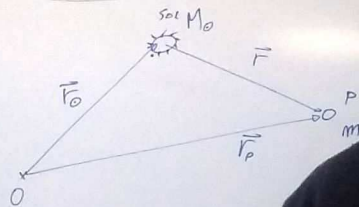
$12^h$  TDB  $1/1000 : JD\ 2451545.0$   
 $MJD = JD - 2400000.0$



$TDB - TT \sim 0.00166$

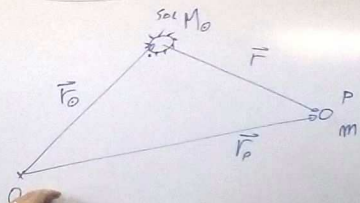
$TT = TAI + 32^s.184$

$\mu = 365.2422$  . días solares  
 $\mu_{app} = 365.2564$   
 $365.2546$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

$$\ddot{\vec{r}}_p = -G \frac{M_\odot m}{r^2}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS



$$M \ddot{\vec{r}}_p = -G \frac{M_0 m}{r^2} \hat{r}$$

$$M_0 \ddot{\vec{r}}_0 = +G \frac{M_0 m}{r^2} \hat{r}$$

---

RESOL:  $\ddot{\vec{r}}_p - \ddot{\vec{r}}_0 = -G \frac{(M_0 + m)}{r^2} \hat{r}$

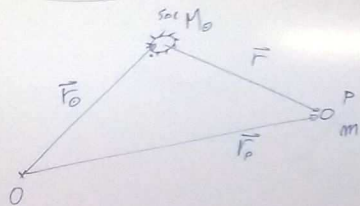
$\ddot{\vec{r}}$

$$\ddot{\vec{r}} = -G \frac{(M_0 + m)}{r^2} \hat{r}$$

EC. MOV. RELATIVO



# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS



$$m \ddot{\vec{r}}_p = -G \frac{M_0 m}{r^2} \hat{r}$$

$$M_0 \ddot{\vec{r}}_0 = +G \frac{M_0 m}{r^2} \hat{r}$$

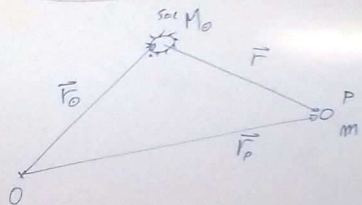
RESOL:  $\ddot{\vec{r}}_p - \ddot{\vec{r}}_0 = -G \frac{(M_0 + m)}{r^2} \hat{r}$

$$\ddot{\vec{r}} = -G \frac{(M_0 + m)}{r^2} \hat{r}$$

EC. MOV. RELATIVO

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \text{INT.}$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS



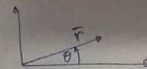
$$m \ddot{\mathbf{r}}_p = -G \frac{M_0 m}{r^2} \hat{\mathbf{r}}$$

$$M_0 \ddot{\mathbf{r}}_0 = +G \frac{M_0 m}{r^2} \hat{\mathbf{r}}$$

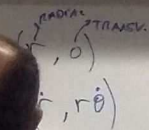
RESOL:  $\ddot{\mathbf{r}}_p - \ddot{\mathbf{r}}_0 = -G \frac{(M_0 + m)}{r^2} \hat{\mathbf{r}}$

$$\ddot{\mathbf{r}} = -G \frac{(M_0 + m)}{r^2} \hat{\mathbf{r}}$$

EC. MOV. RELATIVO



"MOMENTO ANGULAR"

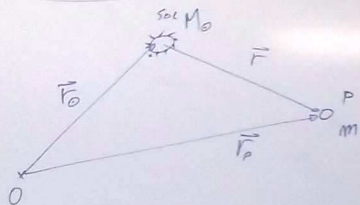


$$\mathbf{r} \wedge \ddot{\mathbf{r}} = 0 \Rightarrow \int \mathbf{r} \wedge \dot{\mathbf{r}} = \text{cte} = \vec{h} = r^2 \dot{\theta} \hat{\mathbf{z}}$$

INTEGRADO

$$\dot{\mathbf{r}} \wedge \dot{\mathbf{r}} + \mathbf{r} \wedge \ddot{\mathbf{r}} = 0$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS



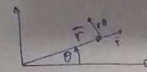
$$m \ddot{\mathbf{r}}_p = -G \frac{M_0 m}{r^2} \hat{\mathbf{r}}$$

$$M_0 \ddot{\mathbf{r}}_0 = +G \frac{M_0 m}{r^2} \hat{\mathbf{r}}$$

RESOL:  $\ddot{\mathbf{r}}_p - \ddot{\mathbf{r}}_0 = -G \frac{M_0 + m}{r^2} \hat{\mathbf{r}}$

$$\ddot{\mathbf{r}} = -G \frac{(M_0 + m) \hat{\mathbf{r}}}{r^2}$$

EC. MOV. RELATIVO



"MOMENTO ANGULAR"

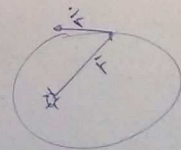
$$\mathbf{r} = (r, \theta)$$

$$\dot{\mathbf{r}} = (\dot{r}, r\dot{\theta})$$

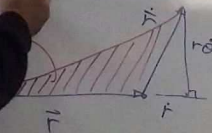
$$\mathbf{r} \wedge \ddot{\mathbf{r}} = 0 \Rightarrow \mathbf{r} \wedge \dot{\mathbf{r}} = \text{cte} = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{\mathbf{z}}$$

INTEGRADO

$$\mathbf{r} \wedge \dot{\mathbf{r}} + \mathbf{r} \wedge \ddot{\mathbf{r}} = 0$$



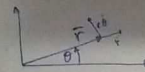
d.



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

$$\ddot{\vec{r}} = -G \frac{(M_0 + m) \hat{r}}{r^2}$$

EC. MOV. RELATIVO



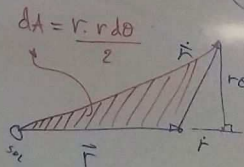
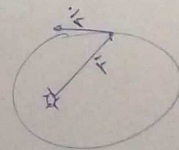
"MOMENTO ANGULAR"

$$\vec{r} = (r, 0)$$

$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \text{INTEGRADO } \vec{r} \wedge \dot{\vec{r}} = \text{cte} = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{z}$$

$$\dot{\vec{r}} \wedge \dot{\vec{r}} + \vec{r} \wedge \ddot{\vec{r}} = 0$$



$$dA = \frac{r \cdot r d\theta}{2}$$

$$\Rightarrow \frac{dA}{dt} = \frac{r^2 \dot{\theta}}{2} \quad \text{VEL. AREOLAR} = \frac{|h|}{2} = \text{cte}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

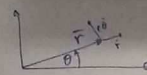
SOLUCION  
EC. NAV  
RELATIVO

$$r =$$

$$\mu = G(M_0 + m) \approx GM_0 = h^2$$

$$\ddot{\vec{r}} = -\frac{G(M_0 + m)\vec{r}}{r^3}$$

EC. NAV. RELATIVO



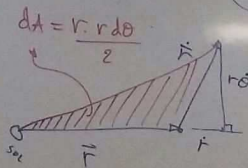
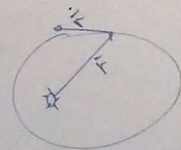
"MOMENTO ANGULAR"

$$\vec{r} = (r, \theta)$$

$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \text{INTEGRADO } (\vec{r} \wedge \dot{\vec{r}}) = \text{cte} = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{z}$$

$$\vec{r} \wedge \dot{\vec{r}} + \vec{r} \wedge \ddot{\vec{r}} = 0$$



$$dA = \frac{r \cdot r d\theta}{2}$$

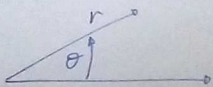
$$\Rightarrow \frac{dA}{dt} = \frac{r^2 \dot{\theta}}{2} \quad \text{VEL. AREOLAR} = \frac{|h|}{2} = \text{cte}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SOLUCIÓN  
EC. MOV  
RELATIVO

$$r = \frac{h^2}{\mu(1 + e \cos \theta)}$$

C. PLANOS  
(r, θ)



$$\mu = G(M_0 + m) \approx GM_0 = h^2$$

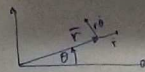
GAUSS

$$h^2 = GM_0 = (0.01720209895)^2$$

UA, días, M<sub>0</sub>

$$\ddot{\vec{r}} = -\frac{G(M_0 + m)}{r^2} \hat{r}$$

EC. MOV. RELATIVO



"MOMENTO ANGULAR"

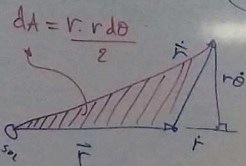
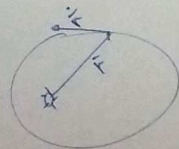
$$\vec{r} = (r, 0)$$

$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \int \vec{r} \wedge \dot{\vec{r}} = \text{cte} = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{z}$$

INTEGRADO

$$\vec{r} \wedge \dot{\vec{r}} + \vec{r} \wedge \ddot{\vec{r}} = 0$$



$$dA = \frac{r \cdot r d\theta}{2}$$

$$\Rightarrow \frac{dA}{dt} = \frac{r^2 \dot{\theta}}{2} \text{ VEL. AREOLAR} = \frac{|h|}{2} = \text{cte}$$

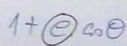
# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª LEY KEPLER

SOLUCIÓN  
EC. MOV  
RELATIVO

C. PLANES  
(r, θ)

$$\frac{h^2}{f}$$



EXCENTRICIDAD

$$\mu = G(M_0 + m) \approx GM_0 = h^2$$

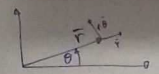
GAUSS

$$h^2 = GM_0 = (0.01720209895)^2$$

UA, días, M<sub>0</sub>

$$\ddot{\vec{r}} = -\frac{G(M_0 + m)}{r^2} \hat{r}$$

EC. MOV. RELATIVO



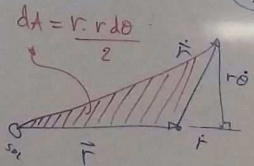
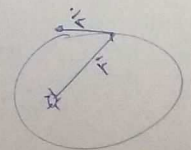
"MOMENTO ANGULAR"

$$\vec{r} = (r, 0)$$

$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \text{INTEGRADO } (\vec{r} \wedge \dot{\vec{r}}) = \text{cte} = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{z}$$

$$\dot{\vec{r}} \wedge \dot{\vec{r}} + \vec{r} \wedge \ddot{\vec{r}} = 0$$



$$\Rightarrow \frac{dA}{dt} = \frac{r^2 \dot{\theta}}{2}$$

2ª LEY KEPLER

$$\text{VEL. AREOLAR} = \frac{|h|}{2} = \text{cte}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1<sup>o</sup> LEY KEPLER

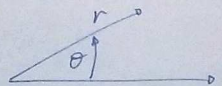
SOLUCIÓN  
EC. MOV  
RELATIVO

$$r = \frac{h^2}{\mu(1 + e \cos \theta)}$$

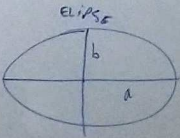
$$\frac{dA}{dt} = CTE = \frac{a \cdot b \cdot \pi}{T} = \frac{a^2 \pi \sqrt{1-e^2}}{T}$$

PERÍODO ORBITAL

C. MARES  
(r, θ)



EXCENTRICIDAD



$$b = a \cdot \sqrt{1-e^2}$$

$$\ddot{\vec{r}} = -\frac{G(M_0 + m)}{r^2} \hat{r}$$

EC. MOV. RELATIVO



$$\vec{r} = (r, \theta)$$

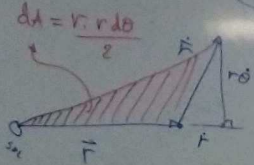
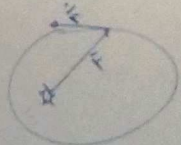
$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$

"MOVIMIENTO ANGULAR"

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \Rightarrow \int \vec{r} \wedge \dot{\vec{r}} = CTE = \vec{h} = r \cdot r\dot{\theta} \cdot \hat{z}$$

INTEGRADO

$$\cancel{\dot{\vec{r}} \wedge \dot{\vec{r}}} + \vec{r} \wedge \ddot{\vec{r}} = 0$$



$$\Rightarrow \frac{dA}{dt} = \frac{r^2 \dot{\theta}}{2}$$

2<sup>o</sup> LEY KEPLER

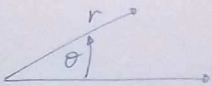
$$VEL. AREOLAR = \frac{|h|}{2} = CTE$$



# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SOLUCIÓN  
EC. NAV  
RELATIVO

C. PLANES  
(r, θ)

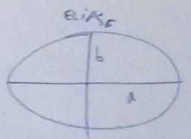


1ª LEY KEPLER

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

EXCENTRICIDAD

1 + e cos θ

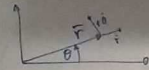


$$b = a \sqrt{1 - e^2}$$

$$\frac{dA}{dt} = \text{cte} = \frac{a \cdot b \cdot \pi}{T} = \frac{a^2 \pi \sqrt{1 - e^2}}{T} = \frac{h}{2} = \frac{\sqrt{\mu \cdot a} \cdot \sqrt{1 - e^2}}{2}$$

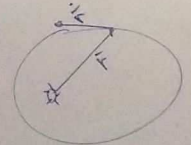
$$h = \sqrt{\mu a (1 - e^2)}$$

$$\frac{a^2 \pi}{T} = \sqrt{\mu}$$



$$\vec{r} = (r, \theta)$$

$$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$$



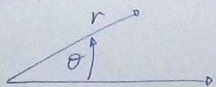
2ª LEY KEPLER

$$|EL. AREOLAR| = \frac{|h|}{2} = \text{cte}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SOLUCION EC. MOV RELATIVO

C. PLANES  $(r, \theta)$

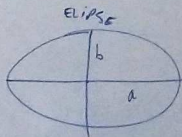


1ª LEY KEPLER

$$r = \frac{h^2/\mu}{1 + e \cos \theta}$$

Labels:  $h^2/\mu$  (circled),  $1 + e \cos \theta$ ,  $a(1-e^2)$

EXCENTRICIDAD



ELIPSE

$$b = a \sqrt{1-e^2}$$

ELIPSE

$$\frac{dA}{dt} = CTE = \frac{b \cdot a \cdot \pi}{T} = \frac{a^2 \pi \sqrt{1-e^2}}{T} = \frac{h}{2} = \frac{\sqrt{\mu \cdot a} \cdot \sqrt{1-e^2}}{2}$$

Labels:  $a \cdot b \cdot \pi$ ,  $T$ ,  $h$ ,  $2$ ,  $\sqrt{\mu \cdot a}$ ,  $\sqrt{1-e^2}$

PERIODO ORBITAL

$$h = \sqrt{\mu a (1-e^2)}$$

$$\frac{a^2 \pi}{T} = \frac{\sqrt{\mu \cdot a}}{2} \Rightarrow \frac{a^4 \pi^2}{T^2} = \frac{\mu a}{4}$$

3ª LEY KEPLER

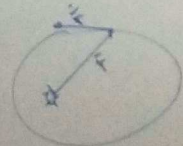
$$\frac{a^3}{T^2} = \frac{\mu}{(2\pi)^2} \Rightarrow \frac{G(M_0 + m)}{(2\pi)^2} = \frac{a^3}{T^2}$$



RADIAL, TANGENTIAL

$$\vec{r} = (r, \phi)$$

$$\vec{r}' = (\dot{r}, r\dot{\phi})$$



# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª LEY KEPLER

$$r = \frac{h^2}{f}$$

EXCENTRICIDAD

The diagram shows an elliptical orbit with a central body (represented by a circle with a cross) and a focus (represented by a circle with a dot). The distance from the central body to the focus is labeled 'EXCENTRICIDAD'. The semi-major axis is labeled 'a' and the semi-minor axis is labeled 'b'. The distance from the central body to the focus is labeled 'f'. The distance from the central body to the orbit at a given angle is labeled 'r'.

$$\frac{G(M_0 + m)}{(2\pi)^2} = \frac{a^3}{T^2}$$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª LEY KEPLER

$$r = \frac{h^2}{\mu}$$

↑

↑

EXCENTRICIDAD

"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑

VEL. ANG. MEDIA

↑

INERCIÓN

$$\frac{G(M_0 + m)}{(2\pi)^2} = \frac{a^3}{T^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{a^3}$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª Ley KEPLER

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

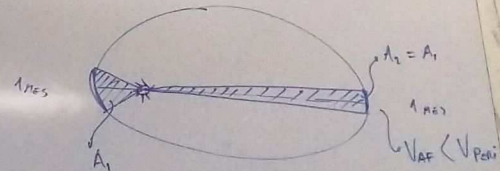
EXCENTRICIDAD

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA

↑  
ROTACION



$$\frac{G(M_0 + m)}{(2\pi)^2} = \frac{a^3}{T^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{a^3}$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª Ley KEPLER

$$r = \frac{h^2}{\mu} (1 + e \cos \theta)$$

EXCENTRICIDAD

$$r(\theta) = \frac{a(1-e^2)}{1 + e \cos \theta}$$

Forma

si  $e < 1$

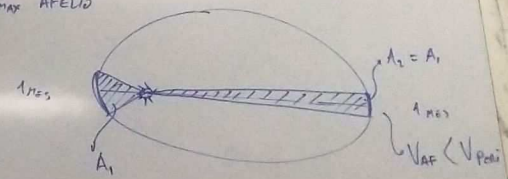
$$r_{max} = r(\theta = 180^\circ) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max} \text{ "AFELIO"}$$

$$r_{min} = r(\theta = 0) \Rightarrow r = a(1-e) = r_{min} \text{ "PERIHELIO"}$$

"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = \dot{m} = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA

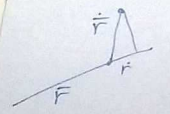
↑  
ROTACION



$$\frac{G(M_0 + m)}{(2\pi)^2} = \frac{a^3}{T^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{a^3}$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª LEY KEPLER



$$r = \frac{h^2 / \mu}{1 + e \cos \theta}$$

$$r(\theta) = \frac{a(1-e^2)}{1 + e \cos \theta}$$

EXCENTRICIDAD

$$\ddot{\vec{r}} = -\frac{\mu}{r^2} \hat{r}$$

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \hat{r} = -\frac{\mu}{r^3} \dot{r} \Rightarrow \dot{\vec{r}} \cdot \ddot{\vec{r}} = -\frac{\mu}{r^2} \dot{r}$$

si  $e < 1$

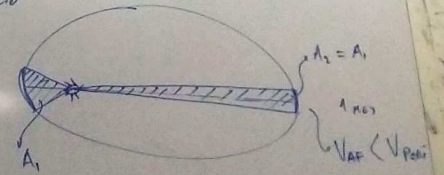
"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA

↑  
NOTACION

$$r_{max} = r(\theta=180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max} \text{ "AFELIO"}$$

$$r_{min} = r(\theta=0) \Rightarrow r = \frac{a(1-e^2)}{1+e} = r_{min} \text{ "PERIHELIO"}$$



ENERGIA ORBITAL

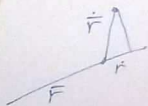
$$\frac{1}{2} v^2 = \frac{\mu}{r} + \mathcal{E} \Rightarrow \mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª Ley KEPLER

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

$$r(\theta) = \frac{a(1-e^2)}{1 + e \cos \theta}$$



EXCENTRICIDAD

$$\ddot{\vec{r}} = -\frac{\mu}{r^2} \hat{r}$$

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} = -\frac{\mu}{r^2} \dot{\vec{r}} \cdot \hat{r}$$

$$\frac{\dot{\vec{r}} \cdot \dot{\vec{r}}}{r^3} = \frac{\dot{r} \dot{r}}{r^3} = \frac{\dot{r}}{r^2}$$

$$\Rightarrow \dot{\vec{r}} \cdot \ddot{\vec{r}} = -\frac{\mu}{r^2} \dot{r}$$

INTEG:

$$\frac{1}{2} \dot{v}^2 = \frac{\mu}{r} + \mathcal{E}$$

$$\mathcal{E} = \frac{1}{2} \dot{v}^2 - \frac{\mu}{r}$$

"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

VEL. ANG. MEDIA

ROTACION

si  $e < 1$

- $r_{max} = r(\theta=180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$  "AFELIO"
- $r_{min} = r(\theta=0) \Rightarrow r = \frac{a(1-e^2)}{1+e} = a(1-e) = r_{min}$  "PERIHELIO"

ENERGIA ORBITAL

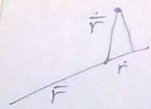
si  $\mathcal{E} < 0 \Rightarrow$  ACOTADO  $\Rightarrow$  LIGADO  $\Rightarrow$  órbita

si  $\mathcal{E} > 0 \Rightarrow$  NO ACOTADO  $\Rightarrow$  NO LIGADO



# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1ª Ley KEPLER



$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

EXCENTRICIDAD

$$V(\theta) = \frac{a(1-e^2)}{1 + e \cos \theta}$$

Forma

si  $e < 1$

"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA

↑  
NOTACIÓN

$r_{max} = r(\theta=180^\circ) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$  "AFELIO"

$r_{min} = r(\theta=0) \Rightarrow r = \frac{a(1-e^2)}{1+e} = a(1-e) = r_{min}$  "PERIHELIO"

$$\ddot{\vec{r}} = -\frac{\mu}{r^2} \hat{r}$$

$$\dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2} \hat{r} \cdot \hat{r} \Rightarrow \dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2}$$

$$\dot{\vec{r}} \cdot \dot{\vec{r}} = \frac{\dot{r}^2}{r^2} = \frac{\dot{r}}{r}$$

$$\Rightarrow \dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2} \Rightarrow \frac{N^2}{2} = \frac{\mu}{r} + \mathcal{E}$$

INTEG:

ENERGÍA ORBITAL

$$\mathcal{E} = \left( \frac{N^2}{2} \right) - \left( \frac{\mu}{r} \right)$$

(KIN) (POT)

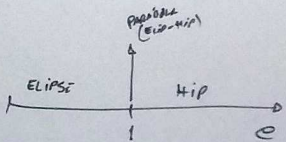
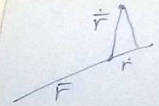
- si  $\mathcal{E} < 0 \Rightarrow$  órbita cerrada
- si  $\mathcal{E} > 0 \Rightarrow$  órbita abierta
- si  $\mathcal{E} = 0 \Rightarrow$  si  $r = \infty \Rightarrow N \rightarrow 0$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

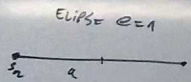
1ª Ley KEPLER

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

$$V(\theta) = \frac{a(1-e^2)}{1 + e \cos \theta}$$



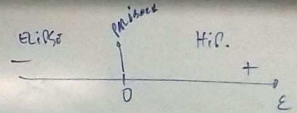
EXCENTRICIDAD



"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

VEL. ANG. MEDIA

NOTACION



si  $e < 1$

$r_{max} = r(\theta=180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$  "AFELIO"  
 $r_{min} = r(\theta=0) \Rightarrow r = \frac{a(1-e^2)}{1+e} = r_{min}$  "PERIHELIO"

ENERGIA ORBITAL

$$\epsilon = \frac{N^2}{2} - \frac{\mu}{r}$$

- si  $\epsilon < 0 \Rightarrow V < V_{esc} \Rightarrow$  LIGADO  $\rightarrow$  Órbita cerrada (ELIPSE)
- si  $\epsilon > 0 \Rightarrow V > V_{esc} \Rightarrow$  NO LIGADO  $\rightarrow$  Órbita abierta (HIPÉRBOLA)
- si  $\epsilon = 0 \Rightarrow V = V_{esc} \Rightarrow N = 0$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

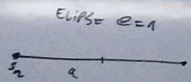
Semi-eje:  $a = \frac{\Delta_{TL}}{2}$

$$V(\theta) = \frac{a(1-e^2)}{1+e \cdot \cos \theta}$$

↑  
Forma

$$T_{caída} = \frac{T}{2}$$

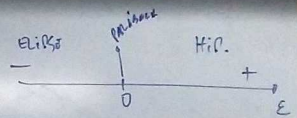
$$\frac{T^2}{a^3} = CTE = \frac{2\pi^2}{\Delta_{TL}^3}$$



"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA

↑  
NOTACIÓN



si  $e < 1$

- $r_{max} = r(\theta=180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$  "AFELIO"
- $r_{min} = r(\theta=0) \Rightarrow r = a(1-e) = r_{min}$  "PERIHELIO"

ENERGIA ORBITAL

$$\Rightarrow \mathcal{E} = \frac{N^2}{2} - \frac{\mu}{r}$$

(N)      (POT)

- si  $\mathcal{E} < 0 \Rightarrow r$  acotado  $\Rightarrow$  LIGADO  $\rightarrow$  órbita cerrada (Elipse)
- si  $\mathcal{E} > 0 \Rightarrow r$  no acotado  $\Rightarrow$  NO LIGADO  $\rightarrow$  órbita abierta (Hipérbola)
- si  $\mathcal{E} = 0 \Rightarrow r = \infty \Rightarrow N \rightarrow 0$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

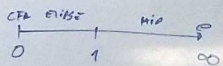
Semi-eje:  $a = \frac{\Delta r_L}{2}$

$e=1$   $\left\{ \begin{array}{l} OL \\ \downarrow \\ O_T \end{array} \right.$  ¿L?

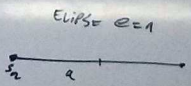
$$r(\theta) = \frac{a(1-e^2)}{1+e \cos \theta}$$

Forma

$T_{orbita} = \frac{T}{2}$

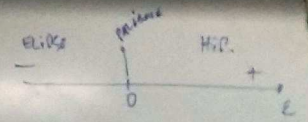


$$\frac{T^2}{a^3} = CTE = \frac{2\pi^2}{\Delta T_L^3} = \frac{(T/2)^2}{(\Delta T_L/2)^3}$$



"MOVIMIENTO MEDIO" =  $\frac{2\pi}{T} = n = \sqrt{\mu/a^3}$

↑  
VEL. ANG. MEDIA  
↑  
INERCIAS



Si  $e < 1$

- $r_{max} = r(\theta=180^\circ) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$  "AFELIO"
- $r_{min} = r(\theta=0) \Rightarrow r = a(1-e) = r_{min}$  "PERIHELIO"

ENERGIA ORBITAL

$$\Rightarrow \mathcal{E} = \frac{N^2}{2} - \frac{\mu}{r}$$

(C) (POT)

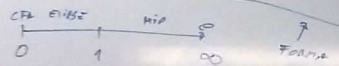
- Si  $\mathcal{E} < 0 \Rightarrow r_{max} < \infty \Rightarrow$  LIGADO  $\rightarrow$  ORBITA CERRADA (ELIPSE)
- Si  $\mathcal{E} > 0 \Rightarrow r_{max} = \infty \Rightarrow$  NO LIGADO  $\rightarrow$  ORBITA ABIERTA (HIPERBOLA)
- Si  $\mathcal{E} = 0 \Rightarrow r_{max} = \infty \Rightarrow N \rightarrow 0$

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

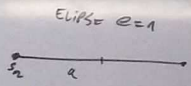
Semi-eje:  $a = \frac{\Delta_{TL}}{2}$

$$V(\theta) = \frac{a(1-e^2)}{1+e \cdot \cos \theta}$$

$T_{caída} = \frac{T}{2}$



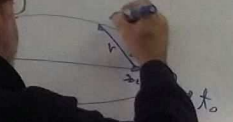
$$\frac{T^2}{a^3} = CTE = \frac{27^2}{\Delta_{TL}^3} = \frac{(T/2)^2}{(\Delta_{TL}/2)^3}$$



DADO  $t \rightarrow$  HALLAR  $\theta, r$

CASO ELIPSE

DEFINIMOS  $M = m \cdot (t - t_0)$   
"ANOMALIA MEDIA"

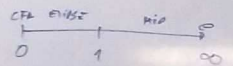


# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

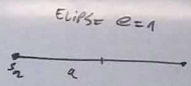
SMALL:  $a = \frac{\Delta_{TL}}{2}$

$$V(\theta) = \frac{a(1-e^2)}{1+e \cos \theta}$$

$T_{caída} = \frac{T}{2}$

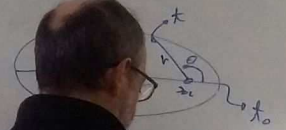


$$\frac{T^2}{a^3} = CTE = \frac{27^2}{\Delta_{TL}^3} = \left(\frac{T}{2}\right)^2 \left(\frac{2}{\Delta_{TL}}\right)^3$$



DADO  $t \rightarrow$  HALLAR  $\theta, r$

## CASO ELIPSE



DEFINIMOS  $M = m \cdot (t - t_0)$

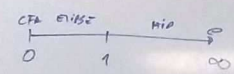
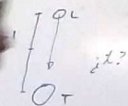
"ANOMALIA MEDIA"

$$\theta = M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

$$M = E - e \cos E$$

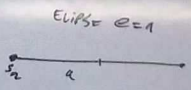
$(\theta(x))$

NTD Y CONFIGURACIONES PLANETARIAS



$$V(\theta) = \frac{a(1-e^2)}{1+e \cos \theta}$$

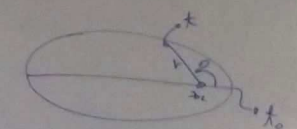
$$\frac{27^4}{\Delta_{TL}^3} = \frac{(T/2)^2}{(\Delta_{TL}/2)^3}$$



ELIPSE  $e=1$

DADO  $t \rightarrow$  HALLAR  $\theta, r$

CASO ELIPSE



DEFINICION  $M = m \cdot (t - t_0)$

"ANOMALIA MEDIA"

$$\theta = M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

$E \sim M$

$$M = E - e \cos E$$

$$r = a(1 - e \cos E)$$

$$E = M + e \cos E$$

DEF. ANOM. EXCENTRICAS

EC DE KEPLER

# MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

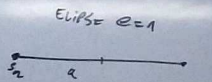
SE PRUEBA QUE

$$E = -\frac{\mu}{2a}$$

$a > 0$  ELIPSES  
 $a = \infty$  PARABOLA  
 $a < 0$  LA LO HIPERBOLAS

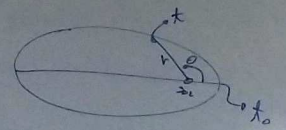
$$V(\theta) = \frac{a(1-e^2)}{1+e \cos \theta}$$

Forma



DADO  $t \rightarrow$  HALLAR  $\theta, r$

CASO ELIPSE



$E \sim M$

DEFINIMOS  $M = m \cdot (t - t_0)$

"ANOMALIA MEDIA"

$$\theta = M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

EC. DE KEPLER

$$M = E - e \cos E$$

DEF "ANOM. EXCENTRICA"

$$r = a(1 - e \cos E)$$

$$E = M + e \cos E$$

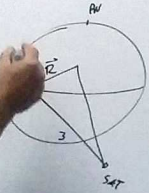
ITERACION

$\Rightarrow$  OBTENGO  $E \rightarrow r$



①

$$\phi = 0$$

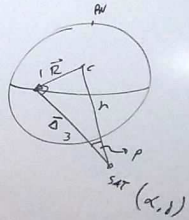


①

$$\phi = 0$$

$$TSL = 6^h$$

$$\alpha = 0^h$$



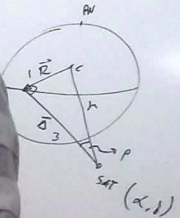
$$r^2 = (1R)^2 + (3R)^2 = 10R^2$$

$$\vec{R} = \text{Pos. OBS} = \begin{pmatrix} \cos \delta \cdot \cos \alpha \\ \cos \delta \cdot \sin \alpha \\ \sin \delta \end{pmatrix} \cdot 3R$$

$$\vec{D} = \begin{pmatrix} \cos \delta \cdot \cos \alpha \\ \cos \delta \cdot \sin \alpha \\ \sin \delta \end{pmatrix} \cdot R$$

$$\vec{F} = \vec{R} + \vec{D} = \begin{pmatrix} 3 \cos 30 \\ 1 \\ 3 \sin 30 \end{pmatrix} = r \begin{pmatrix} \cos \delta \cdot \cos \alpha \\ \cos \delta \cdot \sin \alpha \\ \sin \delta \end{pmatrix}$$

(1)



$$r^2 = (1R)^2 + (3R)^2 = 10R^2$$

$$\vec{R} = \text{pos. obs} = \begin{pmatrix} \cos \alpha \cos \delta_3 \\ \cos \alpha \sin \delta_3 \\ \sin \alpha \end{pmatrix} R$$

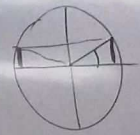
$$\vec{\delta}_3 = \begin{pmatrix} \cos \delta_3 \cos \alpha \\ \cos \delta_3 \sin \alpha \\ \sin \delta_3 \end{pmatrix} 3R$$

$$\vec{F} = \vec{R} + \vec{\delta}_3$$

$$\vec{F} = (3 \cos 30^\circ, 1, 3 \cos 30^\circ) = r \begin{pmatrix} \cos \alpha_6 \cos \delta_6 \\ \cos \alpha_6 \sin \delta_6 \\ \sin \alpha_6 \end{pmatrix}$$

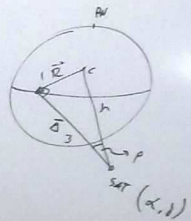
$-90^\circ < \delta < +90^\circ$

$$\sin \alpha_6 = \frac{1}{r \cos \delta_6} = 0.55$$



(1)

$\phi = 0$   
 $TSL = 6^h$   
 $\alpha = 0^h$



$$r^2 = (1R)^2 + (3R)^2 = 10R^2$$

$$\vec{R} = \text{pos. OBS} = \begin{pmatrix} \cos \phi \cos TSL & \cos \phi \sin TSL & \sin \phi \\ \cos \delta \cdot \cos \alpha & \cos \delta \cdot \sin \alpha & \sin \delta \end{pmatrix} R$$

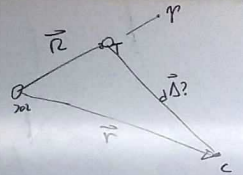
$$\vec{\Delta} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} R$$

$$\vec{F} = \vec{R} + \vec{\Delta}$$

$$\vec{F} = (3. \cos 30, 1, 3. \cos 30) = r \begin{pmatrix} \cos \delta_3 \cos \alpha_3 & \cos \delta_3 \sin \alpha_3 & \sin \delta_3 \end{pmatrix}$$

$-90 < \delta < +90$

$$\sin \alpha_3 = \frac{1}{r \cdot \cos \delta_3} = 0.35$$



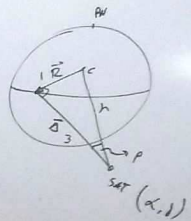
$$\vec{R} = (1, 0, 0) \text{ ua}$$

$$\vec{F} = \text{Sun} (\cos \beta \cos \lambda, \cos \beta \sin \lambda, \sin \beta)$$

$$\vec{\Delta} = \vec{F} - \vec{R}$$

(1)

$\phi = 0$   
 $TSL = 6^h$   
 $\alpha = 0^h$



$$r^2 = (1R)^2 + (3R)^2 = 10R^2$$

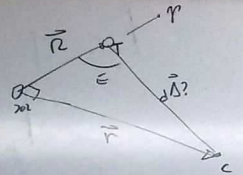
$$\vec{R} = \text{pos. obs} = \begin{pmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{pmatrix} R$$

$$\vec{\Delta} = \dots$$

$$\vec{F} = \vec{R} + \vec{\Delta}$$

$$\vec{F} = (3.630, 1, 3)$$

(2)



$$\vec{R} = (1, 0, 0) \text{ ua}$$

$$\vec{F} = S_{\text{un}} (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta)$$

$$\vec{F} = S (0, \cos \beta, \sin \beta)$$

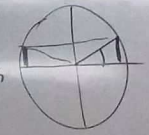
$$\vec{\Delta} = \vec{F} - \vec{R}$$

$$|\Delta| = \sqrt{26}$$

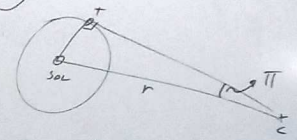
$$E = 78.7$$

$\rightarrow \delta < +90$

$$\sin \alpha_6 = \frac{1}{r \cdot \cos \delta_6} = 0.35$$

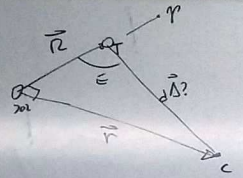


3



$$\sin \alpha = \frac{r_{\text{sol}}}{D} = \frac{1}{20.000}$$

2



$$\vec{R} = (1, 0, 0) \text{ un}$$

$$\vec{F} = S_{\text{un}} (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta)$$

$$\vec{F} = S (0, \cos \beta, \sin \beta)$$

$$\vec{\Delta} = \vec{F} - \vec{R}$$

$$|\Delta| = \sqrt{26}$$

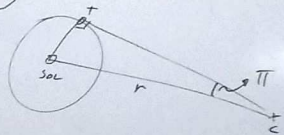
$$E = 78,7$$

$-90 < \delta < +90$

$$\sin \alpha = \frac{1}{r \cdot \cos \beta} = 0,35$$



3

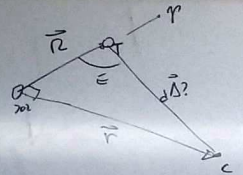


$$\sin \alpha = \frac{r_{\text{Earth}}}{r_{\text{Sun}}} = \frac{1}{20.000}$$

$$\alpha \text{ (arcos)} = 5 \times 10^{-4} \times \frac{360}{2\pi \times 10^5} \times 60 \times 60 = 10,3'' = \alpha$$

$$k = 20''$$

2



$$\vec{R} = (1, 0, 0) \text{ un}$$

$$\vec{F} = S_{\text{un}} (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta)$$

$$\vec{F} = S (0, \cos \beta, \sin \beta)$$

$$\vec{\Delta} = \vec{F} - \vec{R}$$

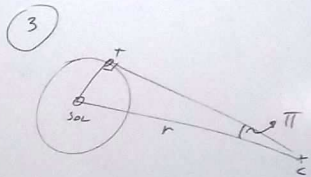
$$|\Delta| = \sqrt{26}$$

$$E = 78,7$$

$-90 < \delta < +90$

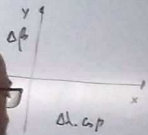
$$\sin \alpha_6 = \frac{1}{r \cdot \cos \delta} = 0,35$$





$$\sin \pi = \frac{1 \text{ ua}}{r_{\text{ua}}} = \frac{1}{20.000}$$

$$\pi \text{ (rads)} = 5 \times 10^{-4} \times \frac{360}{2\pi \cdot 1000000}$$

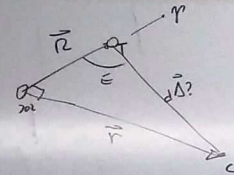


$$x = \pi \cdot \sin(\lambda_0 - \lambda) - K \cos(\lambda_0 - \lambda)$$

$$y = -\pi \sin \beta \cdot \cos(\lambda_0 - \lambda) - K \sin \beta \cdot \sin(\lambda_0 - \lambda)$$

$\lambda_0$

(2)



$$\vec{R} = (1, 0, 0) \text{ ua}$$

$$\vec{F} = S_{\text{un}} (\cos \beta \cos \lambda, \cos \beta \sin \lambda, \sin \beta)$$

$$\vec{F} = S (0, \cos \beta, \sin \beta)$$

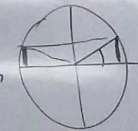
$$\vec{D} = \vec{F} - \vec{R}$$

$$|\vec{D}| = \sqrt{26}$$

$$E = 78,7$$

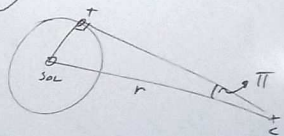
$-90 < \delta < +90$

$$\sin \delta = \frac{1}{r \cdot \cos \beta} = 0,35$$





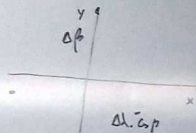
3



$$\sin \pi = \frac{1 \text{ ua}}{r_{\text{ua}}} = \frac{1}{20.000}$$

$$\pi \text{ (Rads)} = 5 \times 10^{-4} \times \frac{360}{2\pi \cdot 101152} \times 60 \cdot 60 = 10''.3 = \pi$$

$$k = 20'' \cdot s$$

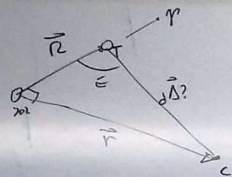


$$x = \pi \cdot \sin(\lambda_0 - \lambda) - k \cos(\lambda_0 - \lambda)$$

$$y = -\pi \sin \beta \cdot \cos(\lambda_0 - \lambda) - k \sin \beta \cdot \sin(\lambda_0 - \lambda)$$

$$\lambda_0 = \lambda$$

2



$$\vec{R} = (1, 0, 0) \text{ ua}$$

$$\vec{F} = S_{\text{un}} (\cos \beta \cos \lambda, \cos \beta \sin \lambda, \sin \beta)$$

$$\vec{F} = S (0, \cos \beta, \sin \beta)$$

$$\vec{\Delta} = \vec{F} - \vec{R}$$

$$|\Delta| = \sqrt{26}$$

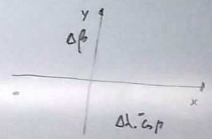
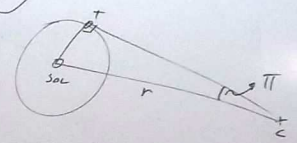
$$E = 78.7$$

$$-90 < \delta < +90$$

$$\mu \alpha_6 = \frac{1}{r \cdot \cos \delta} = 0.35$$



3



$$x = \pi \cdot \sin(\lambda_0 - \lambda) - K \cos(\lambda_0 - \lambda)$$

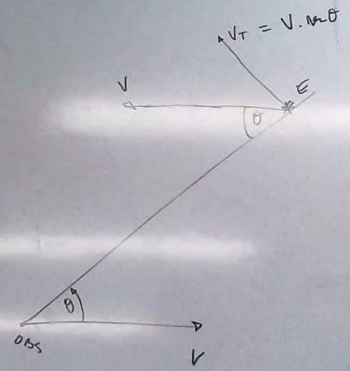
$$y = -\pi \sin \beta \cdot \cos(\lambda_0 - \lambda) - K \sin \beta \cdot \sin(\lambda_0 - \lambda)$$

$$\lambda_0 = \lambda$$

$$\sin \pi = \frac{1 \text{ ua}}{r_{\text{ua}}} = \frac{1}{20.000}$$

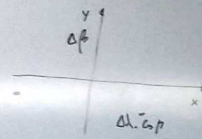
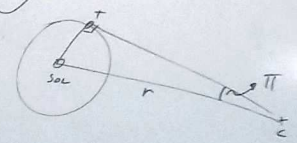
$$\pi (\text{arcsec}) = 5 \times 10^{-4} \times \frac{360}{2 \times 3.14159} \times 60 \times 60 = 10.3 = \pi$$

$$K = 20.5$$



$$\Delta \theta = \frac{V}{c} \cdot \sin \theta$$

3



$$x = \pi \cdot \sin(\lambda_0 - \lambda) - K \cos(\lambda_0 - \lambda)$$

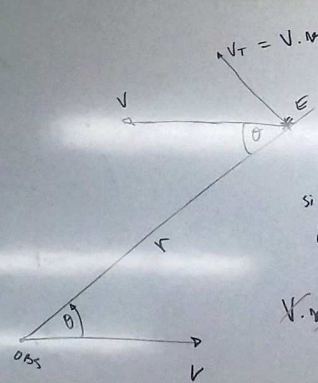
$$y = -\pi \sin \beta \cdot \cos(\lambda_0 - \lambda) - K \sin \beta \cdot \sin(\lambda_0 - \lambda)$$

$$\lambda_0 = \lambda$$

$$\sin \pi = \frac{1 \text{ ua}}{r_{\text{ua}}} = \frac{1}{20.000}$$

$$\pi (2005) = 5 \times 10^{-4} \times \frac{360}{2\pi \times 1000000} \times 60 \times 60 = 10.3 = \pi$$

$$K = 20.5$$



$$V_T = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$$

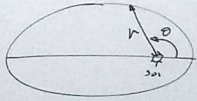
$$\Delta \theta = \frac{V}{c} \cdot \sin \theta$$

$$\sin \Delta t = 1 \text{ a} \approx 0$$

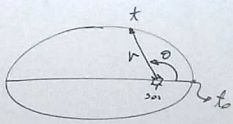
$$d\theta = \frac{V}{c} \cdot \sin \theta$$

$$V \cdot \sin \theta = r \cdot \frac{d\theta}{dt} = \frac{1}{\Delta t}$$

$$V = c \cdot \Delta t \cdot 1 \text{ a} \approx 0$$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

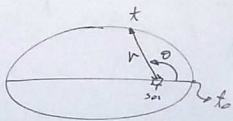


$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$\theta(t)?$

$$\theta(t) \cong M + 2e \cos M + \frac{5}{4} e^2 \cos 2M \dots$$

$$M = \underbrace{m}_{\text{circled}} \cdot (t - t_0)$$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$\theta(t)?$

$$\theta(t) \cong M + 2e \cos M + \frac{5}{4} e^2 \cos 2M \dots$$

$$M = \underbrace{m}_{\sqrt{\mu/a^3}} (t - t_0)$$

DEFINIE TAL RUC

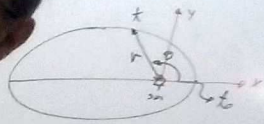
$$r = a(1 - e \cos E)$$

$$M = E - e \cos E$$

EL KEAON

RES. UNIVARIA EL KEAON

$$E = M + e \cos E$$



$$= \frac{a(1-e^2)}{1+e \cos \theta}$$

$$\approx M + 2e \cos \theta + \frac{5}{4} e^2 \cos^2 \theta \dots$$

$$= \frac{m}{\sqrt{1/a^3}} (t - t_0)$$

DEFINIE TAL RUS

$$r = a(1 - e \cos E)$$

$$M = E - e \cos E$$

ET. KOSKOL

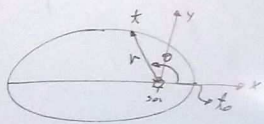
RES. UNĖLIKA ET. KOSKOL

$$(r, \theta) \Rightarrow F = (r \cos \theta, r \sin \theta, 0)$$

$$E = M + e \cos E \quad E_0 = n$$

$$E_1 = M + e \cos(E_0) \rightarrow n$$

$$E_2 = M + e \cos(E_1)$$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$\theta(t)$ ?

$$\theta(t) \cong M + 2e \cos \theta + \frac{5}{4} e^2 \sin 2\theta \dots$$

$$M = \underbrace{m}_{\sqrt{1/a^3}} (t - t_0)$$

DEFINIE TAL RUS

$$r = a(1 - e \cos E)$$

$$M = E - e \sin E$$

ET. KOSON

RES. UNĖNĖLA ET. KOSON

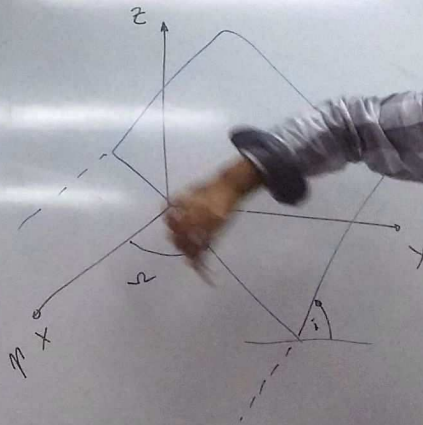
$$E = M + e \sin E$$

$$E_1 = M + e \sin(E_0) \rightarrow n$$

$$E_2 = M + e \sin(E_1)$$

$$(r, \theta) \Rightarrow F = (r \cos \theta, r \sin \theta, 0)$$

PLANO ORBITAL ( $\Omega, i$ )





$a$  = SEMI-EJE ORBITAL  
 $e$  = EXCENTRICIDAD  
 $t_0$  = PASAJE POR AFELIO

DEFINEE TAL QUE

$$r = a(1 - e \cos E)$$

$$M = E - e \sin E$$

EL KEPLER

RES. NUMÉRICA EC. KEPLER

$$E = M + e \sin E \quad E_0 = M$$

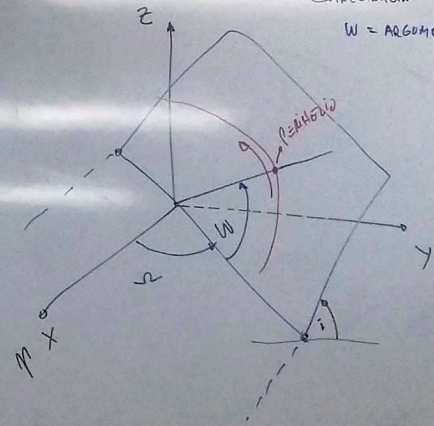
$$E_1 = M + e \sin(E_0) \rightarrow n$$

$$E_2 = M + e \sin(E_1)$$

$$(r, \theta) \Rightarrow F = (r \cos \theta, r \sin \theta, 0)$$

PLANO ORBITAL  $(\Omega, i)$   
 LONGITUD LINDA ASCENDENTE  
 INCLINACION

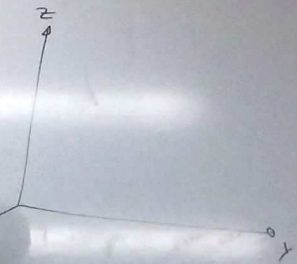
$W$  = ARGUMENTO DEL PERHELIO



$(x, y) \equiv$  PLANO ECLIPTICO

$a =$  SEMI-MAIOR  
 $e =$  EXCENTRICIDADE  
 $f_0 =$  DISTANCIA DO FOCO  
 $i =$  INCLINACAO  
 $\Omega =$  LONGITUDO DO ASCENDENTE  
 $w =$  ARGUMENTO DO PERIHELIO

$\eta = x$



RES. UNIFORME DO CORPO

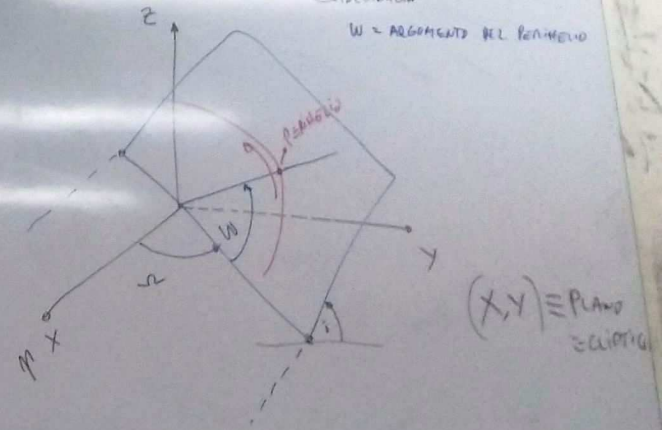
$$E = M + e \cdot \cos E$$

$$E_1 = M + e \cdot \cos E_0 \rightarrow n$$

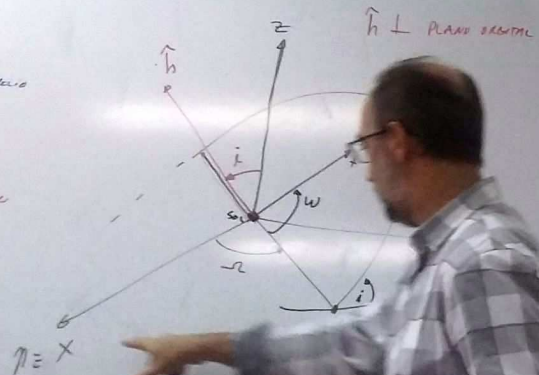
$$E_2 = M + e \cdot \cos E_1$$

$$(r, 0) \rightarrow F = (r \cos B, r \sin B, 0) \text{ PLANO ORBITAL}$$

LONGITUDO DO ASCENDENTE  
 INCLINACAO  
 $W =$  ARGUMENTO DO PERIHELIO



- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LINDA ASC
- $w$  = ARG. DEL PERIFELIO



RES. UNIFORME EC. KEPLER

$$E = M + e \cos E$$

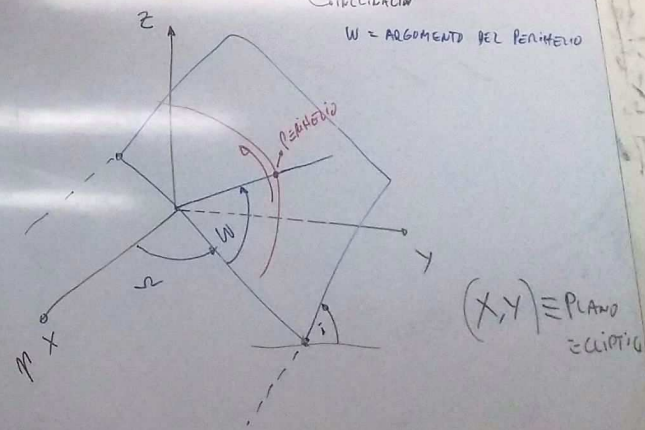
$$E_1 = M + e \cos E_0 \rightarrow M$$

$$E_2 = M + e \cos E_1$$

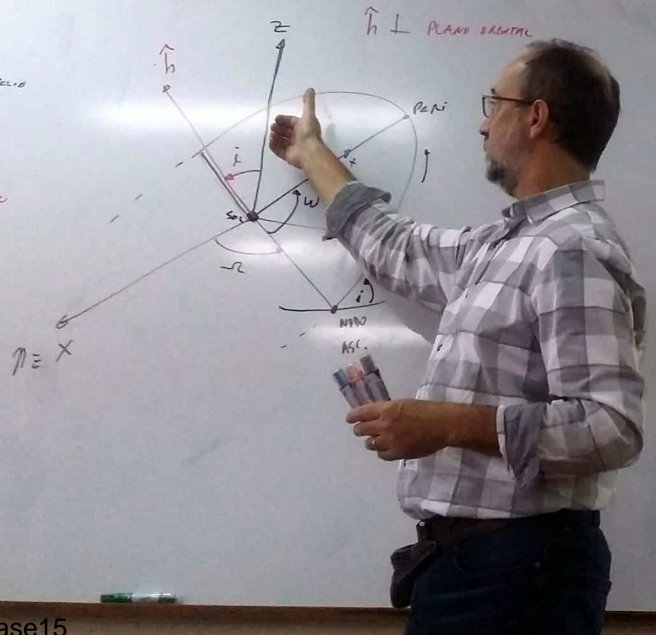
$E_0 = M$

$(r, \theta) \Rightarrow F = (r \cos \theta, r \sin \theta, 0)$  PLANO ORBITAL

LONGITUD UNO ASCENDENTE  
 INCLINACION  
 W = ARGUMENTO DEL PERIFELIO



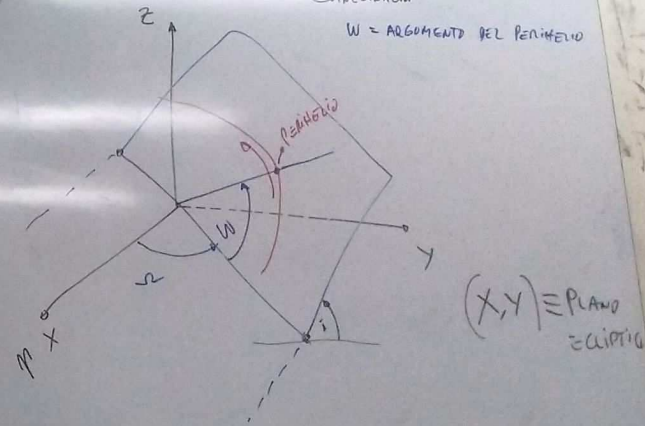
- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LONG. ASC.
- $w$  = ARG. DEL PERIFELIO



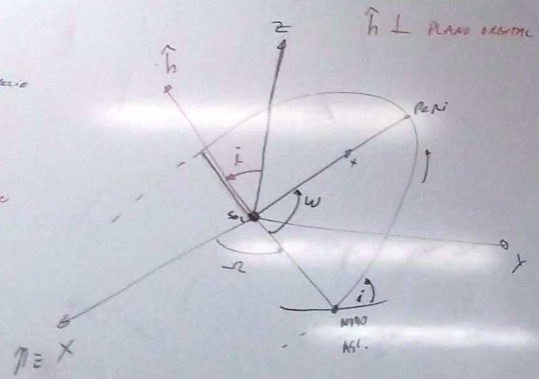
$$R_x(i) \cdot R_z(-w) \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$(r, 0) \Rightarrow \vec{F} = (r \cdot \cos \theta, r \cdot \sin \theta, 0) \text{ PLANO ORBITAL}$$

PLANO ORBITAL  $(\Omega, i)$   
 LONGITUD LONG. ASCENDENTE  
 INCLINACION  
 $w$  = ARGUMENTO DEL PERIFELIO



- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $f_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LONG. ASC.
- $\omega$  = ARG. DEL PERHELIO

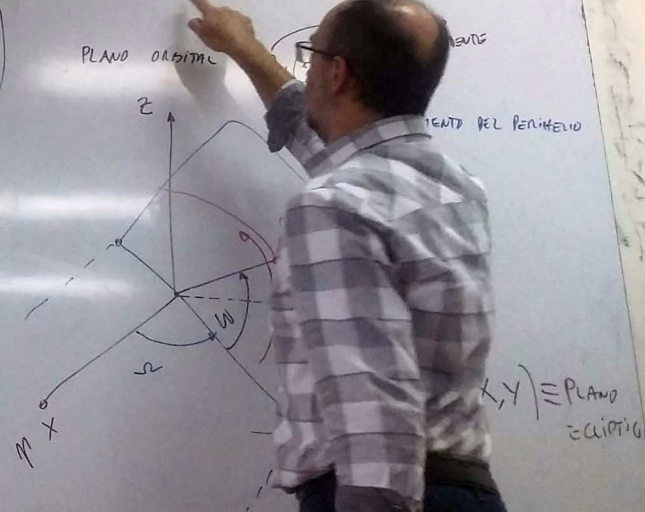


ECLIPSIAS RECTANGULARES

$$(X, Y, Z) = R_z(-\Omega) \cdot R_x(i) \cdot R_z(-\omega) \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$(\alpha, \beta)$

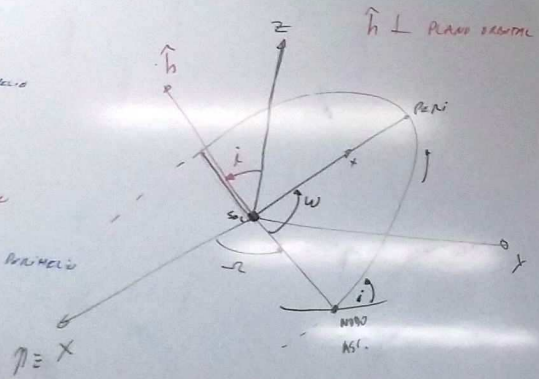
$(r, \theta) \Rightarrow F = (r \cdot \cos \theta, r \cdot \sin \theta, 0)$  PLANO ORBITAL



- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LODS ASC
- $w$  = ARG. DEL PERHELIO

$D = -\Omega + w = \text{LONG. DEL PERHELIO}$

$\lambda = D + M$   
"LONG. MEDIA"



ECLIPSIAS RECTANGULARES

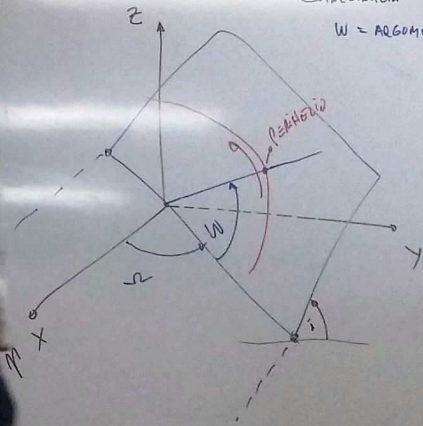
$(x, y, z) = R_z(-\Omega) R_x(i) R_w(\theta) \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

$(\lambda, \beta)$

$\cos(\Omega + w) \rightarrow w$

$(r, \theta) \Rightarrow F = (r \cos \theta, r \sin \theta, 0)$  PLANO ORBITAL

LONGITUD UNO ASCENDENTE  
INCLINACIÓN  
W = ARGUMENTO DEL PERHELIO

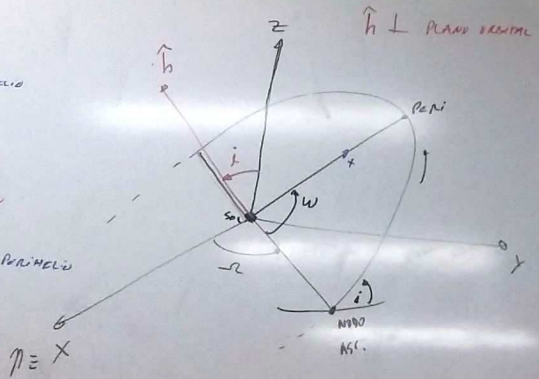


$(x, y) \in \text{PLANO ECLIPTICO}$

- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LONG ASC
- $w$  = ARG. DEL PERHELIO

$\mathcal{W} = \Omega + w = \text{LONG. DEL PERHELIO}$

$\lambda = \mathcal{W} + M$   
"LONG. MED. S"



ECLIPSE RECTANGULAR

$$(X, Y, Z) = R_z(-\Omega) R_x(i) R_z(-w) \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$(\lambda, \beta)$

$\cos(\Omega + w)$

$\mathcal{W}$  = LONGITUD PERHELIO

EFEMÉRIDES

COMETA

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = 0 \end{cases}$$

$(X, Y, Z)$  COMETA

$(X, Y, Z)$  TIERRA

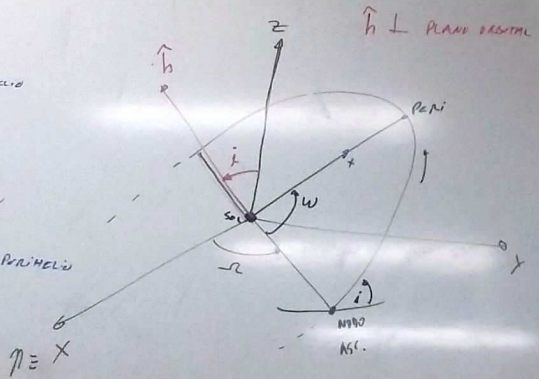
GEOC.

$$\begin{cases} X_G = X_C - X_T \\ Y_G = Y_C - Y_T \\ Z_G = Z_C - Z_T \end{cases}$$

- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LONG ASC
- $W$  = ARG. DEL PERHELIO

$\bar{w} = \Omega + W = \text{LONG. DEL PERHELIO}$

$\lambda = \bar{w} + M$   
"LONG. MED.ª"

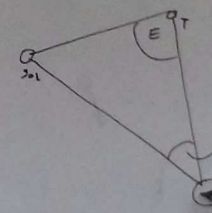


**EFEMÉRIDES**

$x = r \cdot \cos \theta$   
 $y = r \cdot \sin \theta$   
 $z = 0$

$(X, Y, Z)$  COMETA

$(X, Y, Z)$  TIERRA



$\delta = X_C - X_T$   
 $\gamma = Y_C - Y_T$   
 $\beta = Z_C - Z_T$

GEOC.  $\Rightarrow \lambda_\delta, \beta_\delta$

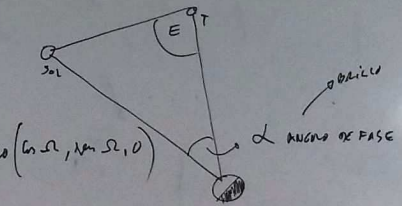
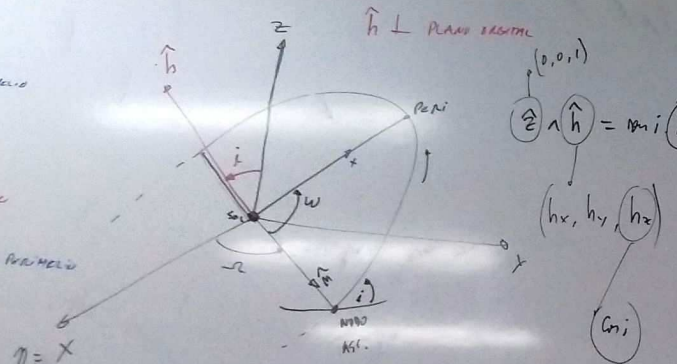
$i \left( \lambda_\delta, \beta_\delta \right)$



- $a$  = SEMI-EJE ORBITAL
- $e$  = EXCENTRICIDAD
- $t_0$  = PASAJE POR AFELIO
- $i$  = INCLINACIÓN
- $\Omega$  = LONG. LONG ASC
- $w$  = ARG. DEL PERHELIO

$D = -\Omega + w = \text{LONG. DEL PERHELIO}$

$\lambda = D + M$   
"LONG. MEDIA"



$\Rightarrow h(\Omega, i)$

EFEMÉRIDES

$x = r \cdot \cos \theta$   
 $y = r \cdot \sin \theta$   
 $z = 0$

COMETA

$(X, Y, Z)$  COMETA

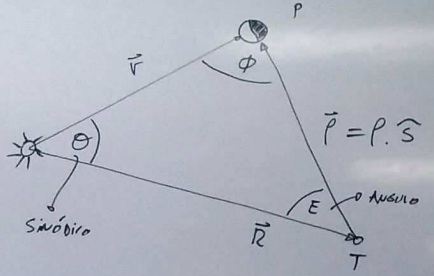
$(X, Y, Z)$  TIERRA

$X_G = X_C - X_T$   
 $Y_G = Y_C - Y_T$   
 $Z_G = Z_C - Z_T$

GEOL.  $\Rightarrow \lambda_G, \beta_G$

$i(\alpha_G, \delta_G)$

MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  $\vec{R} = r_m \cdot \hat{R}$ ,  $\vec{F} = a \cdot \hat{F}$ )



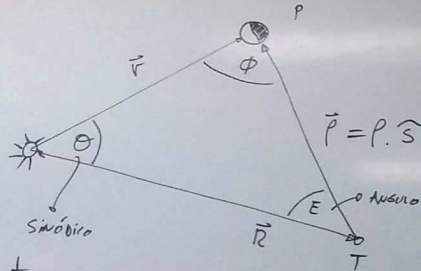
$\phi = \text{ÁNGULO DE FASE}$

$$\vec{P} \cdot \vec{F} = P \cdot r \cdot \cos \phi$$

$$-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$$

$$\vec{P} = P \cdot \hat{S}$$

MOVIMIENTO APARENTE PLANETARIO (ÓRBITAS CIRCULARES)  
 $\vec{r} = r \cdot \hat{r}$



$\phi = \text{ÁNGULO DE FASE}$

$$\vec{P} \cdot \vec{r} = P \cdot r \cos \phi$$

$$-\vec{R} \cdot \vec{P} = R \cdot P \cos E$$

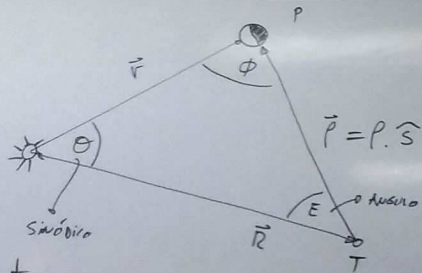
$$\theta(t) \propto t$$

$$M = \sqrt{r^3/a^3} = k/a^2$$

$$\mu = k^2 (M_0 + m_1) \approx k^2$$

↑ Vel. Angular  
RADS/DIA

MOVIMENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{r} = r \cdot \hat{R}$ ,  $\vec{F} = a \cdot \hat{F}$ )



$\phi = \text{ÂNGULO DE PASE}$

$$\vec{P} \cdot \vec{F} = P \cdot r \cdot \cos \phi$$

$$-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$$

$$\vec{P} = P \cdot \hat{S}$$

$$\theta(t) \propto t$$

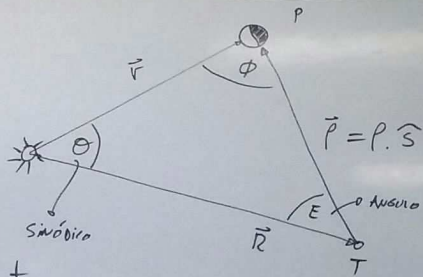
$$\dot{\theta} = \frac{2\pi}{S} = \frac{2\pi}{T_p} - \frac{2\pi}{T_o} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$$

$$M = \sqrt{r/a^3} = \frac{h}{a^2 v} \quad \mu = h^2 (m_0 + m_1) \approx h^2$$

↑ VEL. ANGULAR  
RADS/DIA

S = PERÍODO SINÓDICO

MOVIMIENTO APARENTE PLANETARIO (ÓRBITAS CIRCULARES:  $\vec{r} = r \cdot \hat{R}$ ,  $\vec{F} = a \cdot \hat{F}$ )



$\phi = \text{ÁNGULO DE FASE}$   
 $\vec{P} \cdot \vec{F} = P \cdot r \cdot \cos \phi$   
 $-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$

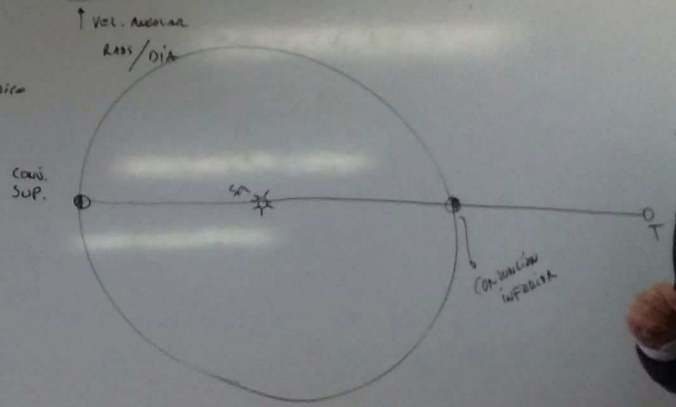
$\vec{P} = P \cdot \hat{S}$

$\theta(t) \propto t$

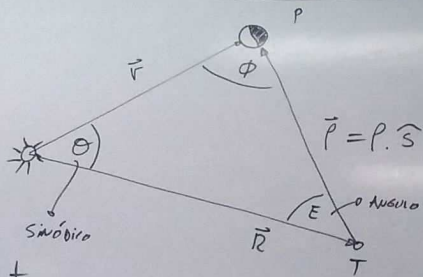
$\dot{\theta} = m_p - m_t = \frac{2\pi}{T_p} - \frac{2\pi}{1 \text{ año}} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$

$m = \sqrt{r/a^3} = k/a^m$       $\mu = k'(m_0 + m) \approx k'$

S = Período sideral



MOVIMENTO APARENTE PLANETARIO (órbitas circulares:  $\vec{r} = r \cdot \hat{r}, \vec{v} = a \cdot \hat{f}$ )



$\phi = \text{ANGULO DE PASE}$

$\vec{P} \cdot \vec{v} = P \cdot v \cdot \cos \phi$

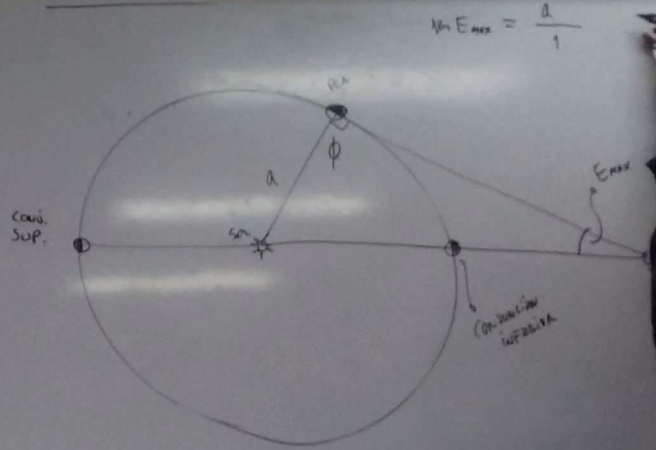
$-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$

$\vec{P} = P \cdot \hat{s}$

$\theta(t) \propto t$

$\dot{\theta} = m_p - m_t = \frac{2\pi}{T_p} - \frac{2\pi}{1 \text{ ano}} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$

PLANETA INTERIOR



$\sin E_{max} = \frac{a}{1}$

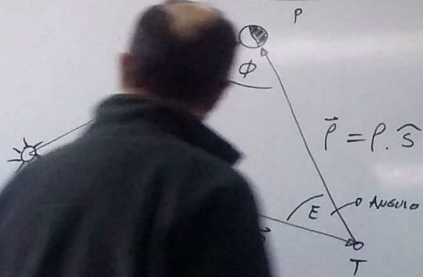
MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{R} = r_m \cdot \hat{R}, \vec{F} = a \cdot \hat{F}$ )

$\phi = \text{ÁNGULO DE FASE}$

$\vec{P} \cdot \vec{F} = P \cdot r \cdot \cos \phi$   
 $-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$

$\vec{P} = P \cdot \hat{S}$

$\Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$



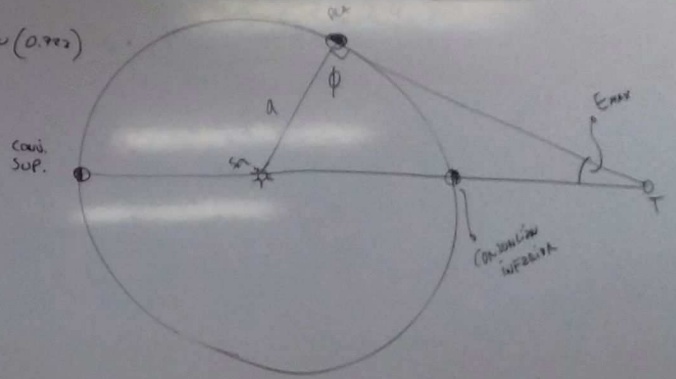
$\theta(t)$   
 $\dot{\theta}$   
 $\frac{2\pi}{S}$

PLANETA INTERIOR

VENUS:  $a = 0.723 \text{ UA}$

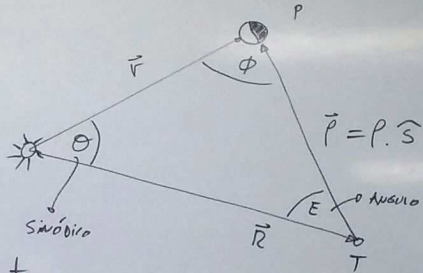
$\Rightarrow E_{\text{max}} = \text{ARSEN}(0.723)$   
 $46^\circ$

$E_{\text{max}} = \frac{a}{1}$



$\frac{2\pi}{S}$

MOVIMIENTO APARENTE PLANETARIO ( ÓRBITAS CIRCULARES:  $\vec{R} = 1 \text{ u.} \cdot \hat{R}, \vec{F} = a \cdot \hat{F}$  )



$\phi = \text{ÁNGULO DE FASE}$   
 $\vec{P} \cdot \vec{F} = P \cdot F \cdot \cos \phi$   
 $-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$

$\vec{P} = P \cdot \hat{S}$   
 $\frac{1}{S} = \frac{1}{T_p} - 1$  (EN RADIOS)

$\theta(t) \propto t$

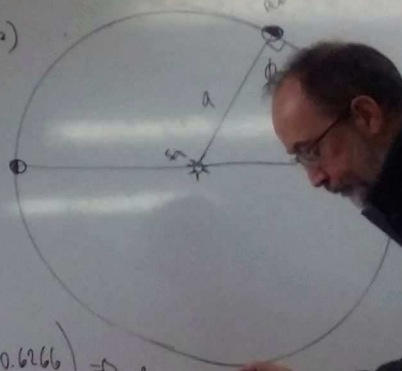
$\dot{\theta} = m_p - m_t = \frac{2\pi}{T_p} - \frac{2\pi}{1 \text{ año}}$

PLANETA INFERIOR

VENUS:  $a = 0.723 \text{ u.}$

$\sin E_{\text{max}} = \frac{a}{1}$

$\Rightarrow E_{\text{max}} = \text{ARCSIN}(0.723)$   
 $46^\circ$

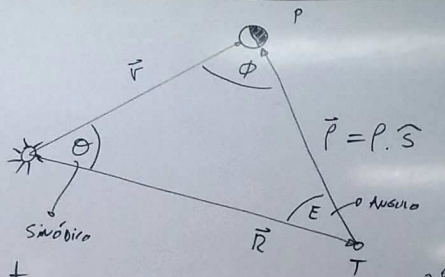


$\frac{2\pi}{S} = \frac{k}{a^3} - \frac{k}{1}$

$\frac{2\pi}{S} = \frac{k}{0.723^3} - k = k(0.6266) \Rightarrow \frac{1}{S} =$



MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  $\vec{R} = r_m \cdot \hat{R}$ ,  $\vec{F} = a \cdot \hat{F}$ )



$\phi = \text{ÁNGULO DE FASE}$

$\vec{P} \cdot \vec{F} = P \cdot r \cdot \cos \phi$

$-\vec{R} \cdot \vec{P} = R \cdot P \cdot \cos E$

$\vec{P} = P \cdot \hat{S}$

$\theta(t) \propto t$

$\dot{\theta} = m_p - m_t = \frac{2\pi}{T_p} - \frac{2\pi}{1 \text{ año}} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$

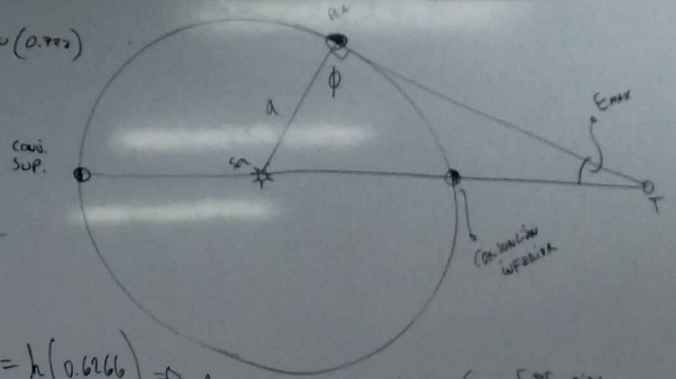
PLANETA INTERIOR

VENUS:  $a = 0.723 \text{ UA}$

$\frac{1}{S} E_{\text{max}} = \frac{a}{1}$

$\Rightarrow E_{\text{max}} = \text{ARCO}(0.723)$

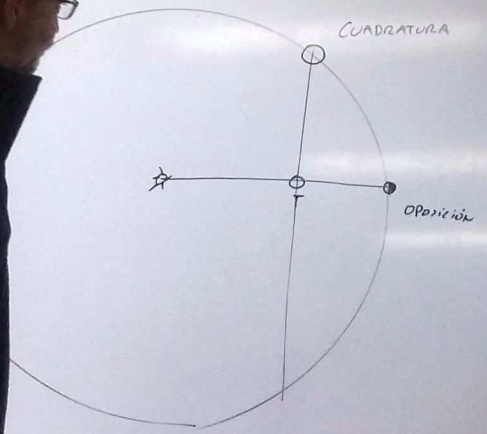
46°



$\frac{2\pi}{S} = \frac{h}{a^2} - \frac{h}{1}$

$\frac{2\pi}{S} = \frac{h}{0.723^2} - h = h(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ día}^{-1} \Rightarrow S = 585 \text{ días}$

EVENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{r} = r_m \cdot \hat{r}, \vec{v} = a \cdot \hat{v}$ )



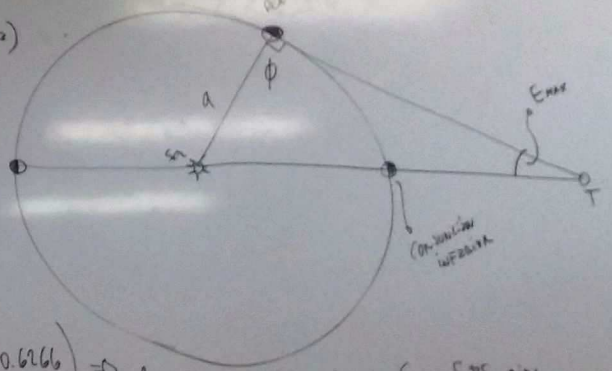
PLANETA INTERIOR

$$\frac{1}{2} E_{max} = \frac{a}{1}$$

VENUS:  $a = 0.723 \text{ ua}$

$$\Rightarrow E_{max} = \text{ARCO}(0.723)$$

46°

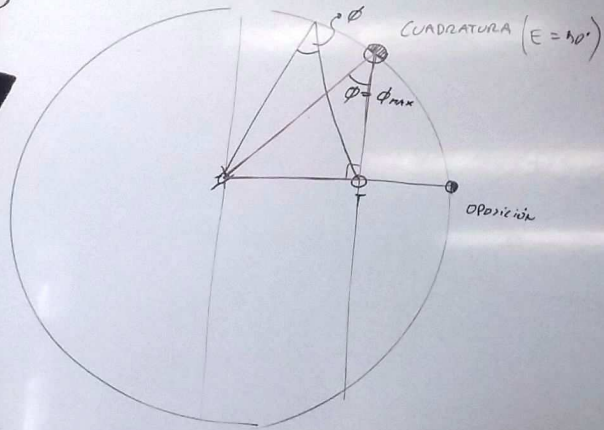


$$\frac{2\pi}{S} = \frac{k}{a^3} - \frac{k}{1}$$

↑  
VENUS

$$\frac{2\pi}{S} = \frac{k}{0.723^3} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ día}^{-1} \Rightarrow S = 585 \text{ días}$$

MOVIMIENTO APARENTE PLANETARIO ( ÓRBITAS CIRCULARES:  
 $\vec{r} = 1 \text{ u. } \hat{r}, \vec{v} = a \cdot \hat{f}$ )



PLANETA INTERIOR

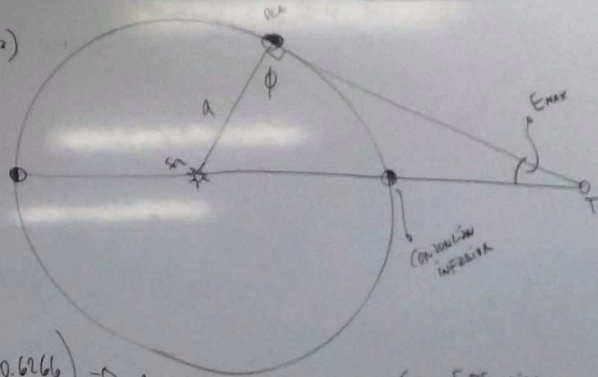
VENUS:  $a = 0.723 \text{ u.}$

$$\sin E_{\max} = \frac{a}{1}$$

$$\Rightarrow E_{\max} = \text{ARCSIN}(0.723)$$

46°

CONJ. SUP.

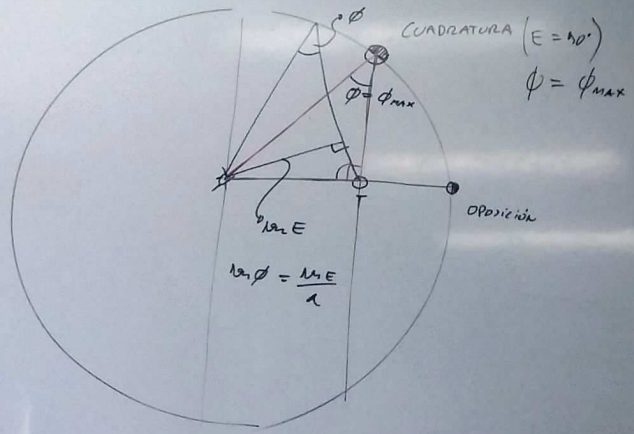


$$\frac{2\pi}{S} = \frac{k}{a^3} - \frac{k}{1}$$

↑  
VENUS

$$\frac{2\pi}{S} = \frac{k}{0.723^3} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ día}^{-1} \Rightarrow S = 585 \text{ días}$$

MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{r} = r_m \cdot \hat{r}, \vec{v} = a \cdot \hat{t}$ )

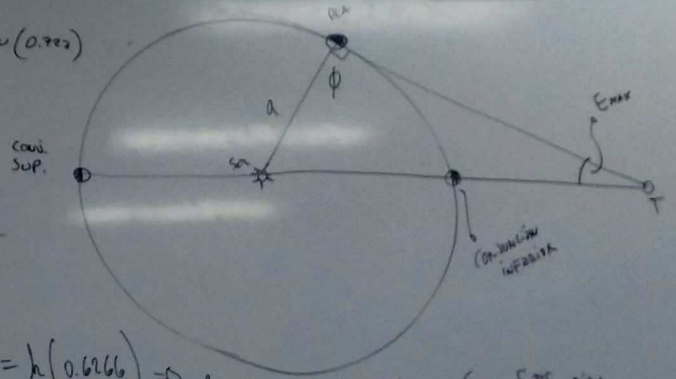


PLANETA INFERIOR

VENUS:  $a = 0.723 \text{ UA}$

$$\alpha_{max} E_{max} = \frac{a}{1}$$

$\Rightarrow E_{max} = \text{ARCSIN}(0.723)$   
 $46^\circ$

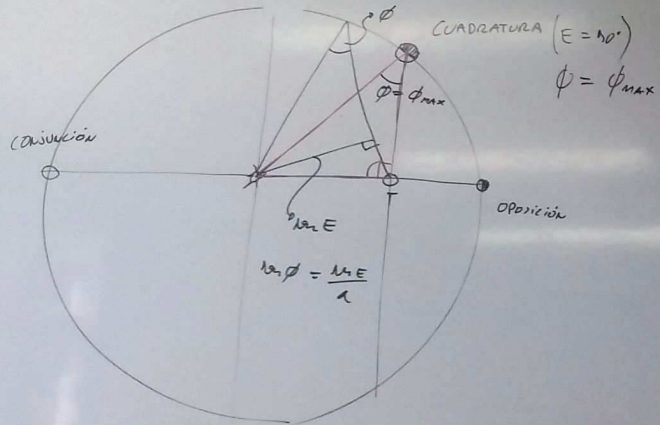


$$\frac{2\pi}{S} = \frac{k}{a^3} - \frac{k}{1}$$

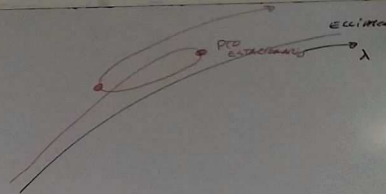
VENUS

$$\frac{2\pi}{S} = \frac{k}{0.723^3} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ día}^{-1} \Rightarrow S \approx 585 \text{ días}$$

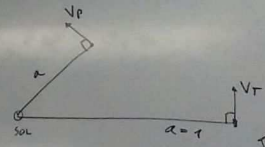
MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{r} = 1 \text{ u. } \hat{r}, \vec{F} = a \cdot \hat{r}$ )



CUADRATURA ( $E = 90^\circ$ )  
 $\phi = \phi_{max}$

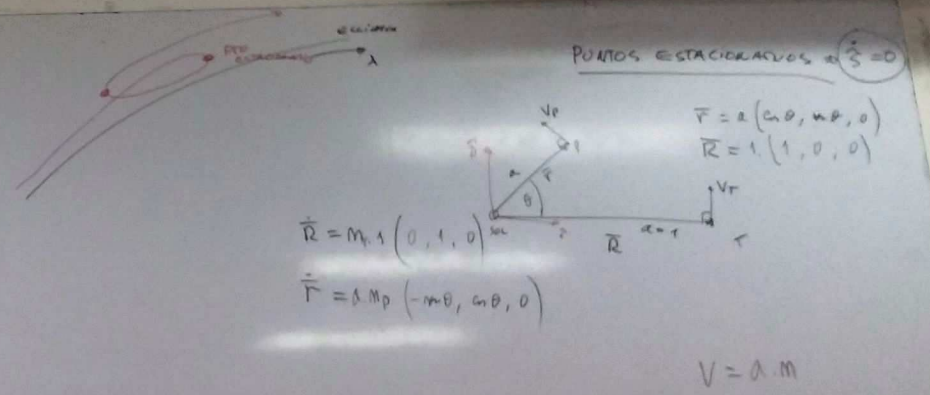
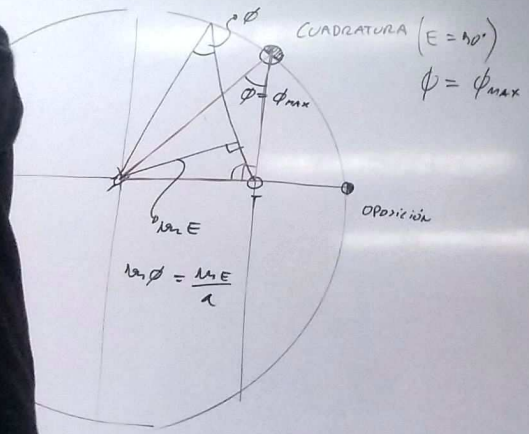


PUNTOS ESTACIONARIOS

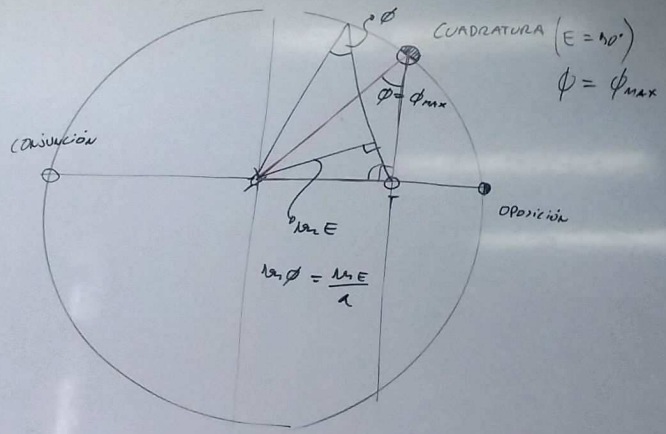


$V = a \cdot \omega$

MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  
 $\vec{r} = a \cdot \hat{r}, \quad \vec{v} = a \cdot \dot{\phi} \cdot \hat{\phi}$ )



MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:  $\vec{R} = 1m \cdot \hat{R}$ ,  $\vec{F} = a \cdot \hat{F}$ )

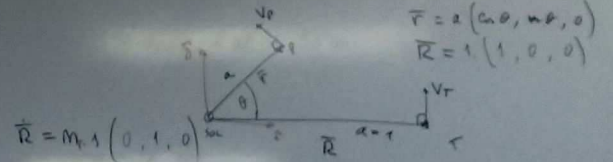


$\vec{P} = \vec{F} - \vec{R}$

$\dot{\vec{P}} \cdot \hat{\zeta} + P \dot{\hat{\zeta}} = \dot{\vec{F}} - \dot{\vec{R}}$

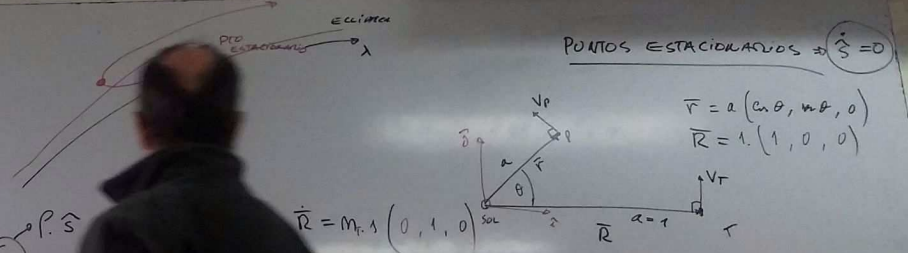
$\hat{\zeta} \wedge \dot{\vec{P}} \hat{\zeta} = \hat{\zeta} \wedge (\dot{\vec{F}} - \dot{\vec{R}}) \Rightarrow$

$0 = 0$



PUEDEN PROBAR

$(\vec{F} - \vec{R}) \wedge (\dot{\vec{F}} - \dot{\vec{R}}) = 0$



PUNTOS ESTACIONARIOS  $\Rightarrow \dot{\hat{s}} = 0$

$$\vec{r} = a(\cos\theta, \sin\theta, 0)$$

$$\vec{R} = 1 \cdot (1, 0, 0)$$

$$\dot{\vec{R}} = m_{\text{r}} \cdot (0, 1, 0)$$

$$= \Delta \cdot M_p (-\sin\theta, \cos\theta, 0)$$

$$V = a \cdot \dot{m}$$

$$\vec{p} \cdot \hat{s}$$

$$\vec{p} = \vec{r}$$

$$\ddot{\vec{s}} + \dots$$

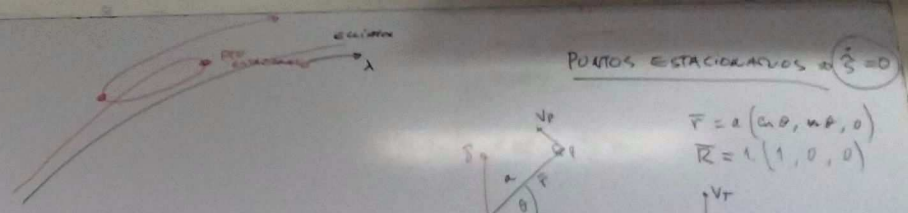
CONDICION PDS

$$(\vec{r} - \vec{R}) \wedge (\ddot{\vec{r}} - \ddot{\vec{R}}) = 0$$





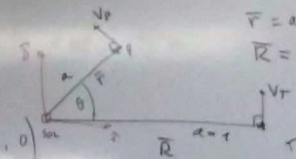
$$= \hat{k} \left[ (k \cos \theta - 1) \sqrt{2 M_p \cos \theta - M_T} + a M_p \sin^2 \theta \right] = 0$$



PUNTOS ESTACIONARIOS  $\dot{z} = 0$

$$\vec{F} = a (\cos \theta, \sin \theta, 0)$$

$$\vec{R} = 1 (1, 0, 0)$$



$$\vec{R} = M_T 1 (0, 1, 0)$$

$$\vec{F} = a M_p (-\sin \theta, \cos \theta, 0)$$

$$V = a \cdot \dot{m}$$

$$\vec{P} = \vec{F} - \vec{R}$$

$$\dot{\vec{P}} = \dot{\vec{F}} - \dot{\vec{R}}$$

$$\hat{s} \wedge \dot{\vec{P}} = \hat{s} \wedge (\dot{\vec{F}} - \dot{\vec{R}})$$

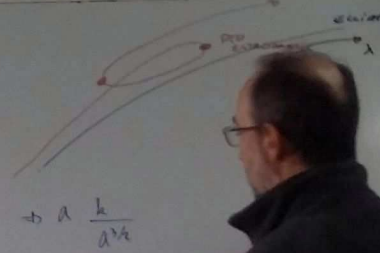
BASE PTO

$$(\vec{F} - \vec{R}) \wedge (\dot{\vec{F}} - \dot{\vec{R}}) = 0$$

$$\begin{bmatrix} a \cos \theta - 1 & m_p a \theta \\ -a m_p \sin \theta & a m_p \cos \theta - m_T \end{bmatrix} = \hat{k} \left[ (a \cos \theta - 1)(a m_p \cos \theta - m_T) + a m_p \sin^2 \theta \right] = 0$$

$$\rightarrow \underbrace{a m_p \cos^2 \theta - a \cos \theta m_T - a m_p \sin \theta + m_T}_{\downarrow} + a m_p \sin^2 \theta = 0$$

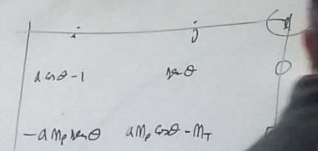
$$a m_p - (m_T + m_p) a \cos \theta + m_T = 0$$



PUNTOS ESTACIONARIOS  $\dot{\lambda} = 0$

$$m = \sqrt{\frac{k}{a}} = \frac{h}{a^{3/2}}$$

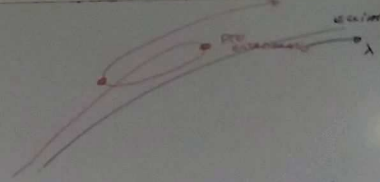
$$\rightarrow a \frac{k}{a^{3/2}}$$



$$-a \left[ (aM_p \cos \theta - M_r) + aM_p \sin^2 \theta \right] = 0$$

$$\rightarrow aM_p \cos^2 \theta =$$

$$aM_p \sin^2 \theta = 0$$



PUNTO ESTACIONARIO  $\dot{\xi} = 0$

$$m = \sqrt{\frac{r}{g}} = \frac{r}{a^2}$$

$$\rightarrow a \frac{k}{a^{1/2}} - \left( \frac{k}{1^m} + \frac{k}{a^k} \right) a \cos \theta + \frac{k}{1^m} = 0$$

$$\Rightarrow a^{-1/2} - (1 + a^{-k}) \cos \theta + 1 = 0$$

$$\Rightarrow (a + a^{-1/2}) \cos \theta = 1 + a^{-1/2} \Rightarrow \cos \theta = \frac{1 + a^{-1/2}}{(a + a^{-1/2})}$$

$$\begin{bmatrix} a_{i\theta-1} & a_{j\theta} \\ -a_{m_p} a_{\theta} & a_{m_p} a_{\theta} - m_T \end{bmatrix} = \hat{k} \left[ (a_{i\theta-1})(a_{m_p} a_{\theta} - m_T) + a_{j\theta} m_p a_{\theta} \right] = 0$$

$$\rightarrow \underbrace{a_{m_p} a_{\theta}}_{\text{circled}} - a_{i\theta-1} m_T - a_{j\theta} m_p + m_T + \underbrace{a_{j\theta} m_p a_{\theta}}_{\text{circled}} = 0$$

$$a_{m_p} - (m_T + m_p) a_{i\theta-1} + m_T = 0$$

PUNTOS ESTACIONARIOS  $\hat{S} = 0$

$$m = \sqrt{\frac{r}{s}} = \frac{r}{a_{m_p}}$$

$$- \left( \frac{r}{1^m} + \frac{r}{a^m} \right) a_{i\theta} + \frac{r}{1^m} = 0$$

$$a^{-m} \cdot a_{i\theta} + 1 = 0$$

$$a_{i\theta} = 1 + a^{1/m} \Rightarrow \boxed{a_{i\theta} = \frac{1 + a^{1/m}}{(a + a^{-1/m})}} > 0 \Rightarrow \theta < 90^\circ$$

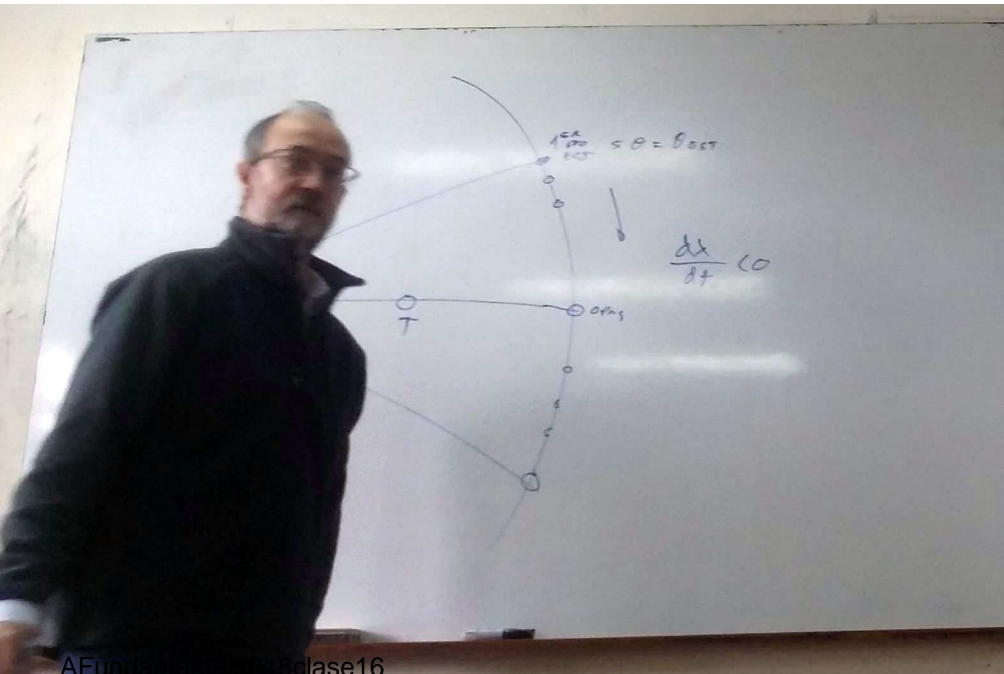


Diagram of an elliptical orbit with a central body 'T' and a point 'C' labeled 'CEN'. A point 'P' is marked on the orbit with the note '1º PTO ESTACIONARIO'. The orbit is labeled 'ELIPSE'. The angle  $\theta$  is shown between the horizontal axis and the line from 'C' to 'P'.

PUNTOS ESTACIONARIOS  $\Rightarrow \dot{\lambda} = 0$

$$n = \sqrt{\frac{\mu}{a^3}} = \frac{h}{a^{3/2}}$$

$$\Rightarrow a^2 \frac{h}{a^{3/2}} - \left( \frac{h}{1^3} + \frac{h}{a^3} \right) a \cos \theta + \frac{h}{1^3} = 0$$

$$\Rightarrow a^{+1/2} - (1 + a^{-3}) \cdot a \cos \theta + 1 = 0$$

$$\Rightarrow (a + a^{-1/2}) \cdot \cos \theta = 1 + a^{+1/2} \Rightarrow \cos \theta_{EST} = \frac{1 + a^{+1/2}}{(a + a^{-1/2})} > 0 \Rightarrow \theta < 90^\circ$$

$\theta = \theta_{EST}$   
 $\theta = 2 \times 31^\circ = 76^\circ$   
 $60^\circ \rightarrow S$   
 $4^\circ \rightarrow \Delta t = 5.7c$

$\frac{dx}{dt} < 0$

**CERES  $a = 2.768 u_a$**

$\theta_{EST} = 37,75$

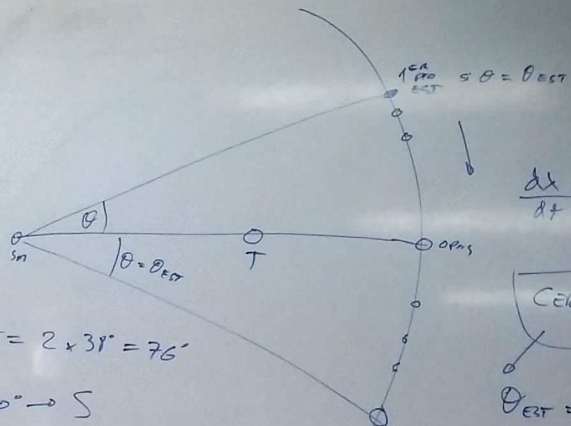
**PUNTOS ESTACIONARIOS  $\dot{\theta} = 0$**

$m = \sqrt{\frac{r}{a}} = \frac{h}{a^2 \dot{\theta}}$

$\rightarrow a^2 \frac{k}{a^{3/2}} - \left( \frac{k}{1^m} + \frac{k}{a^m} \right) a \cos \theta + \frac{k}{1^m} = 0$

$\Rightarrow a^{+1/2} - (1 + a^{-m}) \cdot a \cos \theta + 1 = 0$

$\Rightarrow (a + a^{-1/2}) \cdot \cos \theta = 1 + a^{1/2} \Rightarrow \cos \theta_{EST} = \frac{1 + a^{1/2}}{(a + a^{-1/2})} > 0 \Rightarrow \theta < 90^\circ$



$$\frac{dx}{dt} < 0$$

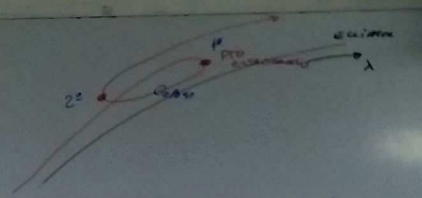
CERES  $a = 2.768 u$

$$\theta_{EST} = 37.75$$

$$\Delta\theta = 2 \times 31^\circ = 76^\circ$$

$$\Delta\theta = 360^\circ \rightarrow S$$

$$76^\circ \rightarrow \Delta t = \frac{5.76}{360} = 18 \text{ días}$$



PUNTOS ESTACIONARIOS  $\dot{\theta} = 0$

$$v = \sqrt{\frac{\mu}{a}} = \frac{h}{am}$$

$$\rightarrow a^2 \frac{h^2}{a^3} - \left( \frac{h^2}{1^3} + \frac{h^2}{a^3} \right) a \cos\theta + \frac{h^2}{1^3} = 0$$

$$\Rightarrow a^{+1/2} - (1 + a^{-1/2}) \cdot a \cos\theta + 1 = 0$$

$$\Rightarrow (a + a^{-1/2}) \cdot \cos\theta = 1 + a^{1/2} \Rightarrow \cos\theta_{EST} = \frac{1 + a^{1/2}}{(a + a^{-1/2})} > 0 \Rightarrow \theta < 90^\circ$$

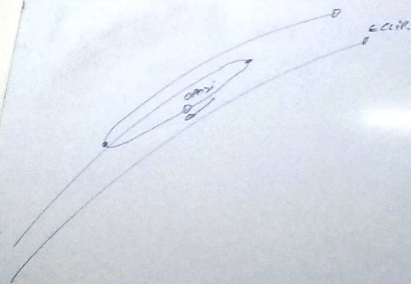
CERES  
 1<sup>o</sup> en el...  
 3 Abril 2019  
 2<sup>o</sup> en el...  
 21 Julio 2019

$\frac{dx}{dt} = 0$   
 CERES  $a = 2.768 u_a$   
 $\theta_{EST} = 37.75^\circ$   
 $\gamma = 76^\circ$   
 $= \frac{5.76}{360} = 1.6$  días

PUNTOS ESTACIONARIOS  $\dot{\zeta} = 0$   
 $m = \sqrt{r/a} = \frac{h}{a^{3/2}}$

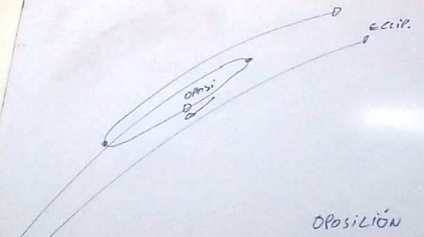
$\rightarrow a^2 \frac{h}{a^{3/2}} - \left( \frac{h}{r^3} + \frac{h}{a^3} \right) a \cos \theta + \frac{h}{r^3} = 0$   
 $\Rightarrow a^{+3/2} - (1 + a^{-3}) \cdot a \cos \theta + 1 = 0$   
 $\Rightarrow (a + a^{-3/2}) \cdot \cos \theta = 1 + a^{3/2} \Rightarrow \cos \theta_{EST} = \frac{1 + a^{3/2}}{(a + a^{-3/2})} > 0 \Rightarrow \theta < 90^\circ$





$$PS = F - R$$

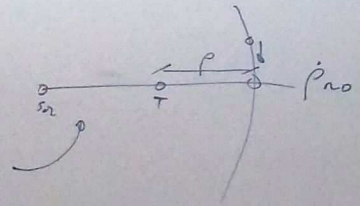
OPosición  
Fases y brillo  
COORD PLANETS  
ANG. POSICIÓN  
PER SÍMBOLO  
OBLICUINAN



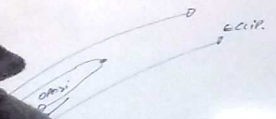
$$P\hat{S} = \vec{F} - \vec{R}$$

$$\cancel{\dot{\hat{S}}} + P\dot{\hat{S}} = \dot{\vec{r}} - \dot{\vec{R}} \Rightarrow \dot{\hat{S}} = \frac{\dot{\vec{r}} - \dot{\vec{R}}}{P}$$

EN OPOSICIÓN =  $r - \delta_{op}$



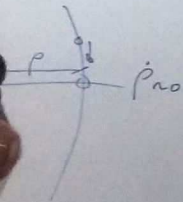
OPOSICIÓN  
 FASE Y BRILLO  
 CÓDIGO PLANETA  
 MUC. POSICIÓN  
 POR SIMBOLO  
 OBSERVACION



$$p\dot{s} = \dot{r} - \dot{R}$$

$$\cancel{\dot{s}} + p\dot{s} = \dot{r} - \dot{R} \Rightarrow \dot{s} = \frac{\dot{r} - \dot{R}}{p}$$

OPOSICIÓN



EN OPOSICIÓN =  $r - \dot{R}$

ÓRB. CIRC.

$$\dot{r} = a \cdot \dot{M} = a \cdot \sqrt{\mu/a^3} = k \cdot a \cdot a^{-3/2} = k \cdot a^{-1/2}$$

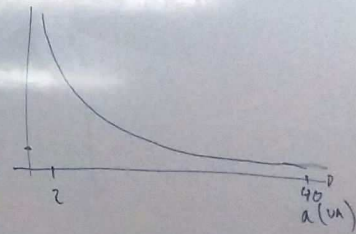
$$\dot{R} = k \cdot (1)^{-1/2} = k$$

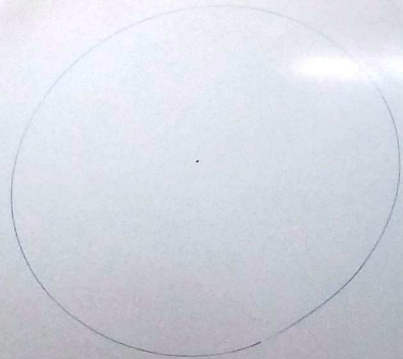
$$\Rightarrow \dot{s} = \frac{k(a^{-1/2} - 1)}{a - 1} < 0$$

OBJETO EXTERIOR  $a > 1$

OPOSICIÓN  
FASE Y BRILLO  
COORD. PLANETA  
MAG. POSICIÓN  
PER. SIMBOL  
OBLICUIDAD

RETROGRADO





dos. circ.

$$\dot{r} = a \cdot M = a \cdot \sqrt{\mu/a^3} = k \cdot a \cdot a^{-3/2} = k \cdot a^{-1/2}$$

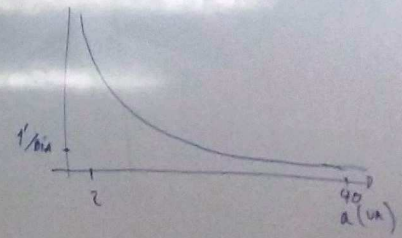
$$\ddot{r} = k \cdot (-1/2) \cdot a^{-3/2} = -k \cdot a^{-3/2}$$

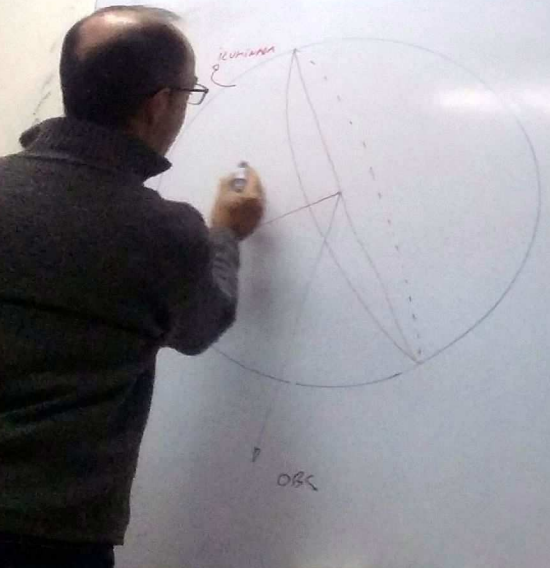
$$\Rightarrow \dot{s} = \frac{k(a^{-1/2} - 1)}{a - 1} < 0$$

OBJETO EXTERIOR  $a > 1$

OPosición  
Fase y brillo  
COORD. PLANEAS  
MUE. POSICION  
Por similitud  
OBLICUIDAD

RETARDADO





Geo. circ.

$$\dot{r} = a \cdot m = a \cdot \sqrt{\mu/a^3} = k \cdot a \cdot a^{-3/2} = k \cdot a^{-1/2}$$

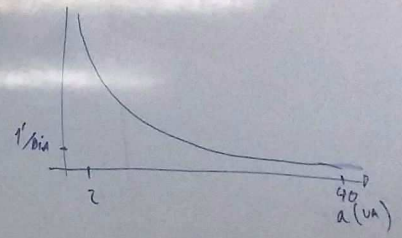
$$\ddot{r} = k \cdot (1)^{-1/2} = k$$

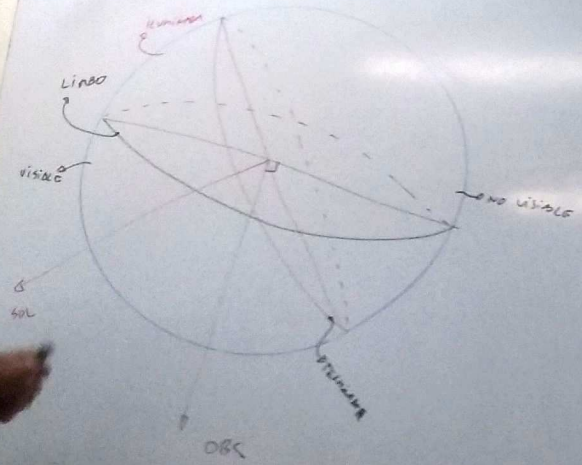
$$\Rightarrow \dot{s} = \frac{k(a^{-1/2} - 1)}{a - 1} < 0$$

OBJETO EXTERIOR  $a > 1$

OPosición  
FASO Y BRILLO  
COORD. PLANETA  
MAG. POSITIVA  
PER SIMBOLA  
OBLICUIDAD

RETROGRADO





Órbita circ.

$$\dot{r} = a \cdot \dot{M} = a \cdot \sqrt{\mu/a^3} = k \cdot a \cdot a^{-3/2} = k \cdot a^{-1/2}$$

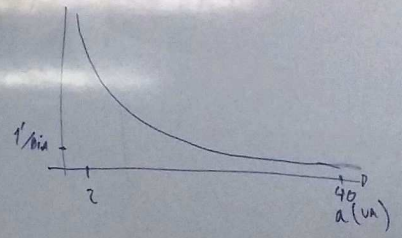
$$\ddot{r} = k \cdot (-1/2) \cdot a^{-3/2} = -k \cdot a^{-3/2}$$

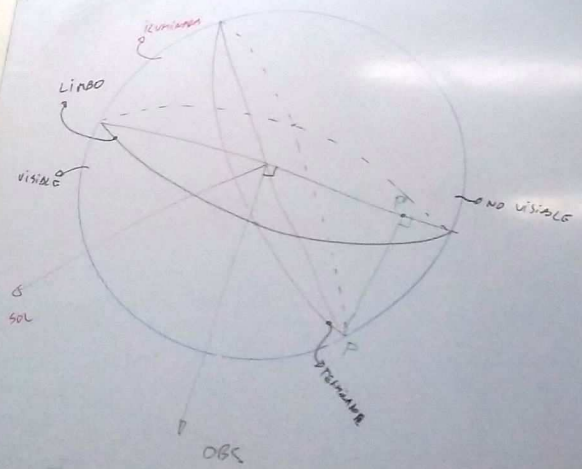
$$\Rightarrow \dot{S} = \frac{k(a^{-1/2} - 1)}{a - 1} < 0$$

OBJETO EXTERIOR  $a > 1$

OPosición  
Fases y brillo  
COORD. PLANEAS  
MAG. POSICION  
PER. SINODIO  
OBLICUINAN

RETROGRADO





GRA. circ.

$$\dot{r} = a \cdot M = a \cdot \sqrt{\mu/a^3} = k \cdot a \cdot a^{-3/2} = k \cdot a^{-1/2}$$

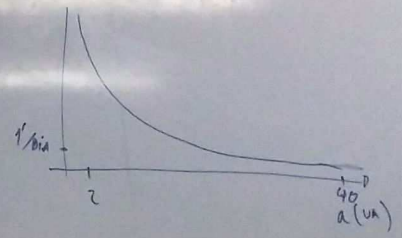
$$\ddot{r} = k \cdot (-1/2) \cdot a^{-3/2} = -k \cdot a^{-3/2}$$

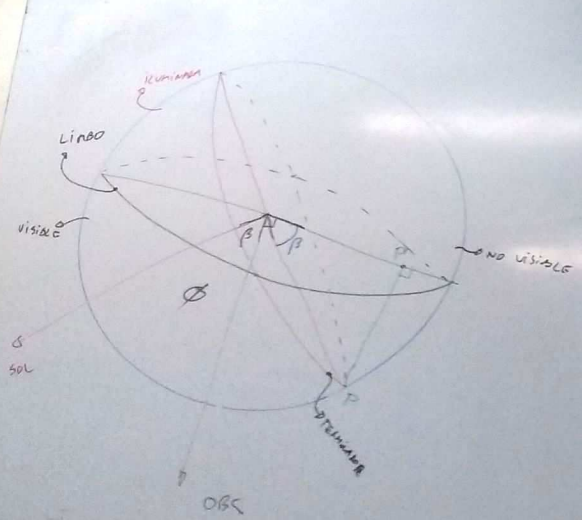
$$\Rightarrow \ddot{S} = \frac{k(a^{-1/2} - 1)}{a - 1} < 0$$

OBJETO EXTERIOR  $a > 1$

OPOSICION  
FASOS Y BARRILLO  
COORD. PLANETAS  
MAG. POSICION  
PER SIMBOLO  
OBLICUINAN

RETROGRADO



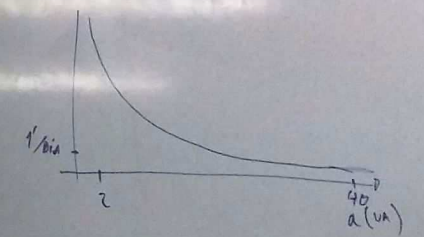


$$\text{AREA iluminada} = \frac{\pi r^2}{2} + \frac{\pi r \cdot r \cos \beta}{2} = \frac{\pi r^2}{2} (1 + \cos \beta)$$

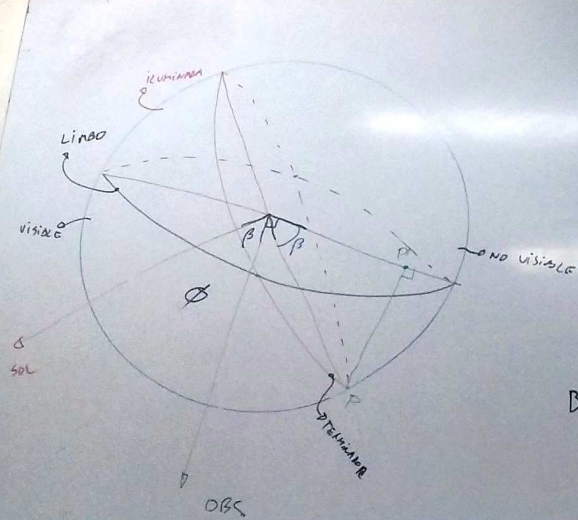
$$\text{"FASE"} = \frac{\text{AREA iluminada}}{\text{AREA TOTAL}} = \frac{\pi r^2 (1 + \cos \beta)}{\pi r^2} = 1 + \cos \beta$$

$$b \cdot a^{-1/2}$$

- OPOSICIÓN
- FASES Y BRILLO
- COORD. PLANETA
- MAG. POSICIÓN
- PER. SINODICO
- OBLICUIDAD







$$\text{AREA iluminada} = \frac{\pi r^2}{2} + \frac{\pi r \cdot r \cos \beta}{2} = \frac{\pi r^2}{2} (1 + \cos \phi)$$

$$\text{"FASE"} = \frac{\text{AREA iluminada}}{\text{AREA TOTAL}} = \frac{\pi r^2 (1 + \cos \phi)}{\pi r^2}$$

$$\text{Brillo} \propto \frac{1 + \cos \phi}{d^2 \cdot p^2}$$

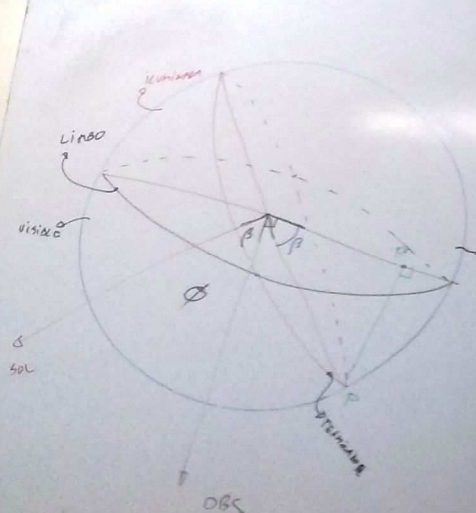
$d^2 \rightarrow \text{HELOS.}$       $p^2 \rightarrow \text{GEOC.}$

$$b \propto a^{-1/2}$$

OPosición  
FASE y BRILLO  
COORD. PLANETA  
MÁS POSICIONES  
POR SIMBOL  
OBLICUIDAD

$$= \frac{1 + \cos \phi}{2}$$

$0 \Rightarrow \phi = 180^\circ$  (NUEVA)  
 $1 \Rightarrow \phi = 0^\circ$  (LLENA)



$$AREA = \pi r^2 + \frac{\pi r^2 \cos \phi}{2} = \frac{\pi r^2}{2} (1 + \cos \phi)$$

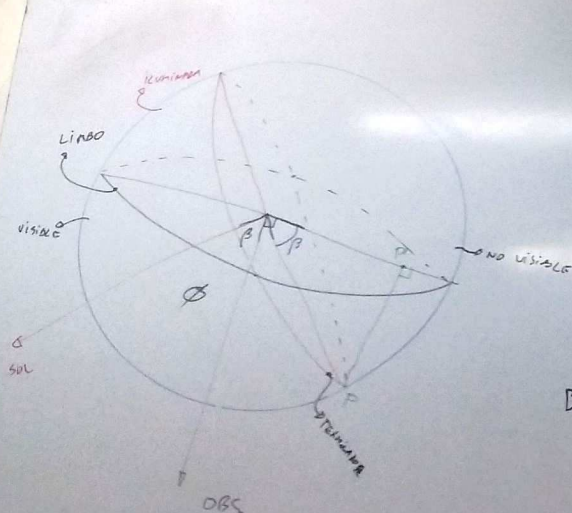
$$= \frac{(1 + \cos \phi)}{2}$$

$0 \Rightarrow \phi = 180^\circ$  (NUEVA)  
 $1 \Rightarrow \phi = 0^\circ$  (LLENA)

$$M = CTE - 2.5 \log B$$

$$b \cdot a^{-1/2}$$

OPOSICION  
 FASES Y BRILLO  
 COORD. PLANETA  
 MUC. POSICION  
 DEL SUELO  
 OBLICUIDAD



$$\text{AREA iluminada} = \frac{\pi r^2}{2} + \frac{\pi r \cdot r \cos \phi}{2} = \frac{\pi r^2}{2} (1 + \cos \phi)$$

$$\text{"FASE"} = \frac{\text{AREA iluminada}}{\text{AREA TOTAL}} = \frac{\pi r^2 (1 + \cos \phi)}{2 \pi r^2}$$

$$\text{BRILLO} \propto \frac{(1 + \cos \phi)}{d^2 \cdot p^2}$$

$\rightarrow d^2$  Helios.  
 $\rightarrow p^2$  Geogr.

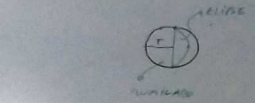
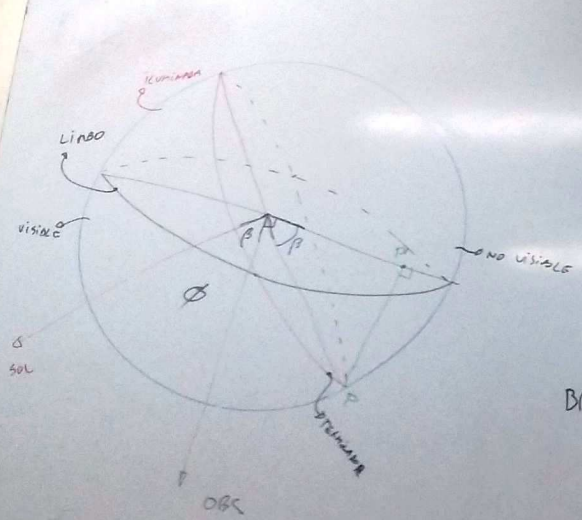
$b \cdot a^{-1/2}$

OPOSICION  
 FASES Y BRILLO  
 COORD. PLANETAS  
 ANG. POSICION  
 PER SIMBOL  
 OBLICUIDAD

$$= \frac{(1 + \cos \phi)}{2}$$

$0 \Rightarrow \phi = 180^\circ$  (NUEVA)  
 $1 \Rightarrow \phi = 0^\circ$  (LLENA)

$$M = \text{CTE} - 2.5 \log B$$



$$\text{AREA iluminada} = \frac{\pi r^2}{2} + \frac{\pi r \cdot r \cos \phi}{2} = \frac{\pi r^2}{2} (1 + \cos \phi)$$

$$\text{"FASE"} = \frac{\text{AREA iluminada}}{\text{AREA TOTAL}} = \frac{1 + \cos \phi}{2}$$

$$\text{Brillo} \propto \frac{1 + \cos \phi}{2}$$

$$d^2 \cdot p^2 \rightarrow \text{Helioc.} \rightarrow \text{Geoc.}$$

b. a<sup>-1/2</sup>

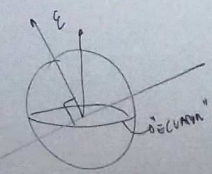
$$\phi \text{ ANG. FASE}$$

$$= \frac{1 + \cos \phi}{2}$$

0 ⇒ φ = 180° (NUEVA)

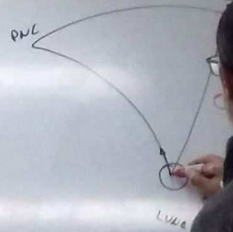
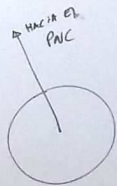
1 ⇒ φ = 0° (LLENA)

OPosición  
Fases y brillo  
COORD. PLANETAS  
MAG. POSICION  
PER SIMBOLA  
OBLICUIDAD



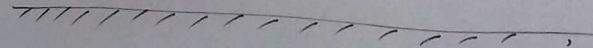
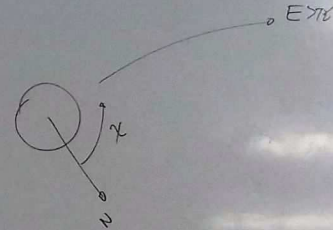
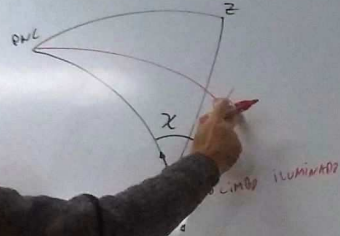
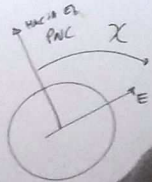
$$M = \text{cte} - 2.5 \log B$$

ÁNGULO DE POSICIÓN  $\chi$



OPUSCULO  
FALSA Y BRILLO  
CORO PLANETA  
MUC. POSICION  
POR SIMILITUD  
OBLICUIDAD

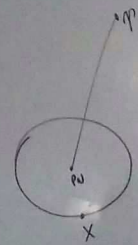
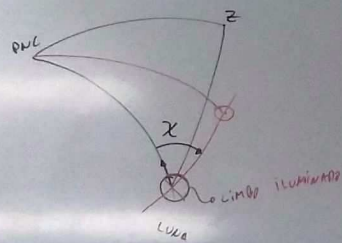
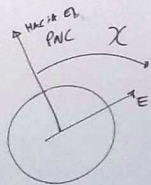
ÁNGULO DE POSICIÓN  $\chi$



• PNL

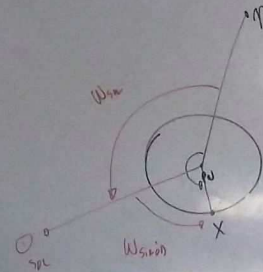
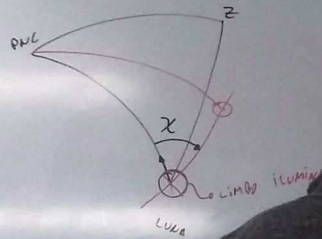
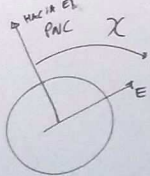
OPOSICIÓN  
FASES Y BRILLO  
COORD. PLANETA  
MAG. POSICIÓN  
DE LA SÍMBOLA  
OBLICUINAN

ÁNGULO DE POSICIÓN  $\chi$

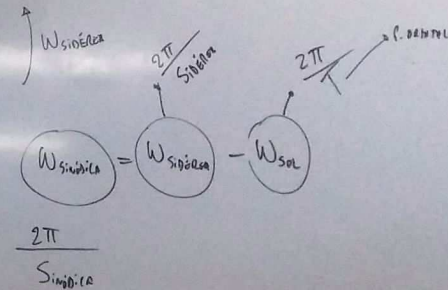


OPOSICIÓN  
 FASE Y BRILLO  
 COORD. PLANETA  
 ANG. POSICIÓN  
 DEL SOMBRA  
 OBLICUIDAD

ÁNGULO DE POSICIÓN  $\chi$

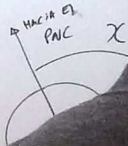


OPOSICIÓN  
 FASES Y BRILLO  
 COORD. PLANETA  
 ANG. POSICIÓN  
 DEL SINOIDA  
 OBLICUIDAD





ÁNGULO DE POSICIÓN  $\chi$

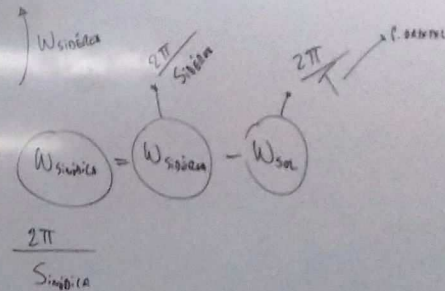
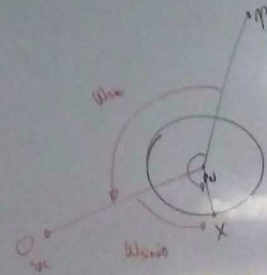


TIERRA  $\left\{ \begin{array}{l} \text{SID: } 23^{\circ} 26' 45'' \\ \text{TORO: } 365.25 \text{ DIAS} \end{array} \right.$

$$\frac{1}{S_{\text{SID}}} = \frac{1}{23^{\circ} 26' 45''} - \frac{1}{365.25} = \frac{1}{24 \text{ MS}}$$

URCIO  $\left\{ \begin{array}{l} \text{SID: } 58.646 \text{ MS} \\ \text{TORO: } \end{array} \right.$

$$\frac{1}{S_{\text{SID}}} = \frac{1}{S_{\text{URCIO}}} - \frac{1}{T_{\text{ORDINE}}}$$



OPORTUNIDAD  
FASE Y BRILLO  
CORRE PLANEOS  
ANG POSICION  
DEL SIDERIAL  
OBLICUIDAD

ÁNGULO DE POSICIÓN  $\chi$



TIERRA  
 Sid:  $23^h 56^m 4^s$   
 $T_{orb} = 365.25$  DIAS

$$\frac{1}{S_{sid}} = \frac{1}{23^h 56^m 4^s} - \frac{1}{365.25 d} = \frac{1}{24^h}$$

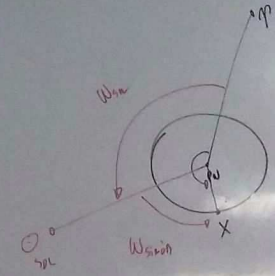
MERCURIO  
 Sid. 58.646 años

$$T_{orb} = a^{3/2} \times 365.25 \text{ dias} = 87.5$$

3841 ua

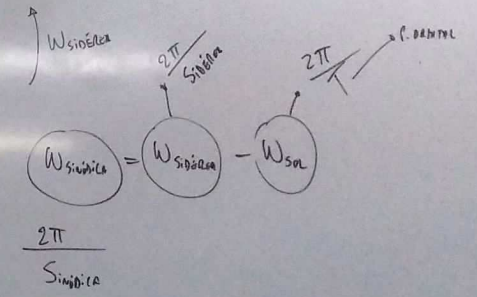
Sinódico =

$$T = a^{3/2}$$



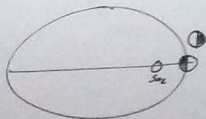
$$\frac{1}{S_{sid}} = \frac{1}{S_{SIDERA}} - \frac{1}{T_{ORDINAL}}$$

OPOSICIÓN  
 FASE Y BRILLO  
 COORD. PLANETA  
 ANG. POSICIÓN  
 DEL SINOÍDICO  
 OBLICUIDAD



PERIODOS SIDÉREO Y SIDERICO

DIA MERCIURIO = 2 AÑOS MERCIURIOS



$$\frac{T^2}{a^2} = 1$$

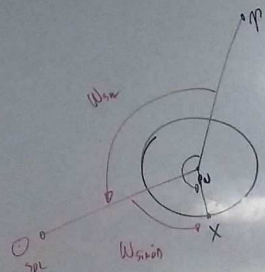
$$T = a^{3/2}$$

TIERRA  
 Sid:  $23^h 56^m 4^s$   
 T<sub>orb</sub>: 365.25 días

$$\frac{1}{S_{sid}} = \frac{1}{23^h 56^m 4^s} - \frac{1}{365.25 \text{ d}} = \frac{1}{24 \text{ h}}$$

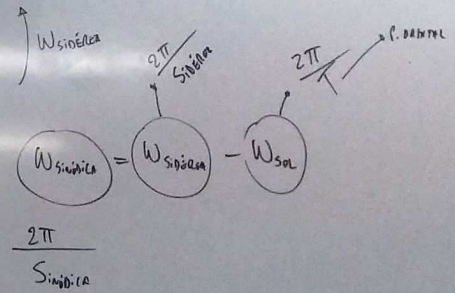
MERCURIO  
 Sid. 58.646 días  
 T<sub>orb</sub> =  $a^{3/2} \times 365.25 \text{ días} = 87.5$   
 a<sub>m</sub> = 0.3841 ua

Siderico = 176 días

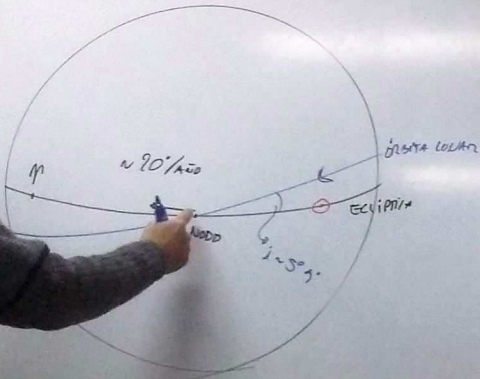


$$\frac{1}{S_{sid}} = \frac{1}{S_{siderico}} - \frac{1}{T_{orbital}}$$

OPOSICION  
 FASE Y BRILLO  
 COORD. PLANETA  
 ANG. POSICION  
 DEL SIDERICO  
 OBLICUIDAD



OCULTACIONES Y ECLIPSES



346.62 días

173.3 días

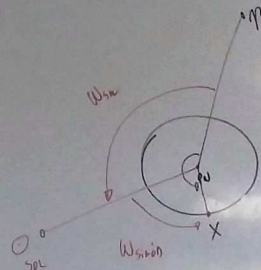
LUNA

PERIODO ORBITAL

SIDEREAL 27<sup>d</sup>.37

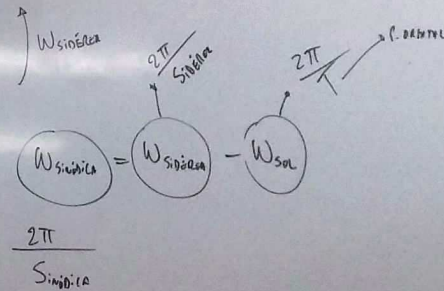
SINODIAL 29<sup>d</sup>.53

DRACÓNICO 27<sup>d</sup>.21

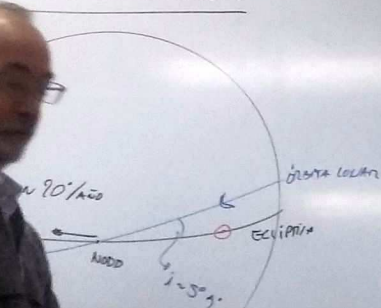


$$\frac{1}{S_{\text{sin}}}} = \frac{1}{S_{\text{siderea}}} - \frac{1}{T_{\text{orbital}}}}$$

OPOSICIÓN  
FASES Y BRILLO  
COORD. PLANETA  
ANG. POSICIÓN  
DEL SINODIAL  
OBLICUIDAD

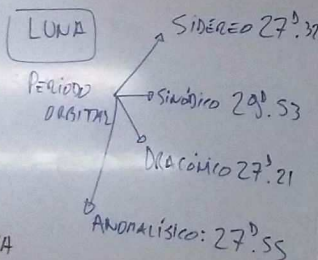


OCULTACIONES Y ECLIPSES

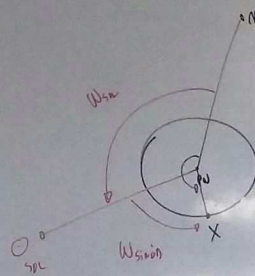


346.62 días

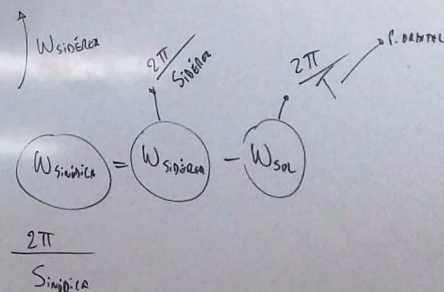
173.3 días



LUNA NUEVA

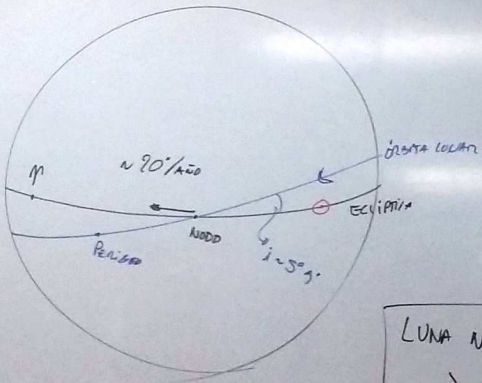


$$\frac{1}{S_{\text{sin}}} = \frac{1}{S_{\text{SIDERA}}} - \frac{1}{T_{\text{ORDINAL}}}$$



OPOSICIÓN  
FASER Y BRILLO  
CONDICIONES PLANEAS  
ANG. POSICION  
PER SINODICO  
OBLICUIDAD

OCULTACIONES Y ECLIPSES



346.62 días

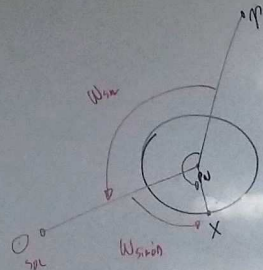
173.3 días

- LUNA**
- PERIODO ORBITAL
    - SIDÉREO 27<sup>d</sup>.37
    - SINÓDICO 29<sup>d</sup>.53
    - DRACÓNICO 27<sup>d</sup>.21
    - ANOMALÍSTICO: 27<sup>d</sup>.55

LUNA NUEVA

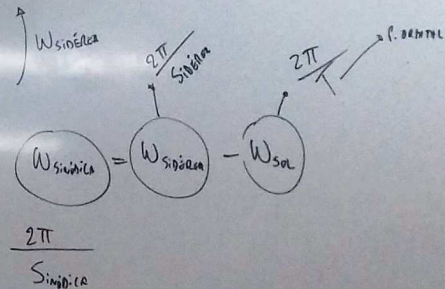
$$\lambda_L = \lambda_S$$

COND. ECLIP.



$$\frac{1}{S_{syn}} = \frac{1}{S_{sidereal}} - \frac{1}{T_{orbital}}$$

- OPOSICIÓN
- FASES Y BRILLO
- COORD. PLANETA
- MUC. POSICIÓN
- PER. SINÓDICO
- OBLICUIDAD



PRÁCTICO VII

$$(3) i_{\text{observada}} = 25^{\circ},17$$

$$(1) r = a(1 - e \cdot \cos E) = 2,2518 \text{ ua}$$

$$M = 1,3569 \text{ rads}$$

$$\text{ITERANDO } E_{n+1} = M + e \cdot \sin E_n \Rightarrow E = 1,46126 \text{ rads}$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALÓN 109

↓  
TRABAJO

PRÁCTICO VII

$$(3) i_{\text{enclava}} = 25^{\circ},17$$

$$(1) r = a(1 - e \cdot \cos E) = 2,2518 \text{ ua}$$

$$M = 1,3569 \text{ rads}$$

$$\text{ITERANDO } E_{\text{int}} = M + e \cdot \sin E_n \Rightarrow E = 1,46126 \text{ rads}$$

OTRA:

$$\theta \approx M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

VIERNES 29  
PARCIAL  
Prácticos 6, 7, 8

LUNES 18  
8:30  
SALON 109

↓  
TRABAJO



## PRÁCTICO VII

$$(3) i_{\text{observada}} = 25^{\circ},17$$

$$(1) r = a(1 - e \cos E) = 2,2518 \text{ ua}$$

$$M = 1,3569 \text{ rads}$$

$$\text{ITERANDO } E_{n+1} = M + e \cos E_n \Rightarrow E = 1,46126 \text{ rads}$$

OTRA:

$$\theta \approx M + 2e \cos M + \frac{5}{4} e^2 \cos 2M + \dots$$

$$M = \sqrt{\frac{\mu}{a^3}} = \frac{k}{a^{3/2}}$$

ua  
Ho  
Día

$$M = m \cdot \Delta t$$

↑  
Días

271,2361 días

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109

↓  
TRABAJO

PRÁCTICO VII

(3)  $i_{\text{observada}} = 25^{\circ},17$

(1)  $r = a(1 - e \cos E) = 2,2518 \text{ ua}$

$M = 1,3569 \text{ rados}$

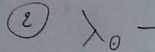
ITERANDO  $E_{\text{next}} = M + e \sin E_n \Rightarrow E = 1,46126 \text{ rados}$

OTRA:

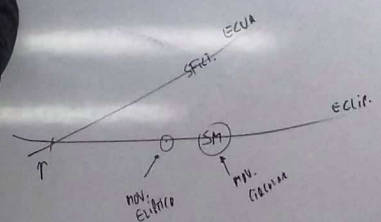
$$\theta \approx M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

$M = \sqrt{\frac{\mu}{a^3}} = \frac{k}{a^{3/2}}$  ua  
Ho  
dia

$M = m \cdot \frac{\Delta t}{\text{Dias}}$  271,2361 dias



☉ y Sol neg



VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRABAJO

PRÁCTICO VII

(3)  $i_{\text{observada}} = 25^{\circ},17$

(1)  $r = a(1 - e \cos E) = 2,2518 \text{ ua}$

$M = 1,3569 \text{ rads}$

ITERANDO  $E_{n+1} = M + e \cos E_n \Rightarrow E = 1,46126 \text{ rads}$

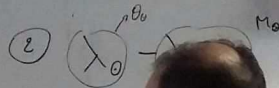
OMEGA:

$$\theta \approx M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

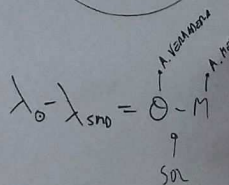
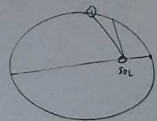
$$M = \sqrt{\frac{\mu}{a^3}} = \frac{k}{a^{3/2}}$$

ua  
Ho  
dia

$M = m \cdot \frac{\Delta t}{\text{Dias}}$   $\rightarrow 271,2361 \text{ dias}$



Y SOL NEG. DIURNAL EN PERIHELIO



Eclit.

VIERNES 29  
PARCIAL  
PRÁCTICOS 6,7,8

LUNES 18  
8:30  
SALON 109

TRABAJO

PRÁCTICO VII

(3) iterativa = 25°, 17

$$① r = a(1 - e \cos E) = 2,2518 \text{ ua}$$

$$M = 1,3569 \text{ rads}$$

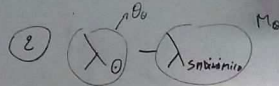
ITERANDO  $E_{n+1} = M + e \cos E_n \Rightarrow E = 1,46126 \text{ rads}$

OTRA:

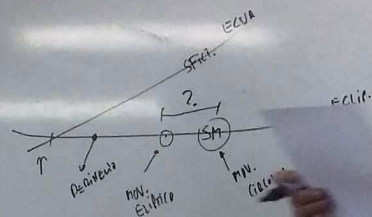
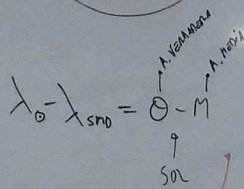
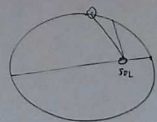
$$\theta \approx M + 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

$$M = \sqrt{\frac{\mu}{a^3}} = \frac{k}{a^{3/2}}$$

UA  
Ho  
Da



Y SOL y MARS COINCIDEN EN PERIHELIO



$$\lambda_0 - \lambda_{Mars} = 2e \sin M + \frac{5}{4} e^2 \sin 2M + \dots$$

ES 29  
CIAL  
S 6, 7, 8

PRÁCTICO VII

(3)  $i_{\text{observada}} = 25^{\circ},17$

(1)  $r = a(1 - e \cos E) = 2,2518 \text{ ua}$

$M = 1,3569 \text{ rads}$

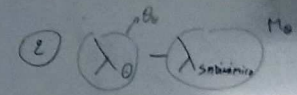
ITERANDO  $E_{m+1} = M + e \cos E_m \Rightarrow E = 1,46126 \text{ rads}$

OTRA:

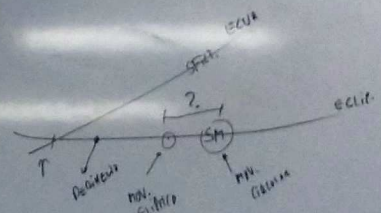
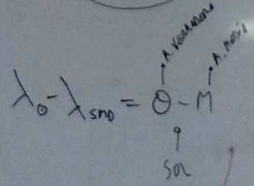
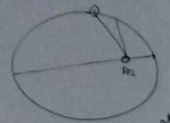
$$\theta \approx M + 2e \cos M + \frac{5}{4} e^2 \cos 2M + \dots$$

$M = \sqrt{\frac{\mu}{a^3}} = \frac{k}{a^{3/2}}$

$M = m \cdot \Delta t$  271,2361 días



☉ y Sol más cercano (COINCIDEN EN PERIHELIO)



VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRABAJO

$e = 0,017$

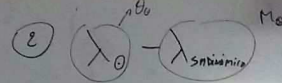
$\lambda_0 - \lambda_{\text{sol}} = 2e \cos M + \frac{5}{4} e^2 \cos 2M < 2e + \frac{5}{4} e^2 \text{ rads}$   
 $\sim 0,034 \text{ rad} \sim 1,96$

PRÁCTICO VII

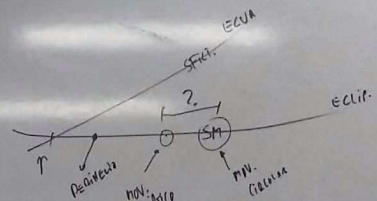
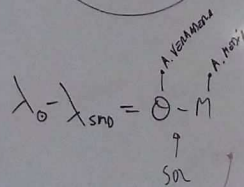
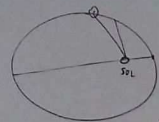
6  $a = 1,5236 \text{ ua}$

$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$

ua  
no  
da



☉ y SOL MED. DIARIO COINCIDEN EN PERIHELIO



VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109

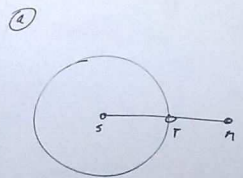
TRABAJO

$e = 0.017$

$\lambda_0 - \lambda_{SMD} = 2e \sin M + \frac{5}{4} e^2 \sin 2M$   $\left( 2e + \frac{5}{4} e^2 \text{ RAD} \right)$   
 $\sim 0.034 \text{ RAD} \sim 1.96$

PRÁCTICO VII

①  $a = 1,5236 \text{ ua}$

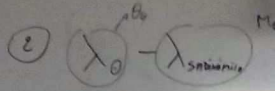


$$\frac{1}{S} = \frac{1}{T_M} - \frac{1}{T_T} \quad 1 \text{ año}$$

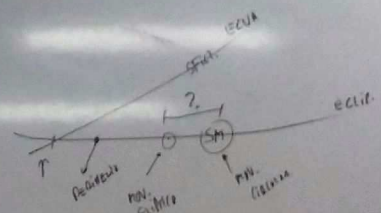
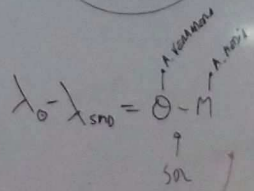
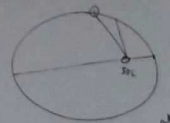
$$T^2 = a^3 \Rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$



○ y Sol por el mismo (COINCIDEN EN PERIHELIO)



$$\lambda_0 - \lambda_{180} = 2e \sin M + \frac{5}{4} e^2 \sin 2M < 2e + \frac{5}{4} e^2 \text{ RAD}$$

$$\sim 0.034 \text{ rad} \sim 1.96^\circ$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

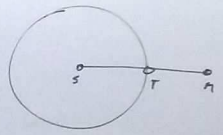
LUNES 18  
8:30  
SALON 105  
↓  
TRABAJO

$e = 0.017$

PRÁCTICA VII

⑥  $a = 1,5236 \text{ ua}$

②



$$\frac{1}{\sin \theta} = \frac{1}{T_M}$$



$$\frac{1}{S} = a^{-3/2} - 1$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

ua  
Mo  
dia

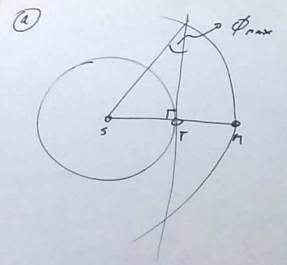
VIERNES 29  
PARCIAL  
PRÁCTICAS 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRABAJO



PRÁCTICO VII

①  $a = 1,5236 \text{ ua}$



$$\frac{1}{S_{\text{inclinada}}} = \frac{1}{T_M} - \frac{1}{T_T} \quad \text{1 año}$$

$$T^2 = a^3 \rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1 \Rightarrow S = 2,136 \text{ años}$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

0.20  
0.20

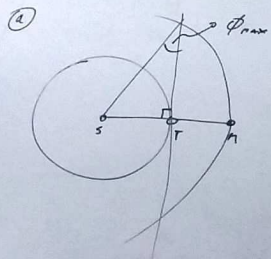
$$B = \text{cte}$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRAMASE

PRÁCTICO VII

①  $a = 1,5236 \text{ ua}$



$$\frac{1}{S \text{ años}} = \frac{1}{T_M} - \frac{1}{T_T} \quad 1 \text{ año}$$

$$T^2 = a^3 \rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1 \Rightarrow S = 2,136 \text{ años}$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

0.23

$$B = \frac{CTE \cdot (1 + e \cos \phi)}{r^2 \cdot p^2}$$

$r^2$  → dist. helioc.  
 $p^2$  → dist. perih.

$B_{max} \rightarrow$  oposición

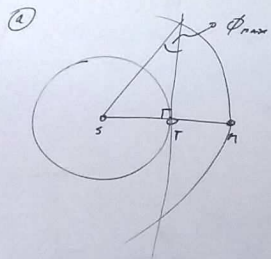
obstr.  $\phi = 0$

VIERNES 29  
 PARCIAL  
 PRÁCTICOS 6, 7, 8

LUNES 18  
 8:30  
 SALÓN 104  
 ↓  
 TRABAJO

PRÁCTICA VII

⑥  $a = 1,5236 \text{ ua}$



$$\frac{1}{S_{\text{inclinada}}} = \frac{1}{T_M} - \frac{1}{T_T} \quad 1 \text{ año}$$

$$T^2 = a^3 \rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1 \Rightarrow S = 2,136 \text{ años}$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

$$B = \frac{\text{CTE} \cdot (1 + e \cos \phi)}{r^2}$$

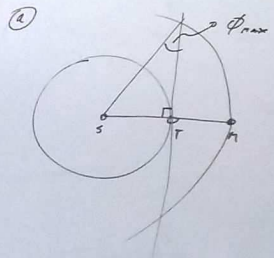
*Dist. Mercurio* (pointing to r)  
*Dist. Saturno* (pointing to phi)

$B_{\text{max}} \rightarrow$  oposición  
 $B_{\text{min}} \rightarrow$  conjunción

VIERNES 29  
PARCIAL  
Prácticas 6, 7, 8  
LUNES 18  
8:30  
10:00  
↓  
CARRASO

PRÁCTICO VII

①  $a = 1,5236 \text{ ua}$



$$\frac{1}{S \sin \phi} = \frac{1}{T_M} - \frac{1}{T_T} \quad \text{1 año}$$

$$T^2 = a^3 \Rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1 \Rightarrow S = 2,136 \text{ años}$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

UA  
Ho  
dia

$$B = \frac{CTE \cdot (1 + e \cos \phi)}{r^2}$$

Dist.  
Helios

Dist.  
Geoc.

oposic.

$\phi = 0$

$$\frac{B_{max}}{B_{min}} = \left( \frac{1+e}{1-e} \right)^2$$

$m = 4$

$$lim \approx \frac{2}{(a-1)^2}$$

$$\approx \frac{2}{(a+1)^2}$$

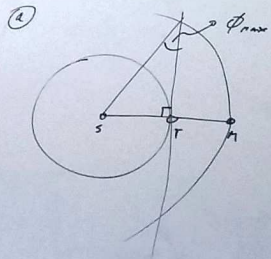
VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109

TRABAJO

PRÁCTICO VII

①  $a = 1,5236 \text{ ua}$



$$\frac{1}{S \text{ años}} = \frac{1}{T_M} - \frac{1}{T_T} \quad \text{1 año}$$

$$T^2 = a^3 \Rightarrow T = a^{3/2}$$

$$\frac{1}{S} = a^{-3/2} - 1 \Rightarrow S = 2,136 \text{ años}$$

$$M = \sqrt{\frac{k}{a^3}} = \frac{k}{a^{3/2}}$$

$$B = \frac{CTE \cdot (1 + e \cos \theta)}{r^2} \cdot p^2$$

*CTE = const. Helio*  
*p = const. orbit.*

$$B_{\text{max}} \rightarrow \text{oposición} \propto \frac{2}{(a-1)^2}$$

$$B_{\text{min}} \rightarrow \text{conjunción} \propto \frac{2}{(a+1)^2}$$

$$\frac{B_{\text{max}}}{B_{\text{min}}} = \left[ \frac{(a+1)}{(a-1)} \right]^2 = 23,5$$

$$m = 4 - 2,5 \log B$$

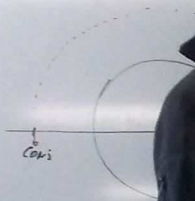
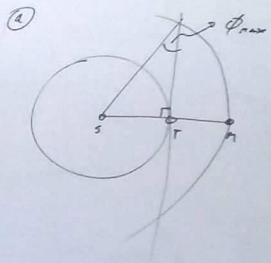
$$M_{\text{max}} - M_{\text{min}} = -2,5 \log \frac{B_{\text{max}}}{B_{\text{min}}}$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

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SALÓN 10º  
↓  
TRABAJO

PRÁCTICO VII

6  $a = 1,5236 \text{ ua}$



$$B = \frac{\text{CTE} \cdot (1 + e \cos \phi)}{r^2 \cdot p^2}$$

$\xrightarrow{\text{Dist. Helio}} r^2$       $\xrightarrow{\text{Dist. Geoc.}} p^2$       $\xrightarrow{\text{oposic.}} \phi = 0$

$$B_{\text{max}} \Rightarrow \text{oposición} \propto \frac{2}{(a-1)^2}$$

$$B_{\text{min}} \Rightarrow \text{conjunción} \propto \frac{2}{(a+1)^2}$$

$$\frac{B_{\text{max}}}{B_{\text{min}}} = \left[ \frac{(a+1)}{(a-1)} \right]^2 = 23,5$$

$$m = \mp 2,5 \log B$$

$$m_{\text{max}} - m_{\text{min}} = -2,5 \log \frac{B_{\text{max}}}{B_{\text{min}}}$$

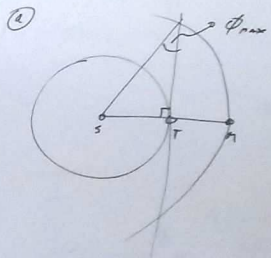
VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALÓN 109

TRABAJO

PRÁCTICO VII

6  $a = 1,5236 \text{ ua}$

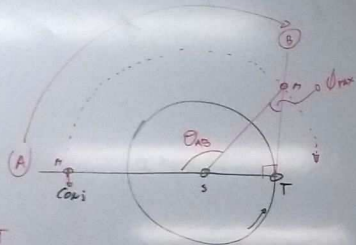


$$\dot{\theta} = \frac{2\pi}{S}$$

$$S \rightarrow 2\pi$$

$$\theta_{\text{ap}} \rightarrow x$$

(d)



$$B = \frac{\text{CTE} \cdot (1 + e \cos \phi)}{r^2 \cdot p^2}$$

$\downarrow$  DIST. HELIOS.       $\downarrow$  DIST. GAL.

$$B_{\text{max}} \Rightarrow \text{oposición} \propto \frac{2}{(a-1)^2}$$

$$B_{\text{min}} \Rightarrow \text{conjunción} \propto \frac{2}{(a+1)^2}$$

$$\frac{B_{\text{max}}}{B_{\text{min}}} = \left[ \frac{(a+1)}{(a-1)} \right]^2 = 23,5$$

$$m = \phi - 2,5 \log B$$

$$M_{\text{max}} - M_{\text{min}} = -2,5 \log \frac{B_{\text{max}}}{B_{\text{min}}}$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

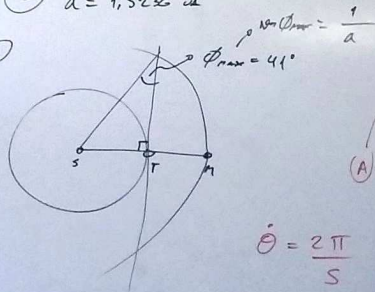
LUNES 18  
8:30  
SALON 109

TRABAJO

PRÁCTICO VII

⑥  $a = 1,5236 \text{ ua}$

②

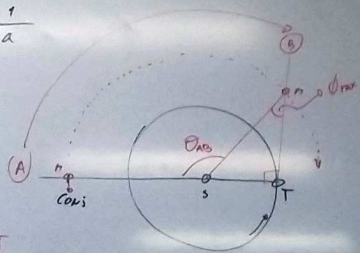


$$\dot{\theta} = \frac{2\pi}{S}$$

$$S \rightarrow 2\pi$$

$$\theta_{AP} \rightarrow x$$

④



$$B = \frac{CTE \cdot (1 + e \cos \theta)}{r^2}$$

*CTE* (circled)  
*r*<sup>2</sup> (circled)  
 Note:  $\theta = 0$  at perihelion

$$B_{max} \rightarrow \text{afélica} \propto \frac{2}{(a-1)^2}$$

$$B_{min} \rightarrow \text{perihélica} \propto \frac{2}{(a+1)^2}$$

$$\frac{B_{max}}{B_{min}} = \left[ \frac{(a+1)}{(a-1)} \right]^2 = 23,5$$

$$m = 4 - 2,5 \log B$$

$$M_{max} - M_{min} = -2,5 \int \frac{dB}{B}$$

VIERNES 29  
 PARCIAL  
 PRÁCTICOS 6, 7, 8

LUNES 18  
 8:30  
 SALON 109  
 ↓  
 TRABAJO

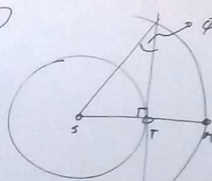


PRÁCTICO VII

6  $a = 1,5236 \text{ ua}$

$v_{max} = \frac{1}{a}$

7  $\phi_{max} = 41^\circ$

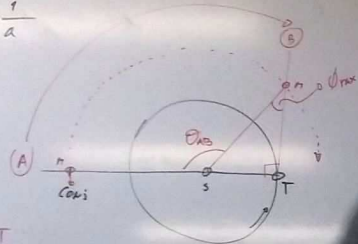


$\dot{\theta} = \frac{2\pi}{S}$

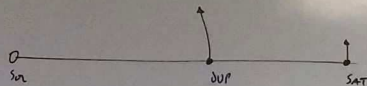
$S \rightarrow 2\pi$

$\theta_{AP} \rightarrow x$

d



10



$$\frac{1}{S} = \frac{1}{T_{SOL}} - \frac{1}{T_{JUP}}$$

VIERNES 29  
PARCIAL  
PRÁCTICOS 6, 7, 8

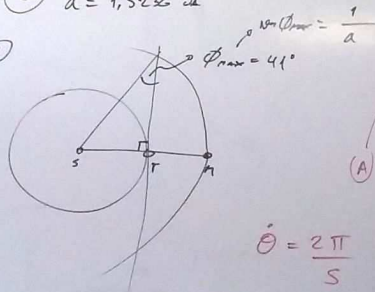
LUNES 18  
8:30  
SALON 109

TRABAJO

PRÁCTICO VII

(6)  $a = 1,5236 \text{ ua}$

(7)

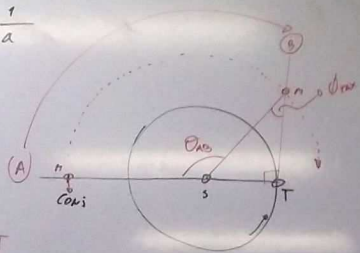


$$\dot{\theta} = \frac{2\pi}{S}$$

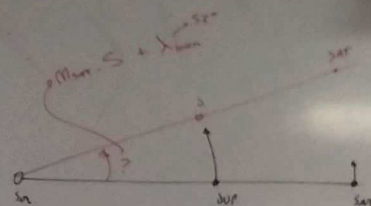
$$S \rightarrow 2\pi$$

$$\theta_{AP} \rightarrow x$$

(d)



(10)



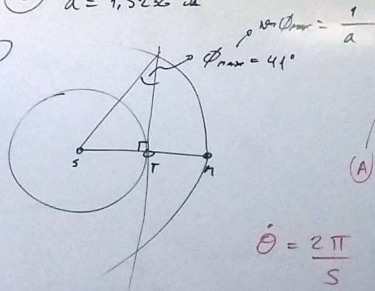
$$\frac{1}{S} = \frac{1}{T_{S_{\text{JUP}}}} - \frac{1}{T_{\text{SAT}}} = a_{\text{SAT}}^{-3/2} - a_{\text{JUP}}^{-3/2} = 10$$

29  
6, 7, 8  
18

PRÁCTICO VII

6  $a = 1,5236 \text{ ua}$

7

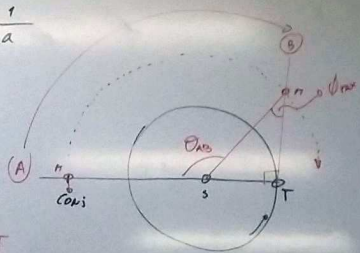


$$\dot{\theta} = \frac{2\pi}{S}$$

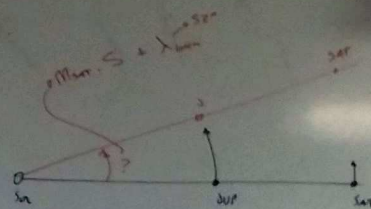
$$S \rightarrow 2\pi$$

$$\theta_{AP} \rightarrow X$$

8



10



$$\frac{1}{S} = \frac{1}{T_{S\text{JUP}}} - \frac{1}{T_{S\text{SAT}}} = a_{S\text{SAT}}^{-2/2} - a_{S\text{JUP}}^{-2/2} = 19,76 \text{ años}$$

$$X_{\text{HELIOC PRXZ ALIBURDIN}} = 52^\circ + X$$

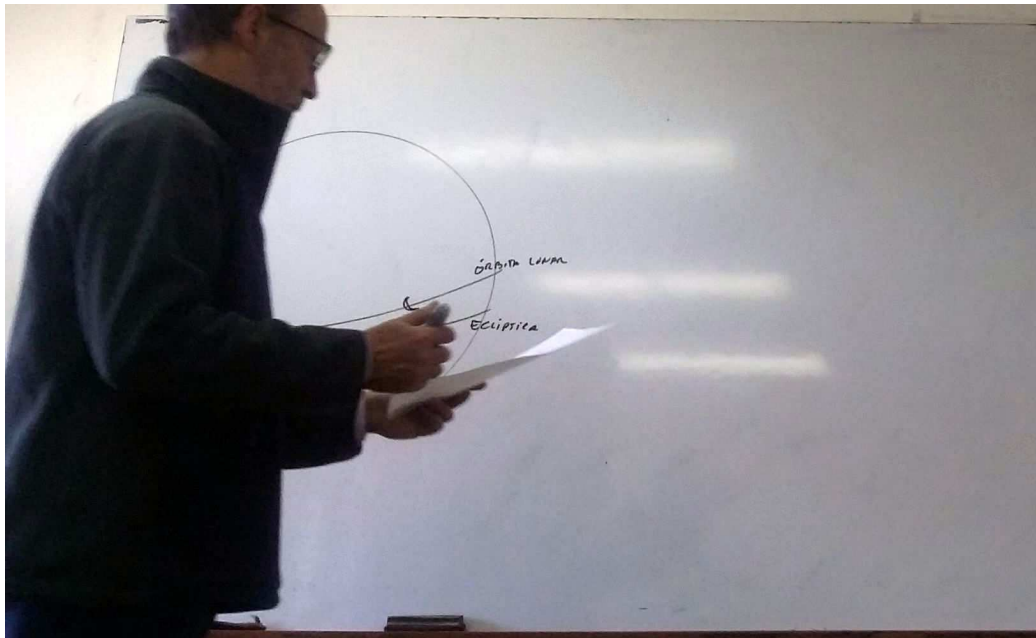
$$\text{SAT: } 360^\circ \rightarrow a_{S\text{SAT}}^{1/2} \text{ años } (29,65 \text{ años})$$

$$X \rightarrow 19,76 \text{ años}$$

VIERNES 29  
PRINCIAL  
PRÁCTICOS 6, 7, 8

LUNES 18  
8:30  
SALON 109

TRAMITE



(10)

VIERNES 29  
PARCIAL  
PRÁCTICAS 6, 7, 8

LUNES 18  
8:30  
SALÓN 10º  
↓  
TRAMITE

$$\frac{1}{S} = \frac{1}{T_{SO}} - \frac{1}{T_{JUP}} = a_{SAT}^{-2/2} - a_{JUP}^{-2/2} = 10,76 \text{ AÑOS}$$

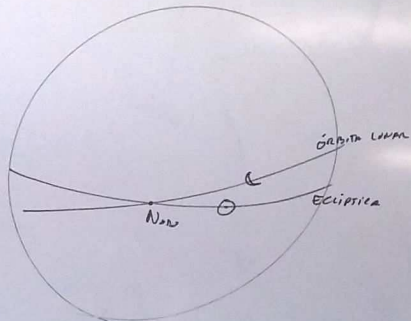
$X_{\text{HELIO}} \text{ PRÓX ALINACIÓN} = 52^\circ + X$

SAT:  $360^\circ \rightarrow a_{SAT}^{1/2} \text{ AÑOS} (29,65 \text{ AÑOS})$   
 $X \rightarrow 10,76 \text{ AÑOS}$

SAROS

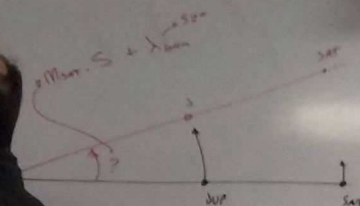
SOL - Júpiter : 346,62 años

LUNA - SOL : 29,5306 años



$$\frac{1}{\text{Mes sideral Sol}} = \frac{1}{\text{Mes sideral Júpiter}}$$

(10)



$$\frac{1}{T_{sol}} - \frac{1}{T_{jup}} = a_{sat}^{-1/2} - a_{jup}^{-1/2} = 10,76 \text{ años}$$

ALTERNAN = 52° + X

SAT : 360° → a<sub>SAT</sub> AÑOS (29,65 años)

X → 10,76 años

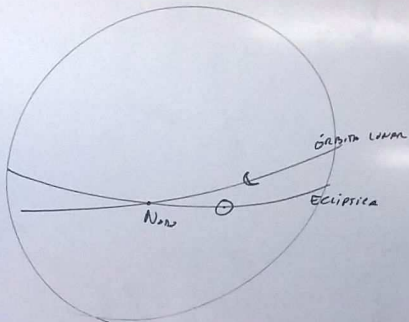
VIERNES 29  
PARCIAL  
PRÁCTICAS 6, 7, 8

---

LUNES 18  
8:30  
SALON 109

↓  
TRABAJO

SAROS



SOL - TIERRA : 366,62 días

LUNA - SOL : 29,5306 días

$$\frac{1}{\text{MES sideral}} = \frac{1}{\text{MES sideral}} - \frac{1}{\text{AÑO}}$$

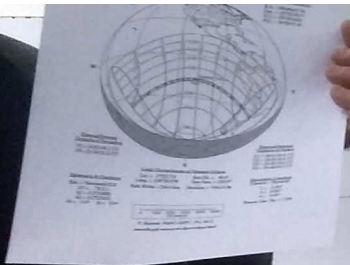
$$\frac{1}{\text{MES sideral}} = \frac{1}{27,32} - \frac{1}{365,242}$$

VIERNES 29  
PRINCIAL  
PRÁCTICAS 6, 7, 8

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LUNES 18  
8:30  
SALON 104

↓  
TRAMITE



346,62 días

29,5306 días

CALCULOS

$$346.62 \times 19 = 6585.8 \text{ días}$$

$$29.5306 \times 223 = 6585.3 \text{ días}$$

= 18 años y 11 días

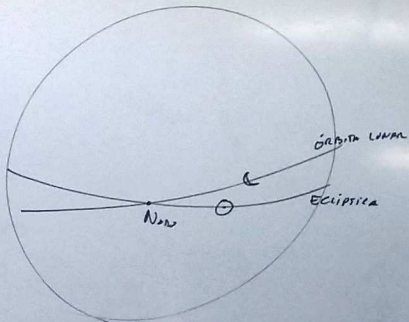
$$\frac{1}{\text{Mes sideral}} = \frac{1}{\text{Mes sideral}} - \frac{1}{\text{AÑO}}$$

$$\frac{1}{\text{MES sid}} = \frac{1}{27.32} - \frac{1}{365.242}$$

VIERNES 29  
PARCIAL  
Prácticas 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRAMITE

SAROS



SOL - LUNA : 346,62 años

LUNA - SOL : 29,5306 años

$$\frac{1}{\text{MES sideral}} = \frac{1}{\text{MES sideral}} - \frac{1}{\text{AÑO}}$$

$$\frac{1}{\text{MES sideral}} = \frac{1}{27,32} - \frac{1}{365,242}$$

CALCULOS

$346,62 \times 19 = 6585,8 \text{ años}$

$29,5306 \times 223 = 6585,3 \text{ años}$

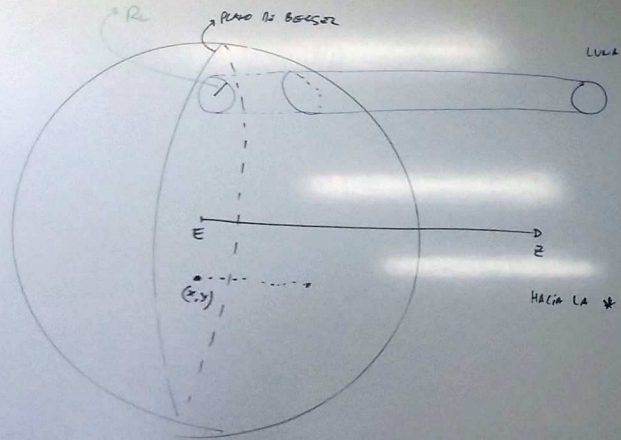
$= 18 \text{ años y } 11 \text{ días}$

VIERNES 29  
PARCIAL  
PRÁCTICAS 6, 7, 8

LUNES 18  
8:30  
SALON 109  
↓  
TRABAJO



# SISTEMA DE BESSEL



VIERNES 29  
PARCIAL

\*  
ESTRELLA

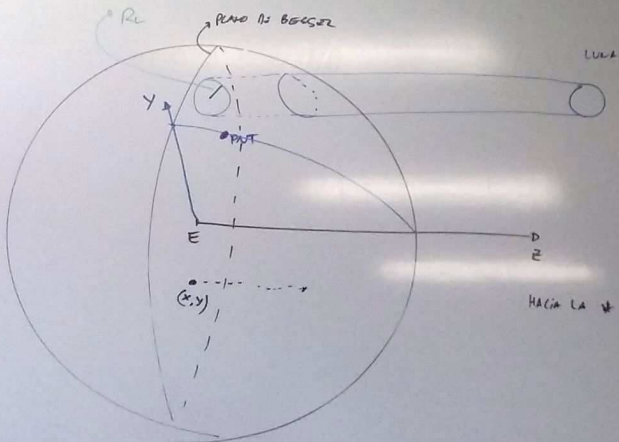




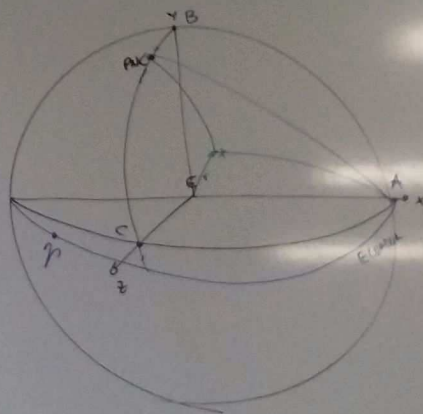
# SISTEMA DE BESSEL

$PQ \in YZ$

$XY \equiv$  PLANO BESSEL



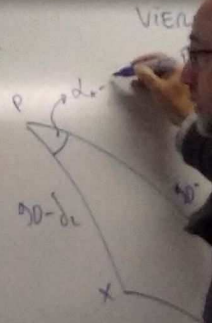
HALLAR COORD. LUNA Y OBSERVADOR (en sist. Bessel)



$r = R_{T-Luna}$

$$\begin{cases} x = r \cos \alpha \\ y = r \sin \alpha \end{cases}$$

$z = r \cos \beta$

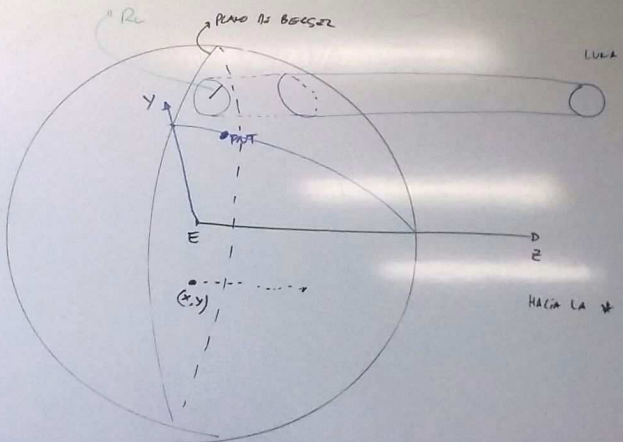


# SISTEMA DE BESSEL

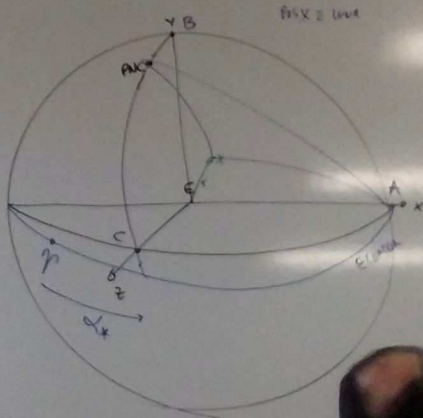


$PQ \in YZ$

$XY \equiv$  PLANO BESSEL



HALLAR COORD. LUNA Y OBSERVADOR (en sist. Bessel)

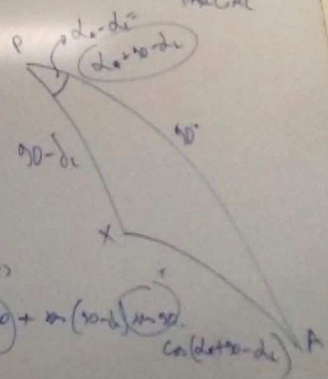


$r = \text{dist. T-Luna}$

$$\begin{cases} x = r \cdot \cos \widehat{AX} \\ y = r \cdot \cos \widehat{BY} \\ z = r \cdot \cos \widehat{CX} \end{cases}$$

VIERNES 29

PARCIAL

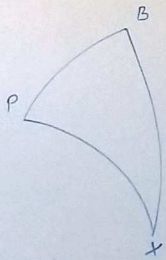


$$\cos \widehat{AX} = \cos(90 - \delta) \cos 90 + \sin(90 - \delta) \sin 90 \cos(\delta + \delta - \delta')$$

$$= \cos \delta \cdot \sin(\delta - \delta')$$

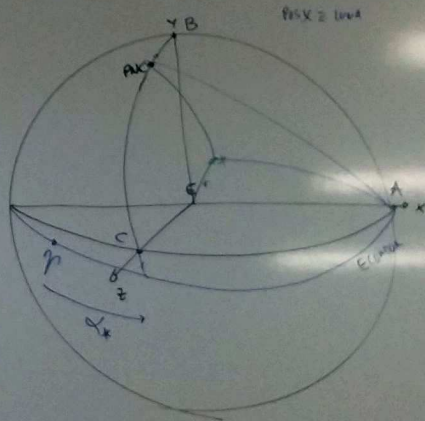
$$\cos \delta \cdot \sin(\delta - \delta')$$

SISTEMA DE BESSEL



$$N \rightarrow \Pi_2 = \frac{R_{\oplus}}{r}$$

HALLAR COORD. LUNA Y OBSERVADOR (en sist. Bessel)

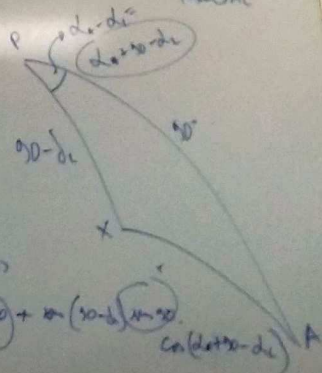


$$r = \text{dist. T-Luna}$$

$$\begin{cases} X = r \cdot \cos \widehat{AX} \\ Y = r \cdot \cos \widehat{BY} \\ Z = r \cdot \cos \widehat{CX} \end{cases}$$

VIENTOS 29

PARCIAL



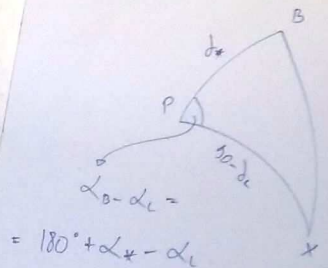
$$\cos \widehat{AX} = \cos(90 - \delta) \cos 90 + \sin(90 - \delta) \sin 90 \cos(\delta - \delta_x)$$

$$\cos \widehat{AX} = -\cos \delta \cdot \sin(\delta - \delta_x)$$

$$X = r \cdot \cos \delta \cdot \sin(\delta - \delta_x) = \cos \delta \cdot \sin(\delta - \delta_x) / \cos T$$

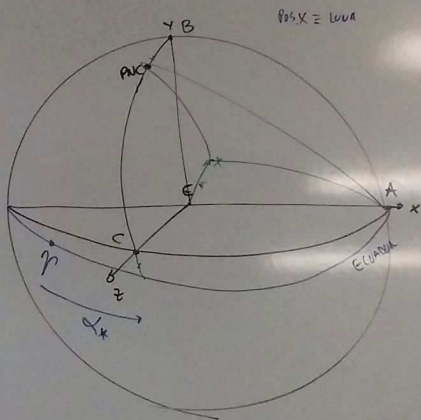
SISTEMA DE BESSEL

$$\sin \pi_2 = \frac{R_{\oplus}}{r}$$



$$\cos B = \frac{\cos b - \cos a \cos c}{\sin a \sin c}$$

HALLAR COORD. LUNA Y OBSERVADOR (en sist. BESSEL)

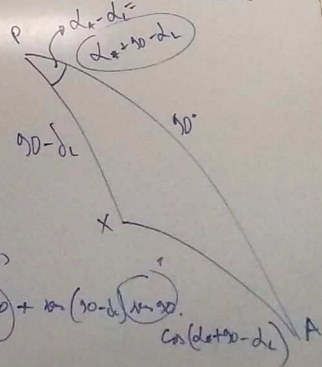


$r = \text{Dist T-Luna}$

$$\begin{cases} x = r \cdot \cos \widehat{AX} \\ y = r \cdot \cos \widehat{BX} \\ z = r \cdot \cos \widehat{CX} \end{cases}$$

VIERNES 29

PARCIAL



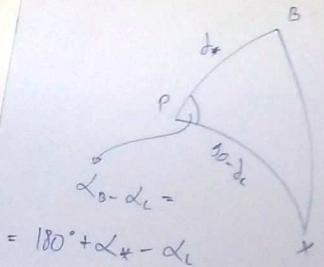
$$\cos \widehat{AX} = \cos(90 - d_L) \cdot \cos 90 + \sin(90 - d_L) \cdot \sin 90 \cdot \cos(\alpha_0 - d_L)$$

$$\cos \widehat{AX} = -\cos d_L \cdot \sin(\alpha_0 - d_L)$$

$$X = (r \cdot \cos d_L \cdot \sin(\alpha_0 - d_L)) = \cos d_L \cdot \sin(\alpha_0 - d_L) / \sin \pi_2$$

SISTEMA DE BESSEL

$$\sin TT_c = \frac{R_\oplus}{r}$$

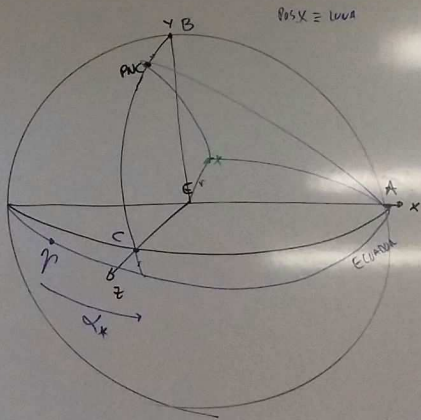


$$\begin{aligned} \cos BX &= \cos d_* \cdot \cos(90-d_c) - \sin d_* \sin(90-d_c) \cdot \cos(d_*-d_c) \\ &= \cos d_* \sin d_c - \sin d_* \cos d_c \cos(d_*-d_c) \end{aligned}$$

EN RADIOS TERRRESTRES:

$$\begin{aligned} X &= \cos d_* \cdot \sin(d_c - d_*) / \sin TT_c \\ Y &= [\cos d_* \sin d_c - \sin d_* \cos d_c \cdot \cos(d_* - d_*)] / \sin TT_c \end{aligned}$$

HALLAR COORD. LUNA Y OBSERVADOR (EN SIST. BESSEL)

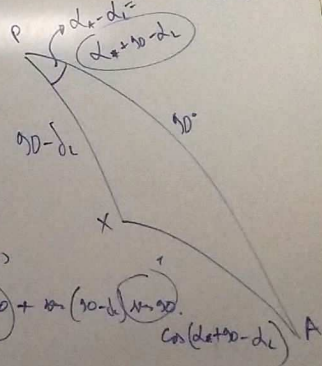


$r = \text{dist T-Luna}$

$$\begin{cases} X = r \cdot \cos \widehat{AX} \\ Y = r \cdot \cos \widehat{BX} \\ Z = r \cdot \cos \widehat{CX} \end{cases}$$

VIERNES 29

PARCIAL



$$\cos AX = \cos(90-d_c) \cdot \cos 90 + \sin(90-d_c) \sin 90 \cdot \cos(d_* + 90 - d_c)$$

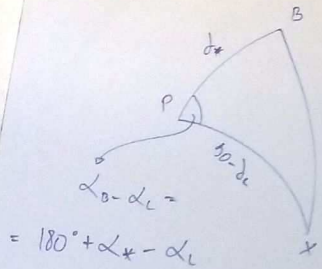
$$\cos AX = -\cos d_c \cdot \sin(d_* - d_c)$$

$$X = r \cdot \cos d_c \cdot \sin(d_c - d_*) = \cos d_c \cdot \sin(d_c - d_*) / \sin TT_c$$



SISTEMA DE BESSEL

$$\sin \pi_i = \frac{R_{\oplus}}{r}$$

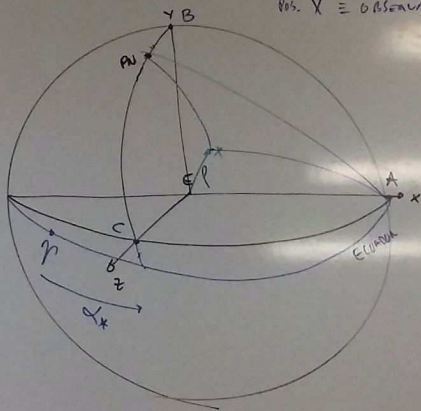


$$\begin{aligned} \cos BX &= \cos d_X \cdot \cos (90 - d_C) - \sin d_X \sin (90 - d_C) \cdot \cos (d_X - d_C) \\ &= \cos d_X \sin d_C - \sin d_X \cos d_C \cos (d_C - d_X) \end{aligned}$$

En radios terrestres:

$$\begin{aligned} X &= \cos d_C \cdot \sin (d_C - d_X) / \sin \pi_i \\ Y &= [\cos d_X \sin d_C - \sin d_X \cos d_C \cdot \cos (d_C - d_X)] / \sin \pi_i \end{aligned}$$

HALLAR COORD. LUNA Y OBSERVADOR (en sist. BESSEL)



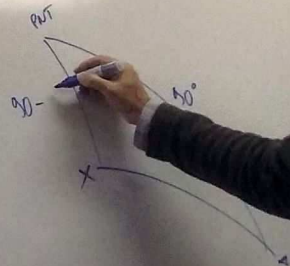
Pos. X  $\equiv$  OBSERVADOR

$$\xi = \rho \cdot \cos \alpha_X$$

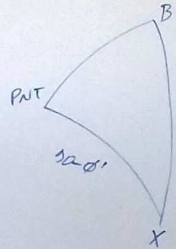
$$\eta = \rho \cdot \sin \alpha_X$$

VIERNES 29

PARCIAL

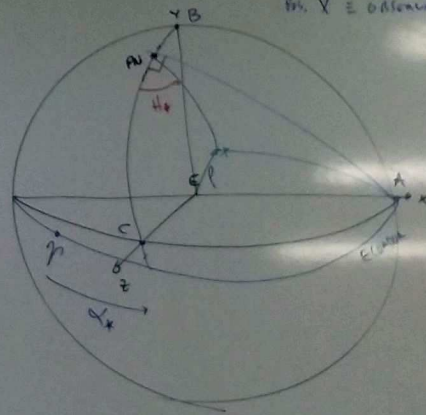


SISTEMA DE BESSEL



$$\sin \Pi_c = \frac{R_\oplus}{r}$$

HALLAR COORD. LUNA Y OBSERVADOR (en sist. Bessel)

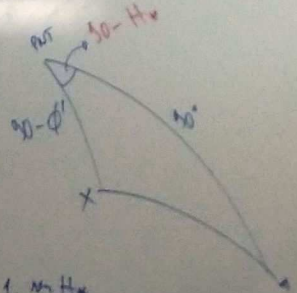


Alt. X = observador

$$\xi = \rho \cdot \cos AX$$

$$\eta = \rho \cdot \cos BX$$

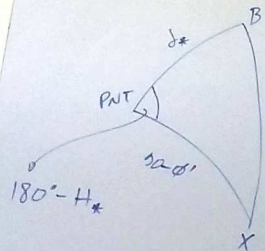
VIERNES 29  
PARCIAL



$$\cos AX = 0 + \cos \phi' \cdot \sin H_0$$

$$\xi = \frac{\rho \cdot \cos \phi' \cdot \sin H_0}{R_\oplus} \quad (\text{en sist. Bessel})$$

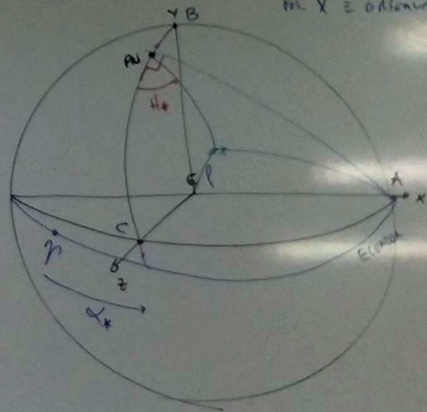
SISTEMA DE BESSEL



$(X, Y)$  LUNA  
 $(\xi, \eta)$  OBSERVADOR  
 } EN FUNCIÓN DE  $t$

$$N = \Pi_2 = \frac{R_{\oplus}}{r}$$

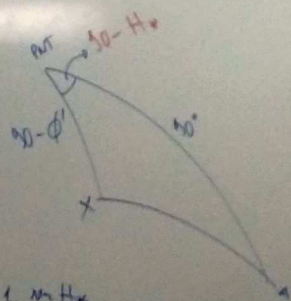
HALCAR COORD. LUNA Y OBSERVADOR (en sist. Bessel)



$(x, y, z)$      $(\xi, \eta, \zeta)$   
 EN X = OBSERVADOR     $\xi = \rho \cdot \cos \alpha X$   
 $\eta = \rho \cdot \cos \beta X$

VIERNES 29

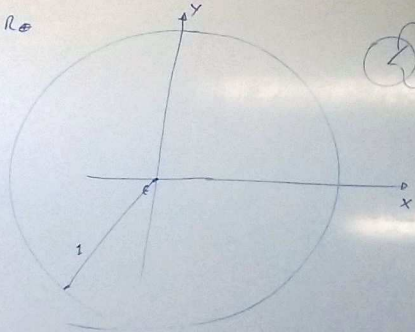
PRECAL



$$\cos \alpha X = 0 + \cos \phi' \cdot \sin H_0$$

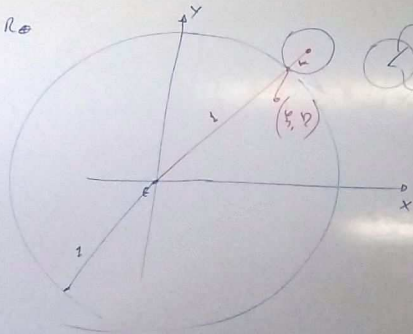
$$\xi = \frac{\rho \cdot \cos \phi' \cdot \sin H_0}{R_{\oplus}} \quad (\text{en sus TOPO})$$

UNIDAD DIST.:  $R_{\oplus}$



$K = \frac{R_L}{R_{\oplus}}$   
 $(x, y)$

VIERNES 29  
MAYO

UNIDAD DIST.:  $R_{\oplus}$ 

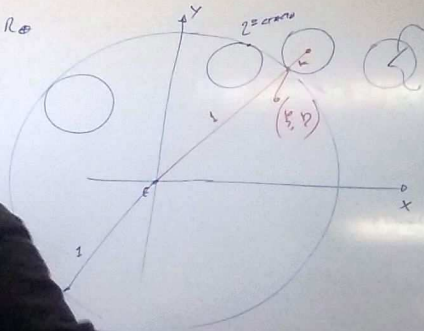
$$K = \frac{R_L}{R_{\oplus}} = 0.2725$$

$$x^2 + y^2 = (1+K)^2 \quad \uparrow \text{1}^{\text{er}} \text{ CONTACTO}$$

$$(\lambda, \varphi) \Rightarrow \uparrow \text{2}^{\text{do}} \text{ CONTACTO}$$

VIERNES 29  
MARCAL

dist.:  $R_{\oplus}$



$$K = \frac{R_L}{R_{\oplus}} = 0.2725$$

$$x^2 + y^2 = (1+k)^2 \quad \text{1º círculo}$$

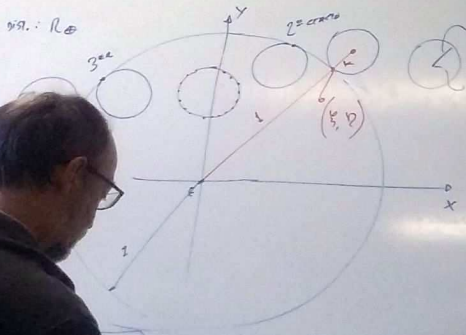
$$(x, \phi) \rightarrow \text{1º círculo}$$

$$x^2 + y^2 = (1-k)^2$$

VIEWES 29  
MERCAL

VIERNES 29  
DARCAI

UNIDAD 99:  $R_E$



$$K = \frac{R_L}{R_E} = 0.2725$$

$$x^2 + y^2 = (1+K)^2 \quad \begin{matrix} 1^{er} \text{ CONTACTO} \\ 4^{to} \end{matrix}$$

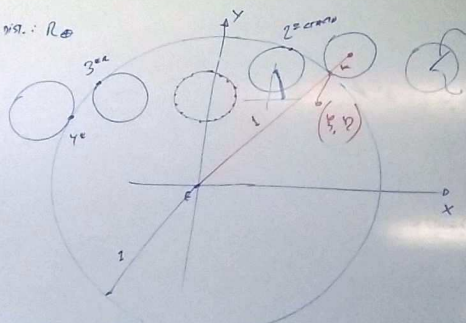
$$(\lambda, \phi) \Rightarrow 1^{er} \text{ CONTACTO}$$

$$x^2 + y^2 = (1-K)^2 \quad \begin{matrix} 2^{do}, 3^{er} \text{ CONTACTO} \end{matrix}$$

1040

2018ESP

UNIDAD DIST.:  $R_{\oplus}$



$$K = \frac{R_L}{R_{\oplus}} = 0.2725$$

$$x^2 + y^2 = (1+K)^2 \quad \text{1er CONTACTO, } 4^{\text{to}}$$

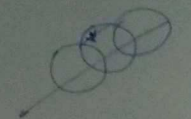
$(x, \phi) \rightarrow 1^{\text{er CONTACTO}}$

$$x^2 + y^2 = (1-K)^2 \quad \text{2do, 3er CONTACTO}$$

LÍNEA DE CENTRALIZACIÓN

$(\xi, \eta, \gamma)$  CORRESP. A  $(x, y)$

VIERNES 29  
MARZO



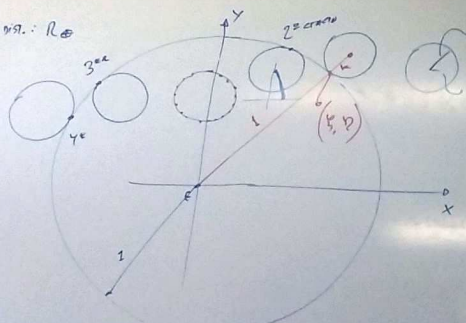
DADO  $(x, \phi) \rightarrow \xi(t), \eta(t)$  y  $\text{condición en } X(t), Y(t)$

$$\text{Si } (\xi-x)^2 + (\eta-y)^2 \leq K^2 \Rightarrow \text{HAY OCULTACIÓN}$$



VIERNES 29  
MARZO

UNIDAD DIST.:  $R_{\oplus}$



$$K = \frac{R_L}{R_{\oplus}} = 0.2725$$

$$x^2 + y^2 = (1 + K)^2 \quad \text{1er CONTACTO, 4to}$$

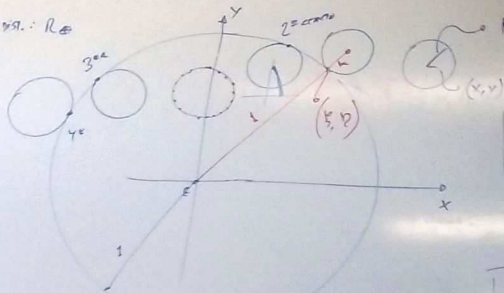
$$(x, \phi) \rightarrow \text{1er CONTACTO}$$

$$x^2 + y^2 = (1 - K)^2 \quad \text{2do, 3er CONTACTO}$$

LÍNEA DE CENTRALIZACIÓN

$$(g, v, y) \text{ CORRESP. A } (x, y)$$

UNIDAD ASTR.:  $R_{\oplus}$



$$K = \frac{R_L}{R_{\oplus}} = 0.2725$$

$$x^2 + y^2 = (1+K)^2 \quad \text{1er CONTACTO} \quad 4^{\text{to}}$$

$$(x, y) \Rightarrow \text{1er CONTACTO}$$

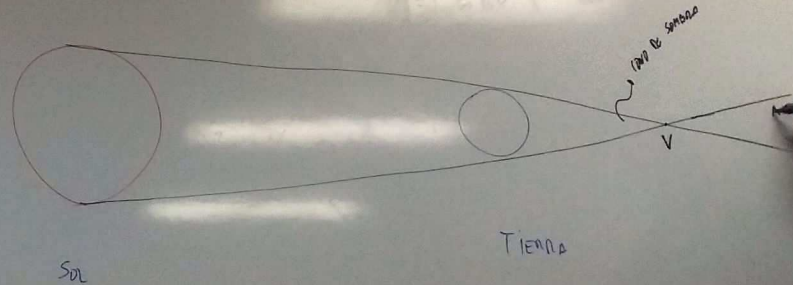
$$x^2 + y^2 = (1-K)^2 \quad \text{2do, 3er CONTACTO}$$

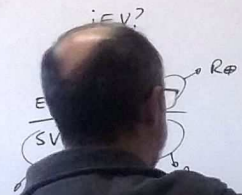
LÍNEA DE CENTRALIDAD

$(xi, yi, \varphi)$  CORRESP. A  $(x, y)$

ECLIPSES DE SOL Y LUNA

VIERNES 29  
PARCIAL

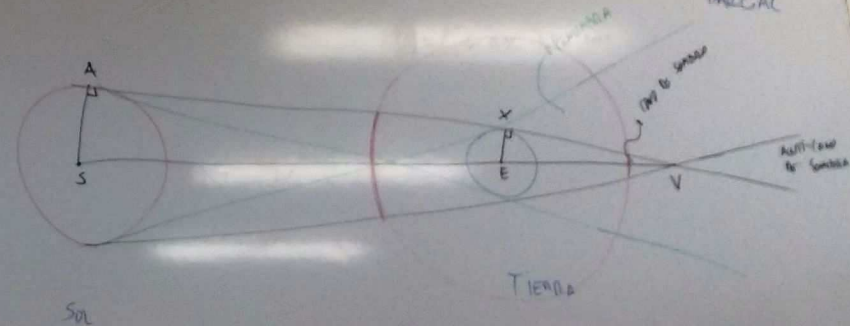




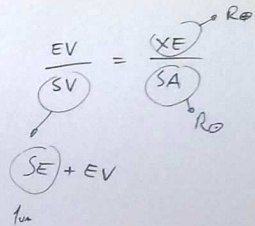
⇒ 1 u

ECLIPSES DE SOL Y LUNA

VIERNES 29  
PARCIAL



¿EV?



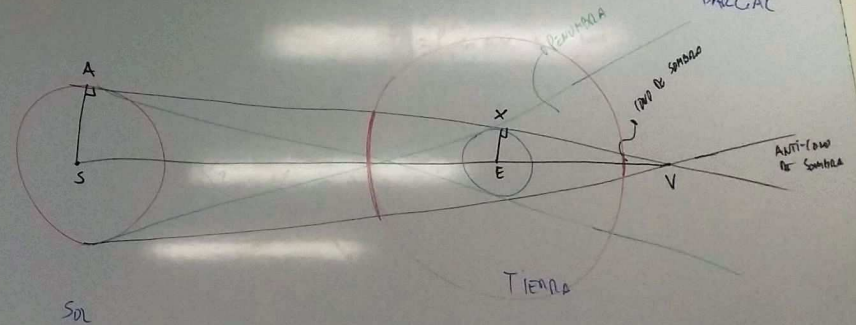
$$\frac{EV}{SV} = \frac{XE}{SA} \Rightarrow \frac{1ua + EV}{EV} = \frac{R_{\odot}}{R_{\oplus}}$$

$$\frac{1ua}{EV} = \frac{R_{\odot}}{R_{\oplus}} - \frac{R_{\oplus}}{R_{\oplus}} \Rightarrow EV = \frac{R_{\oplus}}{\frac{R_{\odot}}{R_{\oplus}} - 1} \text{ uas} \approx 9,3 \times 10^3 \text{ uas}$$

$\frac{R_{\odot} - R_{\oplus}}{\downarrow \quad \downarrow}$   
 $696000 \quad 6400$

ECLIPSES DE SOL Y LUNA

VIERNES 29  
PARCIAL



SOLSTICIO = "S"

12/6 : PUESTA SOL +

29/6 : SALIDA + TARDE

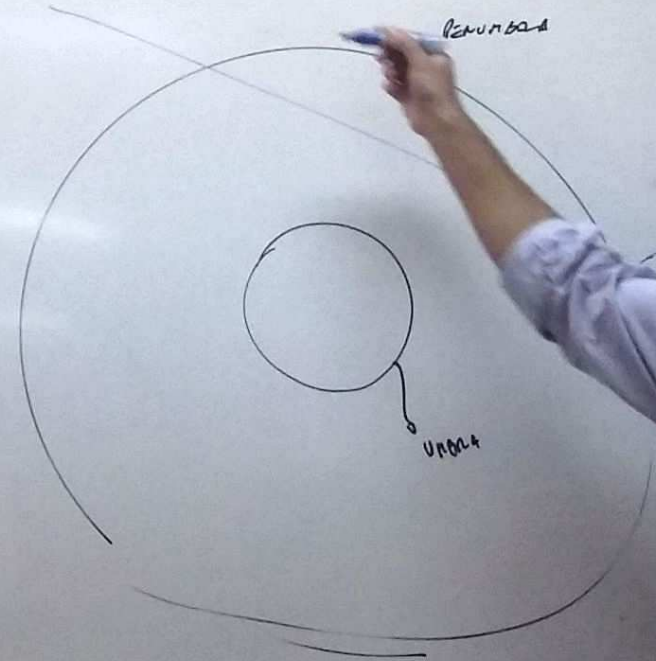
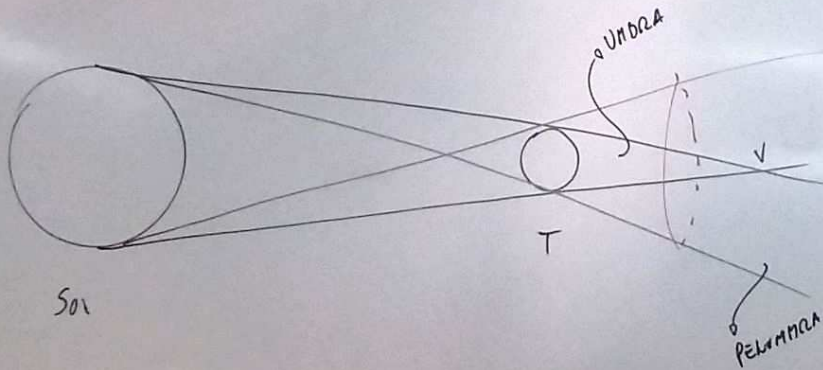
$$TSL = H + \omega$$

↑

25/6 PARCIAL

# ECLIPSES

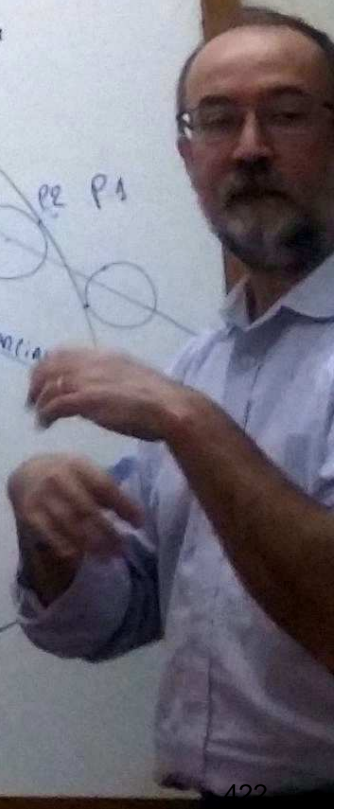
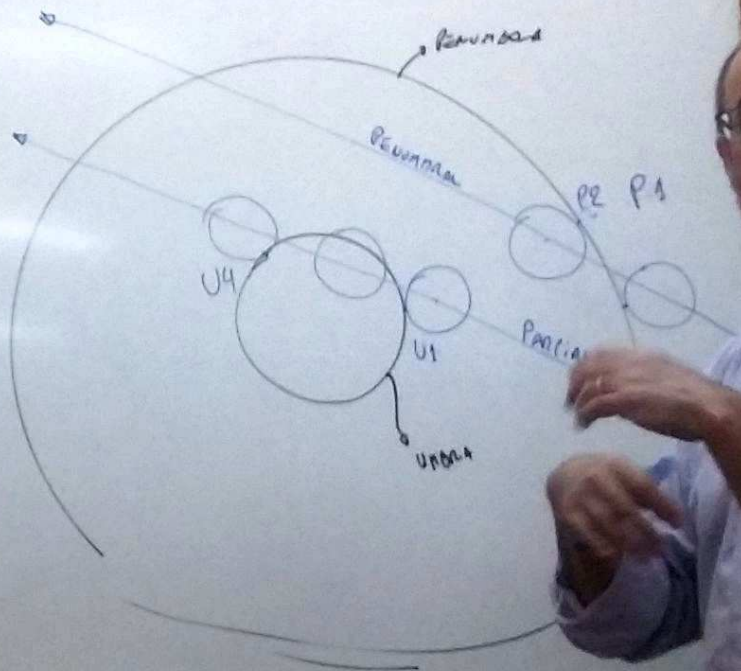
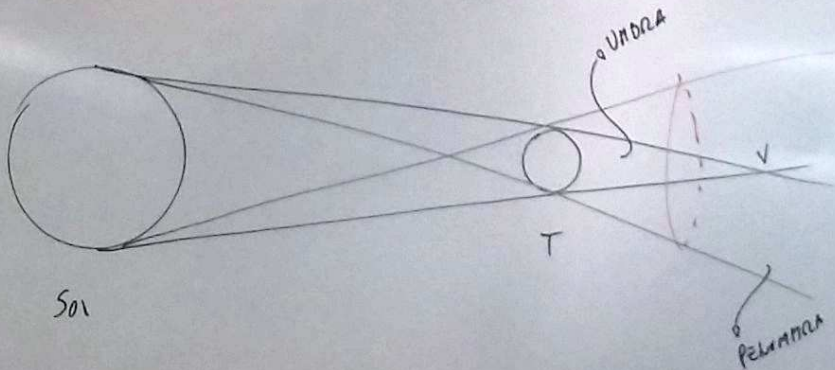
NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$



25/6 PARCIAL

ECLIPSES

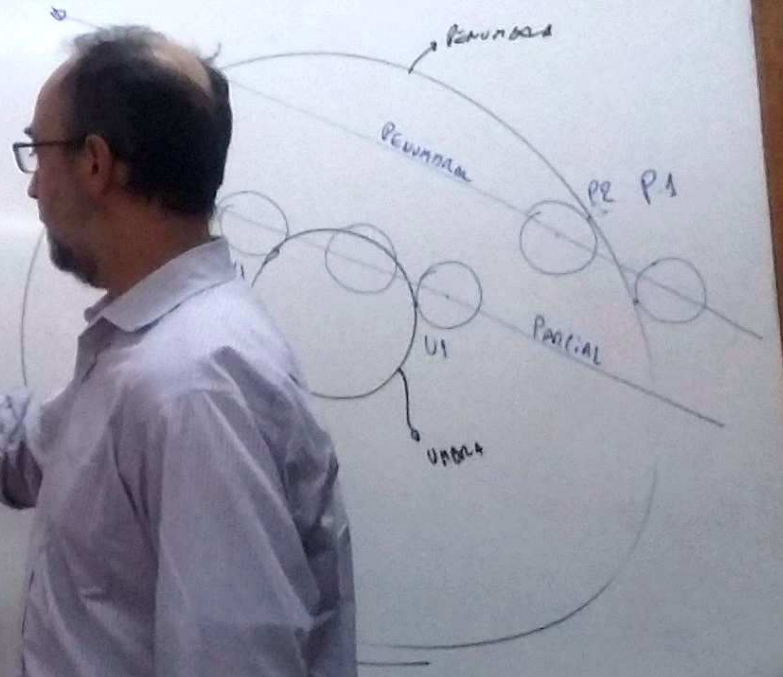
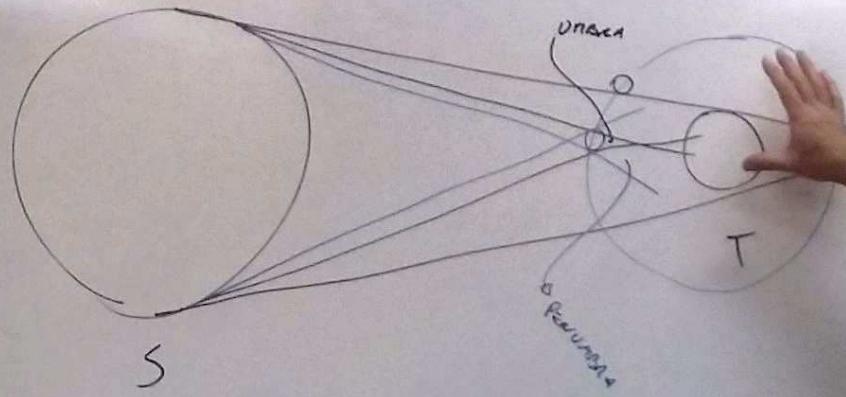
NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$



25/6 PARCIAL

ECLIPSES → LUNA

NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$

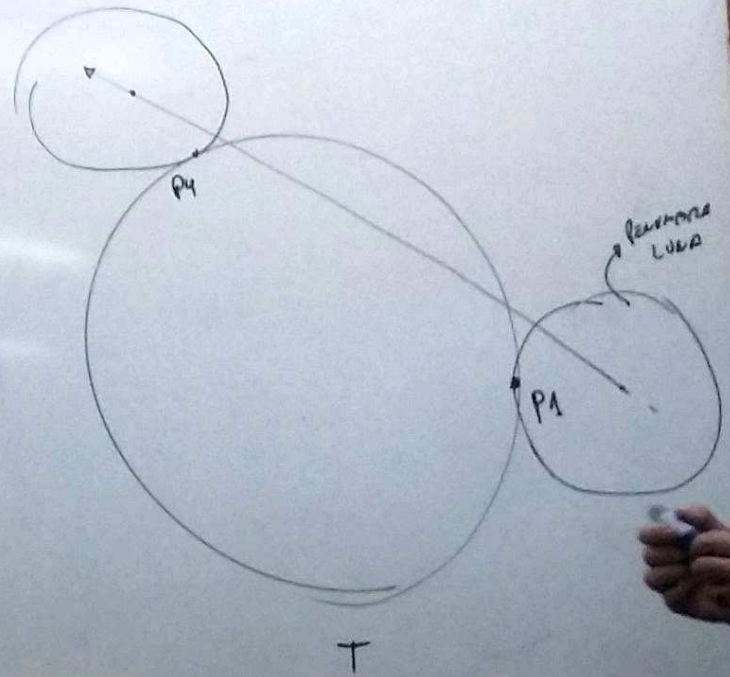
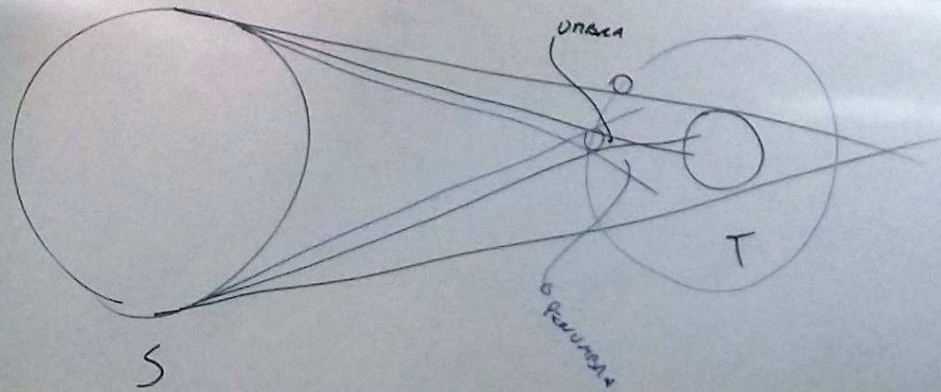




25/6 PARCIAL

NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$

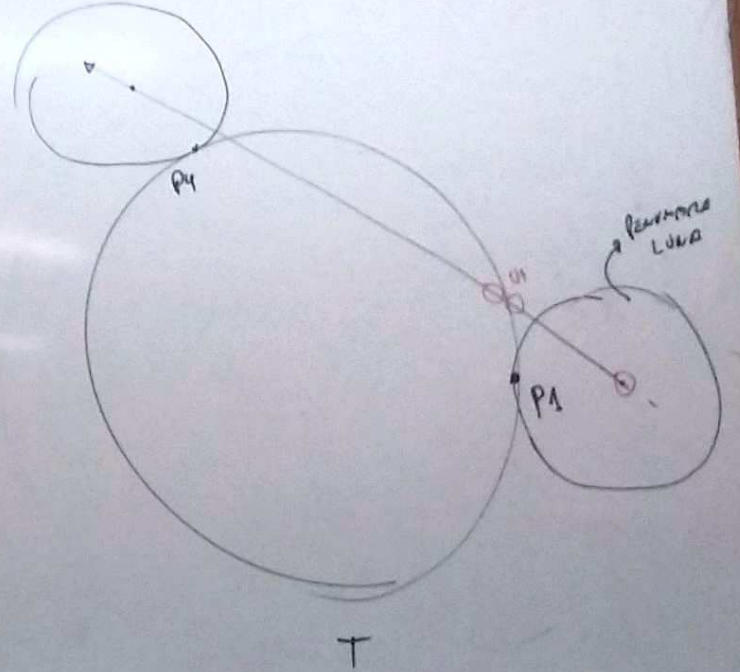
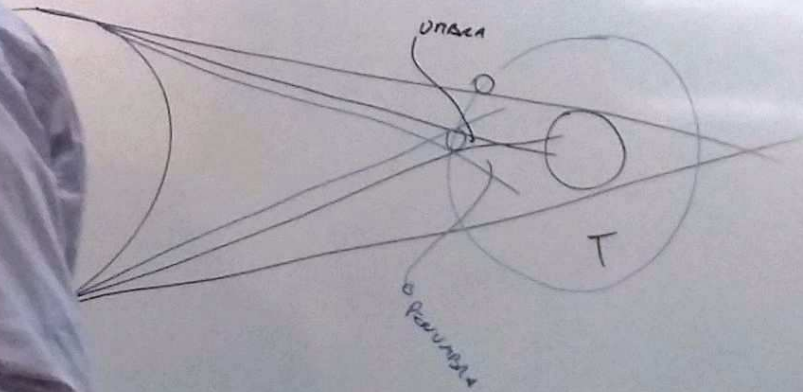
ECLIPSES → LUNA



20/6 PARCIAL

$$\text{FINAL: } \frac{(\text{PUNTAJE} - 150)}{150} \times 3 + 3$$

# ECLIPSES → LUNA

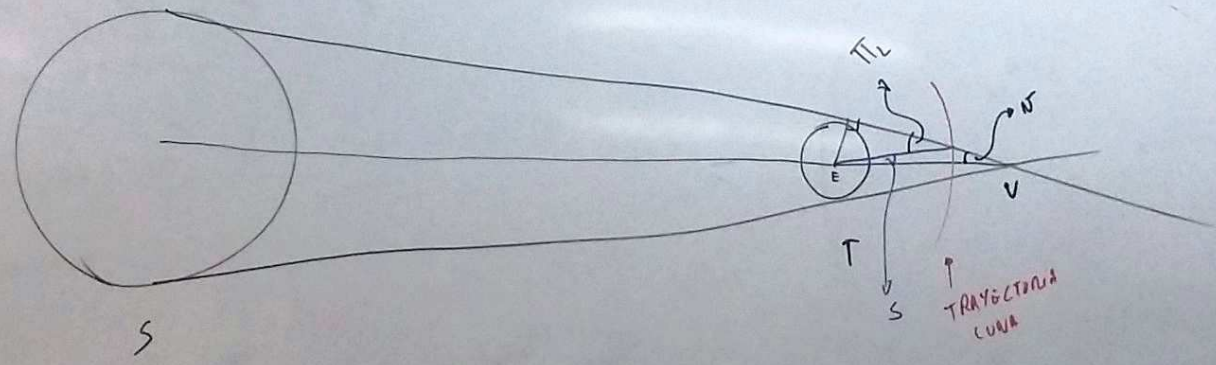


25/6 PARCIAL

NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$

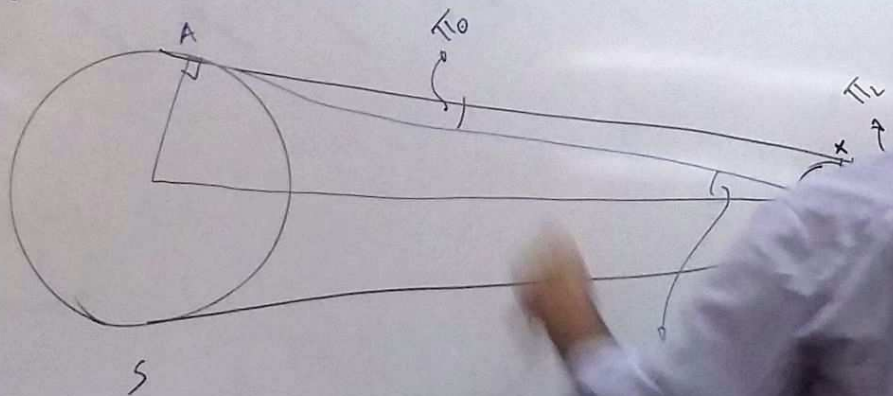
S: SEMIDIÁMETRO CONO SOMBRA  
TIERRA A LA ALTURA DE LA  
LUNA

$\pi_L = S + N$



25/6 PARCIAL

NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$



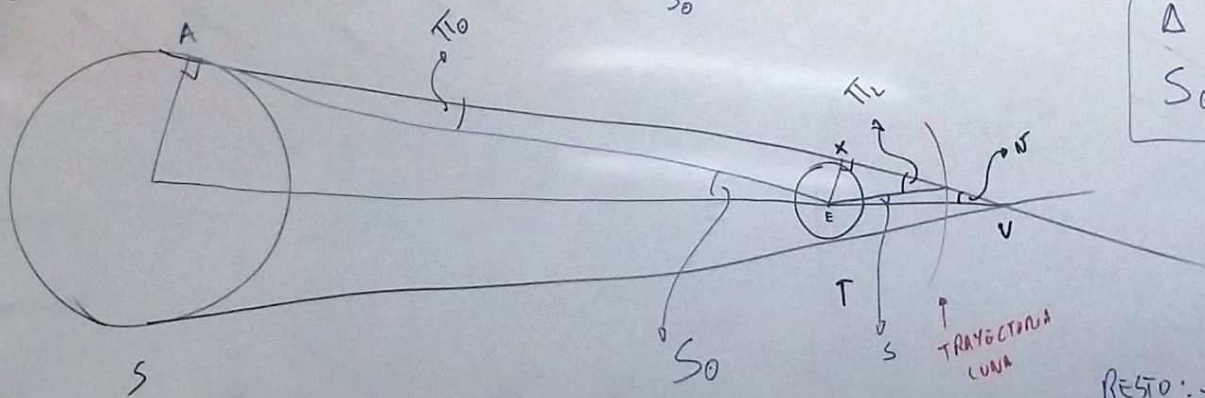
$\Delta EXV:$   
 $S = S + N$

S: SEMIDIÁMETRO COMO SOMBRA  
 TIERRA A LA ALTURA DE LA  
 LUNA

$\Delta AEV:$   
 $S_0 = \pi_0 + N$

25/6 PARCIAL

NOTA FINAL:  $\frac{(\text{PUNTAJE} - 150)}{150} \times 5 + 3$



$\Delta EXV:$   
 $\pi_L = S + N$

S: SEMIDIÁMETRO COMO SOMBRA TIERRA A LA ALTURA DE LA LUNA

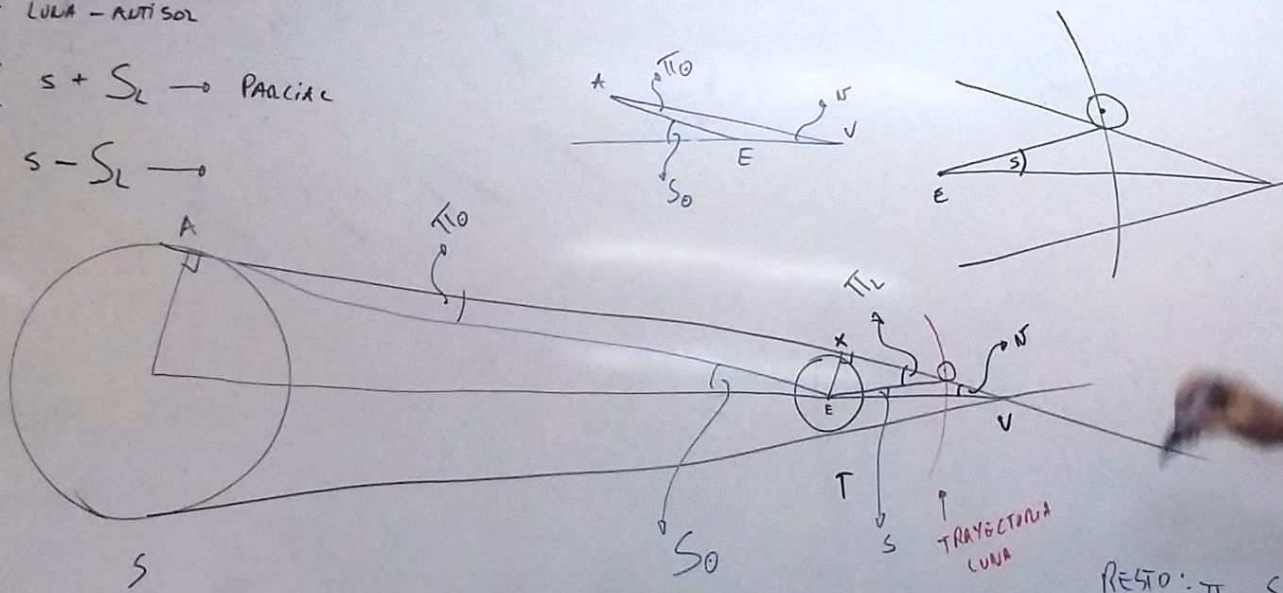
$\Delta AEV:$   
 $S_0 = \pi_0 + N$

$N = S_0 - \pi_0$   
 → 5"  
 ↳ 16'

RESTO:  $\pi_L - S_0 = S - \pi_0$

$\Rightarrow S = \pi_L + \pi_0 - S_0$

$\eta = \text{Dist LUNA - ANTI SOL}$   
 si  $\eta \leq s + S_L \rightarrow$  PAACIAL  
 si  $\eta \leq s - S_L \rightarrow$



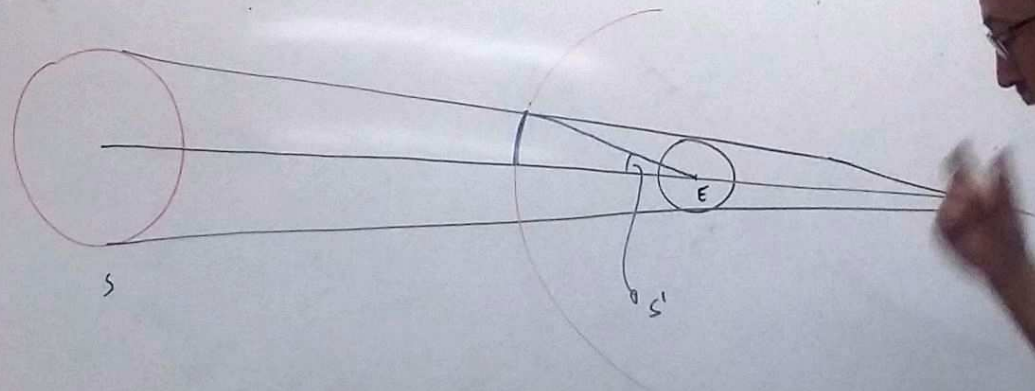
RESTO:  $\pi - S_0 =$   
 $\Rightarrow S = \pi_L +$

CONDICIONES EC. LUNA

$$\eta = \text{DIST LUNA} - \text{ANTI SOL}$$

- si  $\eta \leq S_1,02 + S_L \rightarrow$  PARCIAL
- si  $\eta \leq S_1,02 - S_L \rightarrow$  TOTAL

↑  
ATMÓSFERA



CONDICIONES

EC. LUNA

$$\eta = \text{DIST LUNA} - \text{RADI SOL}$$

$$\text{si } \eta \leq S_{1,02} + S_L \rightarrow \text{PARCIAL}$$

$$\text{si } \eta \leq S_{1,02} - S_L \rightarrow \text{TOTAL}$$

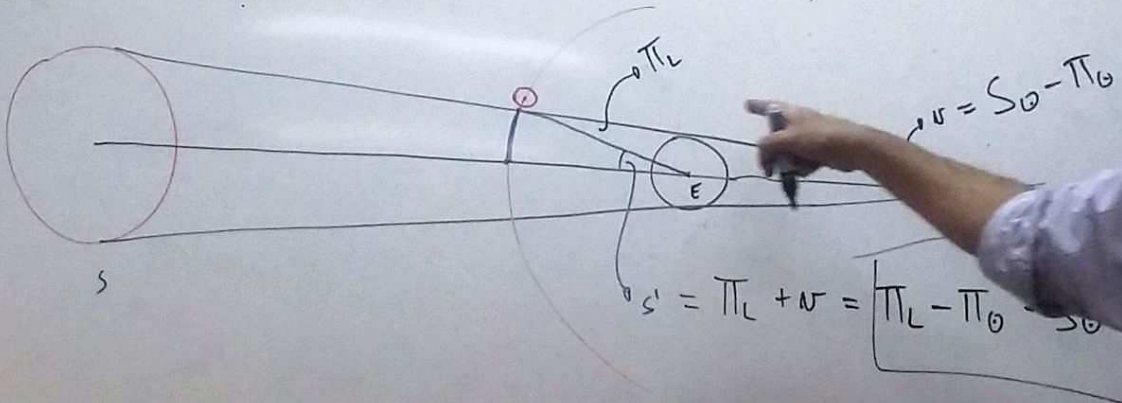
↑  
ATMÓSFERA

CONDICIÓN EC. SOL:

$$\eta = \text{DIST ANUNIM SOL} - \text{LUNA}$$

$$\eta \leq S' + S_L \rightarrow \text{PARCIAL}$$

$$\eta \leq S' - S_L \rightarrow \text{TOTAL}$$



$$s' = \pi_L + u = \pi_L - \pi_0 - S_0$$



CONDICIONES

EC. LUNA

$$\eta = \text{DIST LUNA} - \text{ANTI SOL}$$

$$\text{si } \eta \leq S_{1,02} + S_L \rightarrow \text{PARCIAL}$$

$$\text{si } \eta \leq S_{1,02} - S_L \rightarrow \text{TOTAL}$$

↑  
ATMÓSFERA

CONDICIÓN EC. SOL:

$$\eta = \text{DIST ANULUM SOL} - \text{LUNA}$$

$$\eta \leq s' + S_L \rightarrow \text{PARCIAL}$$

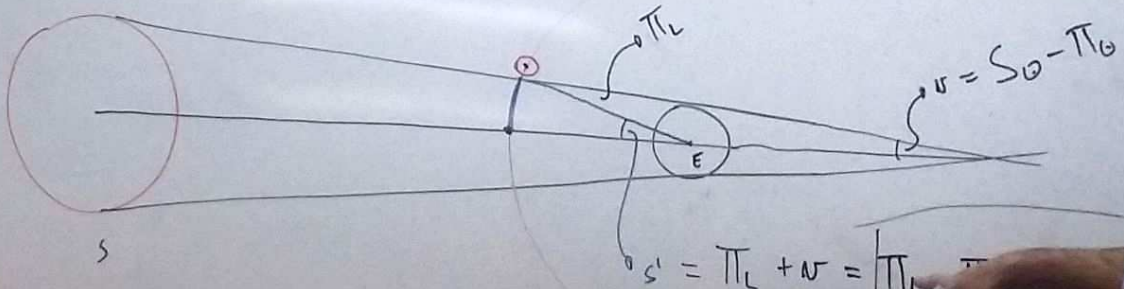
$$\eta \leq s' - S_L \rightarrow \text{TOTAL}$$

$$S_L \sim 15',5$$

$$S_0 \sim 16'$$

$$\pi_0 \sim 9'$$

$$\pi_L \sim 57'$$



CONDICIONES

EC. LUNA

$$\eta = \text{DIST LUNA} - \text{AUTISOL}$$

$$\text{si } \eta \leq s \cdot 1,02 + S_L \rightarrow \text{PARCIAL}$$

$$\text{si } \eta \leq s \cdot 1,02 - S_L \rightarrow \text{TOTAL}$$

↑  
ATMÓSFERA

$$s = \pi_L + \pi_\theta - S_\theta = 41'$$

$$54 \quad - 16'$$

$$\eta \sim 57'$$

CONDICIÓN EC. SOL:

$$\eta = \text{DIST ANUAL SOL} - \text{LUNA}$$

$$\eta \leq s' + S_L \rightarrow \text{PARCIAL}$$

$$\eta \leq s' - S_L \rightarrow \text{TOTAL}$$

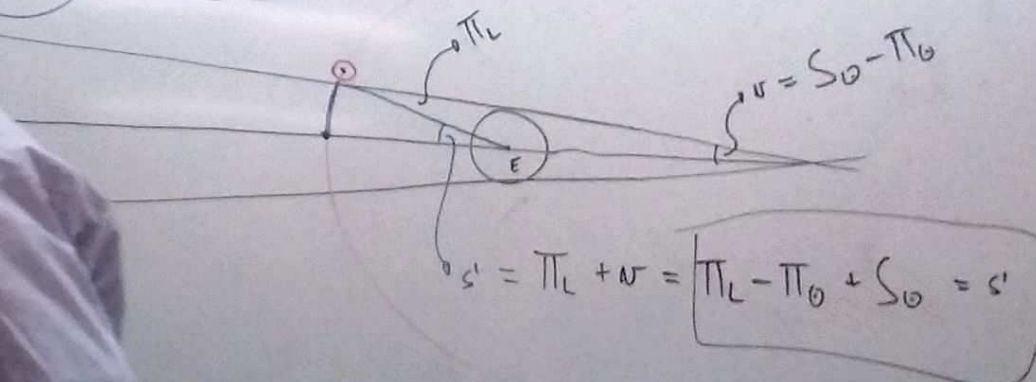
$$S_L \sim 15',5$$

$$S_\theta \sim 16'$$

$$\pi_\theta \sim 9''$$

$$\pi_L \sim 54'$$

~ 89'

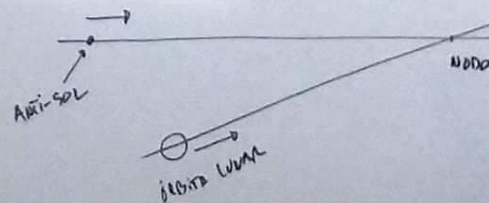




## FRECUENCIA ECLIPSES

$$N = \frac{(P_{\text{Lunas}} - 150)}{150} \times 9 + 3$$

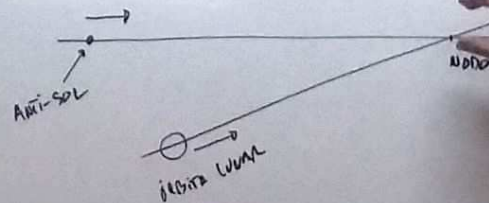
$N \geq 3 \rightarrow \text{AL TEJIDO}$   
 $N < 3 \rightarrow \text{P y T}$



FRECUENCIA ECLIPSES

$$N = \frac{(P_{LUNA} - 150)}{150} \times 9 + 3$$

$N \geq 3 \rightarrow$  AL TERCERO  
 $N < 3 \rightarrow$  P y T

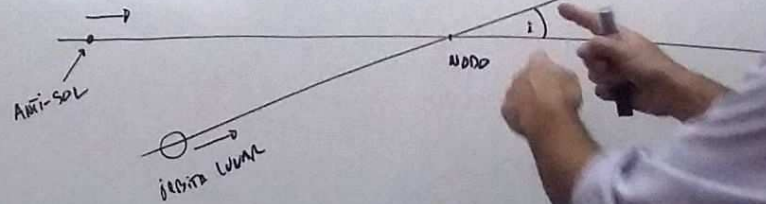


FRECUENCIA ECLIPSES

$$N = \frac{(Puntos - 150)}{150} \times 9 + 3$$

$N \geq 3 \rightarrow$  AL TERCERO  
 $N < 3 \rightarrow$  P y T

$$\omega_{Sol-Luna} \approx \frac{360^\circ}{346 \text{ días}}$$



FRECUENCIA ECLIPSES

$$N = \frac{(Puntos - 150)}{150} \times 9 + 3$$

$N \geq 3 \rightarrow$  AL TERCERO  
 $N < 3 \rightarrow$  P y T

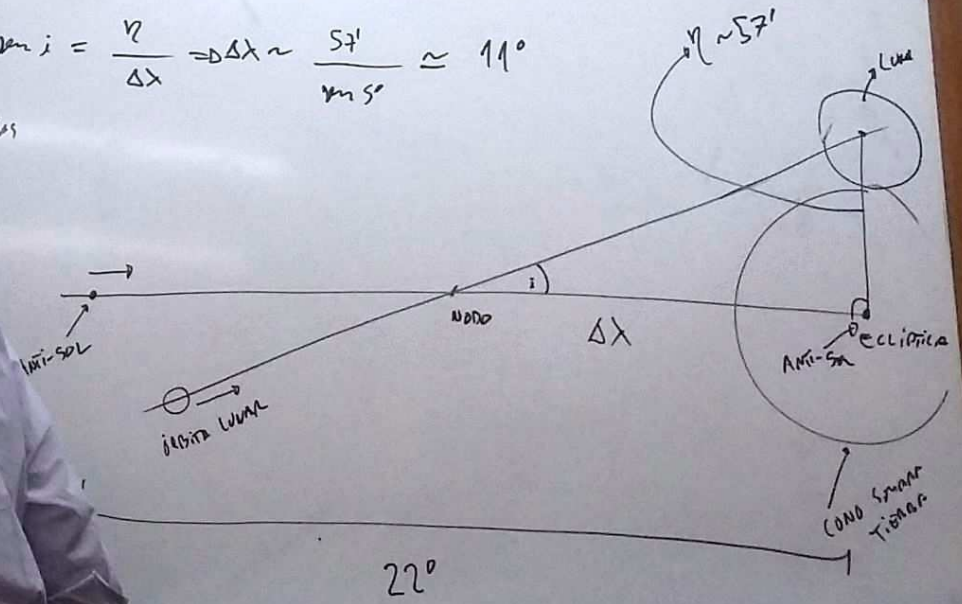
$$\tan i = \frac{r}{\Delta x} \Rightarrow \Delta x \approx \frac{57'}{\tan 5^\circ} \approx 11^\circ$$

A CADA 29 DIAS

$$\omega_{Sol-ano} \approx \frac{360^\circ}{346 \text{ dias}}$$

$$360^\circ \rightarrow 346 \text{ dias}$$

$$27^\circ \rightarrow 21 \text{ dias}$$



FRECUENCIA ECLIPSES

$$N = \frac{(PUNOS - 150)}{150} \times 5 + 3$$

$N \geq 3 \rightarrow$  AL TERCERO  
 $N < 3 \rightarrow$  P Y T

$$\text{sen } i = \frac{r}{\Delta x} \Rightarrow \Delta x \approx \frac{80'}{\text{sen } 5'} \approx 11^\circ$$

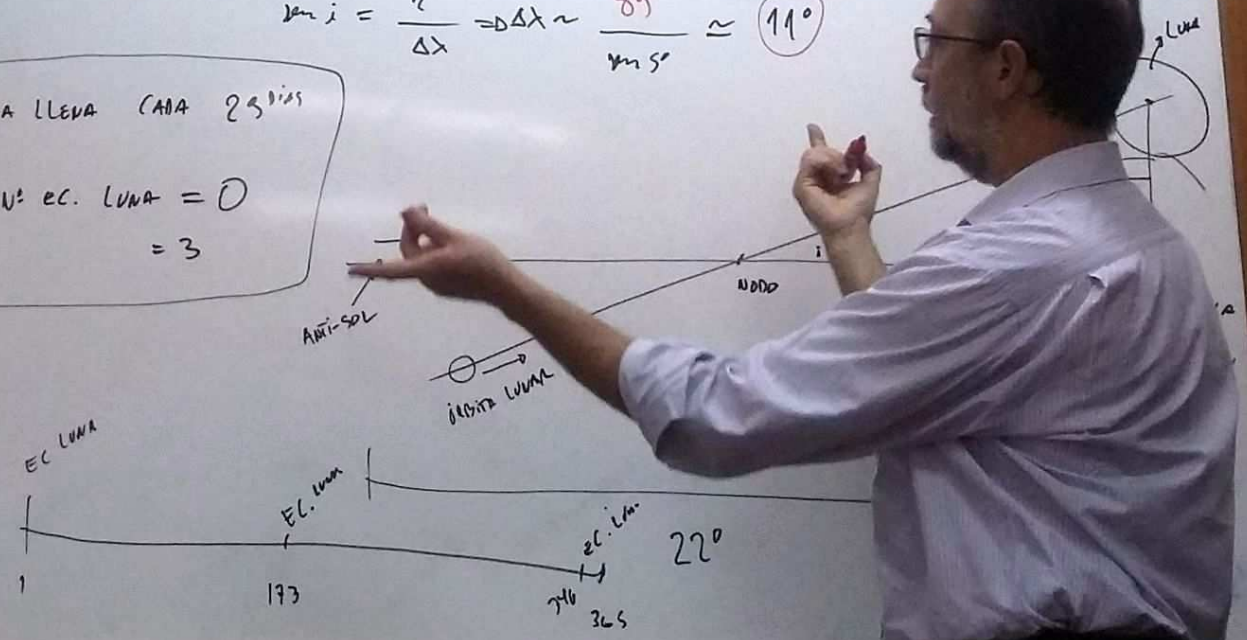
17°

LUNA LLENA CADA 29 DIAS  
 MÍN. N° EC. LUNA = 0  
 MAX. = 3

$$\omega_{\text{Sol-Luna}} \approx \frac{360^\circ}{346 \text{ DIAS}}$$

$$360^\circ \rightarrow 346 \text{ días}$$

$$29^\circ \rightarrow 21 \text{ días}$$





FRECUENCIA ECLIPSES

$$N = \frac{(Puntos - 150)}{150} \times 5 + 3$$

$N \geq 3 \rightarrow$  AL TELÉFONO  
 $N < 3 \rightarrow$  P y T

LUNA LLENA CADA 29 días

MÍN. N° EC. LUNA = 0

MAX. = 3

EC. SOL

MÍN  $\rightarrow$  2

MAX  $\rightarrow$  5

$$\tan i = \frac{r}{\Delta \lambda} \Rightarrow \Delta \lambda \approx \frac{89'}{\tan 50'} \approx 110' \approx 11^\circ$$

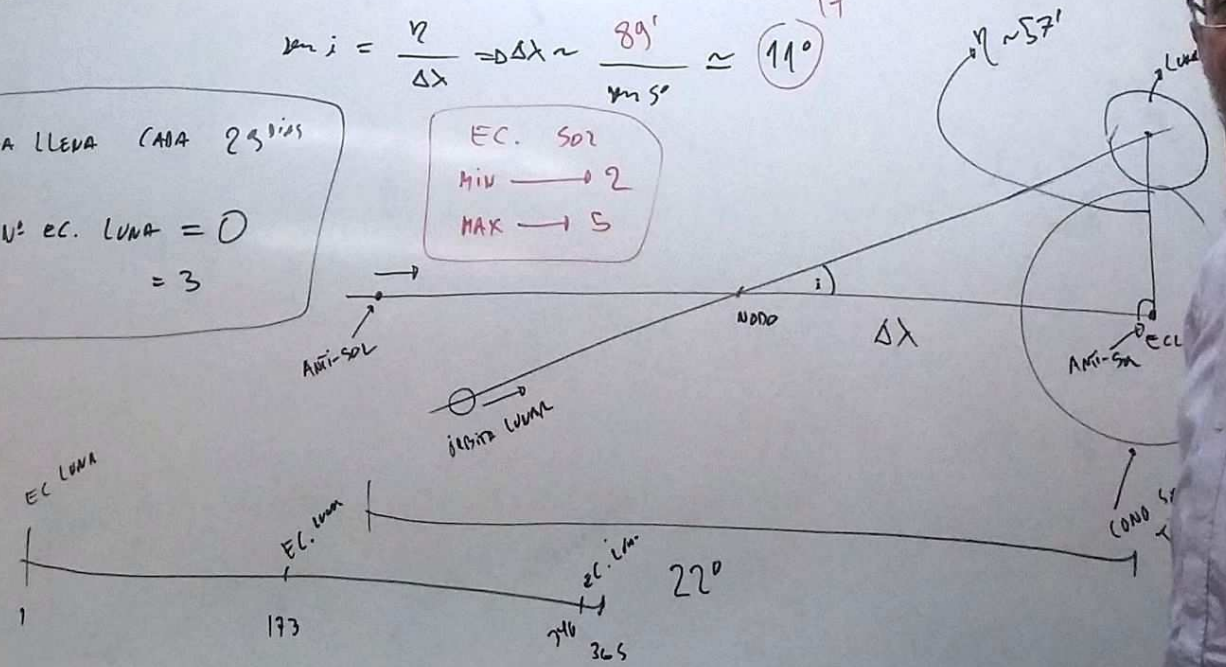
<sup>17°</sup>

$\omega_{Sol-Luna} \approx \frac{360^\circ}{346 \text{ días}}$

360°  $\rightarrow$  346 días

27°  $\rightarrow$  21 días

34°  $\rightarrow$  32 días



$$N = \frac{(P_{\text{LUNA}} - 150)}{150} \times 5 + 3$$

$N \geq 3 \rightarrow$  AL TERCERA  
 $N < 3 \rightarrow$  P y T

$$|\beta_*| \leq 6^\circ 16'$$

FREE ECLIPSES

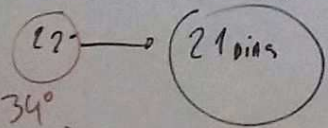
$$v_{\text{LUNA}} = \frac{v}{\Delta x} \Rightarrow \Delta x \sim \frac{8}{v_{\text{LUNA}}} \sim 57'$$

LUNA LLENA CADA 29 DIAS  
 MÍN. N° EC. LUNA = 0  
 MAX. = 3

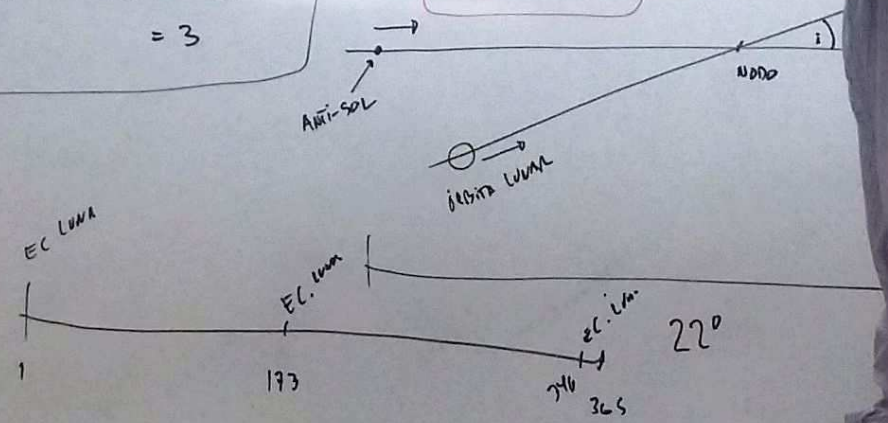
EC. SOL  
 MÍN  $\rightarrow$  2  
 MAX  $\rightarrow$  5

$$W_{\text{Sol-LUNA}} \approx \frac{360^\circ}{346 \text{ DIAS}}$$

360° — 346 días



34° — 32 días



VIII

①  $TSG = 2^h$

$$\alpha_L = 6^h$$

$$d_L = 0.9$$

$$\alpha_* = 6^h 1^m$$

$$d_* = 0^\circ$$

$$\pi_L = 1^\circ$$

VIII

① TSG = 2<sup>h</sup>  
 $\alpha_L = 6^h$   
 $d_L = 0.9$   
 $\alpha_* = 6^h 1^m$   
 $d_* = 0^\circ$   
 $\pi_L = 1^\circ$   
 $i(\phi, \lambda)?$   
 CENTRAL

$$x = \sin d_L \cdot \sin(d_L - d_*) / \sin \pi_L \rightarrow x$$

$$y = (\sin d_L \cos d_* - 0) / \sin \pi_L \rightarrow y$$

$$\left. \begin{matrix} x = x \\ y = y \end{matrix} \right\} \Rightarrow \left( \frac{p}{R_\oplus} \right) \cdot \cos \phi \cdot \sin H_* = x$$

$$1 \cdot \sin \phi \cdot \cos d_* - 0 = y$$

$$\Rightarrow \sin \phi = y \Rightarrow \phi = +5.739$$

$$\sin H_* = \frac{x}{\cos \phi} = -0.2513 \Rightarrow H_* = -14.552$$



VIII

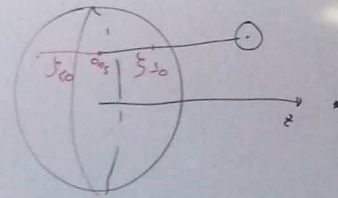
① TSG = 2<sup>h</sup>  
 $\alpha_L = 6^h$   
 $d_L = 0.9$   
 $\alpha_* = 6^h 1^m$   
 $d_* = 0^\circ$   
 $\pi_L = 1^\circ$   
 $i(\phi, \lambda)?$   
 CENTRAL

$$x = \sin d_L \cdot \sin(\alpha_L - \alpha_*) / \sin \pi_L \rightarrow (x)$$

$$y = (\sin d_L \cdot \cos \alpha_* - 0) / \sin \pi_L \rightarrow (y)$$

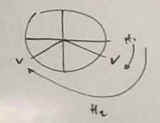
$$\left. \begin{matrix} \xi = x \\ \eta = y \end{matrix} \right\} \Rightarrow \left[ \frac{p}{R_\oplus} \cdot \cos \phi \cdot \sin H_* = (x) \right]$$

$$\rightarrow 1 \cdot \sin \phi \cdot \cos d_* - 0 = (y)$$

$$\Rightarrow \sin \phi = y \Rightarrow \phi = +5,739$$


$$\sin H_* = 0.2513$$

$$\begin{cases} H_* = -14,552 \\ H_* = -165,44 \end{cases}$$



$\zeta =$

VIII

① TSG = 2<sup>h</sup>

$\alpha_L = 6^h$

$d_L = 0.9$

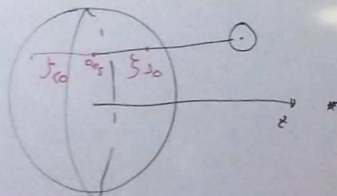
$\alpha_* = 6^h 1^m$

$d_* = 0^\circ$

$\pi_L = 1^\circ$

$i(\phi, \lambda)?$

CENTRAL



$x = \sin d_L \cdot \sin(\alpha_L - \alpha_*) / \sin \pi_L \rightarrow (x)$

$y = (\sin d_L \cos \alpha_* - 0) / \sin \pi_L \rightarrow (y)$

$\left. \begin{matrix} \xi = x \\ \eta = y \end{matrix} \right\} \Rightarrow \left[ \frac{\rho}{R_\oplus} \cdot \cos \phi \cdot \sin H_* = (x) \right]$   
 $\rightarrow 1 \cdot \sin \phi \cdot \cos d_* - 0 = (y)$

$\Rightarrow \sin \phi = \eta \Rightarrow \phi = +5.739^\circ$

$\sin H_* = \frac{x}{\cos \phi} = -0.2513$

$\begin{cases} H_1 = -14.552^\circ \\ H_2 = -165.44^\circ \end{cases}$

$\zeta = (\cos \phi) \cdot 1 \cdot \cos H_* > 0$  (OC. VISIBLE)

$\Rightarrow \cos H_* > 0$

~~$H_2$~~

$H_1 = -14.552^\circ$

$TSL = \alpha_* + H_* = TSG + \gamma \Rightarrow$

VIII

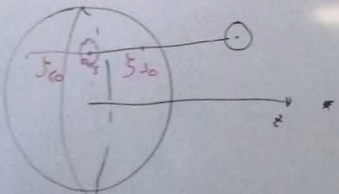
$\delta G = 2^h$   
 $\delta L = 6^h$   
 $\delta L = 0.9$   
 $\delta L = 6^h 1^m$   
 $\delta L = 0^\circ$

$$x = \sin \delta_L \cdot \sin(\alpha_L - \alpha_{\odot}) / \sin \pi_L \rightarrow x$$

$$y = (\sin \delta_L \cos \delta_{\alpha} - 0) / \sin \pi_L \rightarrow y$$

$$\left. \begin{matrix} \xi = x \\ \eta = y \end{matrix} \right\} \Rightarrow \left( \frac{p}{R_{\oplus}} \right) \cdot \cos \phi \cdot \sin H_{\alpha} = x$$

$$\Rightarrow 1 \cdot \sin \phi \cdot \cos \delta_{\alpha} - 0 = y$$



$$\sin \phi = y \Rightarrow \phi = +5.739^\circ$$

$$\sin H_{\alpha} = \frac{x}{\cos \phi} = -0.2513$$

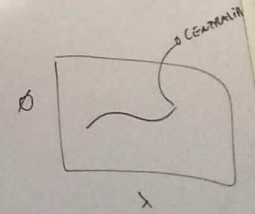
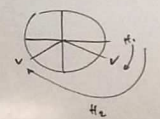
$$\begin{cases} H_1 = -14.552^\circ \\ H_2 = -165.44^\circ \end{cases}$$

$$\zeta = (\cos \phi) \cdot 1 \cdot \cos H_{\alpha} > 0 \quad (\text{OC. VISIBLE})$$

$$\Rightarrow \cos H_{\alpha} > 0 \rightarrow H_2$$

$$H_1 = -14.552^\circ$$

$$TSL = \alpha_{\alpha} + H_{\alpha} = TSG + \gamma \Rightarrow \lambda = 45.699^\circ$$



VIII

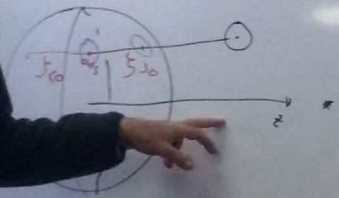
6) ecuaciones

$$x = \sin \delta_c \cdot \sin(\alpha_c - \alpha_*) \rightarrow (x)$$

$$y = (\sin \delta_c \cos \delta_* - 0) / \sin \pi_c \rightarrow (y)$$

$$\left. \begin{matrix} \xi = x \\ \eta = y \end{matrix} \right\} \Rightarrow \left( \frac{p}{R_0} \right) \cdot \cos \phi \cdot \sin H_* = (x)$$

$$\Rightarrow 1 \cdot \sin \phi \cdot \sin \delta_* - 0 = (y)$$



$$\Rightarrow \sin \phi = y \Rightarrow \phi = +5,73^\circ$$

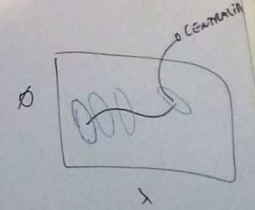
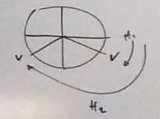
$$\sin H_* = \frac{x}{\cos \phi} = -0,2513$$

$$\begin{cases} H_* = -14,552 \\ H_* = -165,44 \end{cases}$$

$$\zeta = (\cos \phi) \cdot 1 \cdot \cos H_* > 0 \quad (\text{oc. visible})$$

$$\Rightarrow \cos H_* > 0 \rightarrow H_* = -14,552$$

$$TSL = \alpha_* + H_* = TSG + \lambda \Rightarrow \lambda = 45,699$$





VIII

ECLIPSE SOL

= \*

$$= \cos \delta_c \cdot \sin(\alpha_c - \alpha_e) / \sin \pi_c = 0$$

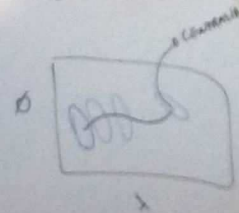
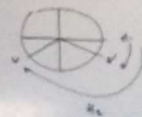
$$\sin(\alpha_c - \alpha_e) / \sin \pi_c$$

$$\rightarrow H_e = \frac{x}{\cos \phi} = -0.2513 \begin{cases} H_e = -14^{\circ}, 552 \\ H_e = -165^{\circ}, 44 \end{cases}$$

$$\zeta = (\cos \phi) \cdot \cos H_e > 0 \quad (\text{OC. VISIBILE})$$

$$\Rightarrow \cos H_e > 0 \rightarrow \begin{cases} H_e \\ H_e = -14^{\circ}, 552 \end{cases}$$

$$TSL = \alpha_e + H_e = TSG + \lambda \Rightarrow \lambda = 45^{\circ}, 698$$



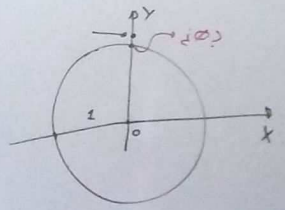
VIII

6 ECLIPSE SOL

SOL = \*

$$X = \cos d_c \cdot \sin(d_c - d_o) / \sin \pi_c = 0$$

$$Y = \sin(d_c - d_o) / \sin \pi_c > 1$$



MAGNITUDE ECLIPSE

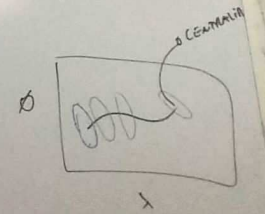
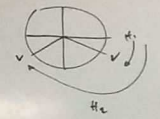


$$\rightarrow \sin H_1 = \frac{x}{\cos \phi} = -0.2513 \begin{cases} H_1 = -14^\circ, 552 \\ H_2 = -165^\circ, 44 \end{cases}$$

$$\zeta = (\cos \phi) \cdot \cos H_1 > 0 \quad (\text{OC. VISIBLE})$$

$$\Rightarrow \cos H_1 > 0 \rightarrow \begin{cases} H_1 = -14^\circ, 552 \\ H_2 = \text{crossed out} \end{cases}$$

$$TSL = \alpha_x + H_x = TSG + \lambda \Rightarrow \lambda = 45^\circ, 699$$



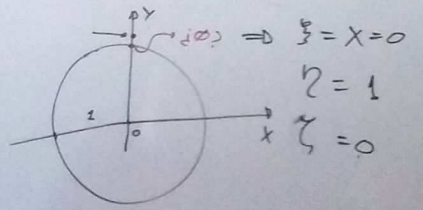
VIII

6 ECLIPSE SOL

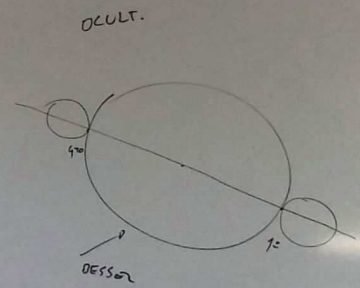
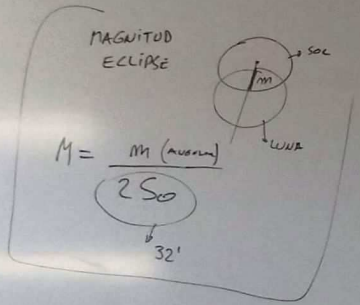
SOL = \*

$$X = \cos d_c \cdot \sin(d_c - d_o) / \sin \pi_i = 0$$

$$Y = \sin(d_c - d_o) / \sin \pi_i > 1$$



$$\left. \begin{aligned} \xi = X = 0 \\ \eta = 1 \\ \zeta = 0 \end{aligned} \right\} \Rightarrow \theta = 67^\circ 15'$$



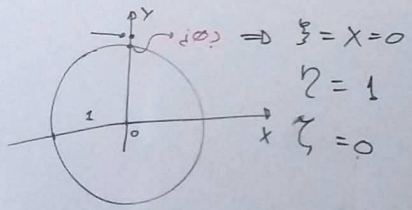
VIII

6) ECLIPSE SOL

SOL = \*

$$X = \cos \delta_L \cdot \sin(\alpha_L - \alpha_S) / \sin \pi_L = 0$$

$$Y = \sin(\alpha_L - \alpha_S) / \sin \pi_L > 1$$



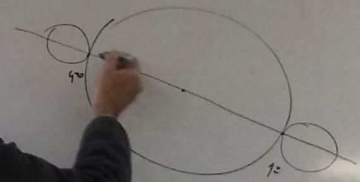
$$\left. \begin{aligned} \xi = X = 0 \\ \eta = 1 \\ \zeta = 0 \end{aligned} \right\} \Rightarrow \theta = 67^\circ 15'$$

MAGNITUD ECLIPSE

$$M = \frac{m(\text{angular})}{2S_{SOL}}$$

32'

OCULT.



DIST ANGULAR LUNA - ESTRELLA

$$\eta \leq \pi_L + S_L$$

CONDICION OCULTACION

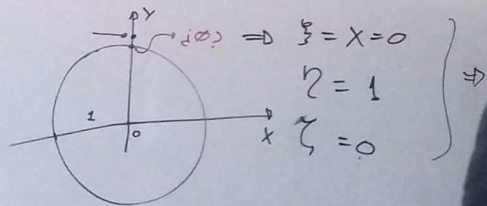
VIII

⑥ ECLIPSE SOL

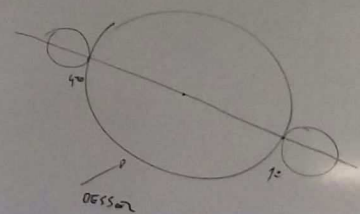
SOL = \*

$$X = \cos d_c \cdot m (d_c - d_0) / m \pi_c = 0$$

$$Y = m (d_c - d_0) / m \pi_c > 1$$



OCULT.



BIGT ANGLE LUNA - ESTRELLA

$$\eta \leq \pi_c + S_c$$

CONDICION OCULTACION

MAX TRAYECTORIA =  $2\eta \approx 2^\circ 25'$

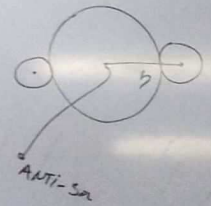
$\Delta t$  PARA QUE  $\Delta \lambda_c \sim 2^\circ 25' ?$

$$27,5 \rightarrow 360^\circ$$

$$X \rightarrow 2^\circ 25'$$

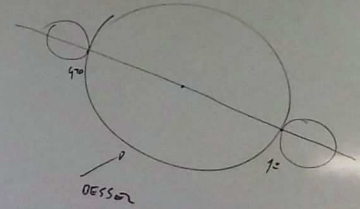
VIII

EC. LUNA :  $\eta \sim 58'$   $\Rightarrow \Delta t$  para recorrer  $2\eta$



$29,5 \rightarrow 360^\circ$   
 $2\eta \rightarrow X$

OCCULT.



DIST ANGULO LUNA - ESTRELLA

$$\eta \leq \pi_L + S_L$$

CONDICION OCCULTACION

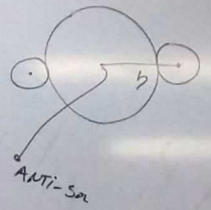
MAX TRAYECTORIA =  $2\eta \cong 2^\circ 25'$

$\Delta t$  PARA QUE  $\Delta \lambda_L \sim 2^\circ 25' ?$

$29,5 \rightarrow 360^\circ$   
 $X \rightarrow 2^\circ 25'$

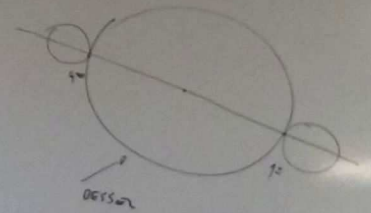
VIII

EC. LUNA :  $\eta \sim 58'$   $\Rightarrow \Delta t$  para recorrer  $2\eta$



$29,5 \rightarrow 360^\circ$   
 $2\eta \rightarrow x$

OCULT.



OC  $\sim 4,4^{\text{H}}$   
 EC. SOL  $\sim 5,8$   
 EC. LUNA  $\sim 3,7$

DIST. ANOM. LUNA - ESTRELLA

$$\eta \leq \pi_L + S_L$$

CONDICION OCULTACION

MAX TRAYECTORIA =  $2\eta \approx 2^\circ 25'$

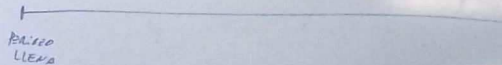
$\Delta t$  para que  $\Delta \lambda_L \sim 2^\circ 25' ?$

$29,5 \rightarrow 360^\circ$   
 $x \rightarrow 2^\circ 25'$

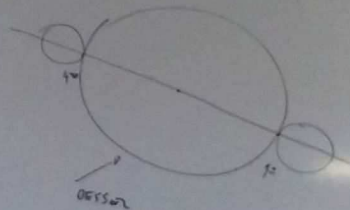
VIII

$$P_{sin} \sim 29.5$$

$$P_{anom} \sim 27.5$$



OCCULT.



$$OC \sim 4.4^{\prime\prime}$$

$$EC. SOL \sim 5.8^{\prime\prime}$$

$$EC. LUNA \sim 3.7^{\prime\prime}$$

DIET ANOMIA LUNA - ESTRELLA

$$\eta \leq \pi_L + S_L \quad \text{CONDICION OCCULTACION}$$

$$\text{MAX TRAYECTORIA} = 2\eta \approx 2^{\circ} 25'$$

$$\Delta t \text{ para que } \Delta \lambda_L \sim 2^{\circ} 25' ?$$

$$27.5 \rightarrow 360^{\circ}$$

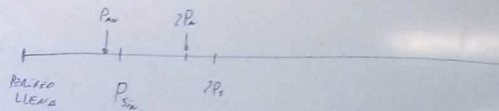
$$x \rightarrow 2^{\circ} 25'$$



VIII

$$P_{sin} \sim 29.5$$

$$P_{anom} \sim 27.5$$



$\sim 14$  Pseudis

$$NP_s \approx (N+1)P_a$$

$$N(P_s - P_a) \approx P_a$$

$$N \approx \frac{P_a}{P_s - P_a}$$

(5) 18 años y 11 días

26/2/2017

DIST ANGLAR LUNA-ESTRELLA

$$\eta \leq \pi_L + S_L$$

CONDICIÓN OCULTACIÓN

$$\text{MAX TRAYECTORIA} = 2\eta \approx 2^\circ 25'$$

$\Delta t$  PARA QUE  $\Delta \lambda_L \sim 2^\circ 25'?$

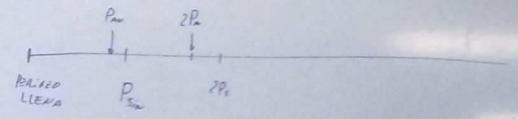
$$27.5 \rightarrow 360^\circ$$

$$x \rightarrow 2^\circ 25'$$

VIII

$P_{sin} \sim 29.5$

$P_{max} \sim 27.5$



$$NP_s \approx (N+1)P_a$$

$$N(P_s - P_a) \approx P_a$$

$$N \approx \frac{P_a}{P_s - P_a} \approx 14$$

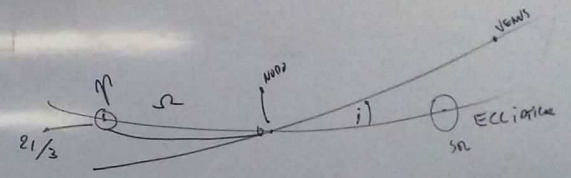
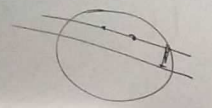
$\sim 14$  P<sub>sin</sub> ciclos

(5) 18 años y 11 días

26/2/2017

15/2/1999

(7) TRANSITO:



$\Omega_v \approx 77^\circ$

$\lambda_0 = \Omega_v + 180$