3. Key Features of Apparatus

Although the Millikan oil drop experiment is very classical, the conventional oil drop apparatus is very hard to operate. To address this issue, we developed this advanced oil drop apparatus consisting of CCD sensing microscope, illumination lamp, LCD monitor display, oil chamber, high-voltage supply, and microprocessor electronics. Hence, this modular oil drop apparatus possesses the following features:

<u>Multiple integrated display technology</u>: by displaying voltage, time, and oil drop images on one monitor screen, the experimental operation is intuitive and simplified.

<u>Electronic scale on screen</u>: this is a dark-field method, i.e. optical illumination on reticle is not needed, so the image of oil drops is bright and clear. It is more accurate than a conventional scale mask without distortion and viewing error.

<u>Multifunctional:</u> a high magnification objective lens is provided for the observation of tiny oil drops that cannot be seen with conventional oil drop apparatus. Using a higher magnification objectives lens, this apparatus can be used to perform "Brownian motion" experiment.

<u>Synchronized voltage and timing operation</u>: voltage applying can be synchronized with the electronic timer for the simplification of manual operation.

<u>Reliable and safe design</u>**:** Microprocessor offers high accuracy for both time and voltage measurements. LED light source has a long lifetime. High voltage protective switch provides safety to users.

4. Apparatus Structure

This oil drop apparatus consists of oil drop chamber, CCD TV microscope, monitor screen, and electrical control unit. Among them, the oil drop chamber is a critical part with a schematic diagram shown in Figure 1.

As seen in Figure 1, the upper and lower electrodes are parallel plates with a bakelite ring placed between them to ensure the parallelism between electrodes with high spacing accuracy (better than 0.01 mm). There is an oil mist hole (0.4 mm in diameter) in the center of the upper electrode. On the wall of the bakelite ring, there are three through holes, one for microscope objective, one for light passing and one as a spare hole.

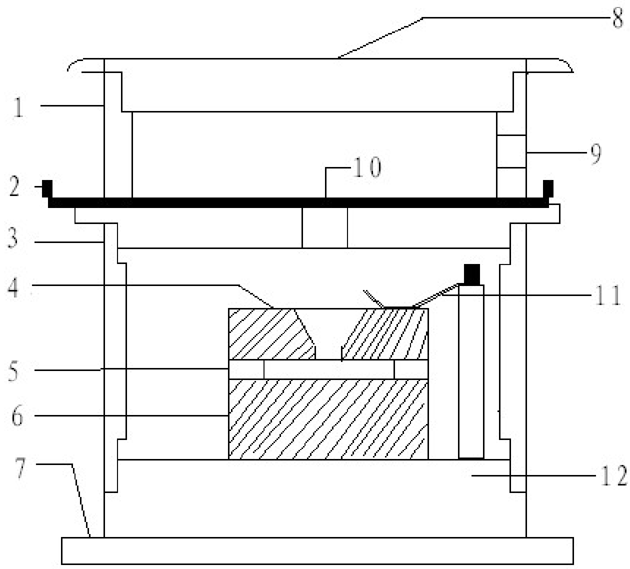

Figure 1 Schematic diagram of oil drop chamber

1–oil mist cup	2–oil shutter	3–windscreen
4–upper electrode	5–oil drop channel	6–lower electrode
7–base stand	8–cover	9–oil spray aperture
10–oil mist aperture	11–electrode clamp	12–oil channel base

The oil drop cavity is covered by a wind proof screen on which a removable oil mist cup is placed. There is an oil drop aperture at the bottom of the oil mist cup and the oil drop aperture can be blocked with a stop.

The upper electrode is secured with a clamp which can be toggled left and right for the removal of the upper electrode. Two illumination lamps are located on the side of the oil channel base with light focusing mechanism. The intersection angle between the illumination light and the microscope objective is about 150° ~ 160° that is designed based on light scattering theory to achieve high brightness. Unlike a conventional Tungsten filament bulb that can be blown out over time, focusing LEDs are used in this apparatus with extended lifetime. The CCD camera microscope is designed in an integral structure with stable and reliable operation. A schematic diagram of the upper panel of the main unit is given in Figure 2. Electronic scale is generated in synchronization with the video signal of the CCD camera, thus ensuring stable scale reading independent of monitor settings.

There are two sets of electronic scales, i.e. A and B. The former has 8 divisions in vertical direction and each division is 0.25 mm when the 60× objective is used; the latter is designed to observe Brownian movement of oil drops, with 15 small divisions in both X and Y directions. The division is 0.08 mm when the 60× objective is used; and 0.04 mm when the 120× objective is used. Scale B can be toggled on or off by pressing button “Timing Start/Stop” for more than 5 seconds.

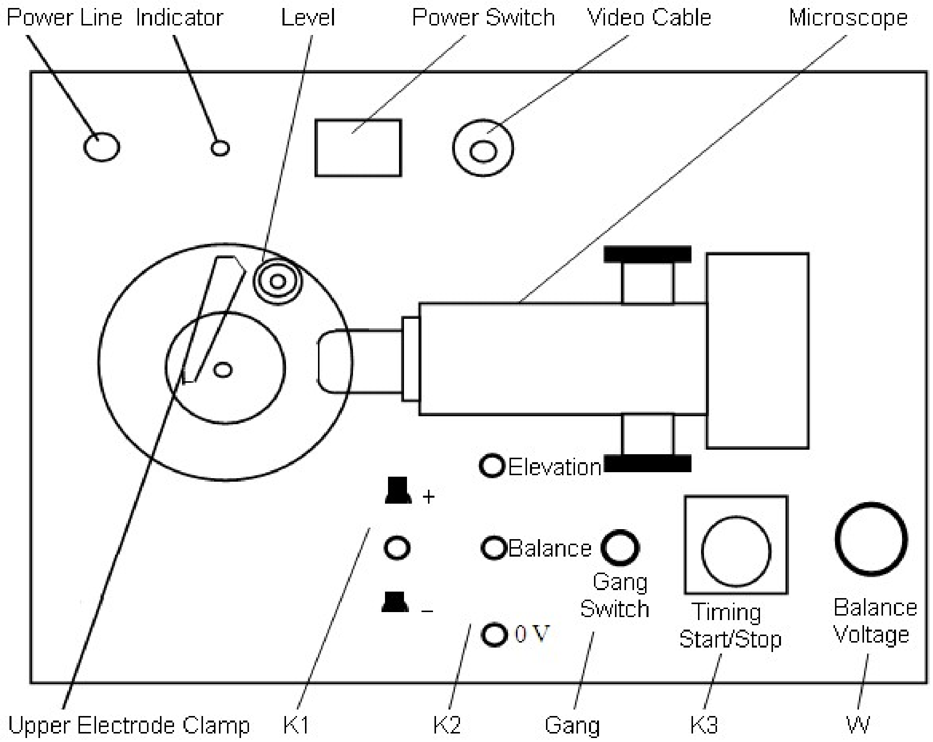

Figure 2 Schematic of the upper panel of main unit

There are two groups of push selector switches (K_2 has 3 selection buttons) for controlling the voltage between the parallel electrodes. K_1 controls the voltage polarity applied to the upper electrode while K_2 controls the voltage value. When "Balance" button is pressed down, the potentiometer W can be used to adjust the balance voltage (adjustable range 0~400 V); when "Elevation" is selected, a fixed elevating voltage between 200 V and 300 V is added to the balance voltage; when "0 V" is selected, voltage on the electrode is set to 0 V.

To increase measurement accuracy, "Balance" and "0 V" can be operated simultaneously with timing switch "Timing Start/Stop" when the "Gang Switch" is pressed down. Under this case, when selector switch K_2 changes from "Balance" to "0 V", oil drop begins to fall at uniform velocity and timing starts simultaneously. When oil drop reaches the preset distance, K_2 should be changed from "0 V" to "Balance". Immediately, the oil drop stops falling and timing ends simultaneously. The distance the oil drop travelled and the time spent are shown on the screen. Alternatively, K_2 and K_3 can be operated independently if "Gang Switch" is released.

Due to air resistance, oil drops fall with variable speed initially before moving at uniform speed. Since the duration of the initial motion is very short (< 0.01 s), oil drops can be considered to undertake motion with uniform speed immediately from a stationary state. On the other hand, when the balance electric field is applied, the motion of oil drops will stop immediately so that oil drops go back to the stationary state.

To time the motion of oil drops, the "Timing" button should be pressed once (reset and start); if the button is pressed again, then timing is stopped.

5. Theory

A. Millikan's oil drop

If an oil drop with mass m and charge e enters the oil chamber, it falls down freely under the force of gravity when no voltage is applied between the parallel polar plates. When the force of gravity is balanced against air resistance (neglecting air buoyancy), the oil drop falls down at a uniform speed of V_g, as described by the following equation:

$$mg = f_a \tag{1}$$

where f_a is the air resistance when the oil drop falls down at a uniform speed of V_g, and g is the gravitational constant. According to Stokes' law, the air resistance exerted on the oil drop is

$$f_a = 6\pi\eta a V_g \tag{2}$$

While the force of gravity exerted on the oil drop is

$$G = mg = \frac{4}{3}\pi a^3 \rho g \tag{3}$$

where η is the coefficient of viscosity for air; ρ is the density of oil drop; and a is the radius of the oil drop. By combining equations (1) to (3), the radius of the oil drop can be derived as:

$$a = 3\sqrt{\frac{\eta V_g}{2g\rho}} \tag{4}$$

When an electric field is applied across the parallel polar plates, the oil drop is subject to the force of the electric field. If the strength and polarity of the electric field are controlled and selected properly, the oil drop can be elevated under the influence of the electric field. If the force of the electric field exerted on the oil drop is balanced against the sum of the force of gravity and the resistance of air exerted on the oil drop, the oil drop will rise at a uniform speed of V_e. Hence, we have

$$mg + f_a^{'} = qE \tag{5}$$

where E is the strength of the electric field applied to the parallel polar plates and q is the electric charge on the oil drop.

From equations (1) and (2), equation (5) can be rewritten as

$$mg + f_a^{'} = f_a + f_a^{'} = 6\pi\eta a(V_g + V_e) = qE = q\frac{V}{d} \tag{6}$$

Thus,

$$q = \frac{6\pi\eta\, ad(V_g + V_e)}{V} \tag{7}$$

where V and d are the voltage and separation between the parallel polar plates, respectively.

Equation (7) is derived assuming the oil drop is in a continuous medium. In this experiment, the radius of the oil drop is as small as 10^{-6} m and therefore it is comparable to air molecules.

Thus, air is no longer considered as a continuous medium, and viscosity coefficient η of the air needs to be corrected as

$$\eta' = \frac{\eta}{1+\frac{b}{pa}} \tag{8}$$

where b is a correction constant; p is the atmospheric pressure; a is the radius of oil drop as given in equation (4). By assuming the falling and rising distance of an oil drop with uniform speed are identical as l and the spent time are t_g and t_e, respectively, we have $V_g = l/t_g$ and $V_e = l/t_e$. By substituting (8) and (4) into (7), we have:

$$q = \frac{18\pi d}{V\sqrt{2\rho g}}\left(\frac{\eta l}{1+\frac{b}{pa}}\right)^{3/2}\left(\frac{1}{t_e}+\frac{1}{t_g}\right)\left(\frac{1}{t_g}\right)^{1/2} = \frac{K}{V}\left(\frac{1}{t_e}+\frac{1}{t_g}\right)\left(\frac{1}{t_g}\right)^{1/2} \tag{9}$$

where $K = \frac{18\pi d}{\sqrt{2\rho g}}\left(\frac{\eta l}{1+\frac{b}{pa}}\right)^{3/2}$

The above formula is used to calculate charge quantity in dynamic (unbalanced) measurement method. In static (balanced) measurement method, we adjust the voltage between the parallel electrodes to hold the oil drop stationary ($V_e = 0$, $t_e \to \infty$). Now equation (9) becomes:

$$q = \frac{K}{V}\left(\frac{1}{t_g}\right)^{3/2} \tag{10}$$

B. Brownian motion (optional)

Particles suspended in a fluid undertaking irregular motion are known as Brownian motion. In theory, the probability of displacement between ξ and $\xi+d\xi$ of a Brownian particle is:

$$\varphi(\xi)\, d\zeta = \frac{1}{\lambda\sqrt{2\pi}} \mathrm{e}^{-\xi^2/2\lambda^2}\, d\xi \tag{11}$$

where ξ is the displacement of a particle rather than the coordinate of a particle at a certain time, and $\varphi(\xi)$ is known as displacement probability density function of a Brownian particle.

If the Brownian motion of a particle is projected in x axis, we have:

$$\varphi(x)\,\Delta_x = \frac{1}{\lambda\sqrt{2\pi}} e^{-x^2/2\lambda^2}\,\Delta_x \tag{12}$$

where $\varphi(x)\Delta_x$ is the probability of the particle located between x and $x+\Delta_x$ within a certain time period; λ^2 is equal to the average value of the square of Brownian particle displacement x, i.e. $\lambda^2 = \overline{x^2}$.

$$\overline{x^2} = \frac{\int_{-\infty}^{\infty} x^2 \varphi(x)dx}{\int_{-\infty}^{\infty} \varphi(x)dx} = \int_{-\infty}^{\infty} x^2 \varphi(x)dx = \int_{-\infty}^{\infty} \frac{x^2}{\lambda\sqrt{2\pi}} e^{-\frac{x^2}{2\lambda^2 dx}} \tag{13}$$

As $\int_{-\infty}^{\infty} e^{-Ax^2} dx = \sqrt{\pi / A}$ and $\int_{-\infty}^{\infty} x^2 e^{-Ax^2} dx = \frac{\sqrt{\pi}}{2}(A)^{3/2}$ while assuming A=1/2λ^2, equation (13) can be rewritten as:

$$\overline{x^2} = \frac{1}{\lambda\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-x^2/2\lambda^2} dx = \frac{1}{\lambda\sqrt{2\pi}} \frac{\sqrt{\pi}}{2} (2\lambda^2)^{3/2} = \lambda^2 \tag{14}$$

Brown particles are impacted by three forces: the collision of irregular motions of molecules, F=(X, Y, Z); the motion resistance in fluid, -βv, which is proportional to the particle speed in reversal direction; the gravity and buoyancy, $+mg(1-\rho_0/\rho)K$, where m and ρ are the mass and density of the Brownian particle, respectively, ρ_0 is the density of the fluid medium, and K is the unit vector in gravity field direction.

The particle motion in this experiment is considered as the projection in horizontal direction x perpendicular to K, so gravity and buoyancy have no effect. Thus, particle motion equation is:

$$m\ \ddot{x} = -\beta \dot{x} + X \tag{15}$$

As $\chi\dot{\chi} = \frac{1}{2}\frac{d}{dt}(x^2)$, we get $\frac{1}{2}\frac{d^2}{dt^2}(mx^2) - m\dot{x}^2 = m\ddot{x}x$. Thus, equation (15) can be rewritten as:

$$\frac{1}{2}\frac{d^2}{dt^2}(mx^2) - m\dot{x}^2 = xX - \frac{1}{2}\beta\frac{d}{dt}x^2 \tag{16}$$

Assuming many particles with identical radius a and mass m, they all share equation (16). By summing these equations divided by the number of particles, we get:

$$\frac{1}{2}\frac{d^2}{dt^2}(m\overline{x^2}) - m\overline{\dot{x}^2} = \overline{xX} — \frac{1}{2}\beta\frac{d}{dt}\overline{x^2} \tag{17}$$

Since the probability of positive and negative signs of same value X are equal, we have $\overline{Xx}$=0. Furthermore, $m\overline{\dot{x}^2}$ of many particles should be equal to that of the average value $m\dot{x}^2$ of a single molecule, i.e., the molecule and a large particle is in thermal equilibrium with each other. In the equilibrium state of irregular movement, the average kinetic energies of molecules and particles are equal, so particles can be considered as giant molecules but the motion velocity is inversely proportional to the square root of its mass. Since $m\overline{\dot{x}^2/2} = kT/2$, equation (17) can be rewritten as:

$$\frac{d^2}{dt^2}\overline{x^2} + \frac{\beta}{m}\frac{d}{dt}\overline{x^2} - \frac{2kT}{m} = 0 \tag{18}$$

Thus

$$\overline{x^2} = \frac{2kT}{\beta}t + C_1 e^{\frac{\beta}{m}} + C_2 \tag{19}$$

where k is Boltzmann's constant, T is temperature in Kelvin, C_1 and C_2 are integral constants, β/m is a great value which can be determined by fluid mechanics. Since Brownian particles are all small, they can be considered as spheres having same radius a. As a result, we can assume that the motion resistance they are subject to is the same as that of a ball slowly moving in a viscous fluid. Based on Stokes law, the force on the particle is: $f=6\pi\eta a v=\beta v$.

Since $a\cong 10^{-4}$ cm and the viscosity of water $\eta\cong 10^{-2}$ P_a, we estimate $\frac{\beta}{m}=\frac{6\pi a\eta}{\frac{4}{3}\pi a^3\rho}=\frac{9\eta}{2a^2\rho}\approx 10^7$.

After a short period of time (e.g. $t>10^{-6}$ s), the exponential term in equation (19) can be ignored ($e^{-\frac{\beta}{m}t}<e^{-10^{7}\cdot 10^{-8}}=e^{-10}$). Thus, equation (19) can be simplified as:

$$\overline{x^2}=\frac{2kT}{\beta}t+C_2 \tag{20}$$

Integral constant C_2 depends on the coordinate location. When $t=0$, $\overline{x_0^2}=C_2$. By assuming $t_0=0$, $x_0=0$, we can rewrite equation (20) as:

$$\overline{x^2}=\frac{2kT}{\beta}t=2Dt \tag{21}$$

This relationship was first derived by Einstein, so equation (21) is called Einstein's formula. It is a rigorous proof of Brownian motion originated from micro molecular motion.

In this experiment, user needs to record displacement of oil drop motion over specific time interval and plot displacement distribution curve $\varphi(x)$ of Brownian particle as seen in Figure 3.

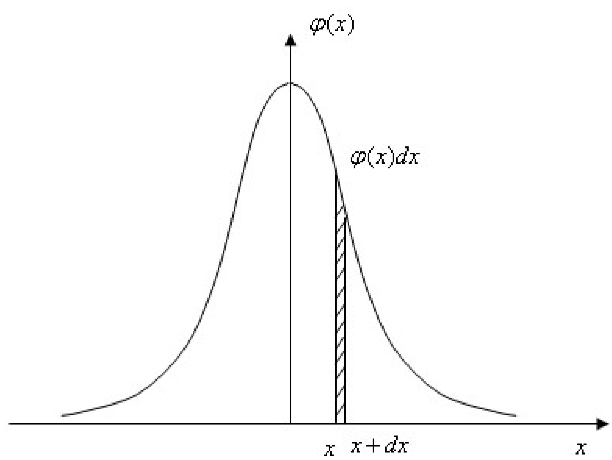

Figure 3 Displacement distribution curve of Brownian particle