

$$\textcircled{1} \underline{A} \quad -\frac{\hbar^2}{2m} \left( \frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} \right) + \left( V + \frac{l(l+1)\hbar^2}{2mr^2} \right) R = ER$$

$$\underline{B} \quad E_n = -\frac{13,6 \text{ eV}}{n^2} = -\frac{e^4 m_e}{2(4\pi\epsilon_0 \hbar)^2} \cdot \frac{1}{n^2}$$

$$\underline{C} \quad \rho(r) = r^2 |R(r)|^2$$

$$\underline{D} \quad R_{nl}(r) = e^{-\frac{Zr}{n a_0}} r^l \cdot \text{pol en } r \text{ de grado } n-1-l$$

$$\underline{E} \quad \langle r \rangle_{2s} = \int_0^{\infty} r^2 \frac{1}{8a_0^3} \left( 4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2} \right) e^{-r/a_0} dr = \left[ u = r/a_0 \right. \\ \left. \frac{du = dr/a_0}{\int_0^{\infty} \left( \frac{u^3}{2} - \frac{u^4}{2} + \frac{u^5}{8} \right) e^{-u} du = \frac{3!}{2} - \frac{4!}{2} + \frac{5!}{8} a_0 = 3 - 12 + 15 = 6 a_0} \right]$$

$$\langle r \rangle_{2p} = \int \frac{r^3}{3} \cdot \frac{r^2}{a_0^2} \cdot \frac{1}{8a_0^3} e^{-r/a_0} dr = a_0 \cdot \frac{5!}{24} = 5a_0$$

$$\underline{F} \quad \langle V \rangle_{2s} = \int \frac{ke^2}{a_0} \frac{r dr}{8a_0^2} \left( 4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2} \right) e^{-r/a_0} = \frac{ke^2}{a_0} \left( \frac{1}{2} - \frac{2!}{2} + \frac{3!}{8} \right) = \frac{ke^2}{4a_0}$$

$$\langle V \rangle_{2p} = \int \frac{ke^2}{3a_0} \frac{r^3 dr}{8a_0^4} e^{-r/a_0} = \frac{6}{3 \cdot 8} \frac{ke^2}{a_0} = \frac{ke^2}{4a_0} = \langle V \rangle_{2s}$$

$$\textcircled{2} \quad E_B = U_c + U_{r,p} + \Delta E$$

D A esa distancia,  $U_{r,p} \approx 0$

$$\text{Buscamos } r / E_B = 0 \Rightarrow \frac{ke^2}{r} = \Delta E \Rightarrow r = \frac{ke^2}{\Delta E} = 9,413 \times 10^{-10} \text{ m} = 9,41 \text{ \AA}$$

$$\begin{aligned} \text{E} \quad & r_{eq} = 0,236 \text{ nm} \\ & E_B = -4,26 \text{ eV} \end{aligned} \left. \vphantom{\begin{aligned} r_{eq} \\ E_B \end{aligned}} \right\} U_{r.p} = E_B - \Delta E - U_c = -4,26 \text{ eV} - 1,53 \text{ eV} + 6,10 \text{ eV} \approx 0,31 \text{ eV}$$

$$\text{F} \quad \text{Estimando } E_B \approx 4 \text{ eV} \rightarrow U_{r.p} = +4 \text{ eV} - 1,53 \text{ eV} + 14,40 \text{ eV} \approx 16,9 \text{ eV}$$