Examen Julio 24'

① **A** $n_1 \longrightarrow n_2 \longrightarrow n=1$ $\qquad \lambda = 486{,}3\,nm$

$\uparrow$

$$E_{n_2} - E_{n_1} = 10{,}2\ eV$$

$$E_{n_2} = -3{,}4\,eV \Rightarrow n_2 = 2$$

$$E_{n_1} = -3{,}4\,eV + \frac{hc}{\lambda} = (-3{,}4 + 2{,}55)\,eV = -0{,}85\,eV$$

$\Rightarrow n_2 = 2 \quad n_1 = 4$

B $E_{inicial} = -0{,}85\,eV \Rightarrow E_b = 0{,}85\,eV$

C $n_1 = 4 \quad n_2 = 2$

D $r_2 = 2^2 a_0 = 4a_0 = 4 \times 0{,}529\,Å = 2{,}12\,Å = 2{,}12 \times 10^{-10}\,m$

$$v_2 = \frac{v_0}{2} = \frac{c}{2\alpha} = 1{,}1 \times 10^6\ \frac{m}{s}$$

E $$N = \frac{\omega \Delta t}{2\pi} = 8{,}3 \times 10^6\ rev = \frac{v \Delta t}{2\pi r} = \frac{1{,}1 \times 10^6 \times 10^{-3}}{12 \times 2{,}1 \times 10^{-10}} \approx 8{,}3 \times 10^6\ rev$$

② **A** $$E_n = -\frac{e^4 m_e}{2(4\pi\varepsilon_0 \hbar)^2} \cdot \frac{1}{n^2} = -\frac{1}{2}\,\frac{\mu c^2 Z^2 \alpha^2}{n^2}$$

B $$-\frac{\hbar^2}{2\mu}\left(\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr}\right) + \left(V + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right)R = ER$$

$$\frac{d}{dr} = \sqrt{8\mu|E|/\hbar^2}\ \frac{d}{dr} \rightarrow -\frac{\hbar^2}{2\mu} d_r^2 R = -\frac{\hbar^2}{2\mu}\frac{8\mu|E|}{\hbar^2} d_\rho^2 R = 4E\, d_\rho^2 R$$

$$\rightarrow -\frac{\hbar^2}{2\mu}\frac{8\mu|E|}{\hbar^2}\frac{2}{\rho} d_\rho R = 8\frac{d_\rho R}{\rho}$$

$|E| = -E$

$$\rightarrow \frac{\ell(\ell+1)}{2\mu\rho^2}\,\frac{8\mu|E|}{\hbar^2}\,\hbar^2 R = -\frac{4\ell(\ell+1)\,E\,R(\rho)}{\rho^2}$$

$$\rightarrow \frac{Ze^2}{4\pi\varepsilon_0}\cdot\sqrt{\frac{8\mu|E|}{\hbar^2}}\,\frac{R(\rho)}{\rho} = \sqrt{\frac{Z^2e^4\mu}{2\hbar^2|E|}}\,\frac{R(\rho)}{\rho}\cdot\frac{4}{\sqrt{4\pi\varepsilon_0}}\,(-E)$$

$$\Rightarrow 4E\,d_\rho^2 R + \frac{8E\,d_\rho R}{\rho} - \frac{4\ell(\ell+1)}{\rho^2}ER - \frac{4E}{\sqrt{4\pi\varepsilon_0}}\frac{\eta R}{\rho} - \frac{ER}{4} = 0$$

C

$$\frac{d^2R}{d\rho^2} = \frac{R}{4} \rightarrow R \sim e^{\pm\frac{\rho}{2}} \rightarrow \boxed{R = A\,e^{-\rho/2}}$$

D $R(\rho) = e^{-\frac{\rho}{2}}G(\rho)$

$$\frac{d^2R}{d\rho^2} + \frac{2}{\rho}\frac{dR}{d\rho} - \frac{\ell(\ell+1)}{\rho^2}R + \frac{\eta}{\sqrt{4\pi\varepsilon_0}}\frac{R}{\rho} - \frac{R}{4} = 0 \qquad \left(\frac{\eta}{\sqrt{4\pi\varepsilon_0}} = \eta^*\right)$$

derivo & simplifico

$$\left(\frac{G}{4} - G' + G''\right) + \frac{2}{\rho}\left(G' - \frac{G}{2}\right) - \frac{\ell(\ell+1)}{\rho^2}G + \eta^*\frac{G}{\rho} - \frac{G}{4} = 0$$

$$\left[G'' + \frac{2G'}{\rho} - \frac{\ell(\ell+1)}{\rho^2}G\right] + \left[\eta^* - 1\right]\frac{G}{\rho} = 0$$

E

$$\rho = \sqrt{\frac{8\mu}{\hbar^2}\cdot\frac{Ze^2}{4\pi\varepsilon_0}\cdot\frac{1}{2n^2 a_0}}\,r = \frac{2r}{na_0} \qquad \text{si } Z=1$$

$\left(\frac{\hbar^2 4\pi\varepsilon_0}{\mu Z e^2} = \frac{a_0}{Z}\right)$