$$\nabla \times B = \frac{4\pi}{c}\vec{J} \longleftrightarrow$$ Oersted 1777-1851

$$-\nabla \times E = \frac{1}{c}\frac{\partial \vec{B}}{\partial t} \longleftrightarrow$$ Faraday 1791-1867

teo. e.m. J.C. Maxwell 1831-1879

ondas e.m., $v \sim c$ H. Hertz 1857-1894

ecs. Maxwell

$$\boxed{\begin{array}{ll} \vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J} & \nabla\cdot E = 4\pi\rho \\ -\nabla\times E = \frac{1}{c}\frac{\partial \vec{B}}{\partial t} & \nabla\cdot B = 0 \end{array}}$$

"en el vacío"

→ no hay cargas magnéticas "monopolos m."

$$\vec{J}(\vec{r},t),\ \rho(\vec{r},t)\ ;\ \underbrace{\vec{B}(\vec{r},t)}_{3},\ \underbrace{\vec{E}(\vec{r},t)}_{3}$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla\cdot\vec{J} = 0$$ ec. continuidad

$$\Rightarrow \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$ ec. continuidad $\longrightarrow$ cons. carga

"Int. radiación materia" : ley Coulomb / Fza. Lorentz

m, q

$\vec{E}, \vec{B}$

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

$m\vec{a} = \vec{F}$

$\Rightarrow$ dinámica de partículas en presencia de $\vec{E}, \vec{B}$

"$\rho$" "$\vec{J}$"

carga puntual

$\vec{J} = \rho \vec{v}$

$\vec{J}(\vec{r},t)$, $\rho(\vec{r},t)$ ; $\underbrace{\vec{B}(\vec{r},t)}_{3}$ , $\underbrace{\vec{E}(\vec{r},t)}_{3}$ magnéticas

"monopolosmo"

$\Rightarrow \dfrac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$ ec. continuidad $\rightarrow$ cons. carga

"Int. radiación materia" : ley Coulomb / Fza. Lorentz $\rightarrow$ H. A. Lorentz, alemán 1835-1928

m, q

$\vec{F} = \underbrace{q}_{\text{"}\rho\text{"}} \left( \vec{E} + \underbrace{\frac{\vec{v}}{c}}_{\text{"}\vec{J}\text{"}} \times \vec{B} \right)$

$\vec{E}, \vec{B}$

$m\vec{a} = \vec{F}$

$\Rightarrow$ dinámica de partículas en presencia de $\vec{E}, \vec{B}$

carga puntual

$\vec{J} = \rho \vec{v}$

Ecuaciones lineales $\Rightarrow$ ppio. superposición

Ecuaciones lineales $\Rightarrow$ ppio. superposición $\Longleftrightarrow$ fenómenos ondulatorios

Leyes conservación
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energía $\quad \vec{J}\cdot\vec{E} + \frac{\partial u}{\partial t} + \nabla\cdot\vec{S} = 0 \qquad u = \frac{E^2+B^2}{8\pi}$, $\vec{S}$ Poynting

impulso lineal $\quad \frac{d}{dt}\left(\vec{P}_{mec} + \vec{P}_{campos}\right) = \oint T_{ij}\, n_j\, da$

momento angular $\quad \frac{\partial}{\partial t}\left(\vec{l}_{mec} + \vec{l}_{campos}\right) = \vec{r}\times\left(\vec{\nabla}\cdot\overleftrightarrow{T}\right)$

$$T_{ij} = \frac{1}{4\pi}\left(E_iE_j + B_iB_j - \frac{\delta_{ij}}{2}\left(E^2+B^2\right)\right)$$

$$\vec{l}_{campos} = \vec{r}\times\vec{g} \quad , \quad \vec{g} = \frac{\vec{E}\times\vec{B}}{4\pi c}$$

# Potenciales

$$\vec{\nabla}\cdot\vec{B}=0 \Rightarrow \underline{\vec{B}=\vec{\nabla}\times\vec{A}} \qquad \vec{A}(\vec{r},t) \text{ potencial vector}$$

$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0, \quad \nabla\times E+\nabla\times\frac{\partial A}{\partial t}=0, \quad \nabla\times\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0 \Rightarrow \underline{\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla\phi}$$

$$\vec{E},\vec{B},\ 6 \longrightarrow \vec{A},\ \phi,\ 4 \quad \text{texto}\Rightarrow \begin{cases} \vec{B}=\nabla\times\vec{A} \\ \vec{E}=-\nabla\phi-\dfrac{\partial\vec{A}}{\partial t} \end{cases}$$

ecs $\vec{A}, \phi$ : ecs. inhom. de maxwell

$$4\ \begin{cases} \nabla\cdot E=4\pi\rho \\ \nabla\times B=\dfrac{1}{c}\dfrac{\partial E}{\partial t}+\dfrac{4\pi}{c}\vec{J} \end{cases} \qquad \begin{aligned} &\nabla\cdot\left(-\nabla\phi-\frac{\partial\bar{A}}{\partial t}\right)=4\pi\rho \\ &\nabla\times(\nabla\times\vec{A})=\frac{1}{c}\frac{\partial}{\partial t}\left(-\nabla\bar{\phi}-\frac{\partial\bar{A}}{\partial t}\right)+\frac{4\pi}{c}\vec{J} \end{aligned}$$

$$\vec{\nabla}\times\vec{\alpha} = \vec{\nabla}(\vec{\nabla}\cdot\vec{\alpha}) - \nabla^2\vec{\alpha}$$

$$\nabla^2\phi + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -4\pi\rho$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = -\frac{1}{c}\nabla\frac{\partial\phi}{\partial t} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} + \frac{4\pi}{c}\vec{J}$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) = -\frac{4\pi}{c}\vec{J}$$

$$\begin{cases} \vec{B} = \nabla \times \vec{A} \\ \vec{E} + \dfrac{\partial \vec{A}}{\partial t} = -\nabla \phi \end{cases}$$

$$\boxed{\begin{aligned} &\vec{\nabla}^2 \phi + \frac{\partial}{\partial t}\left(\vec{\nabla} \cdot \vec{A}\right) = -\rho/\epsilon_0 \\ &\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = -\mu_0 \vec{J} \end{aligned}}$$

MKS

4 ecs, 4 func. escalares incógnitas

## Invariancia gauge

$$\vec{E},\ \vec{B} \longleftrightarrow \vec{A},\ \phi \qquad \begin{cases} \vec{B} = \vec{\nabla}\times\vec{A} \\ \vec{E} = -\dfrac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi \end{cases}$$

$$\vec{A}(\vec{r},t) \longrightarrow \vec{A}'(\vec{r},t) = \vec{A}(\vec{r},t) + \vec{\nabla}\Lambda(\vec{r},t)$$

$\Lambda(\vec{r},t)$ función escalar arbitraria

$$\vec{B}' = \vec{\nabla}\times\vec{A}' = \vec{\nabla}\times\left(\vec{A} + \cancel{\vec{\nabla}\Lambda}\right) = \vec{\nabla}\times\vec{A} = \vec{B} \ !!$$

Transf. gauge
$$\begin{cases} \vec{A} \longrightarrow \vec{A} + \vec{\nabla}\Lambda \\ \phi \longrightarrow \phi' = \phi - \dfrac{\partial\Lambda}{\partial t} \end{cases} \qquad \begin{aligned} \vec{B} &\longrightarrow \vec{B} \\ \vec{E} &\longrightarrow \vec{E} \end{aligned}$$

$$\vec{E} \longrightarrow \vec{E}' = -\frac{\partial\vec{A}'}{\partial t} - \vec{\nabla}\phi' = -\frac{\partial\vec{A}}{\partial t} - \cancel{\frac{\partial}{\partial t}\vec{\nabla}\Lambda} - \vec{\nabla}\phi + \cancel{\vec{\nabla}\frac{\partial\Lambda}{\partial t}} = \vec{E} \ !!$$

elección de gauge: elegir $\phi, \vec{A}$

p. ej $\vec{\nabla} \cdot \vec{A} \neq 0$, busco $\vec{A}' = \vec{A} + \vec{\nabla} \Lambda$

$\vec{\nabla} \cdot \vec{A}' = 0$ : $\vec{\nabla} \cdot \vec{A}' = 0$

$\Rightarrow \nabla \cdot A + \nabla^2 \Lambda = 0$

$\nabla^2 \Lambda = - \nabla \cdot A$

Gauge Coulomb $\vec{\nabla}\cdot\vec{A} = 0$

$$\boxed{\begin{aligned} &\nabla^2\phi = -\rho/\epsilon_0 \\ &\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \frac{1}{c}\vec{\nabla}\left(\frac{\partial\phi}{\partial t}\right) = -\mu_0\vec{J} \end{aligned}}$$

Gauge Lorenz $\nabla\cdot A + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$

$$\boxed{\begin{aligned} &\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho/\epsilon_0 \\ &\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} \end{aligned}}$$

$$\frac{\partial}{\partial}(\nabla\cdot A) = -\frac{1}{c}\frac{\partial^2\phi}{\partial t^2}$$

<u>obs.</u> G. Coulomb $\nabla^2\phi = -\rho/\epsilon_0$ + $\phi(\vec{r},t)$, $\rho(\vec{r},t)$

$$\phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t)}{|\vec{r}-\vec{r}\,'|}\, d^3r'$$

gauge estático

radiación

transverso

$$\left.\right\} \quad \vec{\nabla}\cdot\vec{A} = 0$$

Cond. Lorenz (inv. relat. / covariante), L. V. Lorenz danés

(demostró las pot. retardadas