1

Gauge Coulomb: $\vec{\nabla}\cdot\vec{A}=0$

Lorenz : $\vec{\nabla}\cdot\vec{A}+\frac{1}{c}\frac{\partial\phi}{\partial t}=0$

$$\boxed{\begin{aligned}\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}&=-\mu_0\vec{J}\\ \nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}&=-\rho/\epsilon_0\end{aligned}}\qquad \boxed{\frac{1}{c^2}=\mu_0\epsilon_0}$$

objetivo: sol. ec. ondas c/ fuentes.

$$\boxed{\begin{aligned}\phi(\vec{r},t)&=\frac{1}{4\pi\epsilon_0}\int d^3r'\,\frac{\rho\left(\vec{r}',t-\frac{|\vec{r}-\vec{r}'|}{c}\right)}{|\vec{r}-\vec{r}'|}\\ \vec{A}(\vec{r},t)&=\frac{\mu_0}{4\pi}\int d^3r'\,\frac{\vec{J}\left(\vec{r}',t-|\vec{r}-\vec{r}'|/c\right)}{|\vec{r}-\vec{r}'|}\end{aligned}}$$

Ec. ondas c/ fuentes: $\boxed{\nabla^2\psi(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2\psi(\vec{r},t)}{\partial t^2}=-f(\vec{r},t)}$

$$\underset{\psi}{f(\vec{r},t)}=(2\pi)^{-1/2}\int \underset{\psi_\omega}{f_\omega(\vec{r})}e^{-i\omega t}d\omega\,,\quad \underset{\psi_\omega}{f_\omega(\vec{r})}=\underset{\tilde{\psi}}{\tilde{f}(\vec{r},\omega)}=(2\pi)^{-1/2}\int f(\vec{r},t)e^{i\omega t}dt$$

$$\boxed{\nabla^2\psi_\omega(\vec{r})+\frac{\omega^2}{c^2}\psi_\omega(\vec{r})=-f_\omega(\vec{r})}$$ ec. Helmholtz c/ fuentes

$k^2=\frac{\omega^2}{c^2}$

2

Función de Green: "fuente puntual en $\vec{r}'$"

$$\boxed{\nabla^2G_\omega(\vec{r},\vec{r}')+k^2G_\omega(\vec{r},\vec{r}')=-\delta(\vec{r}-\vec{r}')}$$

$$\boxed{\psi_\omega(\vec{r})=\int f_\omega(\vec{r}')\,G_\omega(\vec{r},\vec{r}')\,d^3r'}$$

$\vec{R}=\vec{r}-\vec{r}'$, $R\neq0$

$R=0$ singular

Sim. esf. alrededor de $\vec{r}=\vec{r}'$ $(\vec{R}=0)$

Coord. esf. : $R=|\vec{r}-\vec{r}'|$

$\frac{1}{R}\frac{d^2}{dR^2}(RG)+k^2G=0$ $\quad(R\neq0)$

$\Longrightarrow$ $\boxed{G=A\frac{e^{\pm ikR}}{R}}$ $\quad k=\sqrt{k^2}=\sqrt{\omega^2/c^2}$

2 sols. linealmente independientes

A? integro esfera, radio a, alrededor de $\vec{r}'$

$$\int_{esf,a}(\nabla^2G+k^2G)\,d^3r=-\int\delta(\vec{r}-\vec{r}')\,d^3r$$

"$a\to0$", $G\sim\frac{A}{R}$, $\nabla^2\frac{1}{|\vec{r}-\vec{r}'|}=-4\pi\delta(\vec{r}-\vec{r}')$

3

$$\int_{esf,a}\left(-4\pi\delta(\vec r-\vec r')A + \frac{k^2A}{|\vec r-\vec r'|}\right)d^3r = -\int_{esf}\delta(\vec r-\vec r')\,d^3r$$

$$\int_{esf}\frac{k^2A}{R}\,d^3r = \int_0^a \frac{k^2A}{R}\,4\pi R^2\,dR = k^2A4\pi\frac{a^2}{2} \xrightarrow[a\to 0]{} 0$$

$$\Rightarrow 4\pi A = 1 \qquad A=\frac{1}{4\pi}$$

$$\boxed{G_w(\vec r,\vec r') = \frac{1}{4\pi}\frac{e^{\pm ikR}}{R}}$$

$$\psi_w(\vec r) = \frac{1}{4\pi}\int f_w(\vec r')\frac{e^{\pm iR}}{R}\,d^3r'$$

$$\psi(\vec r,t) = \frac{1}{4\pi}\frac{1}{\sqrt{2\pi}}\int f_w(\vec r')\,e^{-i(wt\mp kR)}\,dw\,d^3r'$$

$$\boxed{\psi(\vec r,t) = \frac{1}{4\pi}\int\frac{f(\vec r',t')}{|\vec r-\vec r'|}\,d^3r', \quad t' = t\pm R\frac{k}{w} = t\pm R/c}$$

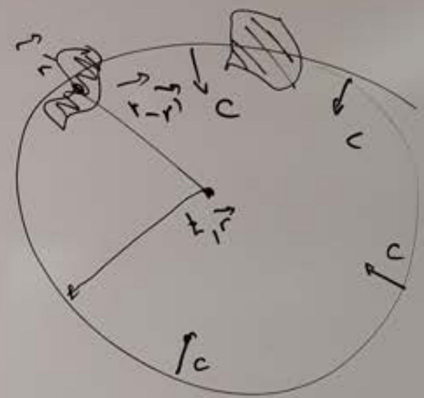
$t - R/c$ ret.

$t + R/c$ avanz.

4

esfera colectora de información

$$\boxed{\vec A(\vec r,t) = \frac{\mu_0}{4\pi}\int\frac{\vec J\left(\vec r', t-\frac{|\vec r-\vec r'|}{c}\right)}{|\vec r-\vec r'|}\,d^3r'}$$



Panofsky / Philips

Class. Electron.

$\vec J\left(\vec r', t-\frac{|\vec r-\vec r'|}{c}\right) = [\vec J(\vec r')]$ "evaluado en $t=t_R$"

$t_R = t - \frac{|\vec r-\vec r'|}{c}$

$J(\vec r,t) = \vec J(\vec r)\,e^{-iwt}$ (conv. compleja)

w: monocromatico → calculo, sumo en w

Conservación → carga eléctrica $\boxed{\nabla\cdot\vec J + \frac{\partial\rho}{\partial t}=0}$

→ otras cons.: energía, momento lineal, angular

Energía $\boxed{\frac{dW}{dt} = -\frac{dU_{em}}{dt} - \oint_{Sup.}\vec S\cdot d\vec a}$ → teorema Poynting

$U_{em} = \int_V u_{em}\,dV = \int_V \frac12\left(\epsilon_0E^2 + B^2/\mu_0\right)dV$, $\frac{dU_{mec}}{dt} = \frac{dW}{dt}$

5

$\vec{S} = \dfrac{\vec{E}\times\vec{B}}{\mu_0}$ vector Poynting

en forma diferencial: $\boxed{\dfrac{\partial}{\partial t}(u_{mec} + u_{em}) + \vec{\nabla}\cdot\vec{S} = 0}$

Impulso lineal / Fuerzas:

$$\vec{F} = \oint_{Sup} \overleftrightarrow{T}\cdot da - \epsilon_0\mu_0 \frac{d}{dt}\int_V \vec{S}\, dV$$

fuerzas em. totales sobre cargas en V.

$(\vec{a}\cdot\overleftrightarrow{T})_j = \sum a_i T_{ij}$

$(\overleftrightarrow{T}\cdot\vec{a})_i = \sum T_{ij} a_j$

T : tensor Maxwell, esfuerzos

$$\boxed{T_{ij} = \epsilon_0\left(E_iE_j - \frac{\delta_{ij}}{2}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{\delta_{ij}}{2}B^2\right)}$$ $i,j = 1,2,3$

T_{ij}: fuerza/area (esfuerzo o tensión) en la dirección $\underline{\underline{i}}$ actuando sobre elto. de sup. orientado $\perp$ a la dir $\underline{\underline{j}}$

$\vec{F} = \dfrac{d\vec{P}_{mec}}{dt}$, $\boxed{\vec{P}_{em} = \mu_0\epsilon_0 \int_V \vec{S}\, dV}$ $\boxed{\vec{p}_{em} = \vec{S}/c^2}$

$$\boxed{\frac{d}{dt}\left(\vec{P}_{em} + \vec{P}_{mec}\right) = \oint_{sup} \overleftrightarrow{T}\cdot d\vec{a}} \quad \boxed{\frac{\partial}{\partial t}\left(\vec{p}_{mec} + \vec{p}_{em}\right) = \vec{\nabla}\cdot\overleftrightarrow{T}}$$

6

Comentario Gauge Coulomb $\vec{\nabla}\cdot\vec{A} = 0$

$\boxed{\nabla^2\phi = -\rho/\epsilon_0}$ $\propto$ Poissón, pero con dep. temporal: $\rho(\vec{r},t)$

"gauge instantáneo"

$$\boxed{\phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t)}{|\vec{r}-\vec{r}'|}\, d^3r'}$$

$$\boxed{\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} - \frac{1}{c^2}\vec{\nabla}\frac{\partial\phi}{\partial t}}$$

$\nabla\times\nabla\dfrac{\partial\phi}{\partial t} = 0$ "irrotacional"

Teorema Helmholtz: campo vector $\vec{V}$

$\left\{\begin{array}{l}\vec{\nabla}\cdot\vec{V} \\ \vec{\nabla}\times\vec{V}\end{array}\right. \rightarrow$ determinan $\vec{V}$

$\boxed{\vec{V} = -\nabla\phi - \vec{\nabla}\times\vec{A}}$, $\phi, \vec{A}$ relacionados c/ $\vec{V}$!

$$\phi = \frac{1}{4\pi}\int \frac{\vec{\nabla}'\cdot\vec{V}(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3r', \quad \vec{A} = \frac{1}{4\pi}\int \frac{\vec{\nabla}'\times V}{|\vec{r}-\vec{r}'|}\, d^3r'$$

$\vec{J} = \vec{J}_l + \vec{J}_t$

t: trans. ó solenoidal, $\vec{\nabla}\cdot\vec{J}_t = 0$

l: longitudinal ó irrotacional, $\nabla\times J_l = 0$

$$\boxed{\vec{J}_l = -\frac{1}{4\pi}\nabla\int \frac{\vec{\nabla}'\cdot\vec{J}'}{|\vec{r}-\vec{r}'|}\, d^3r'}, \quad \boxed{\vec{J}_t = \frac{1}{4\pi}\nabla\times\int \frac{\nabla'\times\vec{J}'}{|\vec{r}-\vec{r}'|}\, d^3r'}$$

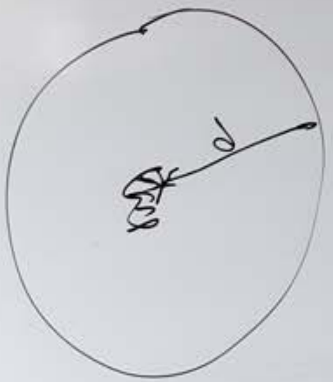
$$+\frac{1}{c^2}\nabla\frac{\partial\phi}{\partial t} = +\frac{1}{c^2}\nabla\frac{\partial}{\partial t}\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec r',t)}{|\vec r-\vec r'|}d^3r' = -\frac{1}{c^2}\frac{1}{4\pi\epsilon_0}\vec\nabla\int\frac{\vec\nabla'\cdot\vec J'}{|\vec r-\vec r'|}d^3r'$$

$$\frac{\partial\rho}{\partial t}+\vec\nabla\cdot\vec J=0 \quad \Big| \quad = + \frac{\vec J_\ell}{c^2\epsilon_0} = +\mu_0\vec J_\ell$$

Ec. ondas para $\vec A$: $-\mu_0\underbrace{\vec J}_{\vec J_\ell+\vec J_t} + \mu_0\vec J_\ell = -\mu_0\vec J_t$

$$\boxed{\nabla^2\vec A - \frac{1}{c^2}\frac{\partial^2\vec A}{\partial t^2} = -\mu_0\vec J_t}$$

Radiación $\quad P=\oint_{sup}\vec S\cdot d\vec a \quad$ potencia total a traves de una sup., esfera



Si para cualq. $\underline{\underline{d}}$ hay $P\neq 0$ emitida

$\Longrightarrow$ radiación

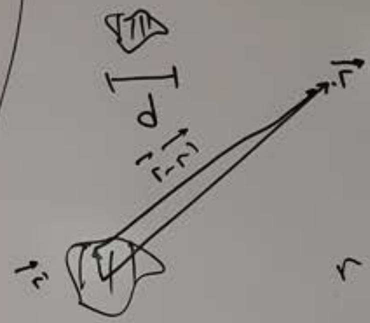
$$\vec S = \frac{\vec E\times\vec B}{\mu_0} \qquad \int\ldots d\vec a \to r^2dr$$

Radiación: $E, B \propto 1/r$

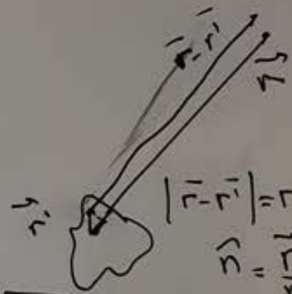
$S\sim 1/r^2$

Rad. de cargas localizadas

Obs. a $\vec r$ $\Big|$ $r=|\vec r|\gg d$



$r\gg d$



$|\vec r-\vec r'| = r-\vec r'\cdot\hat n$

$\hat n = \frac{\vec r}{r}$

Obs. centro de coord. en donde están las cargas.

$$|\vec r-\vec r'| = \sqrt{r^2+r'^2-2\vec r\cdot\vec r'} = r\sqrt{1+\frac{r'^2}{r^2}-\frac{2\hat n\cdot\vec r'}{r}}$$

$$\underset{r\gg|\vec r'|}{\simeq} r\left\{1+\frac12\left(\frac{r'^2}{r^2}-\frac{2\hat n\cdot\vec r'}{r}\right)-\frac14(\ldots)^2\right\}$$

$$= r-\vec r'\cdot\hat n + O(d/r)$$

Gauge Lorenz $\quad \vec A(\vec r,t)=\frac{\mu_0}{4\pi}\int\frac{\vec J(\vec r',t-|\vec r-\vec r'|/c)}{|\vec r-\vec r'|}d^3r'$

radiación $\Longleftrightarrow$ $\vec A$ a orden $1/r$!

$$\vec A(\vec r,t)\simeq\frac{\mu_0}{4\pi r}\int\vec J(\vec r',t-|\vec r-\vec r'|/c)\,d^3r' \quad \left(+\,O(d/r^2)\right)$$

$$\vec J(\vec r',t-|\vec r-\vec r'|/c) = \vec J\Big(\vec r', \underbrace{t-\frac rc}_{\text{tiempo ret. del origen}}+\frac{\vec r'\cdot\hat n}{c}+\ldots\Big)$$

$$\vec B = \vec\nabla\times\vec A = \frac{\mu_0}{4\pi}\int\left(\frac{\vec\nabla\times\vec J}{r} + \cancel{\underbrace{\left(\vec\nabla\frac{1}{r}\right)}_{\sim\frac{1}{r^2}}\times\vec J}\right)d^3r' \dots$$

$$\vec\nabla\times\vec J\left(\vec r', t - \frac{r}{c} + \frac{\vec r'\cdot\hat n}{c} + \dots\right) = \frac{1}{c}\nabla\left(-r + \vec r'\cdot\hat n \dots\right)\times\frac{\partial\vec J}{\partial t}$$

revisar que está ok

$$= \frac{1}{c}\left(-\hat n + \cancel{\underbrace{\nabla(\vec r'\cdot\hat n)}_{\sim 1/r}}\dots\right)\times\frac{\partial \vec J}{\partial t}$$

$$= -\frac{\hat n}{c}\times\frac{\partial\vec J}{\partial t}$$

$$\boxed{\vec B = -\frac{1}{c}\,\hat n\times\frac{\partial\vec A}{\partial t}}$$

$\vec B \perp \hat n$ a la dir. obs.

$$\vec E = -\vec\nabla\phi - \frac{\partial\vec A}{\partial t} = \frac{\hat n}{c}\frac{\partial\phi}{\partial t} - \frac{\partial\vec A}{\partial t} = -c\,\hat n\,\vec\nabla\cdot\vec A - \frac{\partial \vec A}{\partial t}$$

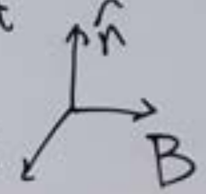
cond. Lorenz

$$\nabla\cdot A + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0$$

$$\vec\nabla \to -\frac{\hat n}{c}\frac{\partial}{\partial t}$$

$$\vec E = +\cancel{c}\,\hat n\left(\frac{\hat n}{\cancel{c}}\cdot\frac{\partial\vec A}{\partial t}\right) - \frac{\partial\vec A}{\partial t} = \hat n\left(\hat n\cdot\frac{\partial \vec A}{\partial t}\right) - \frac{\partial\vec A}{\partial t} = \hat n\times\left(\hat n\times\frac{\partial\vec A}{\partial t}\right)$$

$$\boxed{\vec E = -c\,\hat n\times B}$$



$(\vec E, \vec B, \hat n)$ triedro directo