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Ohanian: Class. Electr.
Panofsky / Philips
Sommerfel

Campos de radiación
(cargas localizadas)

Radiación $\leftrightarrow \vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} \leftrightarrow \begin{matrix}\vec{E}\\ \vec{B}\end{matrix} \sim 1/r$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi r}\int \vec{J}\left(\vec{r}', t - r/c + \frac{\vec{r}'\cdot\hat{r}}{c}\right) d^3r' + O(d/r^2)$$

Gauge Lorenz

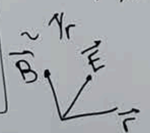
$$\vec{\nabla} \longrightarrow -\frac{\hat{r}}{c}\frac{\partial}{\partial t}$$

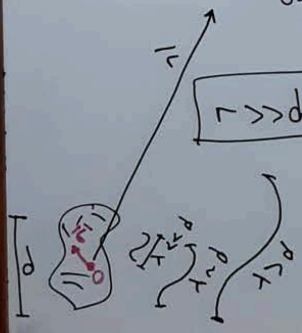
$$\boxed{\begin{aligned}\vec{B}(\vec{r},t) &= -\frac{\hat{r}}{c}\times\frac{\partial\vec{A}}{\partial t}\\ \vec{E}(\vec{r},t) &= -c\,\hat{r}\times\vec{B}\end{aligned}}$$

campos de radiación

$r \gg d$
$r \gg \lambda$

$\sim 1/r$





$\boxed{r \gg d}$

Despreciamos términos $\sim \frac{\mu_0}{4\pi}\int d^3r'\,\frac{\vec{J}'}{r^2} \sim \frac{J}{r^2}$

Frente a $\sim \frac{\mu_0}{4\pi c}\frac{1}{r}\int\frac{\partial\vec{J}'}{\partial t}d^3r' \sim \frac{J}{crT}$

T: escala de tiempo de evol./cambio apreciable de las cargas

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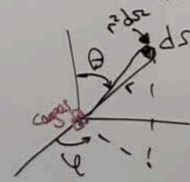
$\frac{J}{r^2} \ll \frac{J}{crT}$

$\boxed{r \gg cT \sim \lambda}$

Campos lejanos / radiación: $\boxed{\begin{matrix} r \gg d\\ r \gg \lambda\end{matrix}}$

Potencia $P(b) = \oint_{\text{esf. } b} \vec{S}\cdot d\vec{a} \xrightarrow[b\to\infty]{} P$ finito : radiación

Distribución angular de potencia:



$d\Omega$, dirección θ, φ

dP?

$$\boxed{dP = \vec{S}\cdot\hat{r}\,r^2 d\Omega}$$

$$\boxed{\frac{dP}{d\Omega} = \vec{S}\cdot\hat{r}\,r^2}$$

$S \sim 1/r^2$
$\Rightarrow S r^2 \sim$ finito
$\Rightarrow P = \int dP$ finito, en principio

$\frac{dP}{d\Omega}(r,\theta,\varphi,t)$; $P(r,t) = \int dP = \int \frac{dP}{d\Omega} d\Omega$

$$\bar{P}(r) = \frac{1}{T}\int_0^T P(r,t)\,dt$$

Desarrollo multipolar de campos de radiación (3)

$r \gg d$

$r \gg \lambda$ $\quad d \overset{?}{} \lambda$. Situación de interés $\underline{d \ll \lambda}$ (átomos, núcleos)

$$\vec{J}\Big(\vec{r}', \underbrace{t - r/c}_{\text{tiempo ret. al origen}} + \vec{r}' \cdot \frac{\hat{r}}{c}\Big) \simeq \vec{J}(\vec{r}', t - r/c) + \vec{r}' \cdot \frac{\hat{r}}{c} \frac{\partial \vec{J}}{\partial t}(\vec{r}', t - r/c) + \dots$$

Desarrollo es válido, en principio, si: $J \gg \frac{d}{c}\frac{J}{T}$, $\quad 1 \gg \frac{d}{cT}$

$$\boxed{cT \sim \lambda \ggg d} \longrightarrow c \gg \frac{d}{T} \simeq v \qquad \boxed{c \gg v}$$

Desarrollo multipolar de campos de radiación: $\boxed{r \gg \lambda \gg d}$

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$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi r} \int \vec{J}(\vec{r}', t - r/c)\, d^3r' + \frac{\mu_0}{4\pi r c} \int \vec{r}' \cdot \hat{r}\, \frac{\partial \vec{J}}{\partial t}(\vec{r}', t - r/c)\, d^3r' + \dots$$

Desarrollo multipolar: $\lambda \gg d$ ó $\underline{\underline{c \gg v}}$!! desarrollo $\underline{\text{no}}$ relativista!

$\underline{1^{er} \text{ término}}$ $\quad \int \vec{J}(\vec{r}', t - r/c)\, d^3r' = -\int \vec{r}'\, \vec{\nabla}' \cdot \vec{J}'\, d^3r'$

$$\int J_x\, d^3r \overset{?}{=} -\int x\, \nabla \cdot J\, d^3r \ \checkmark, \qquad \nabla \cdot (x\vec{J}) = \vec{\nabla} x \cdot \vec{J} + x\, \vec{\nabla} \cdot \vec{J} = J_x + x\, \vec{\nabla} \cdot \vec{J}$$

$$\vec{A}_{de}(\vec{r}, t) = \frac{\mu_0}{4\pi r}\Big(-\int \vec{r}'\, \overbrace{\vec{\nabla}' \cdot \vec{J}'}^{-\partial \rho'/\partial t}\, d^3r'\Big) \qquad \underbrace{\int d^3r\, \nabla \cdot (x\vec{J})}_{=\ \oint x\vec{J} \cdot dS\ =\ 0} = \int J_x\, d^3r + \int x\, \vec{\nabla} \cdot \vec{J}\, d^3r$$

$0 = \oint x \vec{J} \cdot dS = 0$ → esfera grande o todo el espacio, cargas localizadas

$$\vec{A}_{de} = \frac{\mu_0}{4\pi r} \frac{d}{dt} \underbrace{\int \vec{r}' \rho'\, d^3r'}_{\vec{P}\ \text{mom. dip. elec.}}$$

$$\boxed{\vec{A}_{de} = \frac{\mu_0}{4\pi r} [\dot{\vec{P}}]}$$

$[\ ]' \to$ tiempo evaluado en $t - r/c$

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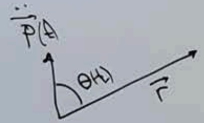
Campos dipolares eléctricos:

$$\boxed{\vec p(t) = \int \vec r\, \rho(\vec r, t)\, d^3r}$$

$$\boxed{\vec B(\vec r, t) = -\frac{\mu_0}{4\pi r c}\,\hat r \times \left[\frac{d^2\vec p}{dt^2}\right]}$$

$$\boxed{\vec E(\vec r, t) = -c\,\hat r \times \vec B(\vec r, t)}$$

$$\boxed{\vec S = \frac{1}{\mu_0}\vec E\times\vec B = \frac{c}{\mu_0}B^2\,\hat r} \text{ d.e.}$$



$$\vec S = |\vec S|\,\hat r$$

$$\boxed{\vec S = \frac{c}{\mu_0}\frac{\mu_0^2}{16\pi^2 c^2 r^2}\left|[\ddot{\vec p}]\right|^2 \sin^2\theta\,\hat r = \frac{\mu_0}{16\pi^2 c r^2}\left|[\ddot{\vec p}]\right|^2 \sin^2\theta\,\hat r}$$

$$\frac{dP}{d\Omega} = \vec S\cdot\hat r\, r^2 = |\vec S|\, r^2 = \boxed{\frac{\mu_0}{16\pi^2 c}\left|[\ddot{\vec p}]\right|^2 \sin^2\theta = \frac{dP}{d\Omega}} \text{ D.E.}$$

$$\int d\Omega \sin^2\theta = \frac{8\pi}{3} \qquad P(t) = \int \frac{dP}{d\Omega}\, d\Omega = \boxed{\frac{\mu_0}{6\pi c}\left|[\ddot{\vec p}]\right|^2 = P(t)} \text{ D.E.}$$

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$$\vec A(\vec r, t) = \frac{\mu_0}{4\pi r}\int \vec J(\vec r\,', t - r/c)\, d^3r' + \frac{\mu_0}{4\pi r c}\int \vec r\,'\cdot\hat r\, \frac{\partial \vec J}{\partial t}(\vec r\,', t - r/c)\, d^3r' + \dots$$

Ejemplo carga oscilando en 1 dirección

$$\vec p = \vec p_0 \cos\omega t \qquad [\ddot{\vec p}] = -\omega^2 \vec p_0 \cos\omega(t - r/c)$$

$$\boxed{\frac{dP}{d\Omega} = \frac{\mu_0 |\vec p_0|^2}{16\pi^2 c}\,\omega^4 \sin^2\theta \cos^2\omega(t - r/c)}$$ distr. ang. de pot. dip. elec. instantánea

$$\boxed{P(t) = \frac{\mu_0 |\vec p_0|^2 \omega^4}{6\pi c}\cos^2\omega(t - r/c)} \qquad \bar P = \int_0^T \frac{P(t)\,dt}{T}, \quad \overline{\cos^2\omega(t - r/c)} = \frac{1}{2}$$

$$\boxed{\bar P = \frac{\mu_0 |\vec p_0|^2 \omega^4}{12\pi c}} \text{ D.E. } \omega^4 !!$$

$$\boxed{\tau = \frac{\hbar\omega}{\bar P}}$$ "vida promedio" dipolar eléctrica, la dominante!

$$\Gamma = \frac{1}{\tau} = \frac{\bar P}{\hbar\omega}$$ tasa de transición "transition rate" $\vec p \to e\int \Psi_f^*\, \vec r\, \Psi_i\, d^3r$

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Campos dipolares $\begin{cases}\text{magnético} \\ \text{cuadrupolar eléctricos}\end{cases}$ del mismo orden.

$$\text{D.E} \sim 1 \qquad \text{D.M.}/\text{Q.E.} \sim d/\lambda \ll 1$$

$$\vec{A}(r,t) = (\text{D.E.}) \cdots\cdots + \underbrace{\frac{\mu_0}{4\pi r c}\frac{d}{dt}\int \vec{r}\,'\cdot\hat{r}\,\vec{J}(\vec{r}\,',t-r/c)\,d^3r'}_{\text{D.M.} + \text{Q.E.}} + \cdots$$

$$\int \vec{r}\,'\cdot\hat{r}\,\vec{J}'\,d^3r', \qquad (\vec{r}\,'\times\vec{J})\times\hat{r} = \underbrace{\hat{r}\cdot\vec{r}\,'\,\vec{J}}\, - \hat{r}\cdot\vec{J}\,\vec{r}\,'$$

$$\int \vec{r}\,'\cdot\hat{r}\,\vec{J}'\,d^3r' = \frac{1}{2}\underbrace{\int(\vec{r}\,'\times\vec{J}')\times\hat{r}\,d^3r'}_{\text{D.M.}} + \frac{1}{2}\underbrace{\int\left(\hat{r}\cdot\vec{r}\,'\,\vec{J}' + \hat{r}\cdot\vec{J}'\,\vec{r}\,'\right)d^3r'}_{\text{Q.E.}}$$

$$\boxed{\vec{m}(t) = \frac{1}{2}\int \vec{r}\times\vec{J}(\vec{r},t)\,d^3r} \qquad \boxed{\vec{A}_{d.m.}(\vec{r},t) = -\frac{\mu_0}{4\pi}\frac{\hat{r}}{cr}\times\left[\dot{\vec{m}}\right]}$$

tiempo retardado del origen

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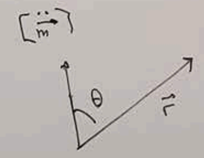
$$\boxed{\vec{B}_{dm}(\vec{r},t) = \frac{\mu_0}{4\pi c^2 r}\,\hat{r}\times\left(\hat{r}\times\left[\ddot{\vec{m}}\right]\right)} = \frac{\hat{r}\times\vec{E}_{dm}}{c}$$

$$\boxed{\vec{E}_{dm}(\vec{r},t) = -c\,\hat{r}\times\vec{B} = \frac{\mu_0}{4\pi c r}\,\hat{r}\times\left[\ddot{\vec{m}}\right]}$$



$$\vec{S}_{dm} = |\vec{S}_{dm}|\,\hat{r}, \qquad \frac{dP}{d\Omega} = |\vec{S}_{dm}|\,r^2$$

$$\boxed{\vec{S}_{dm} = \frac{\mu_0}{16\pi^2c^3r^2}\left[|\ddot{\vec{m}}|^2\right]\operatorname{sen}^2\theta\,\hat{r}} \qquad \frac{dP_{DM}}{d\Omega} = \frac{\mu_0}{16\pi^2c^3r^2}\left[|\ddot{\vec{m}}|^2\right]\operatorname{sen}^2\theta$$

$$\boxed{P_{DM}(t) = \frac{\mu_0}{6\pi c^3}\left[|\ddot{\vec{m}}|^2\right]} \qquad \vec{m}(t) = \vec{m}_0\cos\omega t \Rightarrow \boxed{P_{D.M} = \frac{\mu_0 m_0^2}{12\pi c^3}\,\omega^4}$$

Cuadrupolo eléctrico: $\frac{1}{2}\int\left(\hat{r}\cdot\vec{r}\,'\,\vec{J}' + \hat{r}\cdot\vec{J}'\,\vec{r}\,'\right)d^3r'$

$$\vec{A}_{QE}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{1}{cr}\frac{d}{dt}\frac{1}{2}\int\left(\hat{r}\cdot\vec{r}\,'\,\vec{J}' + \hat{r}\cdot\vec{J}'\,\vec{r}\,'\right)d^3r'$$

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$$\frac{1}{2}\int\left(\hat{r}\cdot\vec{r}\,'\,\vec{J}\,' + \hat{r}\cdot\vec{J}\,'\,\vec{r}\,'\right)d^3r' = \frac{1}{2}\frac{d}{dt}\int \vec{r}\,'(\hat{r}\cdot\vec{r}\,')\rho'\,d^3r'$$

$$\int\left(\hat{r}\cdot\vec{r}\,'\,J'_x + \hat{r}\cdot\vec{J}\,'\,x'\right)d^3r' \overset{?}{=} \frac{d}{dt}\int x'\,(\hat{r}\cdot\vec{r}\,')\rho'\,d^3r'$$

$$\nabla'\cdot\left(x'\,\hat{r}\cdot\vec{r}\,'\,\vec{J}\,'\right) = \left(\vec{\nabla}'x'\,\hat{r}\cdot\vec{r}\,'\right)\cdot\vec{J}\,' + x'\,\hat{r}\cdot\vec{r}\,'\,\vec{\nabla}\cdot\vec{J}$$

$$= \left((\hat{r}\cdot\vec{r}\,',0,0) + x'\hat{r}\right)\cdot\vec{J}\,' + x'\,\hat{r}\cdot\vec{r}\,'\left(-\frac{\partial\rho'}{\partial t}\right)$$

$$\nabla'\cdot\left(x'\,\hat{r}\cdot\vec{r}\,'\,\vec{J}\,'\right) = \hat{r}\cdot\vec{r}\,'\,J'_x + x'\,\hat{r}\cdot\vec{J}\,' - x'\,\hat{r}\cdot\vec{r}\,'\frac{\partial\rho'}{\partial t}$$

$$0 = \int d^3r\,(\quad '' \quad) = 0 = \int\left(\underbrace{\hat{r}\cdot\vec{r}\,'\,J'_x + x'\,\hat{r}\cdot\vec{J}\,'} - \underbrace{x'\,\hat{r}\cdot\vec{r}\,'\frac{\partial\rho'}{\partial t}}\right)d^3r'$$

$$= \oint(\quad)\cdot d\vec{a} = 0$$

cargas localizadas

$$\boxed{\vec{A}_{QE}(\vec{r},t) = \frac{\mu_0}{4\pi c r}\,\frac{1}{2}\frac{d^2}{dt^2}\int \vec{r}\,'(\hat{r}\cdot\vec{r}\,')\rho'\,d^3r'}$$

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$$\vec{Q}(\vec{r},t) = \int d^3r'\left(3\vec{r}\,'\,\hat{r}\cdot\vec{r}\,' - \hat{r}\,r'^2\right)\rho(\vec{r}\,',t), \quad Q_\alpha = \sum_{\beta=1}^{3} Q_{\alpha\beta}\hat{r}_\beta$$

$\alpha = 1,2,3 \rightarrow x,y,z$

$$\boxed{Q_{\alpha\beta} = \int d^3r\left(3r_\alpha r_\beta - \delta_{\alpha\beta}r^2\right)\rho}$$

$Q_{\alpha\beta} = Q_{\beta\alpha}$ tensor cuadrupolar electrico

simetrico

$\sum_\alpha Q_{\alpha\alpha} = 0$ traza nula

$$\boxed{\hat{r}\times\vec{Q} = \int d^3r'\,3\,\hat{r}\times\vec{r}\,'\,\vec{r}\,'\cdot\hat{r}\,\rho'}$$

$$\vec{B}_{QE} = -\frac{\hat{r}}{c}\times\frac{\partial A}{\partial t} = -\frac{\mu_0}{24\pi c^2 r}\,\hat{r}\times\frac{d^3}{dt^3}[\vec{Q}]$$

$$\vec{E}_{QE} = \frac{\mu_0}{24\pi c r}\,\hat{r}\times\left(\hat{r}\times[\dddot{\vec{Q}}]\right)$$

$P_{DE}, P_{DM}, P_{QE}, \ldots$ → complicadas

$P = P_{DE} + P_{DM} + P_{QE}$!!

sin interferencias de los campos que si aparecen en $\frac{dP}{d\Omega}$

$(\vec{E} = \vec{E}_{DE} + \vec{E}_{DM} + \vec{E}_{QE})$

$$\vec{S}_{QE} = |\vec{S}_{QE}|\,\hat{r}, \quad |\vec{S}_{QE}| = \frac{\mu_0}{24^2\pi^2c^3r^2}\left|\hat{r}\times[\dddot{\vec{Q}}]\right|^2$$

$\int d\Omega\,\hat{r}_\alpha\hat{r}_\beta$

$$\boxed{\frac{dP}{d\Omega} = \frac{\mu_0}{576\pi^2c^3}\left|\hat{r}\times[\dddot{\vec{Q}}]\right|^2}$$

$$\boxed{P = \frac{\mu_0}{720\pi c^3}\sum_{\alpha\beta}\left(\dddot{Q}_{\alpha\beta}\right)^2}$$

y si $Q_{\alpha\beta} = Q^0_{\alpha\beta}\cos(\omega t+\delta)$ → $\boxed{\overline{P} = \frac{\mu_0\,\omega^6}{1440\pi c^3}\sum_{\alpha\beta}\left(Q^0_{\alpha\beta}\right)^2}$