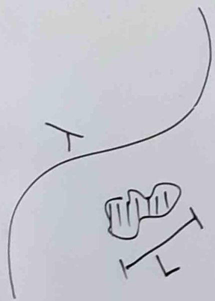
1

Dispersión de energía



$\lambda \gg L$ multipolos orden ppal. $\left(1 \gg \frac{L}{\lambda}\right)$

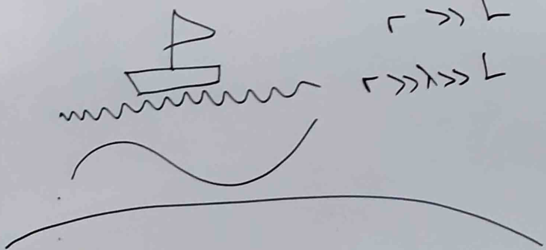
$\lambda \gtrsim L$ todos multipolos

$\lambda \ll L$ óptica geométrica

$\underline{r}$: dist. obs.

$r \gg L$ campos radiación

$r \gg \lambda \gg L$ " " en aprox. multipolar



2

Onda incidente : descom. Fourier en frecuencias ω.

Ecs. Max. lineales : sumo para c/ frecuencia.

$\underline{\omega}$

$$\rho(\vec r,t) = \rho(\vec r)\, e^{-i\omega t} \qquad \{\text{sub. índice } \omega \text{ en } \rho, J \text{ sobreentendidos}\}$$

$$\vec J(\vec r,t) = \vec J(\vec r)\, e^{-i\omega t}$$

$$\vec A(\vec r,t) = \frac{\mu_0}{4\pi}\int d\vec r\,'\, \frac{\vec J(\vec r\,',t_R)}{|\vec r-\vec r\,'|} = \frac{\mu_0}{4\pi}\int d\vec r\,'\, dt'\, \frac{\vec J(\vec r\,',t')}{|\vec r-\vec r\,'|}\,\delta\left(t' - \left(t - \frac{|\vec r-\vec r\,'|}{c}\right)\right)$$

$$t_R = t - \frac{|\vec r-\vec r\,'|}{c}$$

$$\vec J(\vec r,t) = \vec J(\vec r)\, e^{-i\omega t}, \quad \vec A(\vec r,t) = \vec A(\vec r)\, e^{-i\omega t} = \frac{\mu_0}{4\pi}\int d^3r'\, \frac{\vec J(\vec r\,')}{|\vec r-\vec r\,'|}\, e^{-i\omega\left(t - \frac{|\vec r-\vec r\,'|}{c}\right)}$$

$$= \frac{\mu_0}{4\pi}\int d\vec r\,'\, \frac{\vec J(\vec r\,')}{|\vec r-\vec r\,'|}\, e^{i\omega \frac{|\vec r-\vec r\,'|}{c}} \cdot e^{-i\omega t}$$

$$\boxed{\vec A(\vec r) = \int d^3r'\, \frac{\vec J(\vec r\,')}{|\vec r-\vec r\,'|}\, e^{i\omega \frac{|\vec r-\vec r\,'|}{c}}}$$

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$r \gg \lambda \gg L$

$|\vec{r}-\vec{r}\,'| = r - \hat{r}\cdot\vec{r}\,' \ldots\ldots$

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d\vec{r}\,' \, \vec{J}(\vec{r}\,') e^{-ik\hat{r}\cdot\vec{r}\,'}}$$

→ onda esférica e^{ikr}/r

→ modulada x un factor angular (k,θ,φ)

$k = \frac{\omega}{c}$

$\lambda = \frac{2\pi}{k}$

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\sum_{n=0}^{\infty}\frac{(-ik)^n}{n!}\int d^3r'\, \vec{J}(\vec{r}\,')(\hat{r}\cdot\vec{r}\,')^n}$$

$\vec{B} = \vec{\nabla}\times\vec{A}$, $\boxed{\vec{H} = \frac{\vec{\nabla}\times\vec{A}}{\mu_0}}$

$\hat{r}$ (θ,φ)

$\nabla\times H(\vec{r},t) = \cancel{J} + \frac{\partial D(\vec{r},t)}{\partial t}$ en zona sin cargas corrientes $J = 0$

$H(\vec{r},t), D(\vec{r},t) \sim H(\vec{r}), D(\vec{r}), \times e^{-i\omega t}$, $\vec{D} = \epsilon_0\vec{E}$

L

$\frac{\partial}{\partial t} \to -i\omega$ $\quad \nabla\times\vec{H}(\vec{r}) = -i\omega\epsilon_0\vec{E}(\vec{r})$

$$\boxed{\vec{E} = \frac{i}{\omega\epsilon_0}\nabla\times\vec{H}}$$

$\epsilon_0\omega = \epsilon_0 c k = \frac{\epsilon_0 k}{\sqrt{\mu_0\epsilon_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}}k = \frac{k}{Z_0}$

$$\boxed{\vec{E} = \frac{iZ_0}{k}\vec{\nabla}\times\vec{H}}$$

$\boxed{Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}}$ impedancia del vacío

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$Z_0 \simeq 376.73\ldots\,\Omega = 119.917\pi\,\Omega$ $\quad (120\pi\,\Omega \to 0.08\%)$

$\nabla\times E + \frac{\partial B}{\partial t} = 0$, $\nabla\times E - i\omega B = 0$

$\nabla\times E = i\omega B = i\omega\mu_0\vec{H}$, $\boxed{\vec{H}(\vec{r}) = -i\frac{\mu_0}{kc}\vec{\nabla}\times\vec{E}(\vec{r})}$

1er término desarrollo $\vec{A}(\vec{r})$: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int J(\vec{r}\,')d\vec{r}\,' = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(-\int \vec{r}\,' \underbrace{(\vec{\nabla}'\cdot\vec{J}')}_{-i\omega\rho(\vec{r})} d\vec{r}\,'\right)$

"integro x partes"

$\nabla\cdot J(r,t) + \frac{\partial\rho(r,t)}{\partial t} = 0$, $\vec{\nabla}\cdot\vec{J}(\vec{r})e^{-i\omega t} - i\omega\rho(\vec{r})e^{-i\omega t} = 0$

$\vec{A}_{DE}(\vec{r}) = -i\frac{\mu_0\omega}{4\pi}\vec{p}\,\frac{e^{ikr}}{r}$, $\vec{p} = \int \vec{r}\,'\rho(\vec{r}\,')d^3r'$, DE

$\vec{H}_{DE}$, $\vec{E}_{DE}$: $\vec{H}_{DE} = \frac{\nabla\times\vec{A}_{DE}(\vec{r})}{\mu_0} = \boxed{\frac{ck^2}{4\pi}\hat{r}\times\vec{p}\,\frac{e^{ikr}}{r} = \vec{H}_{DE}}$

$1/r$ $\quad 1/r$

$\left\{\begin{array}{l}\frac{\partial}{\partial t} \to -i\omega \\ \vec{\nabla} \to ik\hat{r}\end{array}\right.$

$\vec{E}_{DE} = \frac{iZ_0}{k}\vec{\nabla}\times\vec{H}_{DE} = \boxed{\frac{k^2}{4\pi\epsilon_0}\hat{r}\times(\vec{p}\times\hat{r})\frac{e^{ikr}}{r} = \vec{E}_{DE}(\vec{r})}$

$1/r$

$ck^2\frac{iZ_0}{k}ik\hat{r}\times$

$ck^2 Z_0\hat{r}\times$ $\quad cZ_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{\mu_0}{\epsilon_0}}$ ✓

$r \gg \lambda \gg L$

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$$\langle \vec S \rangle = \frac{1}{2} \mathrm{Re}\left(\vec E \times \vec H^*\right) , \quad \boxed{\frac{d\bar P}{d\Omega} = \frac{1}{2} \mathrm{Re}\left\{ r^2 \, \hat r \cdot \vec E \times \vec H^* \right\}}$$

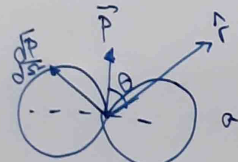
$$\frac{d\bar P_{DE}}{d\Omega} = \frac{1}{2} \mathrm{Re}\left\{ r^2 \, \hat r \cdot \left(\left(\hat r \times (\vec p \times \hat r)\right) \times (\hat r \times \vec p) \right) \right\} \frac{e^{ikr}}{r} \frac{e^{-ikr}}{r} \quad \frac{k^2}{4\pi\epsilon_0} \quad \frac{ck^2}{4\pi}$$

$$\boxed{\frac{d\bar P_{DE}}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 \left| (\hat r \times \vec p) \times \hat r \right|^2} = \frac{c^2 Z_0}{32\pi^2} k^4 |\vec p|^2 \,\mathrm{sen}^2\theta$$

$$\frac{c}{\epsilon_0} = \frac{1}{\sqrt{\mu_0\epsilon_0}\,\epsilon_0} \frac{\mu_0}{\mu_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}\, c^2 = Z_0 c^2$$

rad. cero $\parallel \vec p$

max $\perp \vec p$



$$a \cdot \left((a \times c) \times (-c) \right) = (a \times c) \cdot (-c) \times a = (a \times c) \cdot (a \times c)$$

Diag polar $\mathrm{sen}^2\theta$

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DM

$$A(\vec r) = \vec A_{DE} + \underbrace{\vec A_{DM}}_{\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(-ik)\int \vec J(\vec r')\,\hat r \cdot \vec r'\, d^3 r'} + \vec A_{QE} + \dots \quad \overbrace{\frac{1}{2}\left(\hat r \cdot \vec r' \vec J' + \hat r \cdot \vec J' \vec r'\right)}^{QE} + \overbrace{\frac{1}{2}\left(\vec r' \times \vec J'\right) \times \hat r}^{DM}$$

$$\vec A_{DM}(\vec r) = -\frac{ik\mu_0}{4\pi} \frac{e^{ikr}}{r} \frac{1}{2} \int \left(\vec r' \times \vec J'\right) \times \hat r \, d^3 r' , \quad \vec m = \frac{1}{2} \int \vec r' \times \vec J' \, d^3 r'$$

$$\boxed{\vec A_{DM}(\vec r) = \frac{ik\mu_0}{4\pi} \, \hat r \times \vec m \, \frac{e^{ikr}}{r}}$$

$$\vec H_{DM} = \frac{\vec\nabla \times \vec A_{DM}}{\mu_0}$$

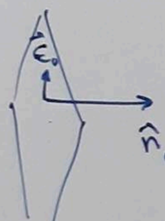
$$\vec E_{DM} = \dots$$

$$\boxed{\begin{aligned} \vec H_{DM}(\vec r) &= \frac{k^2}{4\pi} \left(\hat r \times \vec m\right) \times \hat r \, \frac{e^{ikr}}{r} \\ \vec E_{DM}(\vec r) &= -\frac{Z_0 k^2}{4\pi} \left(\hat r \times \vec m\right) \frac{e^{ikr}}{r} \end{aligned}} \rightarrow \boxed{\frac{d\bar P_{DM}}{d\Omega} = \frac{Z_0 k^4}{32\pi^2} \left| \hat r \times \vec m \right|^2}$$

$\rightarrow$ pol. $\perp$ plano $(\hat r, \vec m)$

Onda plana: incide disperos

ω, pol. $\vec{\epsilon}_0$, dir prop. $\hat{n}_0$



$$\vec{E}_i(\vec{r},t) = \vec{\epsilon}_0 E_0 \, e^{ik\hat{n}_0\cdot\vec{r}} \, e^{-i\omega t}$$

$k = \frac{\omega}{c}$

$$\begin{cases} \vec{E}_i(\vec{r}) = \vec{\epsilon}_0 E_0 e^{ik\hat{n}_0\cdot\vec{r}} \\ \vec{H}_i(\vec{r}) = \dfrac{\hat{n}_0\times\vec{E}_i}{Z_0} \end{cases}$$

(Re: sobrentendido)

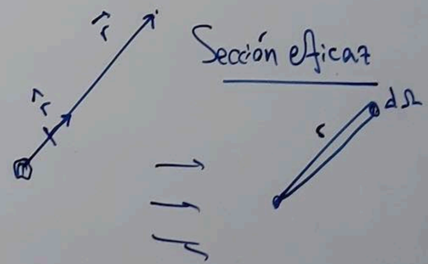
Induce multipolos en el centro dispersor: $\vec{p}$, $\vec{m}$ (para fijar ideas)

Campos de radiación producidos por estos dipolos dispersados

$$\vec{E}_d(\vec{r}) = \frac{1}{4\pi\epsilon_0} k^2 \frac{e^{ikr}}{r}\left(\hat{r}\times(\vec{p}\times\hat{r}) - \frac{\hat{r}\times\vec{m}}{c}\right)$$

$$\vec{H}_d(\vec{r}) = \frac{\hat{r}\times\vec{E}_d(\vec{r})}{Z_0}$$

$\vec{p}$, $\vec{m}$ dependen de $\vec{\epsilon}_0$, E_0, estan evaluados en $t_R = t - r/c$



Sección eficaz

$dP_d = d\sigma \, I_0$

$$\frac{dP}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right) I_0$$

sección eficaz diferencial (por unidad de ángulo sólido)

I_0: intensidad incidente

$$I_0 = \text{"}S_i\text{"} = \langle \vec{S}_i\cdot\hat{n}_0 \rangle, \quad d\sigma = \frac{dP_d}{S_i} = \frac{S_d r^2 d\Omega}{S_i}$$

$r \gg \lambda \gg L$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{S_d}{S_i} r^2}$$

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$$S_i = \langle \vec{S}_i \cdot \hat{n}_0 \rangle = \frac{1}{2} \mathrm{Re}\left\{ \vec{E}_i \times \left(\frac{\hat{n}_0 \times \vec{E}_i}{Z_0} \right)^* \cdot \hat{n}_0 \right\}$$

$$= \frac{1}{2Z_0} \mathrm{Re}\left\{ \left(\vec{E}_i \cdot \vec{E}_i^* \, \hat{n}_0 - \underbrace{\hat{n}_0 \cdot \vec{E}_i}_{0} \, \vec{E}_i^* \right) \cdot \hat{n}_0 \right\}$$

$$\boxed{S_i = \frac{1}{2Z_0} \vec{E}_i \cdot \vec{E}_i^*} \qquad \boxed{S = \frac{1}{2Z_0} \vec{E} \cdot \vec{E}^*}$$ pot. total p.u. área

$$S = \langle \vec{S} \cdot \hat{n} \rangle = \frac{1}{2} \mathrm{Re}\left\{ \vec{E} \times \left(\frac{\hat{r} \times \vec{E}}{Z_0} \right) \cdot \right\} = \cdots = \frac{1}{2Z_0} |E|^2$$

$\hat{r} \cdot \vec{E} = 0$

$$\vec{E} = \vec{E} \cdot \vec{\epsilon}_1 \, \vec{\epsilon}_1 + \vec{E} \cdot \vec{\epsilon}_2 \, \vec{\epsilon}_2$$

$$S_{\epsilon_1} = \langle \vec{S} \cdot \hat{r} |_{\epsilon_1} \rangle = \frac{1}{2Z_0} (\vec{E} \cdot \vec{\epsilon}_1) \cdot (\vec{E}^* \cdot \vec{\epsilon}_1^*)$$

$$\boxed{\langle \vec{S} \cdot \hat{r} |_{\epsilon_1} \rangle = \frac{1}{2Z_0} |\vec{\epsilon}_1 \cdot \vec{E}|^2}$$

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ϵ_1, ϵ_2 lineales $\quad \epsilon_1^2 = \epsilon_2^2 = 1$

$\epsilon_1 \cdot \epsilon_2 = 0$

$$\epsilon_\pm = \frac{\epsilon_1 \pm i \epsilon_2}{\sqrt{2}}$$

$$\epsilon_\pm^* \cdot \epsilon_\pm = 1$$

$$\epsilon_\pm^* \cdot \epsilon_\mp = 0$$

$$\boxed{\vec{E} = \vec{E} \cdot \epsilon_+^* \, \vec{\epsilon}_+ + E \cdot \epsilon_-^* \, \vec{\epsilon}_-}$$

$$\langle S \cdot \hat{r} \rangle |_{\epsilon_+} = \frac{1}{2Z_0} (\vec{E} \cdot \epsilon_+^*) \cdot (\vec{E}^* \cdot \vec{\epsilon}_+) = \boxed{\frac{|\epsilon_+^* \cdot \vec{E}|^2}{2Z_0} = \langle S \cdot \hat{r} \rangle |_{\epsilon_+}}$$

En gral. tenemos $\vec{S} \cdot \hat{r} |_{\epsilon} = \frac{1}{2Z_0} |\vec{\epsilon}^* \cdot \vec{E}|^2$

$$\frac{d\sigma}{d\Omega} = \frac{S_d}{S_i} r^2$$

$$\frac{d\sigma}{d\Omega}(\hat{r},\vec{\epsilon};\hat{n}_0,\vec{\epsilon}_0) = \frac{r^2 \frac{1}{\cancel{2Z_0}} |\vec{\epsilon}^* \cdot \vec{E}_d|^2}{\frac{1}{\cancel{2Z_0}} |\vec{\epsilon}_0^* \cdot \vec{E}_i|^2}$$

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$\vec{\epsilon} \cdot \hat{r} = 0$

$\vec{\epsilon}_0 \cdot \hat{n}_0 = 0$

$\ldots \left\{ \hat{r} \times (p \times \hat{r}) - \hat{r} \times \frac{\vec{m}}{c} \right\} \cdot \vec{\epsilon}^*$

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2} \left| \vec{\epsilon}^* \cdot \vec{p} + \hat{r} \times \vec{\epsilon}^* \cdot \frac{\vec{m}}{c} \right|^2$$

$\vec{p}, \vec{m}$: dep. $\epsilon_0, n_0, E_0, \ldots \underline{\underline{\omega}}$

$k^4 \sim \omega^4$ ley Raileigh

límite grandes long. onda

Si $\vec{p}=0$, $\vec{m}=0 \Rightarrow$ QE, octupolar....

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ϵ_1, ϵ_2 lineales $\quad \epsilon_1^2 = \epsilon_2^2 = 1$

$\epsilon_1 \cdot \epsilon_2 = 0$

$$\epsilon_\pm = \frac{\epsilon_1 \pm i\epsilon_2}{\sqrt{2}}$$

$\epsilon_\pm^* \cdot \epsilon_\pm = 1$

$\epsilon_\pm^* \cdot \epsilon_\mp = 0$

$$\boxed{\vec{E} = \vec{E} \cdot \epsilon_+^* \vec{\epsilon}_+ + E \cdot \epsilon_-^* \vec{\epsilon}_-}$$

$$\langle S \cdot \hat{r} \rangle \Big|_{\epsilon_+} = \frac{1}{2Z_0} (\vec{E} \cdot \epsilon_+^*) \cdot (\vec{E}^* \cdot \vec{\epsilon}_+) = \boxed{\frac{|\epsilon_+^* \cdot \vec{E}|^2}{2Z_0} = \langle S \cdot \hat{r} \rangle \Big|_{\epsilon_+}}$$

En gral. tenemos $\vec{S} \cdot \hat{r} \Big|_{\epsilon} = \frac{1}{2Z_0} |\vec{\epsilon}^* \cdot \vec{E}|^2$