The van Cittert–Zernike theorem in pulse echo measurements

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A classical theorem of statistical optics, the van Cittert–Zernike theorem, is generalized to pulse echo ultrasound. This theorem fully describes the second-order statistics of the spatial fluctuations (the spatial covariance) of the field produced by an incoherent source. As a random scattering medium is insonified, it behaves as an incoherent source. The van Cittert–Zernike theorem can thus predict the spatial covariance of the pressure field backscattered by a random medium. It is shown that this spatial covariance and the incident energy diagram are Fourier pairs. In the case of a focused illumination, the spatial covariance of the backscattered pressure field is proportional to the autocorrelation of the transmitting aperture function. This is independent of frequency and of $F/$ number. Experimental results obtained with a linear array are in good agreement with theoretical expectations. The implications of this theorem in speckle reduction and in focusing in nonhomogenous media are discussed.

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INTRODUCTION

In recent papers,[1,2] we presented the random phase screen approach to speckle reduction. In order to predict the signal-to-noise ratio improvement, we introduced the information grain theory. In this paper, we show the link between the concept of information grain and a classical theorem of statistical optics namely, the van Cittert–Zernike theorem. This theorem fully describes how the spatial covariance of a field produced by an incoherent source propagates.[3] In other words, it can predict the autocorrelation function of the wave field emanating from a scattering medium illuminated by any acoustic (or optical) source. This autocorrelation function depends only on the incident beam, the wavelength and the distance from the scattering medium.

In this paper, we present a set of experimental and theoretical results that show how this classical theorem of statistical optics can be generalized to pulsed ultrasound and how the coherence of the field backscattered by a random medium can be predicted. We discuss the underlying physical principles, as well as the implications of this theorem in pulse-echo imaging. This theorem sets the limit to the signal-to-noise ratio improvement in incoherent processing. The experiments are performed with a linear array working at 3.5 MHz. The measured spatial covariances at different frequencies and for different illumination beams are in good agreement with theoretical predictions.

In addition, the usefulness of this theorem in the study of correction of phase aberrations is shown. The experimental backscattered pressure field spatial covariance, measured with and without phase aberrations, will be presented. These results explain many results observed and not totally understood in phase aberration correction and in speckle reduction.

I. THEORETICAL BACKGROUND

In this section, we first present the classical theorem of statistical optics in intuitive terms and explain its physical significance. We then present the hypothesis that lead to its extension to the acoustics field and more precisely to pulsed ultrasound. We emphasize the differences between the two fields. We show that the result is more general in acoustics, mainly because it does not require the continuous-wave assumption.

A. The van Cittert–Zernike theorem

The van Cittert–Zernike theorem is one of the most important theorems of modern optics. It describes the propagation of the spatial covariance of a wave field produced by an incoherent source such as a thermal source of light. A very complete discussion of this theorem can be found, for example, in Ref. 3. The concept of spatial covariance of a field can be explained in its relation to the well-known Young's slit experiment. This experiment is fully described in Ref. 3. In a word, it consists of an incoherent light source radiating toward an observation screen. An opaque screen with two tiny holes in it is placed between the source and the screen. Since the source is incoherent, the field is spatially homogenous in the opaque screen plane. Now, although the source is incoherent, interference fringes can be observed on the observation screen. As the holes are moved apart or as the source is brought nearer to the opaque screen, the visibility of the fringes decreases. The ability of producing interference fringes in Young's experiment is described by the spatial covariance of the wave field. When the holes are close, the interference fringes are quite visible. This means that *the wave field at the two holes is correlated, or has spatial covariance.* As the distance between the holes increases, the spatial covariance decreases. It is thus possible to define the spatial covariance as a function of the distance between the holes, or more generally, considering two points X_1 and X_2 on the plane of the holes, as a function of $(X_1 - X_2)$. The notable physical phenomenon is that the spatial covariance of the field is infinitely narrow at the incoherent source and becomes wider and wider as the wave field propagates. This phenomenon is described by the van Cittert–Zernike theorem: the spatial covariance of the field at points X_1 and X_2 of an observation plane is equal to the Fourier transform of the source aperture function taken at spatial frequency $(X_2 - X_1)/\lambda z$, where λ is the wavelength and z the distance between the source and the observation plane. It can be verified easily that the larger z, the wider the spatial covariance and that the wider the source, the narrower the spatial covariance.

B. Spatial covariance in pulsed ultrasound

As we have seen, the van Cittert–Zernike theorem describes the propagation of an incoherent field. Pulsed ultrasound is generally associated with coherence and it may seem irrelevant to consider the field produced by an incoherent source. In this section, we first explain the transition from coherence to incoherence in pulsed ultrasound, we then present precisely the framework in which we then derive the generalization of the van Cittert–Zernike theorem to acoustics.

Let us consider a transducer placed in front of a totally incoherent scattering medium such as biological tissue, composite material or grain steel. As the transducer illuminates the scattering medium, a pressure field is scattered back toward the transducer. At any given time, the scattered field sensed by the transducer originates from a limited region of the scattering medium, namely, the isochronous volume.[4] Note, however, that the isochronous volume depends on the considered time instant. At any given time, the scattered pressure field can be considered to be produced by a volumic source occupying the entire isochronous volume. The aperture function of this equivalent source is thus related to the transmitted energy diagram. The statistical properties of the equivalent source depend on those of the scattering medium. If the scatterers are randomly distributed in the medium and the isochronous volume contains a great number of them, the equivalent source will be incoherent. If the distribution of scatterers is not isotropic, as for example in some composite materials, the coherence of the source will be anisotropic. With this point of view, one can generalize some interesting results from optics. However, one has to be aware of the differences. Because of the correspondence between time and depth in pulsed ultrasound, the time dependence does not play the same role in the two domains. The temporal incoherence of the source results from this correspondence between time and depth and from the incoherence of the scattering medium. Moreover, because of time varying (i.e., depth dependent) diffraction effects, the statistical properties of the source vary in time. As a consequence, the signals are not stationary and, hence, cannot be considered as ergodic. This means that, in general, temporal averages, ensemble averages, and spatial averages are not equivalent as it is some times the case in optics.

C. Backscattered pressure field

Before studying the spatial covariance of the pressure field produced by the equivalent source, we derive an analytic expression of this field in an impulsional formalism. Let us consider a transducer working in piston mode embedded in a rigid baffle. We call $O(\mathbf{X})$, $\mathbf{X} = (x,y,0)$, its aperture function. Its diffraction impulse response at a point $\mathbf{X}_1$ is given by Stepanishen's formalism as[4]

$$h(\mathbf{X}_1,t) = \int\int_O O(\mathbf{X})\,\frac{\delta(t - r/c)}{r}\,d\mathbf{X};\quad r = |\mathbf{X}_1 - \mathbf{X}|. \tag{1}$$

This impulse response is described in the frequency domain by its Fourier transform:

$$H(\mathbf{X}_1,f) = \int\int_O O(\mathbf{X})\,\frac{e^{j2\pi fr/c}}{r}\,d\mathbf{X}. \tag{2}$$

This frequency response represents the incident pressure field at point $\mathbf{X}_1$ and at frequency f.

Now suppose that the transducer illuminates an uniform scattering medium with scattering function $\chi(\mathbf{X},f)$. Additionally, assume that the scattering medium is incoherent, that is, that the medium microstructure is random and finer than the smallest wavelength we use. This implies that the medium details cannot be seen (or resolved) at our working frequencies. At those frequencies where the medium is unresolved, the autocorrelation of the scattering function is a separable function of frequency and space: $\langle\chi(\mathbf{X}_1,f)\chi(\mathbf{X}_2,f)\rangle = \chi_0(f)\delta(\mathbf{X}_1 - \mathbf{X}_2)$. At every such frequency, the autocorrelation of the scattering function is proportional to a Dirac function.

The pressure field scattered by an individual scatterer located at point $\mathbf{X}_1$ is a spherical wave. At a point $\mathbf{X}_0$, this pressure field is given by

$$P(\mathbf{X}_0,\mathbf{X}_1,f) = \chi(\mathbf{X}_1,f)H(\mathbf{X}_1,f)\,\frac{e^{j2\pi f(r_{01}/c)}}{r_{01}};\quad r_{01} = |\mathbf{X}_0 - \mathbf{X}_1|. \tag{3}$$

The pressure field backscattered from the whole medium in response to a Dirac pulse and sensed at a point $\mathbf{X}_0$ is obtained by integrating over the whole space the above equation:

$$P(\mathbf{X}_0,f) = \int\int\int_V \chi(\mathbf{X}_1,f)H(\mathbf{X}_1,f)\,\frac{e^{j2\pi f(r_{01}/c)}}{r_{01}}\,d^3\mathbf{X}_1. \tag{4}$$

D. Spatial covariance of the scattered pressure field

The spatial covariance of the pressure field at two points $\mathbf{X}_1$ and $\mathbf{X}_2$ is defined as the correlation between the field sensed at the two points. This definition is a consequence of the statistical nature of the medium. Hence, the statistics implied by this definition are ensemble statistics over an ensemble of scattering media. The spatial covariance is a function of $\mathbf{X}_1$, $\mathbf{X}_2$, frequency and, as we will see later, of time (i.e., depth). We call $R_p(\mathbf{X}_1,\mathbf{X}_2,f)$ the spatial covariance function at points $\mathbf{X}_1$ and $\mathbf{X}_2$ and at frequency f. We have

$$R_p(\mathbf{X}_1,\mathbf{X}_2,f) = \langle P(\mathbf{X}_1,f),P^*(\mathbf{X}_2,f)\rangle, \tag{5}$$

where the brackets represent an average over an ensemble of scattering media and the * represents the complex conjugate. From the expression of the pressure field, we have

$$P(\mathbf{X}_1,f)P^*(\mathbf{X}_2,f) = \int\int\int_V\int\int\int_V \chi(\mathbf{X}_1',f)\chi^*(\mathbf{X}_2',f) \times H(\mathbf{X}_1',f)H^*(\mathbf{X}_2',f)\frac{e^{j2\pi f[(r_1'-r_2')/c]}}{r_1' r_2'}\,d^3\mathbf{X}_1'\,d^3\mathbf{X}_2',$$

$$r_i' = |\mathbf{X}_i - \mathbf{X}_i'|. \tag{6}$$

Now, since the only random variable is the scattering function and since the scattering medium is incoherent, the spatial covariance of the pressure fluctuations is

$$R_p(\mathbf{X}_1,\mathbf{X}_2,f) = \chi_0(f)\int\int\int_V |H(\mathbf{X},f)|^2 \times \frac{e^{j2\pi f[(r_1-r_2)/c]}}{r_1 r_2}\,d^3\mathbf{X},$$

$$r_i = |\mathbf{X}_i - \mathbf{X}|. \tag{7}$$

Note that this formula is valid in a very general case and that it is based on a impulsional formalism.

In order to probe further, we introduce the Fresnel approximation. With this approximation, we restrict the domain of validity of the formula, specifically, the following will not be valid in the theoretical case where the transducer transmits a Dirac pressure pulse. However, we stress the fact that the Fresnel approximation is widely valid in pulsed ultrasound because the ultrasound beam is collimated around the transducer axis and because the spectra of ultrasound signals are limited by the acoustoelectric response of the transducer. Therefore, if the Fresnel approximation is verified for the smallest wavelength, it is also verified for the complete spectrum.

Under the Fresnel (or paraxial) approximation, it is assumed that the depth is large compared to the transverse distances. In that condition, the distance r between two points $\mathbf{X}_1 = (\mathbf{x}_1,0)$ and $\mathbf{X}_2 = (\mathbf{x}_2,z)$ ($r = |\mathbf{X}_1 - \mathbf{X}_2| = [z^2 + (\mathbf{x}_1 - \mathbf{x}_2)\cdot(\mathbf{x}_1 - \mathbf{x}_2)]^{1/2}$) is approximated by the first two terms of its series expansion:

$$r \approx z[1 + (1/2z)(\mathbf{x}_1 - \mathbf{x}_2)\cdot(\mathbf{x}_1 - \mathbf{x}_2)]. \tag{8}$$

This approximation only requires that z be about ten times larger than $|\mathbf{x}_1 - \mathbf{x}_2|$. In the Fresnel approximation, whenever r appears in a phase term, it is replaced according to the above equation, and whenever it appears in an amplitude term, it is replaced by z.

Under this approximation, Eq. (7) becomes

$$R_p(\mathbf{X}_1,\mathbf{X}_2,f) = R_p(\mathbf{x}_1,\mathbf{x}_2,f) = \chi_0(f)\int\int\int_V \frac{|H(\mathbf{x},z,f)|^2}{z^2} \times \exp\left(j2\pi f\frac{1}{2zc}[(\mathbf{x}-\mathbf{x}_1)(\mathbf{x}-\mathbf{x}_1) - (\mathbf{x}-\mathbf{x}_2)\cdot(\mathbf{x}-\mathbf{x}_2)]\right)d^2\mathbf{x}\,dz \tag{9}$$

$$R_p(\mathbf{x}_1,\mathbf{x}_2,f) = \chi_0(f)\int\int\int_V \frac{|H(\mathbf{x},z,f)|^2}{z^2} \times \exp\left(j\pi f\frac{[\mathbf{x}_1\cdot\mathbf{x}_1 - \mathbf{x}_2\cdot\mathbf{x}_2 - 2\mathbf{x}\cdot(\mathbf{x}_1 - \mathbf{x}_2)]}{zc}\right) \times d^2\mathbf{x}\,dz$$

$$= \chi_0(f)\int\int\int_V \frac{|H(\mathbf{x},z,f)|^2}{z^2} \times \exp\left(j\pi f\frac{(\mathbf{x}_1\cdot\mathbf{x}_1 - \mathbf{x}_2\cdot\mathbf{x}_2)}{zc}\right) \times \exp\left(-j2\pi f\frac{[\mathbf{x}\cdot(\mathbf{x}_1 - \mathbf{x}_2)]}{zc}\right)d^2\mathbf{x}\,dz. \tag{10}$$

This last equation is the mathematical formulation of the van Cittert–Zernike theorem in pulse echo measurements.

One very useful property of pulsed ultrasound is the shortness of the pulse duration and the subsequent correspondence between time and depth. With the use of a narrow time window, it is possible to isolate the pressure field emanating from scatterers located at a particular depth z. It is thus possible to define the spatial covariance of that pressure field. This function will also be a function of depth z or of time $t(t = 2z/c)$. It can be obtained by imposing z to be a constant in Eq. (10) and by dropping the corresponding integration:

$$R_p(\mathbf{x}_1,\mathbf{x}_2,z,f) = \frac{\chi_0(f)}{z^2}\exp\{j\pi f[(\mathbf{x}_1\cdot\mathbf{x}_1 - \mathbf{x}_2\cdot\mathbf{x}_2)/zc]\} \times \int\int_S |H(\mathbf{x},z,f)|^2 \times \exp\left(-j2\pi f\frac{[\mathbf{x}\cdot(\mathbf{x}_1 - \mathbf{x}_2)]}{zc}\right)d^2\mathbf{x}. \tag{11}$$

where S is the integration surface defined as the plane orthogonal to the transducer axis, placed at distance z.

This approximation is indeed a strong one. A more correct approach would be to integrate over the isochronous volume of interest, rather than on a surface. However, one could instead introduce another approximation that is described in Ref. 5. This approximation is somewhat complex and for that reason it will not be used here. However, it would give similar results to the very simple one we use here. In a few words, this approximation consists in integrating over the isochronous volume. In the Fresnel approximation, the scatterers contributing to the backscattered pressure field are concentrated around the transducer axis. Since the isochronous volume is not very thick around the axis, we can assume that the transducer impulse response does not vary too much along a radius. Hence, this volume integral can be replaced by a surface integral. The integration domain has to be a surface that is included in the isochronous volume. A sphere centered on the transducer center is one such surface. The difference between the two approximations is the surface over which we integrate. In our simpler approximation, we integrate over a plane surface rather than over a spherical one.

In a first approach, we will consider that the two points are symmetrical with respect to the center of the transducer ($\mathbf{x}_1 = -\mathbf{x}_2$). The phase term outside the integral drops and $R_p(\mathbf{x}_1,\mathbf{x}_2,z,f)$ is simply the *two-dimensional Fourier transform* of $|H(\mathbf{x},z,f)|^2$ taken at spatial frequency $(\mathbf{x}_1 - \mathbf{x}_2)f/zc = (\mathbf{x}_1 - \mathbf{x}_2)/\lambda z$. We thus find here the classical theorem of statistical optics. In order to probe further, we now consider the incident pressure field, i.e., $H(\mathbf{x},z,f)$. Let us apply the Fresnel approximation to Eq. (2). We have

$$\begin{aligned} H(\mathbf{x}_1,z,f) &= \int\int_O O(\mathbf{x}) \exp\left[j2\pi f \frac{z}{c}\left(1 + \frac{1}{2} \times \frac{(\mathbf{x}-\mathbf{x}_1)(\mathbf{x}-\mathbf{x}_1)}{z^2}\right)\right] (z)^{-1}\, d\mathbf{x} \\ &= \frac{1}{z} e^{j2\pi f(z/c)} e^{(j\pi f/zc)\mathbf{x}_1\cdot\mathbf{x}_1} \int\int_O O(\mathbf{x}) \times e^{(j\pi f/zc)\mathbf{x}\cdot\mathbf{x}} e^{(j\pi f/zc)\mathbf{x}\cdot\mathbf{x}_1}\, d\mathbf{x}. \end{aligned} \tag{12}$$

This formula can be interpreted as follows: The first phase term corresponds to a propagation delay from the transducer to the surface S at depth $z(t = z/c)$. The second phase term is due to the fact that the surface S is plane rather than spherical. However, since in the estimation of the spatial covariance we use its squared modulus, this phase term can be ignored. The integral is a two-dimensional Fourier transform of $O(\mathbf{x})e^{(j\pi f/zc)\mathbf{x}\cdot\mathbf{x}}$ at spatial frequency $\mathbf{x}_1/\lambda z$. For an aperture focusing at depth z, the aperture function $O(\mathbf{x})$ is not simply a circle or rectangle function. It contains a phase term $e^{(-j\pi f/zc)\mathbf{x}\cdot\mathbf{x}}$. Therefore, $O(\mathbf{x})e^{(j\pi f/zc)\mathbf{x}\cdot\mathbf{x}}$ is real. In the following, we will note it $\tilde{O}(\mathbf{x}) = O(\mathbf{x})e^{(j\pi f/zc)\mathbf{x}\cdot\mathbf{x}}$. We thus have

$$|H(\mathbf{x},z,f)|^2 = \frac{1}{z^2} \int\int_O \int\int_O \tilde{O}(\mathbf{x}_1')\tilde{O}^*(\mathbf{x}_2') \times \exp\left(j2\pi f \frac{\mathbf{x}\cdot(\mathbf{x}_1' - \mathbf{x}_2')}{zc}\right) d^2\mathbf{x}_1'\, d^2\mathbf{x}_2'. \tag{13}$$

We now combine this last equation with Eq. (11) to obtain

$$\begin{aligned} R_p(\mathbf{x}_1,\mathbf{x}_2,z,f) &= \frac{\chi_0(f)}{z^4} \int\int_S \int\int_O \int\int_O \tilde{O}(\mathbf{x}_1')\tilde{O}^*(\mathbf{x}_2') \exp\left(-j2\pi f \frac{\mathbf{x}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}{zc}\right) \exp\left(j2\pi f \frac{\mathbf{x}\cdot(\mathbf{x}_1' - \mathbf{x}_2')}{zc}\right) d^2\mathbf{x}\, d^2\mathbf{x}_1'\, d^2\mathbf{x}_2' \\ &= \frac{\chi_0(f)}{z^4} \int\int_O \int\int_O \tilde{O}(\mathbf{x}_1')\tilde{O}^*(\mathbf{x}_2') \int\int_S \exp\left(-j2\pi f \frac{\mathbf{x}\cdot(\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{x}_1' + \mathbf{x}_2')}{zc}\right) d^2\mathbf{x}\, d^2\mathbf{x}_1'\, d^2\mathbf{x}_2' \\ &= \frac{\chi_0(f)}{z^4} \int\int_O \int\int_O \tilde{O}(\mathbf{x}_1')\tilde{O}^*(\mathbf{x}_2')\delta(\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{x}_1' + \mathbf{x}_2')\, d^2\mathbf{x}_1'\, d^2\mathbf{x}_2' \\ &= \frac{\chi_0(f)}{z^4} \int\int_O \tilde{O}(\mathbf{x}_1')\tilde{O}^*[\mathbf{x}_1 - (\mathbf{x}_1' - \mathbf{x}_2')]\, d^2\mathbf{x}_1' = [\chi_0(f)/z^4] R_{\tilde{O}}(\mathbf{x}_1 - \mathbf{x}_2). \end{aligned} \tag{14}$$

The spatial covariance is thus a separable function of $\mathbf{x}_1 - \mathbf{x}_2$ and of frequency. Its spatial dependance is proportional to the autocorrelation of the transmitter aperture function. For example, in the case where the transmitter is a 1-D aperture such as a linear array, its aperture function is a rectangle function. Its autocorrelation is thus a triangle function whose base is twice as wide as the rectangle. The van Cittert–Zernike theorem in pulse echo measurements is summarized in Fig. 1.

Note that the above derivation is valid for any frequency and, as a consequence, *the spatial dependence of the spatial covariance function is independent of frequency.* This is an interesting property because it allows a simple definition of the normalized spatial covariance function in which the frequency dependence is omitted. This new spatial covariance function will then be the same for continuous wave as well as for pulsed ultrasound. Moreover, the (normalized) spatial covariance function of the field only depends on the transmitter aperture function. It is independent of focal depth or $F/$ number.

The above results are valid when the phase term in Eq. (10) drops. We have seen that this is true when the two points are symmetrical with respect to the transducer center. More generally, it is also true when the backscattered pressure field is sensed on a focusing surface that exactly compensates the phase term. Note that the focal plane of such a surface is at depth z.

II. CONSEQUENCES IN ULTRASONIC IMAGING

A. Speckle reduction

Speckle noise is one serious limitation of ultrasonic imaging and tissue characterization. It reduces the contrast resolution and thus the detectability of small targets. Moreover, it introduces random errors in estimates of qualitative parameters.

This speckle noise is linked to the spatial coherence of ultrasound beams as well as to the stochastic nature of the explored media. It is the consequence of random interferences, at the receiver, between echoes produced by randomly

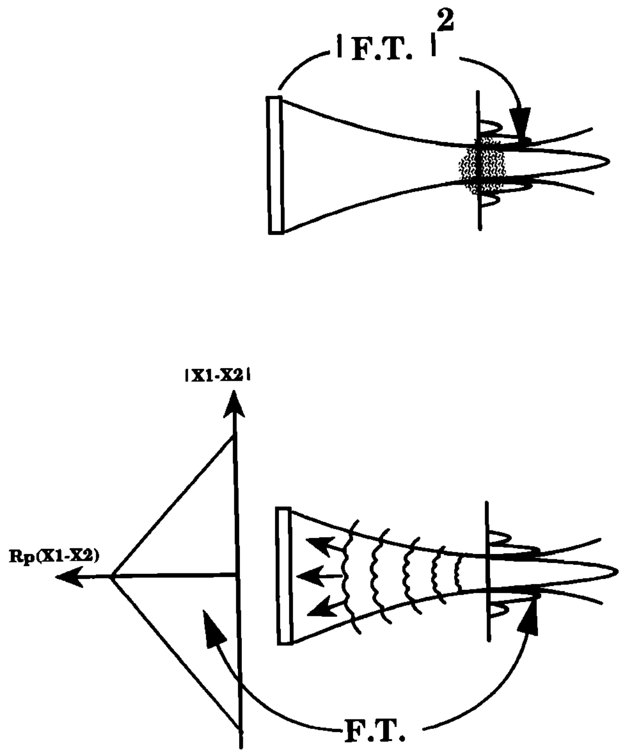

FIG. 1. The van Cittert–Zernike theorem in pulse echo measurements. A focused aperture illuminates a scattering medium (top). The illuminated medium behaves as an incoherent source that radiates a backscattered field (bottom). The spatial covariance of the scattered field is the Fourier transform of the incident energy diagram. For a 1-D illumination, it is a triangle function whose base is twice as wide as the transmitter.

located reflectors within a resolution cell.

The problem of speckle reduction has been approached by the use of incoherent processing techniques.

1. Principle of incoherent processing

One form of incoherent processing consists in breaking the coherence of the ultrasonic beam by the use of phase insensitive transducers. Such transducers exist, as was shown by Heyman[6] with CdS acoustoelectric transducers, or as we have shown in Refs. 1 and 2 with the random phase transducer. A similar approach was proposed by Smith *et al.*[7] with the Maltese Cross processor. They used a circular transducer divided into a number of sectors. The signal sensed by the sensors were combined after envelope detection, hence, providing a phase insensitive processing of the backscattered ultrasound. More generally, Johnston *et al.*[8] have proposed incoherent processing schemes where the backscattered pressure field is sampled by small coherent subapertures and where the individual signals are combined in a phase insensitive way (after envelope detection). In all these studies, rather low signal-to-noise ratio (SNR) improvements were obtained, except when dramatic losses of resolution were allowed.

2. SNR improvement in incoherent processing

One success of the van Cittert–Zernike theorem is the interpretation of these results. We already presented in Refs. 1 and 2 the SNR improvement that can be achieved with such techniques. That presentation was not backed by the complete van Cittert–Zernike formalism, however, the basic concepts are stated there. Therefore, we will omit the lengthy derivation here.

The idea is to consider the phase insensitive transducer as a collection of pointlike sensors that sense the intensity of the pressure field. The incoherent processing is then viewed as an averaging of the data sensed by all these sensors. Now, with the averaging point of view in mind, it is easy to realize that the more correlated the data, the lower the SNR improvement. In other words, the wider the spatial covariance of the backscattered pressure field, the lower the SNR improvement. Following this reasoning, we show in Ref. 1 that the SNR improvement is given by

$$\mathrm{SNR} = A_R A_T \Big/ \left(\int [R_P(\mathbf{x})]^2 R_{O_R}(\mathbf{x}) d\mathbf{x} \right)^{1/2}, \qquad (15)$$

where A_R is the area of the receiver, A_T is the area of the transmitter, and R_{O_R} is the receiver autocorrelation function. This formula depends only on the receiving aperture function and the spatial covariance of the backscattered pressure field. The SNR improvement is low when the denominator is large. That is, when the spatial covariance of the backscattered pressure field is wide. This means that in order to increase the SNR, we have to receive a large number uncorrelated data (or information grains). For this, we can either make the spatial covariance narrower or use a wider receiver.

3. Influence of transmitting aperture size on SNR improvement

The above equation shows the relationship between SNR improvement and spatial covariance of the pressure field. The van Cittert–Zernike theorem shows that the latter is a function of the transmitting aperture as long as it is focused. The SNR improvement is thus a function of the transmitting and of the receiving aperture functions. As a rule of thumb, it is given by the relative size of the information grain (or coherence area of the scattered field) and of the receiving aperture. In Ref. 1, we compute the numerical value of the SNR improvements that can be expected with such techniques for several transmitter sizes. For example, with 1-D apertures, if the transmitter and the receiver have the same size, the SNR improvement is limited to $2^{1/2}$.

III. EXPERIMENTS

We now investigate the experimental aspect of the van Cittert–Zernike theorem. The goal is to define a measurement procedure and then to validate the previous theoretical analysis.

In these experiments, we are interested in the spatial statistics of the backscattered pressure field. We want to measure the scattered pressure field rather than a pulse echo signal. We therefore need a spatial and temporal sampling of the pressure field in the plane of the transmitter. For this, we have to place tiny pressure sensors wherever we want to sample the pressure field. This could be done, for example, with a very small receiver, such as a hydrophone, that scans the transmitter plane on a sufficiently fine grid. However, such an approach is delicate because the presence of the hydrophone in the beam of the transmitting transducer alters the incident field in an unpredictable way and because the hydrophone cannot be placed in the plane of the transmitter. For these reasons, it is more suitable to use a linear array that serves both as the transmitter and as the receiver. The linear array will properly sample the field provided that its pitch is finer than the field fluctuations and that it does not perturb the field. These two conditions are met in practice because the field fluctuations are wide (the spatial covariance is expected to be as wide as the transmitter) and because the acoustic impedance of the array is matched to that of the propagation medium (the PZT transducer array utilizes a $\lambda/4$ adapting layer). The signal received on each array element is a true electric transcription (except for a temporal convolution with the acoustoelectric response of the transducers) of the pressure at the corresponding point.

A programmable multichannel transceiver was designed for these experiments. It allows the flexible programmation of any 0-200V binary signal independently on any transmitter channel with a time resolution of 20 ns. In a preferred working mode, it is programmed to transmit a focused pulse. Each transmitter is associated with a receiver preamplifier. The received signals are multiplexed and fed to a LeCroy 8 bits, 32-MHz digitizer. Processing is performed on the AT 386 computer that also drives the transceiver. The array is a 128 elements PZT linear array working at 3.5 MHz, with a 0.75-mm pitch from Philips Ultrasound Inc. Particular care was taken to minimize any cross talk between the array elements. This setup is shown in Fig. 2. Typical transmitted pulses are displayed in Fig. 3.

A. Estimation of the spatial covariance

We have seen that the spatial covariance of the field is the correlation of the fluctuations of the pressure field as a function of distance. In our experiments, we end up with a two-dimensional signal that is a function of time and of space. The time dependence is obvious, the spatial dependence comes from the spatial sampling of the field. At any time instant, the spatial covariance of the field can be viewed as the autocorrelation function of the corresponding one-dimensional function of space. The spatial covariance of the field at time t can thus be estimated for all distances, n, given in number of array elements as

$$\hat{R}_p(n,t) = \frac{N}{N-n}\,\frac{\sum_{i=1}^{N-n} p(i,t)p(i+n,t)}{\sum_{i=1}^{N} p(i,t)^2}. \tag{16}$$

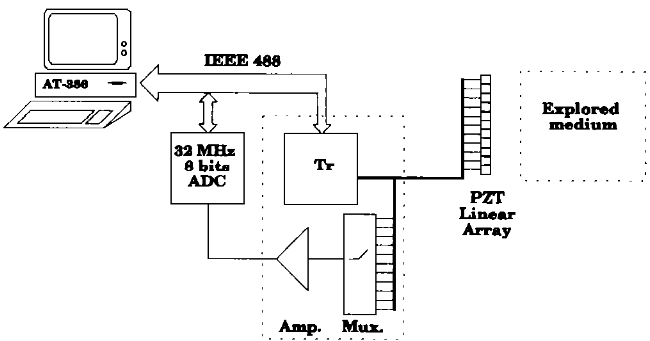

FIG. 2. Schema of the experimental setup used in the experiments. The transducer array is excited by an AT-386 controlled programmable transmitter. On receive, the signals are multiplexed, amplified, and digitized. Processing is performed on the AT-386 personal computer.

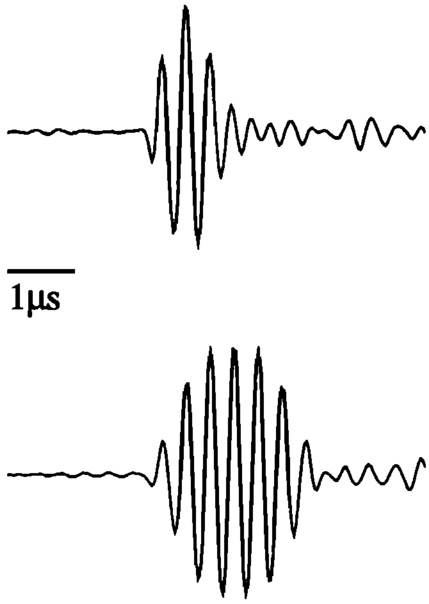

FIG. 3. Typical pulses transmitted by the array. Wide- and narrow-band pulses are used in the following.

In autocorrelation measurements, it is often necessary to perform some kind of averaging to reduce random errors. This implies the estimation of the autocorrelation function of the field for different realizations of the scattering medium. However, it is simpler to perform averaging in the time domain. The rational for time domain averaging is that the pressure fields sensed at different time instants correspond to different zones of the scattering medium. However, this kind of averaging requires that the signals are ergodic and that the spatial covariance is independent of depth. Unfortunately, this is not true, mainly because the transmitted beam varies with depth. However, since the beamwidth is almost constant around the focal point, it is possible to specify a time window for which the spatial covariance is expected not to vary and to use it for averaging. In our experiments, we used time windows of between 2.0 and 4.0 μs. The spatial covariance is therefore estimated according to:

$$\hat{R}_p(n) = \frac{N}{(N-n)}\,\frac{\sum_{i=1}^{N-n}\int_{T_f-T/2}^{T_f+T/2} p(i,t)p(i+n,t)\,dt}{\sum_{i=1}^{N}\int_{T_f-T/2}^{T_f+T/2} p(i,t)^2\,dt}, \tag{17}$$

where T_f is the transmit focus time and T the time window duration. If we omit, in Eq. (17), the normalization denominator, this estimator can be viewed as an average of the cross-correlation coefficient for all pairs of signals $\{p(i,t),p(i+n,t)\}$. However, notice that the cross-correlation coefficient we consider here does not involve the estimation of the cross-correlation function between $p(i,t)$ and $p(i+n,t)$ for all time lags. It is simply the value of the cross-correlation function for zero lag.

Because of this estimation procedure, an important issue in these measurements is the phase relationship between the time-dependent signals. In the derivation of the van Cittert–Zernike theorem, we assumed that the correlation is estimated for points symmetrical with respect to the array axis or that the field was sampled on a focusing surface. Here we sample the field on a flat (nonfocusing) surface. The effect of such a sampling can be approximated by a time shift of the signals which relates to focusing delays in a sampled aperture imager (and to the time of flight differences from the focal point to the array elements). Therefore, we need to shift the signals in order to compensate for these focusing delays. In doing so, the signals are brought into phase. This is illustrated in the following section.

B. One example: Needle target

In a first set of experiments, we did not use a random scattering medium. Instead, we used a needle target located at a known distance on the axis of the linear array. This allows a proper understanding of the necessity to shift the signals.

Since the needle tip is small with respect to the wavelength, we expect a spherical wave to emanate from it. This means that the field is uniform on the plane where it is sampled and as a consequence all the array elements receive the same signal except for a delay. (This delay can be viewed as a focusing delay.) For this reason, a flat correlation function is expected.

This example can be used to demonstrate the importance of the phase in the van Cittert–Zernike theorem. Let us consider the spatial covariance estimated *with* and *without* correction for the aforementioned focusing delays. We see from Fig. 4 that the expected flat correlation function is obtained only if the signals are corrected for the delays. In other words, the signals are to be in phase before the estimation of the correlation function.

Further insight on the importance of the phase is gained in the following experiment in which the same needle target is illuminated through an aberrating layer. Since the target is pointlike, the reflected wave front is spherical. Therefore, we expect a flat spatial covariance. This is indeed obtained only if the delays introduced by the aberrator are corrected in addition to the focusing delays (Fig. 5).

These two results are also illustrated qualitatively in Fig. 6 that uses a display format introduced by Flax and O'Donnell.[9] In this format, the rf signals are displayed as they are received on the array elements. Each line represents an array element, time is the horizontal axis of the figure and the signal amplitude is represented by the gray level. Negative signal values are in black while positive signal values are

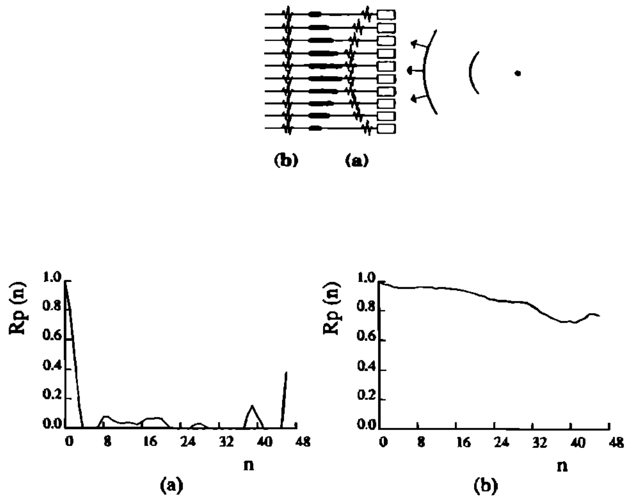

FIG. 4. Spatial covariance of the field reflected on a needle point. Estimated (a) with and (b) without correction for the focusing delays. The schema represents the experimental configuration.

in white. With this format, it is easy to understand the importance of the delay correction and to see that all the elements receive approximately the same signal, indicating that a spherical wave is reflected by the target.

C. Uniform scattering function

The analysis we developed earlier assumes a medium with uniform scattering function. In order to simulate such a medium, we used a Nuclear Associates tissue mimicking phantom. Particular care was taken to choose a region of the phantom free of resolution wires or contrast cysts.

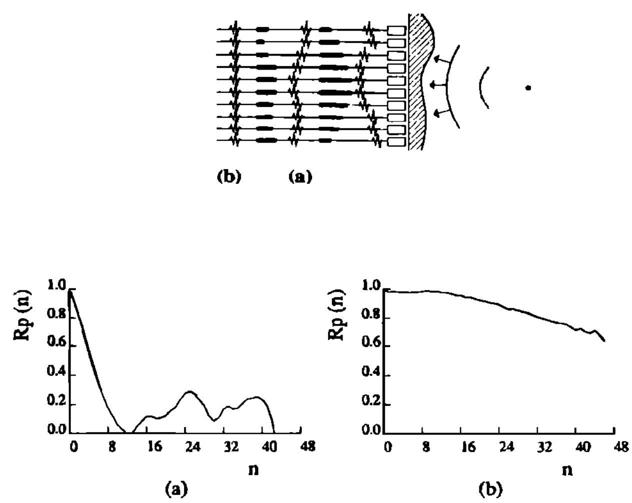

FIG. 5. Spatial covariance of the field reflected on a needle point illuminated throught an aberrating layer. Estimated (a) with and (b) without correction for the aberrating delays. The schema represents the experimental configuration.

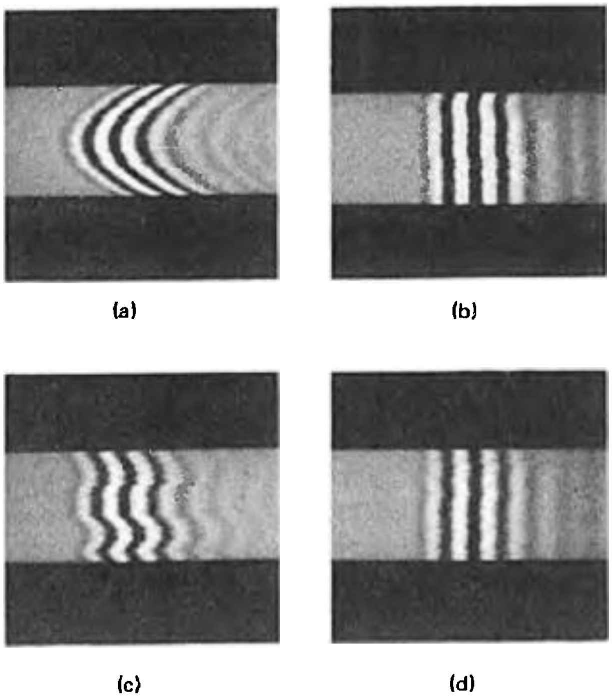

FIG. 6. Representation of the rf signals used in the needle target experiments. (a) Nonaberrating medium, (b) same as (a) after correction for the focusing delays. (c) Aberrating medium, (d) same as (b), after correction for the aberrating delays.

A first difficulty in these experiments arises, as we have already seen, from the curvature of the wave fronts emanating from the medium. These can easily be corrected by appropriate focusing. Figure 7 shows the rf lines recorded at the array elements using the Flax/O'Donnell format. From this figure, it is clear that although the signals have been delayed for focusing, they are in phase on a limited range. From this observation, we anticipate a bias in the spatial correlation estimate. More precisely, the spatial correlation is expected to be underestimated because the signals are out of phase at both ends of the record.

In an attempt to minimize this problem, one would argue that tracking focusing should be performed over the region of interest. However, we should also keep in mind that our theoretical derivation assumes that the depth of observation is fixed (and is the focal distance of the transmitter). This has two effects. First, the spatial correlation of the scattered field is a function of depth [Eq. (11)]. Second, the incident field also depends on depth. Hence, so does the spatial covariance. Outside of the focal zone the incident pressure field is wider and gives rise to a scattered field with a narrow spatial covariance.

This suggests that in order to estimate the spatial correlation of the scattered pressure field, one has to be careful to consider a slice of medium located around the focus and not wider that the depth of focus.

1. Influence of focal depth

In a first set of scattering medium experiments, we study the influence of the focal length. We used a 48 elements transmitting aperture and focused it at 60 and 90 mm corresponding to $F/$ numbers of 1.7 and 2.5. In both cases, the expected triangle function was obtained for the spatial covariance of the backscattered field (Fig. 8). This indicates that the van Cittert–Zernike theorem correctly predicts the spatial covariance of a field scattered by a random medium and that this spatial covariance is indeed independent of focal length and of $F/$ number.

2. Influence of frequency

To study the influence of frequency, we excited the linear array with five-period-long pulse trains at frequencies of 2.5 and 3.5 MHz. The focal length was set at 90 mm and the aperture size to 48 elements. Here also, good agreement was obtained between experiments and theory (Fig. 9). That is to say that the spatial covariance of the backscattered pressure field is independent of frequency.

3. Influence of aperture size

Another interesting result is obtained when we change the size of the transmitter. We have seen that the spatial

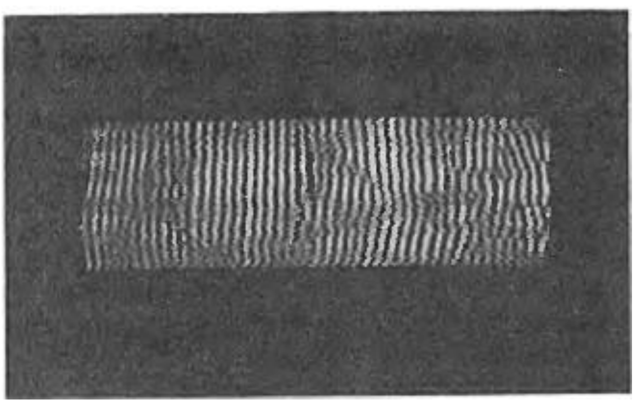

FIG. 7. Representation of the rf signals used in the scattering medium experiments. Note the curvature of the wave front at both ends of the record.

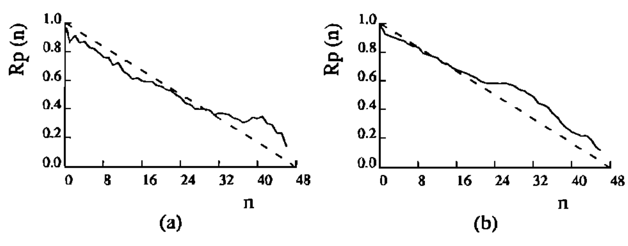

FIG. 8. Spatial covariance of the field scattered by a random medium. Influence of focal length: (a) 60 mm and (b) 90 mm. The dashed line represents the theoretical prediction.

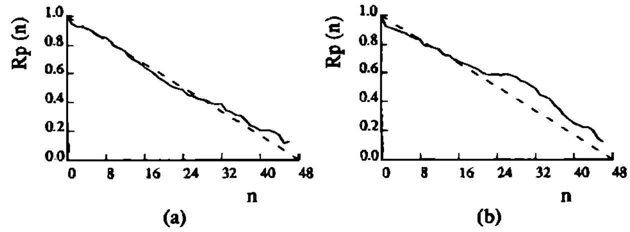

FIG. 9. Spatial covariance of the field scattered by a random medium. Influence of frequency: (a) 2.5 MHz and (b) 3.5 MHz. The dashed line represents the theoretical prediction.

covariance of the field is proportional to the autocorrelation of the transmitting aperture. If we use the full available aperture, we expect a triangle function that is twice as wide as the transmitter. If we use a smaller transmitting aperture, we still expect a triangle function, but with a narrower base. This is indeed obtained experimentally as shown in Fig. 10 where the transmitting apertures used were 31 and 16 elements wide. This result is easily explained in the light of the van Cittert–Zernike theorem: The smaller transmitter produces a wider beam which in turn produces a backscattered field whose spatial covariance is narrower.

This observation is very interesting because it shows how the field coherence can be varied with the choice of the transmitting aperture and how the size of the transmitter can influence the signal-to-noise ratio improvement in incoherent processing of pulse echo signals. As always, higher SNR improvement are expected at the cost of a loss of geometric resolution.

D. Nonhomogenous medium

Similar effects can be observed when the explored medium is not homogenous. More specifically, if the medium is illuminated through an aberrating layer, the transmitted beam is spread. The wider beam, in turn, produces a backscattered field whose spatial covariance is narrow. In this experiment, we used a 31 elements aperture and placed a rubber layer of nonuniform thickness in front of the array. The delays introduced by the layer were corrected in receive before the estimation of the spatial covariance (Fig. 11).

This result is of special importance in incoherent processing of pulse echo signals because the same receiver can

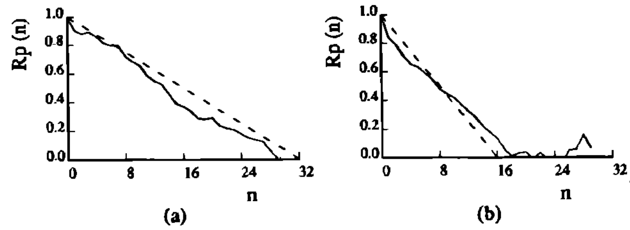

FIG. 10. Spatial covariance of the field scattered by a random medium. Influence of the transmitting aperture size: (a) 31 elements and (b) 16 elements. The dashed line represents the theoretical prediction.

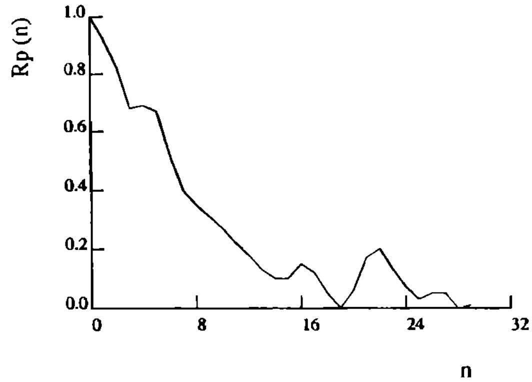

FIG. 11. Spatial covariance of the field scattered by a random medium illuminated through an aberrating layer.

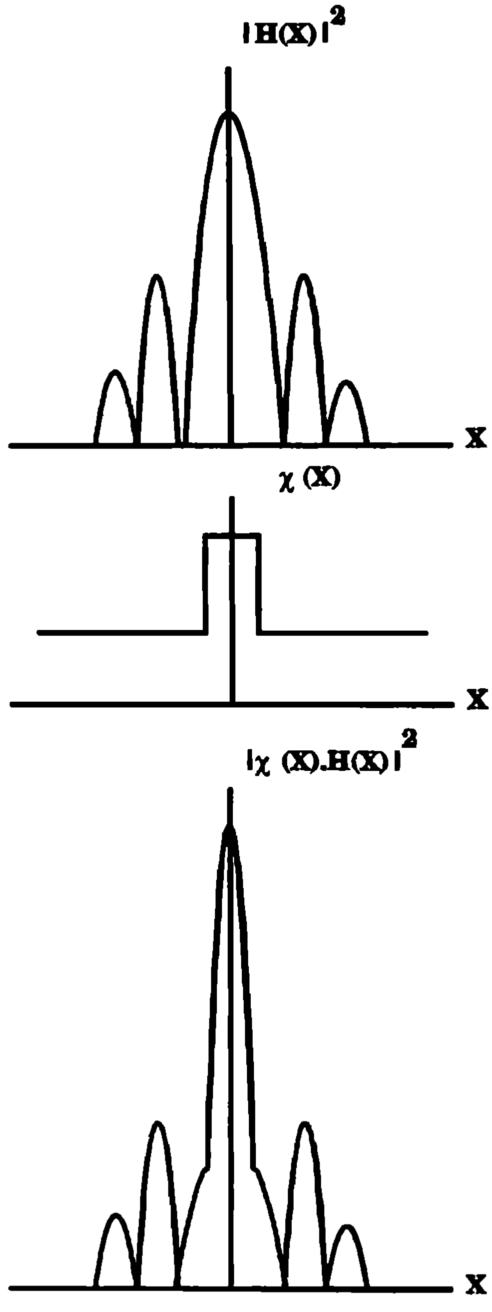

FIG. 12. Equivalent incoherent source aperture function. The incident energy diagram (top) is modulated by the scattering function of the medium (middle) to form the equivalent incoherent source aperture function (bottom).

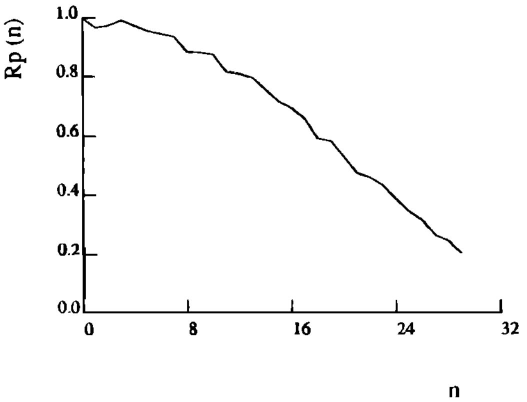

FIG. 13. Spatial covariance of the field scattered by a nonuniform random medium containing a small positive contrasted region.

sense more information grains. Also, the reduced coherence of the field is a concern in some phase aberration correction schemes where the time delay between rf signals are to be estimated.[9] Indeed, the robustness of the time delay estimates is a function of the correlation between the signals.

E. Nonuniform scattering function

Some interesting results can also be obtained in a medium with a nonuniform scattering function. We have already seen the case of the pointlike target. Let us consider now the case of the positive contrasted region embedded in an uniform scattering medium (a region where the scattering function is higher than in the surrounding tissue). We have defined earlier the equivalent incoherent source aperture function as the energy diagram of the transmitter. Since the medium is not uniform, the equivalent incoherent source aperture function is no longer proportional to the incident energy diagram. This is shown schematically in Fig. 12. The energy diagram is modulated by the scattering function of the medium to form the aperture function of the incoherent source. In our example, the new source is smaller. Therefore, the spatial covariance of the backscattered field is wider. This increase of the field coherence is actually observed in an experiment where the incident beam was produced with a 31 elements aperture (Fig. 13).

IV. CONCLUSION

We have developed a theoretical formalism that allows the prediction of the coherence of the field backscattered by a random medium illuminated by a coherent pulsed ultrasound beam.

The coherence of the field is described by its spatial covariance. It only depends on the transmitted beam and on the distance of the considered slice of medium. For a focused illumination, the spatial covariance is the autocorrelation of the transmitter aperture function. This is independent of focal length, $F/$ number and frequency. These results set the limits of signal-to-noise ratio improvement in incoherent processing of pulse echo signals to a rather low value and explains some experimental results obtained in this domain.

The van Cittert–Zernike theorem is well supported by the experimental results presented here. We have shown experiments that demonstrate how the coherence of the backscattered field varies with the nature of the explored medium and of the illumination beam.

The formalism developed here could be used in the studies involving focusing in aberrating scattering media. The coherence of the backscattered field could be used to obtain information on the illuminating beamwidth.

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