Física de Radiaciones I

Parcial 1 - 2025

1 a) Momento del fotón

$$p_\nu = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{2\pi\hbar c}{\lambda c} = 10{,}6 \ eV/c$$

Energía del fotón: $E_\nu = h\nu = p_\nu c = 10{,}6 \ eV$

b) $E_4 = -R_H \frac{Z^2}{4^2}$, $E_3 = -R_H \frac{Z^2}{3^2}$

$$E_\nu = \Delta E_{43} = -R_H Z^2 \left(\frac{1}{16} - \frac{1}{9} \right) = 13.6 \cdot \frac{7}{144} Z^2$$

$$\Rightarrow Z^2 = \frac{10{,}6}{13{,}6} \frac{144}{7} \simeq 16 \Rightarrow Z = 4$$

De la tabla, $Z=4$ es Be $\Rightarrow$ el ion es Be^{+++}

c) Capa K $\Rightarrow n=1$

$$E_1 = -16 \cdot 13{,}6 = -217{,}76 \ eV$$

$\Rightarrow$ la energía de ligadura de la capa K es

$$E_B(K) = 217{,}76 \ eV$$

d) $E_1 = -217{,}76 \ eV$

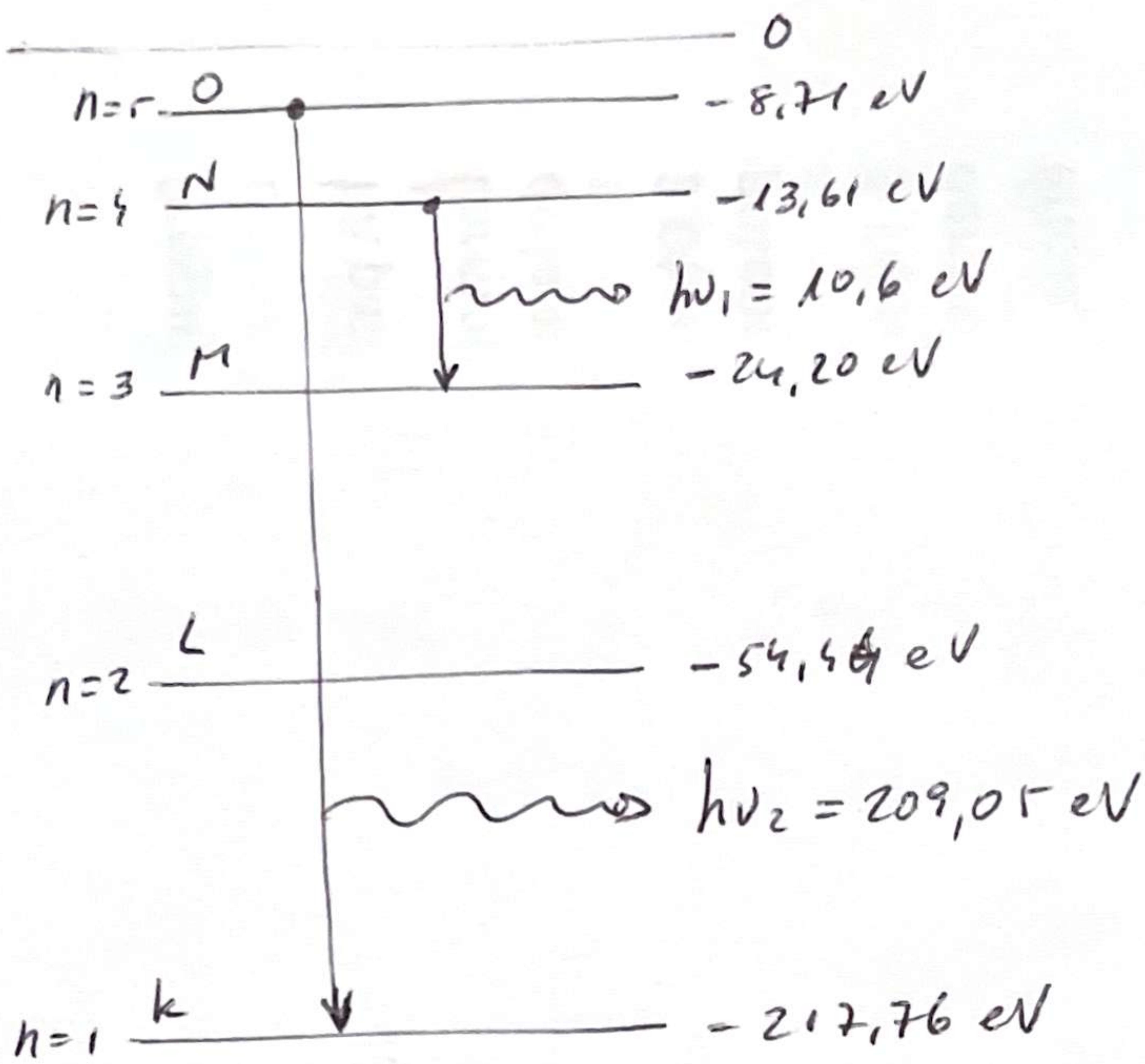
$E_2 = -54{,}44 \ eV$

$E_3 = -24{,}20 \ eV$

$E_4 = -13{,}61 \ eV$

$E_5 = -8{,}71 \ eV$

e)



$$\Delta E_{51} = -8{,}71 + 217{,}76 = 209{,}05 \text{ eV} = E_{\nu_2}$$

$$\lambda = \frac{hc}{E_{\nu_2}} = \frac{2\pi\hbar c}{E_{\nu_2}} = \frac{2\pi \cdot 1973}{209{,}05} = 59{,}3 \text{ Å}$$

(2) Ecuación de Schrödinger para n-p

a) $$\nabla^2\psi(r,\theta,\varphi) + \frac{2\mu}{\hbar^2}\left[E' - V(r)\right]\psi(r,\theta,\varphi) = 0$$

Simetría esférica , $\mu = \frac{m_n m_p}{m_n + m_p} = \frac{m_p}{2} = \frac{M}{2}$

$m_n \simeq m_p$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)\psi(r) + \frac{M}{\hbar^2}\left[E' - V(r)\right]\psi(r) = 0$$

Como $\psi(r) = \frac{u(r)}{r} \Rightarrow$

$$\Rightarrow \frac{d^2u}{dr^2} + \frac{M}{\hbar^2}\left[E' - V(r)\right]u = 0$$

$$V(r) = \begin{cases} -P_0 , & r \leq R \\ 0 , & r > R \end{cases}$$

; E' total = energía de ligadura $= -E$

$r \leq R$: $$\frac{d^2u}{dr^2} + \frac{M}{\hbar^2}\left[P_0 - E\right]u = 0$$

$$k^2 = \frac{M(P_0 - E)}{\hbar^2} \Rightarrow \frac{d^2u}{dr^2} + k^2u = 0$$

$$\Rightarrow u_1(r) = A \operatorname{sen} kr + B \cos kr$$

Para $r \to 0$, $u_1 \to 0 \Rightarrow B = 0 \Rightarrow u_1(r) = A \operatorname{sen} kr$

$r > R$: $$\frac{d^2u}{dr^2} - \frac{M}{\hbar^2}E\,u = 0$$

$$\xi^2 = \frac{M}{2\hbar^2}E \Rightarrow \frac{d^2u}{dr^2} - \xi^2u = 0$$

$$\Rightarrow u_2(r) = Ce^{-\xi r} + De^{-\xi r}$$

b) Para $r \to \infty$, $u_2 \to 0 \Rightarrow D = 0$; $u_2(r) = Ce^{-\xi r}$

CC en $r = R \Rightarrow$ $u_1(R) = u_2(R)$

$$\left.\frac{du_1}{dr}\right|_{r=R} = \left.\frac{du_2}{dr}\right|_{r=R}$$

$$\begin{cases} A \sin kR = Ce^{-\xi R} \\ Ak\cos kR = -\xi C e^{-\xi R} \end{cases} \Rightarrow \frac{k\cos kR}{\sin kR} = -\xi$$

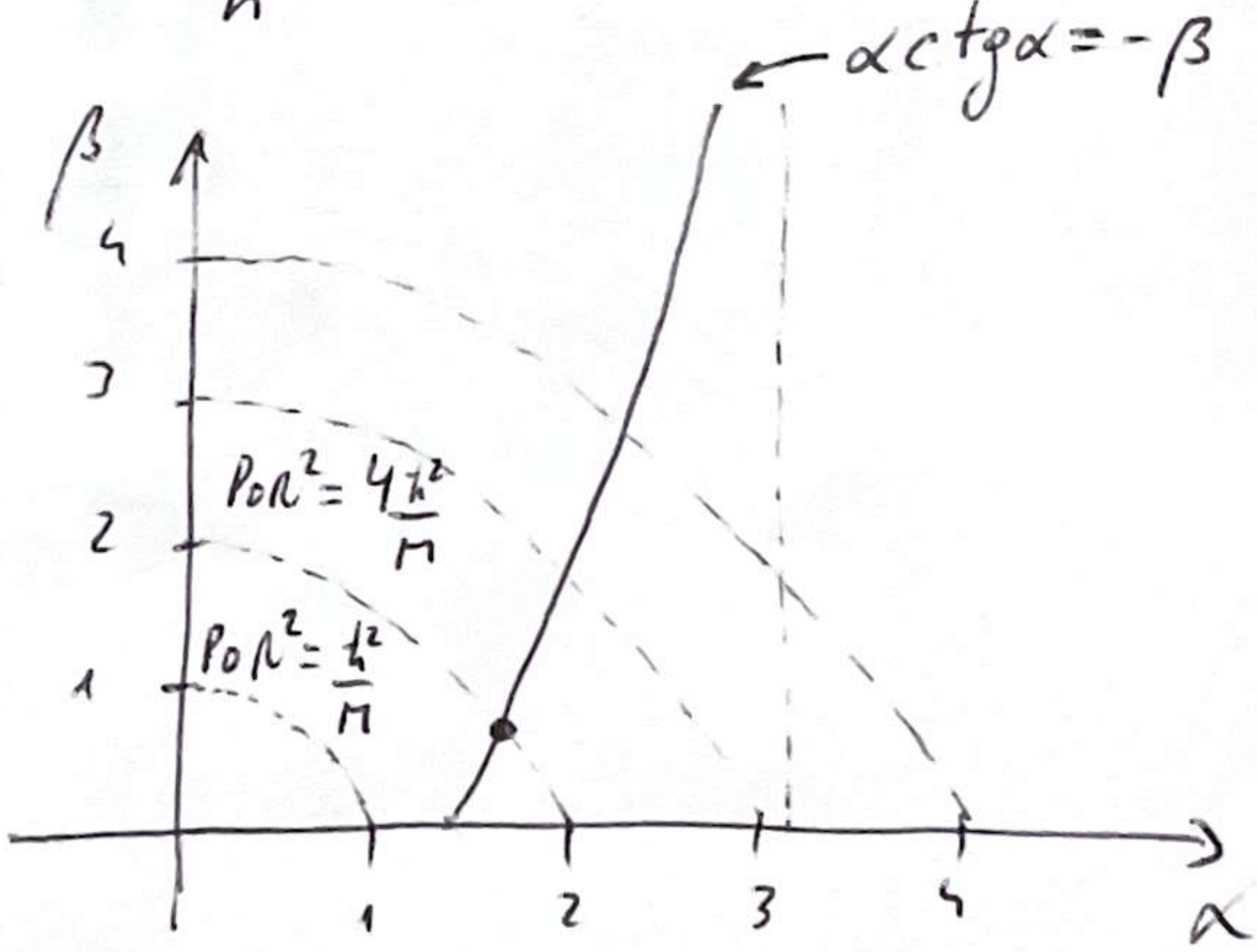
$$\Rightarrow k \,\mathrm{ctg}\, kR = -\xi$$

Sustituyo: $\alpha = kR$, $\beta = \xi R$ $\Big\}$ $\alpha^2 + \beta^2 = R^2 \frac{M(P_0 - E)}{\hbar^2} + \frac{R^2 EM}{\hbar^2}$

$$= \frac{R^2 M P_0}{\hbar^2} = cte = \eta^2$$

$$\alpha^2 + \beta^2 = \eta^2$$

$$\alpha \,\mathrm{ctg}\, \alpha = -\beta$$



c) Si $P_0 \gg E$

$$\Rightarrow \mathrm{ctg}\, kR = \frac{-\xi}{k} \to 0$$

$$\Rightarrow kR = \frac{\pi}{2} \Rightarrow k^2R^2 = \frac{\pi^2}{4} \Rightarrow \frac{M(P_0 - E)}{\hbar^2}R^2 = \frac{\pi^2}{4}$$

$$\overset{P_0 \gg E}{\Longrightarrow} \frac{MP_0R^2}{\hbar^2} \simeq \frac{\pi^2}{4} \Rightarrow P_0R^2 \simeq \frac{\pi^2}{4}\frac{\hbar^2}{M} = \frac{\cancel{\pi^2}}{4}\frac{h^2}{\cancel{4\pi^2}}\frac{1}{M}$$

$$P_0R^2 \simeq \frac{h^2}{16M}$$

b

③

d) $R = 2\,fm$

$$E_0 = \frac{\pi^2 \hbar^2 c^2}{4 m c^2 R^2} = \frac{\pi^2}{4} \frac{(1{,}973 \cdot 10^8)^2}{9{,}4 \cdot 10^8 \cdot 4} \simeq 25{,}5\ MeV$$

$\hbar c = 1973\ eV \cdot \mathring{A} = 1{,}973 \cdot 10^8 eV \cdot fm.$

$mc^2 = 940\ MeV = 9{,}40 \cdot 10^8\ eV$

~~[illegible]~~