Física de Radiaciones I

Parcial 2 - 2025



1) a) N° de átomos de Al por $cm^3$

$$n = \frac{N_A}{A}\rho = 6{,}022\cdot 10^{23}\,\frac{2{,}7}{27} = 6{,}022\cdot 10^{22}\ cm^{-3}$$

N° átomos por $cm^2$:

$$nt = 6{,}022\cdot 10^{22}\cdot 10^{-4} = 6{,}022\cdot 10^{18}\ cm^{-2}$$

b) La masa reducida es $\mu = \dfrac{m_{Al}\, m_\alpha}{m_{Al} + m_\alpha} =$

$$= \frac{27u\ 4u}{27u + 4u} = 3{,}48\ u$$

La energía en CM:

$$E_{cm} = \frac{\mu}{m_\alpha} T_\alpha = \frac{3{,}48}{4}\cdot 10 = 8{,}71\ MeV$$

$$D = \frac{Z_\alpha Z_{Al} e^2}{4\pi\varepsilon_0 E_{cm}} = \underbrace{\frac{e^2}{4\pi\varepsilon_0}}_{1{,}44\ MeV\cdot fm} \frac{2\cdot 13}{8{,}71} = 4{,}3\ fm$$

c) En LAB: $\tan\theta_{LAB} = \dfrac{\operatorname{sen}\theta_{cm}}{\dfrac{m_\alpha}{m_{Al}} + \cos\theta_{cm}}$

$$\theta_{cm} = 34{,}2° \Rightarrow \tan\theta_{LAB} = 0{,}576 \Rightarrow \theta_{LAB} \simeq 30°$$

$$D_{min} = \frac{D}{2}\left(1 + \frac{1}{\operatorname{sen}\theta_{LAB}/2}\right) = \frac{4{,}3}{2}\left(1 + \frac{1}{\operatorname{sen}15°}\right) = 10{,}46\ fm$$

Comparando con el radio del Al: $R_{Al} = 1{,}3\,A^{1/3} = 1{,}3\cdot 27^{1/3}$

$$= 3{,}9\ fm$$

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⟹ son del mismo orden de magnitud ⟹ se esperarían algunas diferencias menores con la fórmula de Rutherford.

d) $\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{D^2}{16}\frac{1}{\sin^4\theta/2} = \frac{4{,}3^2}{16}\frac{1}{\sin^4 34{,}2/2} = 1{,}54\ b$

$$\left(\frac{d\sigma}{d\Omega}d\Omega\right)_{CM} = \left(\frac{d\sigma}{d\Omega}d\Omega\right)_{LAB}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{LAB} = \left(\frac{d\sigma}{d\Omega}\right)_{CM}\frac{d\Omega_{CM}}{d\Omega_{LAB}} = \left(\frac{d\sigma}{d\Omega}\right)_{CM}\frac{\sin\theta_{CM}}{\sin\theta_{LAB}}\frac{d\theta_{CM}}{d\theta_{LAB}}$$

$$\tan\theta_{LAB} = \frac{\sin\theta_{CM}}{\frac{m_\alpha}{m_{Al}} + \cos\theta_{CM}} \Rightarrow$$

$$\Rightarrow (1+\tan^2\theta_{LAB})\,d\theta_{LAB} = \frac{\cos\theta_{CM}\left(\cos\theta_{CM}+\frac{m_\alpha}{m_{Al}}\right)+\sin^2\theta_{CM}}{\left(\cos\theta_{CM}+\frac{m_\alpha}{m_{Al}}\right)^2}d\theta$$

$$= \frac{1+m_\alpha/m_{Al}}{\cos\theta_{CM}+\frac{m_\alpha}{m_{Al}}}d\theta_{CM}$$

$$\frac{\sin\theta_{CM}}{\sin\theta_{LAB}}\frac{d\theta_{CM}}{d\theta_{LAB}} = \frac{\sin\theta_{CM}}{\sin\theta_{LAB}}\frac{(1+\tan^2\theta_{LAB})\cdot\left(\cos\theta_{CM}+\frac{m_\alpha}{m_{Al}}\right)^2}{1+\frac{m_\alpha}{m_{Al}}}$$

$$= \frac{\sin 34{,}2^\circ}{\sin 30^\circ}\frac{(1+1/3)(\cos 34{,}2^\circ + 4/27)^2}{1+4/27} = \frac{\sin 34{,}2^\circ}{\sin 30^\circ}1{,}1 = 1{,}23$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{LAB} = 1{,}54\cdot 1{,}23 = 1{,}89\ b$$

e) $P = nt\left(\frac{d\sigma}{d\Omega}\right)_{LAB} d\Omega_{LAB} = 6{,}022\cdot 10^{18}\cdot 1{,}89\cdot 10^{-24}\cdot 10^{-6} = 1{,}1\cdot 10^{-11}$

f) $10^{12}$ partículas $\alpha$ ⟹ llegan $10^{12}\cdot 1{,}1\cdot 10^{-11} = 11$

ó $10^{12}\ s^{-1}$ ⟹ llegan $10^{12}\cdot 1{,}1\cdot 10^{-4} = 11\ s^{-1}$

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(2) $V(r) = \begin{cases} -V_0 & , r < R \\ 0 & , r > R \end{cases}$ , $x = qR = 2kR \sin\theta/2$

a) En la aprox. de Born:

$$f(\theta) = \frac{-m}{2\pi\hbar^2} \int e^{-i\vec{k}\cdot\vec{r}} V(\vec{r}) e^{i\vec{k}'\vec{r}} d^3r$$

$$\vec{q} = \vec{k} - \vec{k}' \quad ; \quad |\vec{k}| = |\vec{k}'| = k \Rightarrow q = 2k \sin\theta/2$$

$$f(\theta) = -\frac{m}{2\pi\hbar^2} \int e^{-i\vec{q}\vec{r}} V(\vec{r}) d^3r =$$

$$= -\frac{m}{2\pi\hbar^2} \int_0^\infty V(r) r^2 dr \int_0^\pi e^{-iqr\cos\theta'} 2\pi \sin\theta' d\theta' =$$

$$= -\frac{2m}{\hbar^2} \int_0^\infty V(r) r^2 dr \underbrace{\int_0^\pi e^{-iqr\cos\theta'} \sin\theta' d\theta'}_{\frac{\sin(qr)}{qr}} =$$

$$= -\frac{2m}{\hbar^2} \int_0^\infty V(r) \frac{\sin qr}{q\cancel{r}} r^{\cancel{2}} dr = \frac{2mV_0}{\hbar^2 q} \int_0^R r \sin qr \, dr =$$

$$= \frac{2mV_0}{\hbar^2 q^3} \left[ \sin qR - qR \cos qR \right]$$

$$f(x) = \frac{2mV_0}{\hbar^2 q^3} \left[ \sin x - x \cos x \right]$$

$$\frac{d\sigma}{d\Omega} = |f(x)|^2 = \frac{4m^2 V_0^2 R^6}{\hbar^4} \left( \frac{\sin x - x\cos x}{x^3} \right)^2$$

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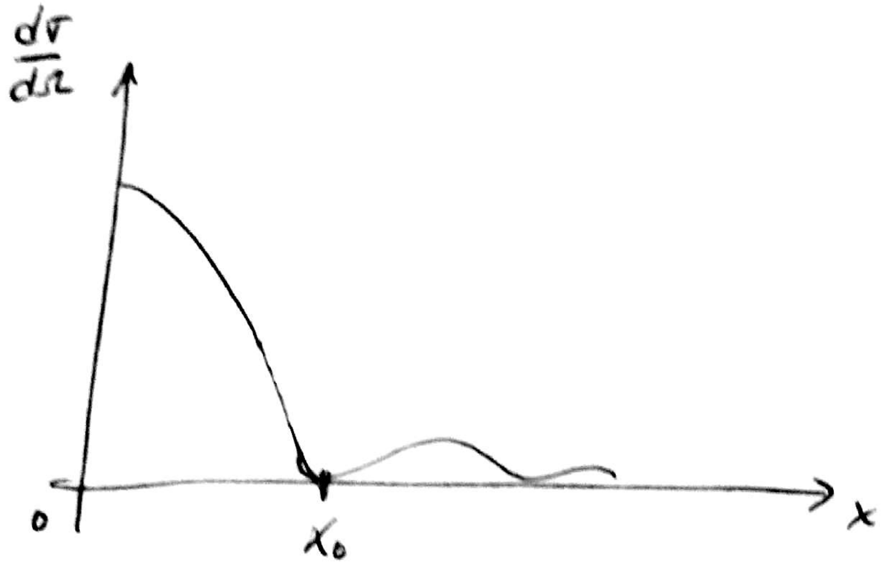


El primer cero de $\frac{d\sigma}{d\Omega}$ tiene lugar para

$$\operatorname{sen} x_0 - x_0 \cos x_0 = 0 \Rightarrow x_0 = \tan x_0$$

$$\Rightarrow x_0 = 4{,}49 = 1{,}43\pi$$

b) $x_0 = 1{,}43\pi = 2kR \operatorname{sen} \theta_1/2 \Rightarrow R = \dfrac{1{,}43\pi}{2k \operatorname{sen} \theta_1/2}$

~~$\Rightarrow \theta_1 = 2 \operatorname{arcsen} \frac{1{,}43\pi}{2kR}$~~

c) $R = 5\,10^{-13}$ cm

El método es válido si el valor máximo de $x$ es mayor que $1{,}43\pi$

$$2kR \geq 1{,}43\pi \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\sqrt{2mE} = \frac{1{,}43\pi\hbar}{2R}$$

$$E \geq \left(\frac{1{,}43\pi}{2R}\right)^2 \frac{\hbar^2}{2m} \frac{c^2}{c^2}$$

$$E \geq \frac{(1{,}43\pi)^2}{8(5.10^{-13})^2} \frac{(6{,}58.10^{-22})^2}{938} (3.10^{10})^2$$

$$E \geq 4{,}3 \text{ MeV}$$

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③ ③

a) La energía para separar un protón o un neutrón de un núcleo estable es:

$$S_p = B(A,Z) - B(Z-1, A-1)$$
$$S_n = B(A,Z) - B(Z, A-1)$$

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_a\left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2}$$

$$\cancel{S_n = a_V A - a_S A^{2/3} - a_C Z^2}$$

$$S_p - S_n = \cancel{B(A,Z)} - B(Z-1, A-1) - \cancel{B(A,Z)} + B(Z, A-1) =$$
$$= \cancel{-a_V(A-1)} + \cancel{a_S(A-1)^{2/3}} + a_C(Z-1)^2(A-1)^{-1/3} + a_a\left[\frac{A-1}{2} - (Z-1)\right]^2(A-1)^{-1} -$$
$$\cancel{+ a_p \delta (A-1)^{-1/2}} + \cancel{a_V(A-1)} - \cancel{a_S(A-1)^{2/3}} - a_C(A-1)^{-1/3}Z^2 -$$
$$- a_a\left(\frac{A-1}{2} - Z\right)^2 (A-1)^{-1} + \cancel{a_p\delta(A-1)^{-1/2}} =$$

$$= a_C\left[(Z-1)^2 - Z^2\right](A-1)^{-1/3} + a_a(A-1)^{-1}\left[\left(\frac{A-2Z+1}{2}\right)^2 - \left(\frac{A-1-2Z}{2}\right)^2\right]$$
$$= a_C\left[\cancel{Z^2} - 2Z + 1 - \cancel{Z^2}\right](A-1)^{-1/3} + \frac{a_a(A-1)^{-1}}{4}\left\{\cancel{(A-2Z)^2} + 2(A-2Z) + \cancel{1} \atop - \cancel{(A-2Z)^2} + 2(A-2Z) - \cancel{1}\right\} =$$
$$= a_C(1-2Z)(A-1)^{-1/3} + a_a(A-2Z)(A-1)^{-1}$$

Para núcleos estables: $Z = \dfrac{A}{2 + \frac{2a_C}{a_a}A^{2/3}}$

Como $a_C \ll a_a \implies Z \simeq \dfrac{A}{2}\left(1 - \dfrac{a_C}{a_a}A^{2/3}\right)$

$$S_p - S_n = a_C\left[1 - A\left(1 - \frac{a_C}{a_a}A^{2/3}\right)\right](A-1)^{-1/3} +$$
$$+ a_a\left[A - A\left(1 - \frac{a_C}{a_a}A^{2/3}\right)\right](A-1)^{-1} =$$
$$= \left\{a_C\left[1 - A + \frac{a_C}{a_a}A^{5/3}\right](A-1)^{2/3} + \cancel{a_a}\left[\frac{a_C}{\cancel{a_a}}A^{5/3}\right]\right\}(A-1)^{-1}$$

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$$S_p - S_n = \frac{a_c}{A-1}\left[ A^{5/3} + \frac{a_c}{a_a} A^{5/3}(A-1)^{2/3} - (A-1)^{5/3} \right]_{4}$$

b) $A \gg 1 \Rightarrow A-1 \simeq A$

$$S_p - S_n = \frac{a_c}{A}\left[ \cancel{A^{5/3}} + \frac{a_c}{a_a} A^{5/3} A^{2/3} - \cancel{A^{5/3}} \right]$$

$$S_p - S_n = \frac{a_c^2}{a_a} A^{4/3} = \frac{0{,}714^2}{92{,}8} A^{4/3} \simeq 5{,}5 \cdot 10^{-3} A^{4/3} \text{ MeV}$$

A medida que A aumenta se necesita más energía para arrancar un protón que un neutrón.

c) $^{56}Fe \longrightarrow A = 56 \Rightarrow S_p - S_n = 1{,}18$ MeV

$^{208}Pb \longrightarrow A = 208 \Rightarrow S_p - S_n = 6{,}78$ MeV.

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