EXPERIMENTAL MULTISTABILITY IN MACKEY-GLASS MODEL

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Experimental results, obtained by means of an electronic implementation of the model, are contrasted with numerical simulations showing an excellent agreement. The existence of multistability, i.e. the coexistence the more than one attractor for a common set of parameter values, is found in broad regions. The different possible solutions are characterized using the number (and the order) of peaks. The frequency of appearance of the solutions is compared numerically and experimentally.

Mackey-Glass Model

In Mackey-Glass model P represents the density population of mature circulating white blood cells. Where the nonlinearity represents the regulatory mechanisms in hematopoiesis, and the delay between initiation of cellular production in the bone marrow and the release of mature cells.

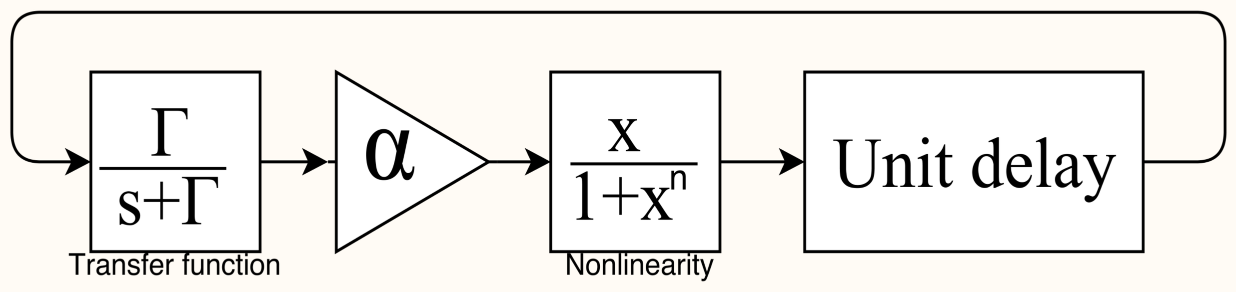
Mackey-Glass equation

$$\frac{dP}{dt} = \frac{\beta_0 \Theta^n P_\tau}{\Theta^n + P_\tau^n} - \gamma P \quad \text{with} \quad P_\tau(t) = P(t-\tau)$$

Dimensionless equation

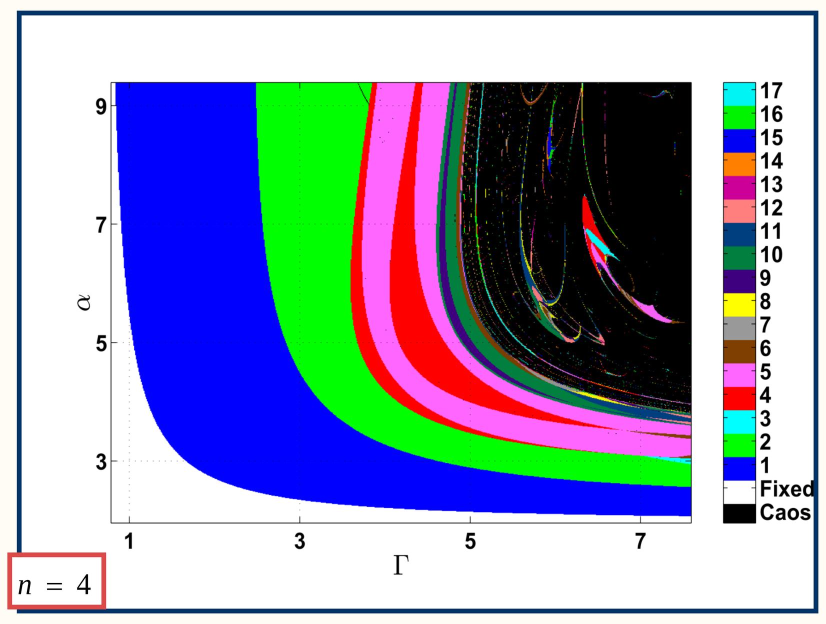
$$\begin{cases} x \equiv P/\Theta \\ \rho \equiv t/\tau \\ x_1(\rho) = x(\rho - 1) \end{cases}, \begin{cases} \Gamma \equiv \gamma\tau \\ \alpha \equiv \beta_0/\gamma \end{cases} \Rightarrow \quad \frac{dx}{d\rho} = \Gamma\alpha \frac{x_1}{1 + x_1^n} - \Gamma x$$

Block implementation

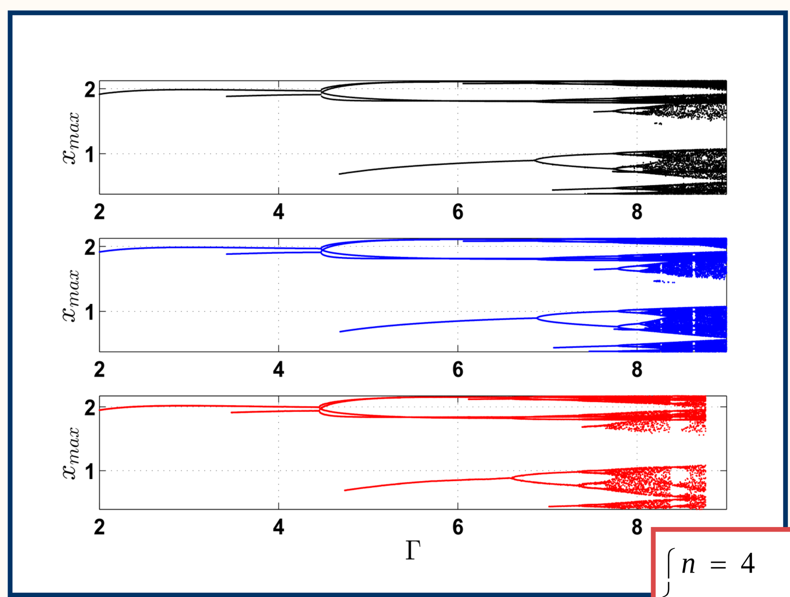


Parameter Space

With low values of the parameters α and Γ, the system doesn't show multistability.

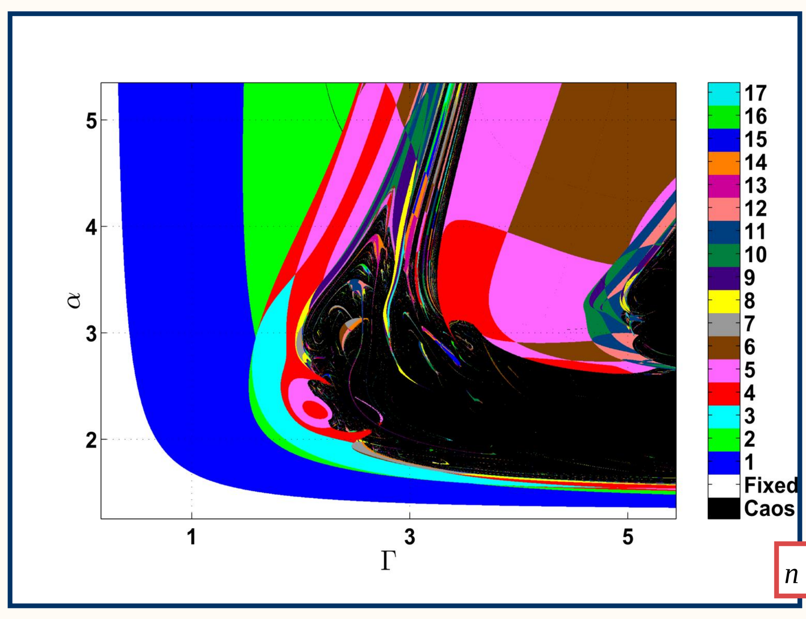


$n = 4$

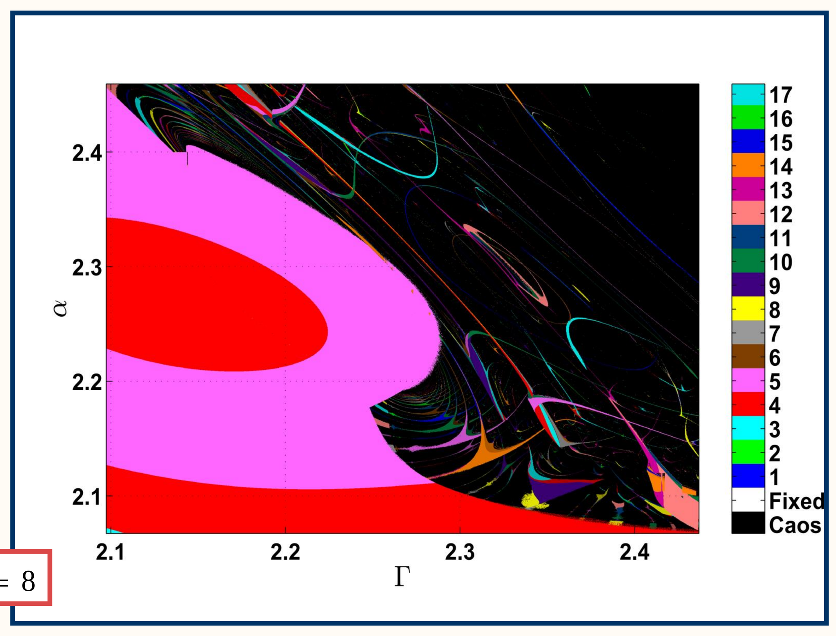


$$\begin{cases} n = 4 \\ \alpha = 3.73 \end{cases}$$

Three bifurcations diagrams were made. Continuous-time simulation (black), discrete-time simulation (blue) and electrical implementation (red).

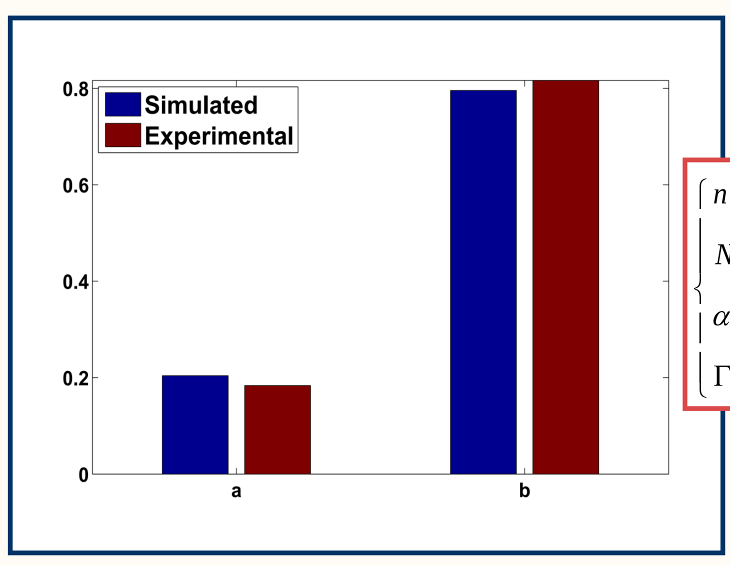


$n = 8$

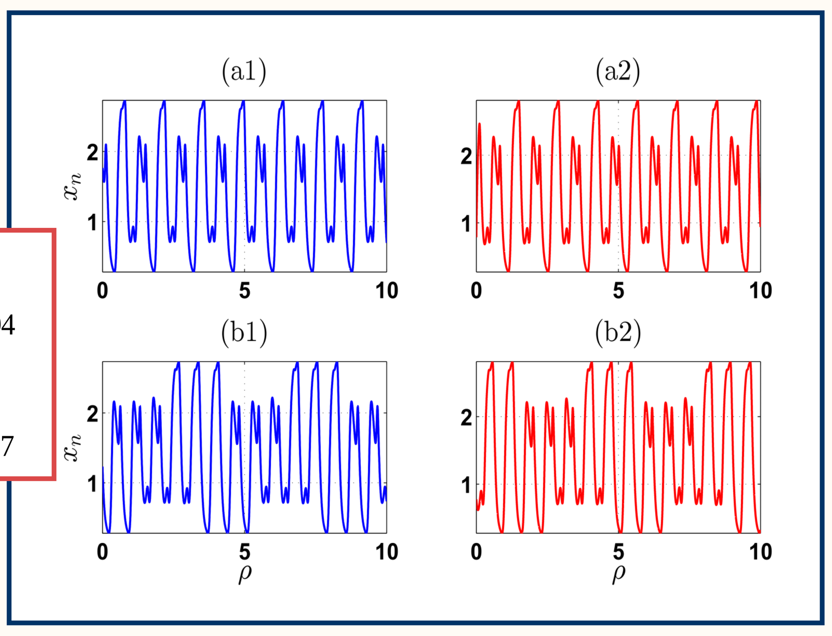


Random Phase Study of Multistability

A random phase sine wave was used as initial condition

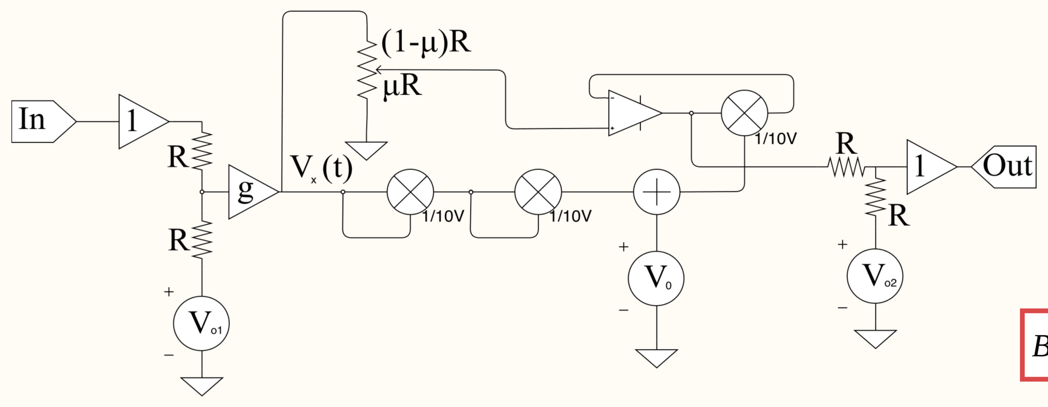


$$\begin{cases} n = 4 \\ N = 1194 \\ \alpha = 4.9 \\ \Gamma = 15.7 \end{cases}$$



Electronic Implementation

Gain and nonlinear block for n=4



$$\begin{cases} x_l \equiv v_x(lT_s)/\sqrt[4]{(10V)^3 v_0} \\ \rho \equiv t/\tau \end{cases}, \begin{cases} \Gamma \equiv \tau/RC \\ \alpha \equiv \frac{(10V)}{v_0}\frac{gg_d\mu}{4} \end{cases}$$

$$BBD_{out}(t) = g_d BBD_{in}((\lfloor t/T_s \rfloor - N + 1)T_s) + V_{o3}$$

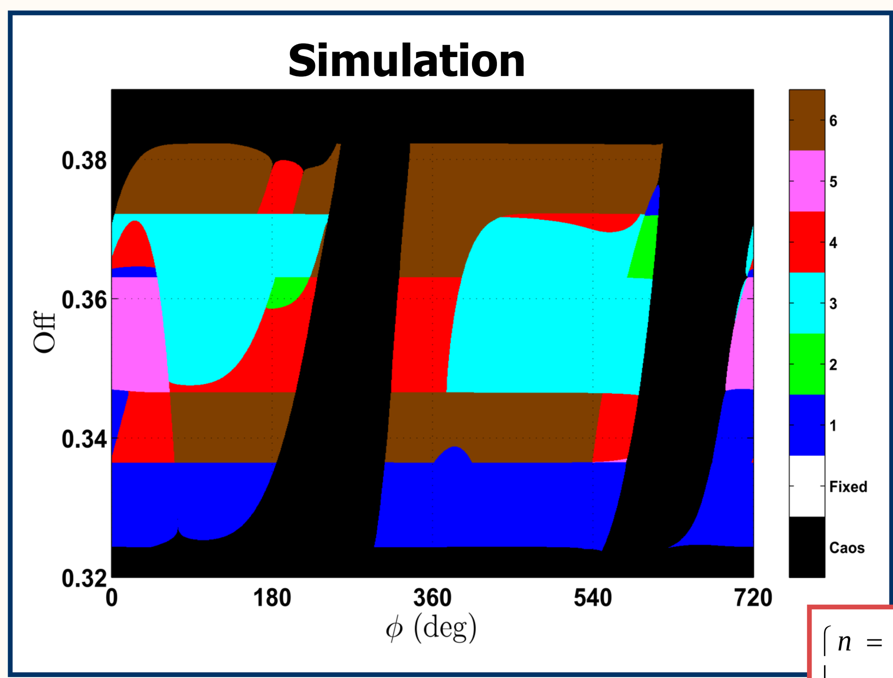
The delay block was implemented using a BBD and set to $\tau = NT_s = 10\ mS$, this device also added a gain (g_d), and an offset voltage.

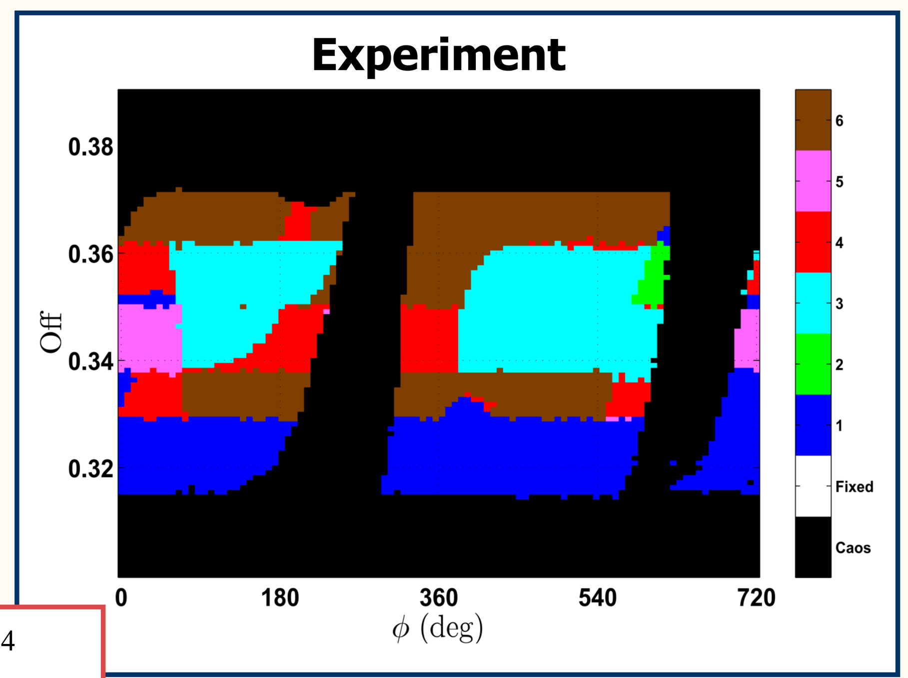
The transfer function, being a one pole lowpass, was implemented with an RC-Circuit, which was varied to obtain different values of Γ.

Setting Γ, α, ρ and x appropiatelly, the circuite aproximate the dimensionless equation with the discrete-time equation:

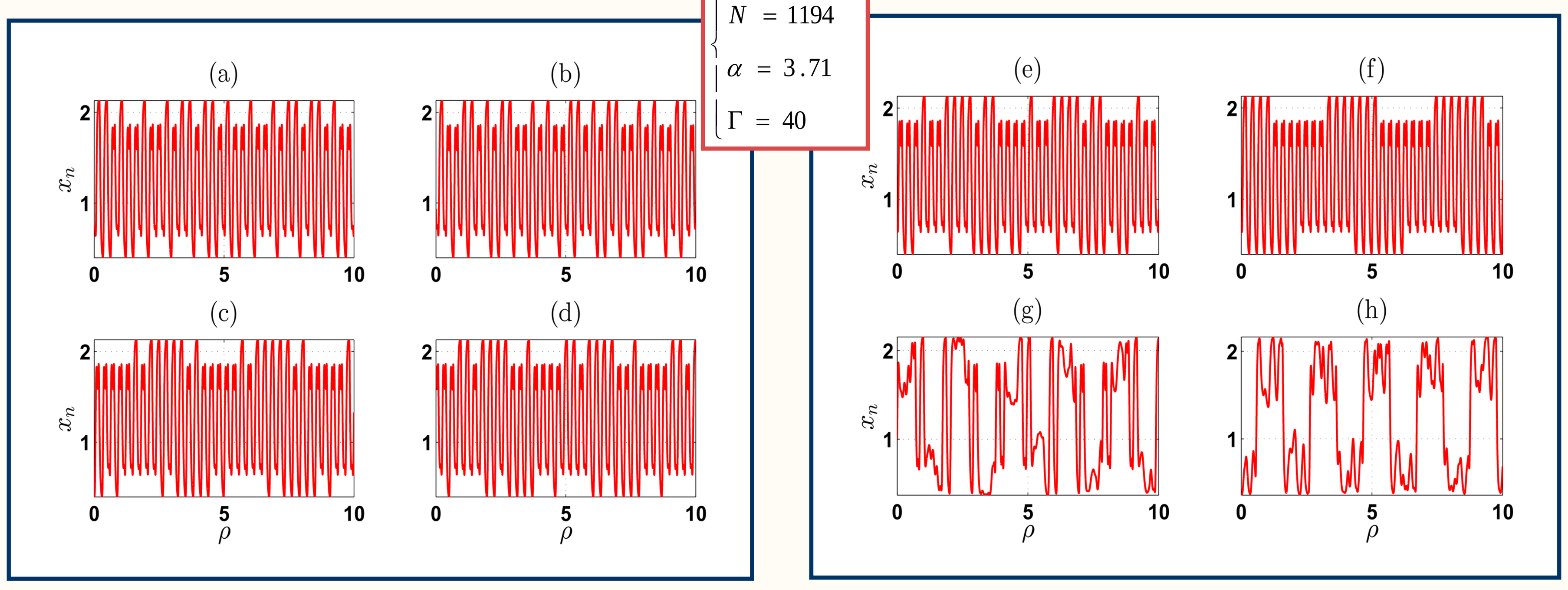
$$x_{l+1} = x_l e^{-\Gamma/N} + \left(1 - e^{-\Gamma/N}\right)\alpha \frac{x_{j-N+1}}{1 + (x_{j-N+1})^4}$$

Initial Condition Space Mapping Multistability



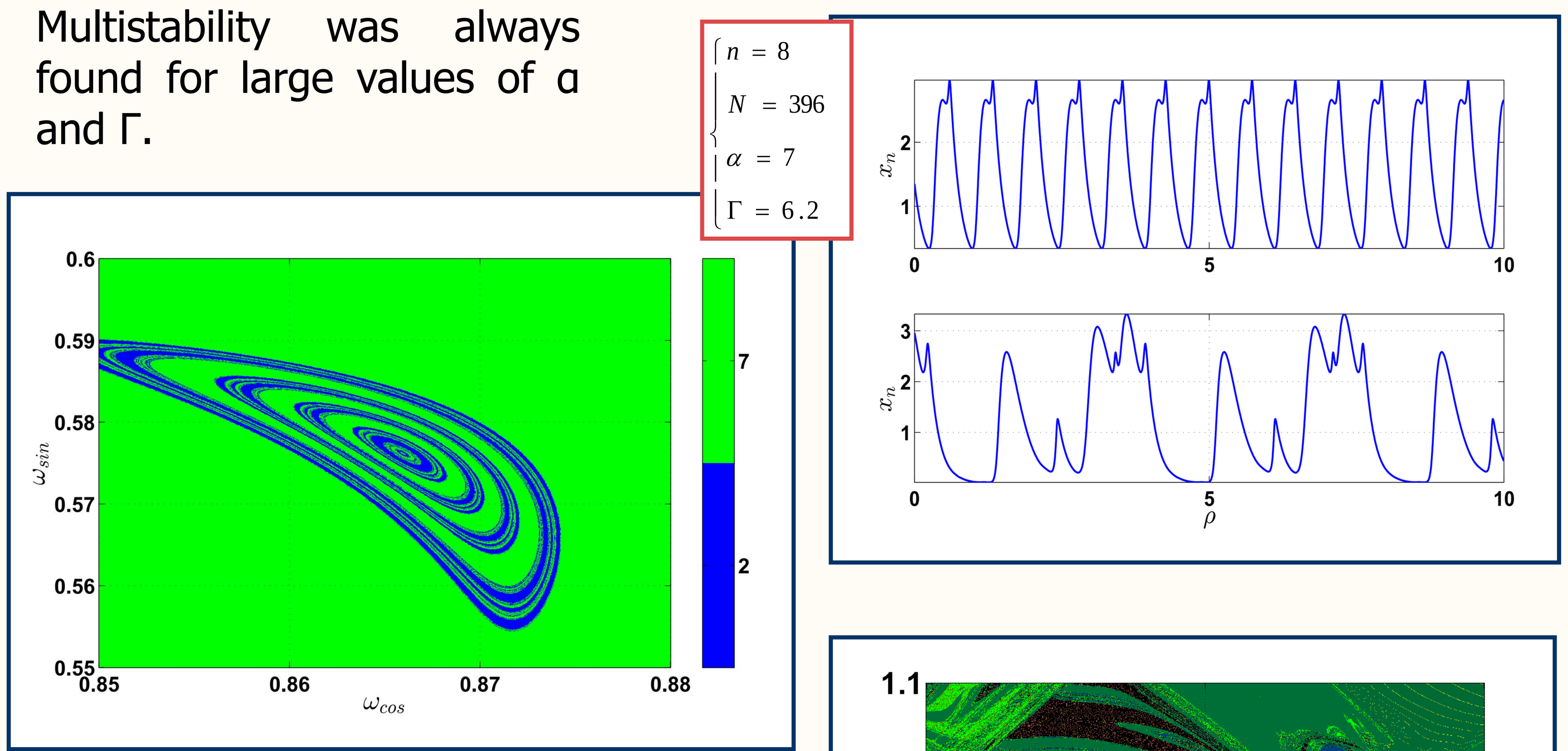


$$\begin{cases} n = 4 \\ N = 1194 \\ \alpha = 3.71 \\ \Gamma = 40 \end{cases}$$



Multistability was always found for large values of α and Γ.

$$\begin{cases} n = 8 \\ N = 396 \\ \alpha = 7 \\ \Gamma = 6.2 \end{cases}$$



$$\begin{cases} n = 8 \\ N = 396 \\ \alpha = 8 \\ \Gamma = 23 \end{cases}$$