Practico 5

(7)

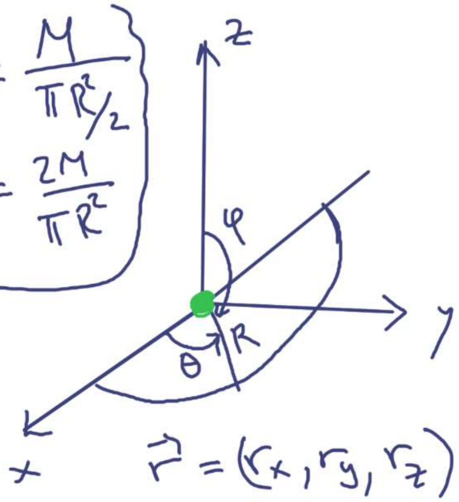
$$\boxed{\mathbb{I}_{ij} = \sum_{\alpha=1}^{N} m_\alpha \left(r_\alpha^2 \delta_{ij} - x_{\alpha,i}\, x_{\alpha,j} \right)}$$

$$T_{rot} = \frac{1}{2} \sum_{i,j} \mathbb{I}_{ij}\, \omega_i \omega_j$$

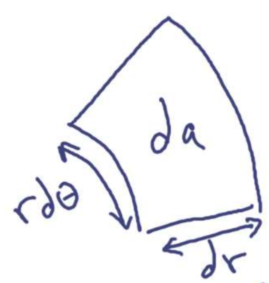
$$\mathbb{I} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \quad \Big\} i,j = x,y,z$$

$$\boxed{\mathbb{I}_{ij} = \int_V \rho(\vec{r}) \left(r^2 \delta_{ij} - r_i r_j \right) dv}$$

$$\sigma = \frac{M}{\pi R^2/2} = \frac{2M}{\pi R^2}$$



$$\vec{r} = (r_x, r_y, r_z)$$



$$I_{xx} = \int_S \sigma \underbrace{(y^2 + z^2)}_{r^2 - r_x r_x} \underbrace{da}_{r\,d\theta\,dr}$$

$$= \sigma \int_0^R \int_0^\pi r^2 \cos^2\theta \; r\, d\theta\, dr$$

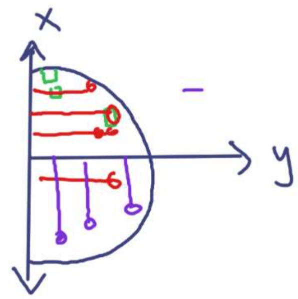
$$= \sigma \frac{R^4}{4} \int_0^\pi \cos^2\theta \, d\theta$$

$$\boxed{I_{xx} = \sigma \frac{R^4}{4} \frac{\pi}{2} = \frac{\pi}{8} \sigma R^4}$$

$$I_{yy} = \int_0^R \int_0^\pi \sigma \overbrace{r^2 \operatorname{sen}^2\theta}^{x^2+z^2} \; r\, d\theta\, dr = \boxed{\frac{\pi}{8} \sigma R^4 = I_{yy}}$$

$$I_{zz} = \int_S \sigma\,(y^2+x^2)\,da = \int_0^R\int_0^\pi \sigma\,\overbrace{\left(r^2\operatorname{sen}^2\theta + r^2\cos^2\theta\right)}^{x^2+y^2}\, r\,d\theta\,dr$$

$$= \sigma\int_0^R\int_0^\pi r^3\,d\theta\,dr = \boxed{\sigma\,\frac{R^4}{4}\,\pi = I_{zz}}$$



$$I_{zz} = 2\,I_{xx} = 2\,I_{yy}$$

$$I_{xx} + I_{yy} = I_{zz}$$

$$I_{xy} = -\int_S \sigma\,da\,xy = -\sigma\int_0^R\int_0^\pi r\,d\theta\,d\varphi\; r^2\operatorname{sen}\theta\cos\theta$$

$$= -\sigma\,\frac{R^4}{4}\,\underbrace{\int_0^\pi \operatorname{sen}\theta\cos\theta\,d\theta}_{0} = 0$$

$$I_{xz} = -\int_S \sigma\,da\,xz = 0$$

$$I_{yz} = 0$$

$$\boxed{\mathbb{I} = \sigma\pi\,\frac{R^4}{4}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$