V e.v. , $W \subset V$ s.e.v. $\pi : V \longrightarrow V/W$

V/W

(A) $\{\bar{v}_1, \dots, \bar{v}_n\}$ l.i. en V/W

(B) $\{v_1, \dots, v_n\}$ l.i en V y $\langle v_1, \dots, v_n \rangle \cap W = \{0\}$.

$\bar{v} = [v]$

$\underline{(A) \Leftrightarrow (B)}$

$(A) \Rightarrow (B)$: $\underline{\alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n = \bar{0}} \Leftrightarrow \alpha_i = 0 \ \forall i.$

$\{v_1, \dots, v_n\}$ es l.i.

$- \ \alpha_1 v_1 + \dots + \alpha_n v_n = 0$

$\overline{\alpha_1 v_1 + \dots + \alpha_n v_n} = \bar{0}$

$\alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n = \bar{0} \rightarrow \alpha_i = 0$ ($\{\bar{v}_1, \dots, \bar{v}_n\}$ es l.i.)

$\langle v_1, \dots, v_n \rangle \cap W = \{0\}$

$\cdot \ \bar{w} = \bar{0}$

$\alpha_1 v_1 + \dots + \alpha_n v_n = w \in W$

$\overline{\alpha_1 v_1 + \dots + \alpha_n v_n} = \bar{0}$

$\underline{\alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n = \bar{0} \rightarrow \alpha_i = 0}$

- $[v] = [v'] \Leftrightarrow v \sim v'$

$\Leftarrow$: $v = v' + w$ para algún w.

$[v] \subset [v']$

$u \in [v']$ queremos ver que $u \in [v]$

$u - v' = \hat{w}$

$u \in [v]$

$v' = u - \hat{w} \rightarrow v = u - \hat{w} + w \rightarrow v - u = w - \hat{w} \in W$

$\Rightarrow$ $[v] = [v']$ $\qquad v \sim v \qquad v - v = 0 \in W$

$v \in [v'] \Leftrightarrow v \sim v'$

$\{u : u \sim v'\}$

$(B) \Rightarrow (A)$: $\quad \alpha_1 \bar{v}_1 + \dots + \alpha_n \bar{v}_n = \bar{0} \qquad (\{\bar{v}_1, \dots, \bar{v}_n\}$ es l.i.$)$

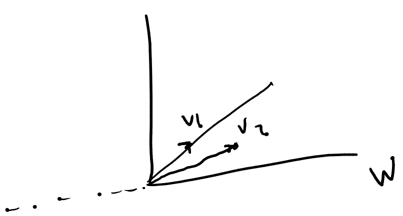
$$\overline{\alpha_1 v_1} + \dots + \overline{\alpha_n v_n} = \bar{0}$$

$$\overline{\alpha_1 v_1 + \dots + \alpha_n v_n} = \bar{0}$$

$$\alpha_1 v_1 + \dots + \alpha_n v_n \sim 0$$

$$\left.\begin{array}{l} u = \alpha_1 v_1 + \dots + \alpha_n v_n \in W \\ u \in \langle v_1, \dots, v_n \rangle \end{array}\right\} \to u = 0$$

$$\alpha_1 v_1 + \dots + \alpha_n v_n = 0 \longrightarrow \alpha_i = 0 \quad \forall i.$$



$\langle v_1, v_2 \rangle \cap W = W$

$\{v_1, v_2\} \qquad \pi(\{v_1, v_2\})$ no es l.i.

$T : V \longrightarrow W$

$\{v_1, \dots, v_n\}$ es l.i. y $\langle v_1, \dots, v_n \rangle \cap N(T) = \{0\}$

¿es l.i. $\{T(v_1), \dots, T(v_n)\}$?

b) $V = \mathbb{R}[x]$, $W = \{ x^3 p(x) : p \in V \}$

$A = \{ \overline{x^3 - x^2}, \overline{x^2 - x}, \bar{x} \}$

$B = \{ \overline{x^2 - x}, \overline{x - 1}, \overline{-x^2 + 1} \}$

$C = \{ \overline{1 + x^4 + x^5}, \overline{x + x^7}, \overline{x^2 + x^3 + x^4} \}$

¿ C es base de V/W ?

(I) A: $\{ x^3 - x^2, x^2 - x, x \}$ es l.i. sí

$\langle x^3 - x^2, x^2 - x, x \rangle \cap W \neq \{0\}$

$x^3 \cancel{- x^2} \cancel{+ x^2} \cancel{- x} \cancel{+ x} \in W$ A no es l.i.

B $\{ x^2 - x, x - 1, -x^2 + 1 \}$

$(x^2 - x) + (x - 1) + (-x^2 + 1) = 0 \to$ no es l.i.

$\to$ B no es l.i.

C $\{ \underset{p_1}{1 + x^4 + x^5}, \underset{p_2}{x + x^7}, \underset{p_3}{x^2 + x^3 + x^4} \}$ es l.i.

$\langle p_1, p_2, p_3 \rangle \cap W \ni q$

$\downarrow$

$\{0\}$

$\underbrace{\alpha p_1 + \beta p_2 + \gamma p_3}_{q} = \underbrace{x^3 p(x)}_{q}$ para algún p.

$\beta p_2 + \gamma p_3 = x^3 p(x)$

$\gamma p_3 = x^3 p(x)$

$x^3 p(x) = 0 \to p = 0 \to q = 0$

$\to$ C es l.i. $C = \{ \bar{p}_1, \bar{p}_2, \bar{p}_3 \}$.

$W = \{ x^3 P(x) : P \in \mathbb{R}[x] \} = \langle x^3, x^4, \dots, \dots \rangle$

Prop $\dim V/W = 3$, vamos a ver que

$\{\bar{1}, \bar{x}, \bar{x^2}\}$ es l.i. maximal.

- $\{\bar{1}, \bar{x}, \bar{x^2}\}$ es l.i. (mismo argumento que para C)

- $\{\bar{1}, \bar{x}, \bar{x^2}, \overline{P(x)}\}$ no es l.i. para ningún P:

$\{1, x, x^2, P(x)\}$ no es l.i.

$\uparrow$

$P = \underbrace{ax^2 + bx + c}_{} + w$

$w = 0$: $\{\bar{1}, \bar{x}, \bar{x^2}, \overline{P(x)}\}$ no es l.i. $\rightarrow$ no es base.

$w \neq 0$: $\overline{P - ax^2 - bx + c} = w \neq 0$

$\uparrow \qquad \uparrow$

$\langle 1, x, x^2, P \rangle \cap W \neq \{0\}$ $\rightarrow$ $\{\bar{1}, \bar{x}, \bar{x^2}, \overline{P(x)}\}$

Ej 3

$V = \{ f : [a,b] \rightarrow \mathbb{K} \}$

$W = \{ f \in V : f(a) = 0 \}$

$V/W \cong \mathbb{K}$

$f \in V \quad f(a) = c \quad f_c(x) = c.$

V $\mathbb{K}$-e.v. $\dim V = 1$ $\quad V \simeq \mathbb{K}$

$f - f_c \in W$

$f \sim f_c$

$\rightarrow V/W = \{ [f_c] : c \in \mathbb{R} \} =$

$= \{ c[f_1] : c \in \mathbb{R} \} = \langle [f_1] \rangle \cong \mathbb{K}$