Clase de consulta

48 | $C = \{v, T(v), \ldots, T^K(v)\}$

correspondiente a $\lambda \in \mathbb{K}$.

$$C = \{v, (T-\lambda I_d)(v), \ldots, (T-\lambda I_d)^K(v)\}$$

$$v \in \mathrm{Ker}(T-\lambda I_d)^p / \mathrm{Ker}(T-\lambda I_d)^{p-1} \quad \forall p \geq 1$$

para $p = K+1$

λ es val. p. si $T(v) = \lambda v$

$(T-\lambda I_d)(v) = 0$

si $v \in \mathrm{Ker}(T-\lambda I_d)$

Si $p = 1$, $v \in \mathrm{Ker}(T-\lambda I_d) / \mathrm{Ker}(T-\lambda I_d)^0$

$(T-\lambda I_d)^0 = I_d$

$\Rightarrow \mathrm{Ker}(T-\lambda I_d)^0 = \{0_V\}$

Si $p = 1$ $v \in \mathrm{Ker}(T-\lambda I_d) \setminus \{0_V\}$

implica $v \neq 0_V$ $\Rightarrow$ $T(v) = \lambda v$

El vector inicial de C es $(T-\lambda I_d)^K(v) =: w$

w es un vector propio de T asociado a λ

si $T(w) = \lambda w$

$\hookrightarrow (T-\lambda I_d)(w) = 0$

$$(T-\lambda I_d)(w) = (T-\lambda I_d)(T-\lambda I_d)^K(v)$$

$= (T - \lambda Id)^{K+1}(v) = 0$

$v \in Ker(T - \lambda Id)^p / Ker(T - \lambda Id)^{p-1}$

$\hookrightarrow$ para $p = K+1$

$v \in Ker(T - \lambda Id)^{K+1} / Ker(T - \lambda Id)^K$:

$v \neq 0_v$, $(T - \lambda Id)^{K+1}(v) = 0$

49. $C = \{(T - \lambda Id)^{p-1}(v), \ldots, (T - \lambda Id)(v), v\}$

$v \in Ker(T - \lambda Id)^p / Ker(T - \lambda Id)^{p-1}$

C es L.I. si $\alpha_{p-1}(T - \lambda Id)^{p-1}(v) + \cdots + \alpha_1(T - \lambda Id)(v) + \alpha_0 v = 0$

implica que $\alpha_j = 0 \ \forall j$.

Supongamos que es L.D, existen $\alpha_0, \alpha_1, \ldots, \alpha_{p-1}$ no todos nulos tal que

$\underbrace{\alpha_{p-1}(T - \lambda Id)^{p-1} v + \cdots + \alpha_1(T - \lambda Id) v + \alpha_0 v}_{\text{hay } p \text{ terminos}} = 0$

en particular hay algun $\alpha_j \neq 0$ $(j > 0)$

$(T - \lambda Id)^j (v) = -\frac{1}{\alpha_j}\left[\alpha_{p-1}(T - \lambda Id)^{p-1} v + \cdots + \alpha_0 v\right]$

Aplico $(T-\lambda Id)^{p-j}$ al vector $(T-\lambda Id)^j v$

$$0 = (T-\lambda Id)^p (v) = -\frac{1}{\alpha_j}\left[\alpha_{p-2}(T-\lambda Id)^{p-j+(p-2)} v + \dots + \alpha_0 (T-\lambda Id)^{p-j} v\right]$$

Entonces $\underbrace{\alpha_{p-2}(T-\lambda Id)^{p-j+p-2} v + \dots + \alpha_0 (T-\lambda Id)^{p-j}}_{\text{Hay } p-2 \text{ terminos}} v = 0$

Repitiendo el argumento, en algún paso (a lo sumo p), llegamos

$$\alpha_p (T-\lambda Id)^p v = 0 \quad \text{con } \alpha_\ell \neq 0, \ p-2 \leq \ell \leq 0$$

Esto es absurdo ↯.

OBS: Si $v \in Ker\, S^p \Rightarrow S^{p+j}(v) = S^j \circ S^p(v) = S^j(0) = 0$

$C = \{v\}$ ~~$(T-3Id)v, \dots, (T-3Id)^{p-2} v$~~ $\}$

$T - 2\, Id = L_A$ para alguna matriz A

$Ker(T-2Id)^2 = Ker(L_A)^2 = Ker\, A^2$