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12.b) $\langle T(v), v \rangle = \langle v, v \rangle' = (\|v\|')^2 > 0$

si $v \neq 0_V$

11.a) $\langle\, , \,\rangle$, $\langle\, , \,\rangle'$

$\forall\, v \in V$, $\exists!\, v_0$ tq $\langle u, v \rangle' = \langle u, v_0 \rangle \quad \forall u \in V$

Unicidad: Sean v_0, v_2 tq $\langle u, v \rangle' = \langle u, v_0 \rangle = \langle u, v_2 \rangle$

Sea $B = \{w_1, \ldots, w_n\}$ una b.o.n.

Luego $v_0 = \sum_{i=1}^{n} \overline{\langle v_0, w_i \rangle}\, w_i$

$$\overline{\langle v_0, w_i \rangle} \;\|\; \overline{\langle v_2, w_j \rangle}$$

$v_2 = \sum_{j=1}^{n} \overline{\langle v_2, w_j \rangle}\, w_j$

$$\langle u, v_0 \rangle = \left\langle u, \sum_{i=1}^{n} \langle v_0, w_i \rangle\, w_i \right\rangle$$

$$= \sum_{i=1}^{n} \overline{\langle v_0, w_i \rangle}\, \langle u, w_i \rangle \qquad \delta_{ij} \qquad w_j \qquad \forall u \in V$$

$$\langle u, v_2 \rangle = \sum_{j=1}^{n} \overline{\langle v_2, w_j \rangle}\, \langle u, w_j \rangle$$

$$\langle w_i, v_0\rangle = \langle w_i, v_2\rangle$$

$$\overline{\langle v_0, w_i\rangle} = \overline{\langle v_2, w_i\rangle} \quad \forall\, w_i$$

Entonces $\langle v_0, w_i\rangle = \langle v_2, w_i\rangle \quad \forall\, w_i$ en la base.

$\langle u, v_0\rangle = \langle u, v_2\rangle$

$\langle u, v_2 - v_0\rangle = 0 \quad \forall\, u \in V.$

Tomo $u = v_2 - v_0 \Rightarrow \langle v_2 - v_0, v_2 - v_0\rangle = \|v_2 - v_0\|^2 = 0$

sii $v_2 - v_0 = 0_V$

sii $v_2 = v_0$

$\boxtimes$

<u>Existencia</u>: Sean $u, v \in V$, queremos ver que existe $v_0 \in V$ tq $\langle u, v\rangle' = \langle u, v_0\rangle$

Sea $\mathcal{B} = \{w_1, \dots, w_n\}$ bon para $\langle\,,\,\rangle_{\text{usual}}$

Luego $v_0 = \sum_{i=1}^{n} \underbrace{\langle v_0, w_i \rangle}_{\langle v, \cdot \rangle'} w_i$

$\langle v_0, w_i \rangle = \overline{\langle w_i, v_0 \rangle} := \boxed{\overline{\langle w_i, v \rangle'}} = \langle v, w_i \rangle'$

Definimos v_0 para que esto se cumpla.

$$\boxed{v_0 = \sum_{i=1}^{n} \langle v, w_i \rangle' w_i}$$

Conocemos $V, \langle \, , \rangle, \langle \, , \rangle'$.
$\mathcal{B} = \{w_1, \dots, w_n\}$

Hay que ver que $\langle u, v \rangle' = \langle u, v_0 \rangle$

$\langle u, v_0 \rangle = \langle u, \sum_{i=1}^{n} \langle v, w_i \rangle' w_i \rangle$

$= \sum_{i=1}^{n} \overline{\langle v, w_i \rangle'} \langle u, w_i \rangle$

Concluir

$\overline{\langle v, w_i \rangle'} = \overline{\langle v_0, w_i \rangle}$

7.1. b)

$V = \mathbb{R}_1[x],$ $T(p) = p'$ $\Big\}$ Hallar T^*

1) Hallar una $\mathcal{B}$ bon ^{P}

2) Hallar $\text{Coord}_{\mathcal{B}}(ax+b)$

3) Sea $A = [T]_{\mathcal{B}}$

4) $\boxed{[T^*]_{\mathcal{B}} = \underbrace{[T]^*_{\mathcal{B}}}_{P}}$ $\longrightarrow$ Muy importante

5) $\text{Coord}_{\mathcal{B}}(T^*(ax+b)) = [T^*]_{\mathcal{B}} \, \text{Coord}_{\mathcal{B}}(ax+b)$

6) $\text{Coord}_{\mathcal{B}}(T^*(ax+b)) = (\lambda_1, \lambda_2)$

$\mathcal{B} = \{v_1, v_2\}$

$T^*(ax+b) = \lambda_1 v_1 + \lambda_2 v_2.$

2.c) $\langle T[\overbrace{T^*(v)}^{w}], v \rangle$

$\langle T \circ T^*(v), v \rangle$

$\|$

$\langle T^*(v), T^*(v) \rangle = \|T^*(v)\|^2 \geq 0 \quad \forall v$

$T(iw) = i\,T(w)$

$T(w)\left[1 - \underset{\pi_2}{i}\right] = 0$

c) $T(x,y) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \rightsquigarrow$ Rotación $\frac{\pi}{2}$ en $\mathbb{R}^2$

$\langle T(v), v \rangle = 0 \quad \forall v \,, \quad T \neq O_T$

b) $(\Rightarrow)$ T autoadjunto $\langle T(v), v\rangle = \langle v, T(v)\rangle$

Por otro lado, $\langle T(v), v\rangle \underset{\text{P.I.}}{=} \overline{\langle v, T(v)\rangle}$

$$\langle v, T(v)\rangle = \overline{\langle v, T(v)\rangle} \quad \forall v \in V \quad \text{sii} \quad \langle v, T(v)\rangle \in \mathbb{R}$$

$(\Leftarrow)$ $\langle T(v), v\rangle \in \mathbb{R} \quad \forall v \in V$

Queremos ver que $T = T^*$ sii $T - T^* = 0$

Alcanza probar que $\underline{\langle (T - T^*)(v), v\rangle = 0} \quad \forall v \in V$

$$\begin{aligned}
\langle (T - T^*)(v), v\rangle &= \langle T(v), v\rangle - \langle T^*(v), v\rangle \\
&= \langle T(v), v\rangle - \overline{\langle v, T(v)\rangle} \\
&= \langle T(v), v\rangle - \overline{\langle T(v), v\rangle} \\
&\underset{\langle T(v), v\rangle \in \mathbb{R}}{=} \langle T(v), v\rangle - \langle T(v), v\rangle \\
&= 0 \quad \forall v \in V
\end{aligned}$$

En $\mathbb{C}$: $\left.\begin{array}{l}\langle T(v),v\rangle \geq 0\\ \langle T(v),v\rangle > 0\end{array}\right\}$ implican T autoadjunto.

En $\mathbb{R}$: $\left.\begin{array}{l}\langle T(v),v\rangle \geq 0\\ \langle T(v),v\rangle > 0\end{array}\right\}$ No implica T autoadjunto

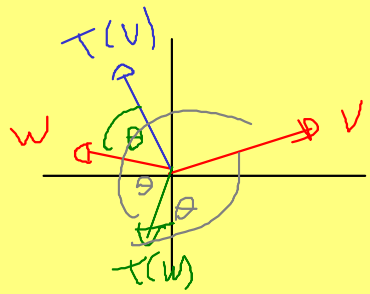
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Ej: $T(x,y) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (-y, x)$

$\langle T(v),v\rangle = 0 \quad \forall v$

$\langle T(v),w\rangle \neq \langle v, T(w)\rangle$

Queremos.



$$\left.\begin{array}{l} v = \left(\frac{1}{2}, \frac{1}{2}\right) \\ w = \left(-1, \frac{1}{10}\right)\end{array}\right\} \quad T(v) = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$\langle T(v),w\rangle = \left\langle\left(-\frac{1}{2}, \frac{1}{2}\right), \left(-1, \frac{1}{10}\right)\right\rangle = \frac{1}{2} + \frac{1}{20} = \frac{11}{20}$$

$$T(w) = \left(-\frac{1}{10}, -1\right) \to \langle v, T(w)\rangle = \left\langle\left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{10}, -1\right)\right\rangle = -\frac{1}{20} - \frac{1}{2} = -\frac{11}{20}$$

$\langle T(v),w\rangle \neq \langle v, T(w)\rangle$

Para encontrar T que no sea auto adjunta,

alcanza con encontrar T tq $(T)_\varphi \neq (T)^*_\varphi$

$$(T)^*_\varphi = (T^*)_\varphi$$