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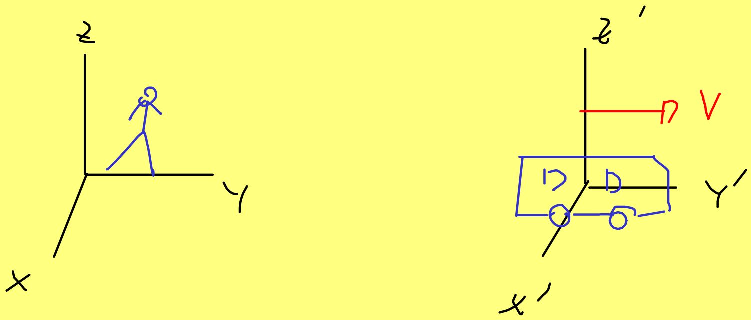

$|\nu| = \left|\frac{v}{c}\right| < 1$

12) $T(x,y,z,t) = \left(\frac{x - vt}{\sqrt{1-v^2}}, y, z, \frac{t - vx}{\sqrt{1-v^2}}\right)$

a) $\Phi(x,y,z,t) = x^2+y^2+z^2 - t^2$

Una manera de hacerlo:

$\varphi((x,y,z,t),(x',y',z',t')) = xx'+yy'+zz'-tt'$

$\varphi((1,0,0,0),(0,1,0,0)) = 0+0+0+0 = 0$

$\varphi(e_i,e_j)=0 \quad \forall i\neq j$, $\quad \varphi(e_i,e_i) = \Phi(e_i) = \begin{cases}1 & \text{si } i=1,2,3\\ -1 & \text{si } i=4\end{cases}$

Sea $\mathcal{C}$ la base canónica:

$$M_{\mathcal{C}}(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

El rango de φ es $4 = \dim(V)$, entonces

$\dim(V_0) = 0$ sii $V_0 = \{0_V\}$

rad (φ)

Entonces la forma es no degenerada.

b) $$T_v(x,y,z,t) = \left(\frac{x - vt}{\sqrt{1-v^2}}, y, z, \frac{t - vx}{\sqrt{1-v^2}}\right)$$

$$\Phi(x,y,z,t) = x^2+y^2+z^2-t^2$$

$$\Phi(T_v(w)) = \Phi(w) \qquad \forall w \in \mathbb{R}^4$$
$$w = (x,y,z,t)$$

$$\Phi\left(\left(\frac{x-vt}{\sqrt{1-v^2}}, y, z, \frac{t-vx}{\sqrt{1-v^2}}\right)\right) =$$

$$= \left(\frac{x-vt}{\sqrt{1-v^2}}\right)^2 + y^2 + z^2 - \left(\frac{t-vx}{\sqrt{1-v^2}}\right)^2$$

$$= y^2+z^2 + \frac{x^2+v^2t^2 \cancel{-2xvt}}{1-v^2} - \left[\frac{t^2+v^2x^2 \cancel{-2xvt}}{1-v^2}\right]$$

$$= y^2+z^2 + \frac{x^2-t^2-x^2v^2+t^2v^2}{1-v^2}$$

$$= y^2 + z^2 + \frac{x^2 - t^2 - v^2\left[x^2 - t^2\right]}{1 - v^2}$$

$$= y^2 + z^2 + x^2 - t^2 \, \frac{\cancel{\left[1 - v^2\right]}}{\cancel{1 - v^2}}^{\,1}$$

$$= x^2 + y^2 + z^2 - t^2 = \Phi(x, y, z, t) = \Phi(w)$$

$\forall (u,v) \in V \times V$

Camino corto: $\quad \varphi(u, v) = \dfrac{\Phi(u+v) - \Phi(u) - \Phi(v)}{2}$

$$\varphi\big(\overbrace{T_v(u)}^{u}, \overbrace{T_v(w)}^{v}\big) = \frac{\Phi\big(T_v(u) + T_v(w)\big) - \overbrace{\Phi\big(T_v(u)\big)}^{\Phi(u)} - \overbrace{\Phi\big(T_v(w)\big)}^{\Phi(w)}}{2}$$

$$T_v \in L(\mathbb{R}^4) \rightarrow T_v(u) + T_v(w) = T_v(u+w)$$

$$\Rightarrow \Phi\big(T_v(u) + T_v(w)\big) = \Phi\big(T_v(\overbrace{u+w}^{z})\big) = \Phi(\overbrace{u+w}^{z})$$

$$\varphi(T_v(u), T_v(w)) = \frac{\Phi(u+w) - \Phi(u) - \Phi(w)}{2}$$

Por polares: $\dfrac{\Phi(u+w) - \Phi(u) - \Phi(w)}{2} = \varphi(u,w)$

Entonces $\varphi(T_v(u), T_v(w)) = \varphi(u,w)$.

<u>Otra forma</u>: Sabemos que $\varphi((x,y,z,t),(x',y',z',t')) = xx' + yy' + zz' + tt'$

entonces podemos calcular $\varphi(T_v(u), T_v(w))$ con $u = (x,y,z,t)$, $w = (x',y',z',t')$

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c) $T_v \in \mathcal{L}(\mathbb{M}^4)$

$$T_v(x,y,z,t) = \left(\frac{x - vt}{\sqrt{1-v^2}}, y, z, \frac{t - vx}{\sqrt{1-v^2}}\right)$$

Si $\mathcal{B}$ es base canónica:

$$A = (T_v)_\beta = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} & 0 & 0 & \frac{-v}{\sqrt{1-v^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-v}{\sqrt{1-v^2}} & 0 & 0 & \frac{1}{\sqrt{1-v^2}} \end{pmatrix} \quad B$$

$$\det(A) = \frac{1}{\sqrt{1-v^2}} \cdot \frac{1}{\sqrt{1-v^2}} - \left(\frac{-v}{\sqrt{1-v^2}}\right)\left(\frac{-v}{\sqrt{1-v^2}}\right)$$

$$= \frac{1}{1-v^2} - \frac{v^2}{1-v^2} = \frac{1-v^2}{1-v^2} = 1 \neq 0$$

Entonces es invertible.

$$T_{-v}(x,y,z,t) = \left(\frac{x+vt}{\sqrt{1-v^2}},\ y,\ z,\ \frac{t+vx}{\sqrt{1-v^2}} \right)$$

$$\underbrace{T_{-v} \circ T_v}_{Id} (x,y,z,t) = (x,y,z,t)$$

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$$\varphi(T_v(u), T_v(w)) = \varphi(u, T_v^\circ \circ T_v(w))$$

$\|$ ← Parte b

$$\varphi(u,w)$$

⇒ Sii $w = \underbrace{\boxed{T_v^\circ \circ T_v}}_{Id}(w)$ $\forall w \in V$

Sii $T_v^\circ = T_v^{-1}$