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6) $T(x,y,z) = (-y-z,\ x+y,\ y+2z)$

$\mathcal{E}$ base canónica $\Rightarrow$ $(T)_{\mathcal{E}} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

$$\chi_T(\lambda) = (-\lambda)\left[(1-\lambda)(2-\lambda)\right] - 1\left[-(2-\lambda)+1\right]$$
$$= -\lambda\left[\lambda^2 - 3\lambda + 2\right] + \left[2-\lambda-1\right]$$
$$= -\lambda^3 + 3\lambda^2 - 3\lambda + 1 \rightarrow \text{Raíz evidente } 1$$

| | | | | |
|---|---|---|---|---|
| | $-1$ | $3$ | $-3$ | $1$ |
| $1$ | | $-1$ | $2$ | $-1$ |
| | $-1$ | $2$ | $-1$ | $0$ |

$\rightarrow -\lambda^3 + 3\lambda^2 - 3\lambda + 1 = (\lambda-1)(-\lambda^2+2\lambda-1)$

$\vdots$

$$\chi_T(\lambda) = (\lambda-1)^3$$

$\chi_T(T) = (T-I)^3 \rightarrow$ probando:

$T - I \neq 0$

$(T-I)^2 \neq 0$

$(T-I)^3 = 0$

Entonces $m_T(t) = (t-1)^3$

$\rightarrow$ Hay un solo bloque de tamaño $3\times 3$.

$$J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\{0\} \neq Ker(T-I) \subsetneq Ker([T-I]^2) \subsetneq Ker([T-I]^3) = V$$

$$\rightarrow [(0,-1,1),\ (1,0,-1)]$$

Sea $v=(1,0,0) \in Ker([T-I]^3) \setminus Ker([T-I]^2)$

Como el bloque de Jordan tiene tamaño 3, una base para ese bloque es $\{(T-I)^2(v), (T-I)(v), v\}$.

$\boxed{2}$

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7) $B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$$\chi_B(t) = (-t)[(-t)(-1-t)] - [-1-t]$$
$$= -t[t^2+t] + t + 1$$
$$= -t^3 - t^2 + t + 1$$

$(t=-1)$ $+1$ $-1$ $-1$ $+1$

$(t=1)$ $-1$ $-1$ $1$ $1$

| | | | | |
|---|---|---|---|---|
| | $-1$ | $-1$ | $1$ | $1$ |
| $1$ | | $-1$ | $-2$ | $-1$ |
| | $-1$ | $-2$ | $-1$ | $0$ |

$$\left.\begin{array}{l} -t^2 - 2t - 1 = 0 \\ t^2 + 2t + 1 = 0 \\ (t+1)^2 \end{array}\right\} \text{ $m_3$ /:}$$

Queda así

$$\boxed{\chi_B(t) = -(t-1)(t+1)^2}$$

$m_1 = (t-1)(t+1)^2$

$\boxed{m_2 = (t-1)(t+1)} \rightarrow$ Minimal

$$\boxed{m_B(t) = t^2 - 1}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \longrightarrow \begin{matrix} (0,0,1) & \lambda = 1 \\ (1,1,0) & \lambda = 1 \\ (1,-1,0) & \lambda = -1 \end{matrix}$$

---

$$\begin{pmatrix} 0 & -1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

→ Ejercicio 6

$J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

↳ $J = A - I$

→ Diagonalizable

$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$$B = \{ ((T-I)^2(v),0,0,0),\ ((T-I)(v),0,0,0),\ (v,0,0,0),\ (0,0,0,1,1,0),\ (0,0,0,0,0,1),\ (0,0,0,1,-1,0) \}$$

Luego $\boxed{Q = (Id)_{\mathcal{E}B}}$

---

b) $0 = T^4 - 5T^3 + 3T^2 + 9T$

a)

$0 = t\,[\underbrace{t^3 - 5t^2 + 3t}_{-1 \text{ raíz}} + 9)$ → Raíz 0

| | 1 | -5 | 3 | 9 |
|---|---|---|---|---|
| -1 | | -1 | 6 | -9 |
| | 1 | -6 | 9 | 0 |

$t^2 - 6t + 9 = 0 \to (t-3)^2$

$\underbrace{t^2 - 6t + 9 = 0}_{3 \text{ raíz}}$

| | 1 | -6 | 9 |
|---|---|---|---|
| 3 | | 3 | -9 |
| | 1 | -3 | 0 |

$t - 3 = 0 \to \boxed{t = 3}$

$$t^4 - 5t^3 + 3t^2 + 9t = t(t+1)(t-3)^2$$

Como T es sobre $\Rightarrow$ T es iny. $\Rightarrow$ 0 no puede ser raíz del mínimo)

Entonces las posibilidades son: $(t+1)(t-3)$

$(t+1)(t-3)^2$

b) Como T no es diagonalizable:

$$\boxed{m_T(t) = (t+1)(t-3)^2}$$

Lo único que sabemos es que hay un bloque de tamaño 2 asociado a $\lambda = 3$

$$J = \begin{pmatrix} \boxed{2\times 2} & \\ & \boxed{\boxed{2} \quad \boxed{2}} \end{pmatrix}$$

$$\to \left\{ \begin{array}{l} \begin{pmatrix} \boxed{-1} & 0 \\ 0 & \boxed{-1} \end{pmatrix} \\ \\ \begin{pmatrix} \boxed{3} & 2 \\ 0 & \boxed{-2} \end{pmatrix} \end{array} \right.$$

$$\cancel{\begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix}}$$ No puede ser.

---

9) $V = [\underline{e^x}, \underline{xe^x}, \underline{x^2e^x}, \underline{e^{2x}}] \subseteq \mathcal{F}$

$T: V \to V$,
$f \mapsto f'$

| $f$ | $f'$ |
|---|---|
| $e^x$ | $e^x$ |
| $xe^x$ | $e^x + xe^x$ |
| $x^2e^x$ | $2xe^x + x^2e^x$ |
| $e^{2x}$ | $2e^{2x}$ |

$$(T)_\beta = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\chi_T(t) = (1-t)[(1-t)(1-t)(2-t)]$$
$$= (1-t)^3(2-t)$$

$m_1 = (1-t)(2-t)$
$m_2 = (1-t)^2(2-t)$ $\Big\}$ No anda

$m_3 = (1-t)^3(2-t)$. $\to$ Tendría que ser este.

Entonces hay un bloque $3\times3$ y uno $1\times1$
Asociado a $x=1$ | Asociado $x=2$