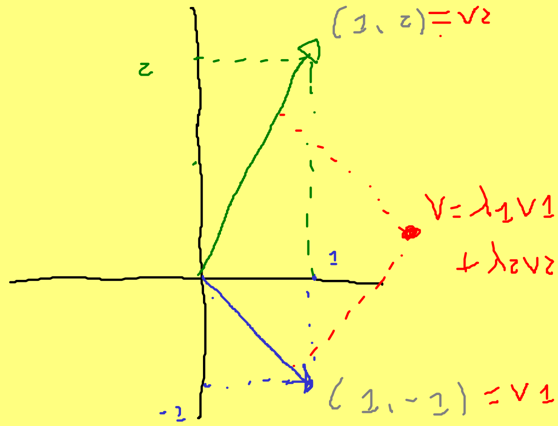
2/9:

b)



$P:\mathbb{R}^2 \to \mathbb{R}^2$

$Im(P) = \langle (1,-1) \rangle$

$Ker(P) = \langle (1,2) \rangle$

Obs: $\{(1,-1),(1,2)\}$ ⟵ Es L.I., y es base.

$$\mathbb{R}^2 = \underbrace{\langle (1,-1) \rangle}_{V_1} \oplus \underbrace{\langle (1,2) \rangle}_{V_2}$$

$Im(P) = V_1$, $\forall v \in \mathbb{R}^2$, $\exists!\ v_1, v_2$ tq $v = v_1 + v_2$

$Ker(P) = V_2$

$$P(v) = P(v_1) + \overset{0}{\overset{\shortparallel}{P(v_2)}} = v_1 \in V_1$$

Estamos buscando $\quad P(1,-1) = (1,-1) = 1(1,0) - 1(0,1)$

$P(1,2) = (0,0)$

$\mathcal{B} = \{(1,-1),(1,2)\}$ base de $\mathbb{R}^2$.

$$_{\mathcal{B}}(P)_{\mathcal{B}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

⟶ $\text{coord}_{\mathcal{B}}(P(1,-1))$

$P(1,-1) = 1 \cdot (1,-1) + 0\,(1,2)$

$P(1,2) = 0 \cdot (1,-1) + 0\,(1,2)$

Hallar matriz asociada.

1) Dado $v=(x,y)\in\mathbb{R}^2$, hallar $\mathrm{coord}_{\mathcal{B}}(x,y)$

2) $\mathrm{Coord}_{\mathcal{B}}(P(x,y)) = {}_{\mathcal{B}}(P)_{\mathcal{B}}\cdot \mathrm{coord}_{\mathcal{B}}(x,y)$

3) Usar $\mathcal{B}=\{(1,-1),(1,2)\}$, y $\mathrm{Coord}_{\mathcal{B}}(P(x,y))$

escribimos la transformación.

Otra forma: Como $\mathbb{R}^2 = \underbrace{V_2}_{v}\oplus \underbrace{V_2}_{w}$, entonces

dado $(x,y)=\lambda_1(1,-1)+\lambda_2(1,2)$

Hacemos $P(x,y)=\lambda_1(1,-1)$

Todo se resume a hallar $\mathrm{coord}_{\mathcal{B}}(x,y)$.

$$\begin{cases} \lambda_1+\lambda_2 = x & (i)\\ -\lambda_1+2\lambda_2 = y & (ii)\end{cases}$$

$(i)+(ii)$: $3\lambda_2 = x+y \rightarrow \boxed{\lambda_2=\dfrac{x+y}{3}}$

De (i): $\lambda_1 = x-\lambda_2 = x-\left(\dfrac{x+y}{3}\right) = \dfrac{3x-x-y}{3} = \dfrac{2x-y}{3}$

$$\boxed{\lambda_1=\dfrac{2x-y}{3}}$$

$$(x,y) = \overbrace{\frac{2x-y}{3}(1,-1)}^{v} + \overbrace{\frac{x+y}{3}(1,-2)}^{w} \qquad P(v+w) = v$$

$$P(x,y) = \frac{2x-y}{3}(1,-1) = \left(\frac{2x-y}{3}, \frac{-2x+y}{3}\right)$$

- ¿P es lineal? ✓

- ¿P proyección? $P^2(x,y) = P(P(x,y)) = P\left(\overbrace{\frac{2x-y}{3}}^{\tilde{x}}, \overbrace{\frac{-2x+y}{3}}^{\tilde{y}}\right)$

Queremos

$$\frac{2x-y}{3}(1,-1) = \frac{2\tilde{x}-\tilde{y}}{3}(1,-1)$$

$$= \frac{\frac{4x-2y}{3} + \frac{2x-y}{3}}{3}(1,-1)$$

$$= \frac{\frac{6x-3y}{3}}{3}(1,-1)$$

$$= \frac{\overbrace{\frac{3}{3}}^{1}(2x-y)}{3}(1,-1)$$

$$= \frac{2x-y}{3}(1,-1) = P(x,y)$$

Habría que verificar que $\operatorname{Im}(P) = \langle (1,-2) \rangle$

$\operatorname{Ker}(P) = \langle (1,2) \rangle$

$$v = \cancel{\lambda_1(1,-1)} + \lambda_2(1,2) \quad \Big\} \quad P(v) = 0 \quad \Big| \quad \underbrace{\langle(1,2)\rangle}_{1} \subset \underbrace{\ker(P)}_{1}$$

0

$$v = \lambda_1(1,-1) + \lambda_2(1,2) \rightarrow P(v) = \lambda_1(1,-1) \in \langle(1,-1)\rangle$$

$$\overbrace{\langle(1,-1)\rangle}^{1} \subset \overbrace{\mathrm{Im}(P)}^{1}$$

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9) $V = V_1 \oplus \dots \oplus V_n$

$T_i(v) = v_i$ donde $v = v_1 + \dots + v_n$

• ¿ T_i lineal? $T_i(\lambda v + w) = T_i(\overbrace{\lambda v_1 + w_1 + \dots + \lambda v_n + w_n}^{\lambda v_1 + \lambda v_2 + \lambda v_3 \dots + w_1 + w_2 + \dots + w_n})$

$= \lambda v_i + w_i = \lambda T_i(v) + T_i(w)$

$T_i(v) = T_i(0 + 0 + \dots + 0 + v_i + 0 + \dots + 0) = v_i$

• ¿ T_i es proyección? $T_i^2(v) = T_i(T_i(v)) = T_i(v_i) = v_i$

$\rightarrow \overset{v_1}{0} + \overset{v_2}{0} + \dots + \overset{v_i}{v_i} + 0 + \dots 0$

Entonces T_i es proyección $\forall i = 1, \dots, n$

$v_j = 0 + \overset{v_i}{0} + 0 + v_j + 0 + \dots 0$

d) $T_i \circ \overbrace{T_j(v)}^{v_j} = T_i(v_j) = \begin{cases} v_j & i = j \\ 0 & i \neq j \end{cases}$

$v = v_1 + \dots + v_i + \dots + v_j + \dots + v_n$

b) $T_1 + \dots + T_n = Id$

$$(T_1 + \dots + T_n)(v) \underset{def}{=} T_1(v) + T_2(v) + \dots + T_n(v)$$

$$= v_1 + v_2 + \dots + v_n = v = Id(v)$$

c) ¿$Im(T_i) = V_i$?

$w \in Im(T_i)$, $\iff$ $w = T_i(v)$

para algún $v \in V$. $T_i(v) = v_i$ $\iff$ $\underbrace{w = v_i}$

$\Rightarrow w \in V_i$

11) $V = \mathbb{R}^4$

$\overset{z}{x, t} \rightarrow (x, x, x, t)$

$B = \{(1,1,1,0), (0,0,0,1)\}$

$(x, x, x, t) = x(1,1,1,0) + t(0,0,0,1)$

$V_1 = \{ (x, y, z, t) : x = y = z \}$

$V_2 = \langle (2,1,1,1) \rangle$

$V_3 = \langle (2,2,1,1) \rangle$

a) Probar $V = V_1 \oplus V_2 \oplus V_3$. Basta ver que

$\dim V = \dim V_1 + \dim V_2 + \dim V_3$ y que

$\{v_1, v_2, v_3\}$ sea independiente

Sea $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = (0,0,0,0)$

con $v_i \in V_i$.

$\lambda_1(x,x,x,t) + \lambda_2(2,1,1,1) + \lambda_3(2,2,1,1) = (0,0,0,0)$

$$\begin{cases} \lambda_1 x + 2\lambda_2 + 2\lambda_3 = 0 \\ \lambda_1 x + \lambda_2 + 2\lambda_3 = 0 \\ \lambda_1 x + \lambda_2 + \lambda_3 = 0 \\ \lambda_1 t + \lambda_2 + \lambda_3 = 0 \end{cases}$$

Queremos ver que $\lambda_1 x, \lambda_1 t, \lambda_2, \lambda_3$ son todos nulos.

Como el sistema es homogeneo $\Rightarrow$ Es compatible.

Hay que ver que es determinado.

$$\begin{pmatrix} 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 x \\ \lambda_1 t \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Alcanza con ver que A tiene $\det \neq 0$.

b) Hallar $T_i : V \to V$ tq $T_i(v) = v_i$

$v = v_1 + v_2 + v_3$

Hay que hallar

$v = (x, y, z, t) = \vec{\lambda_1} v_1 + \overline{\lambda_2} v_2 + \vec{\lambda_3} v_3$

$T_1(v) = \lambda_1 v_1$, $T_2(v) = \lambda_2 v_2$, $T_3(v) = \lambda_3 v_3$