25/9

$T: \mathbb{R}^2 \to \mathbb{R} \subset \mathbb{R}^2, \quad T(a_n) = a_{n-1} + a_{n-2}$

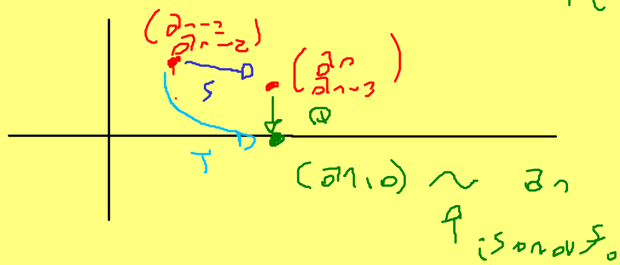
$S: \mathbb{R}^2 \to \mathbb{R}^2, \quad \boxed{S\begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix} = A \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix}} = \begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} \rightsquigarrow \begin{pmatrix} a_n \\ 0 \end{pmatrix} = a_n$

$Q: \mathbb{R}^2 \to \mathbb{R} \subset \mathbb{R}^2$

$(x, y) \mapsto (x, 0)$

$\to \quad T: Q \circ S$

$P\begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} = a_n$



$\|v\| = \sqrt{\langle v, v \rangle} \to \quad v \in \mathbb{C}^n \quad \circ \quad v = (v_1, v_2, v_3)$

$v \in \mathbb{C}^3$; $\quad \|v\| = \sqrt{v_1 \overline{v_1} + v_2 \overline{v_2} + v_3 \overline{v_3}}$

¿Como hallar bases ortogonales?

1) Elegir una base cualquiera de V

$\mathcal{B} = \{v_1, \dots, v_n\}$ L.I y además $[\{v_1, \dots, v_n\}] = \mathcal{B}$

2) Aplico Gram-Schmidt.

$\mathcal{B} \to \boxed{G-S} \to \mathcal{C}$ ortogonal L.I

además $[\mathcal{C}] = [\mathcal{B}] = V$

3) Convertir la base $\mathcal{C} = \{w_1, \dots, w_n\}$ en una base

ortonormal : $\tilde{\mathcal{B}} = \left\{ \frac{w_1}{\|w_1\|}, \dots, \frac{w_n}{\|w_n\|} \right\}$

Otra forma:

1) Agarro cualquier $v \in V$ $v \neq 0$..

2) Busco $w \in V$ tq $\langle v, w \rangle = 0$ (Hay infinitos w)

3) Eligimos un w

4) $\mathcal{B} = \left\{ \frac{v}{\|v\|}, \frac{w}{\|w\|} \right\}$

$\langle , \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$, $\langle (x,y), (x',y') \rangle = 2xx' + \underline{yx' + xy'} + yy'$

a) —

b) 1) $v = (1,0)$

2) Buscar w tq $\langle v, w \rangle = 0$

$\langle (\overset{x}{1}, \overset{y}{0}), (x', y') \rangle = 2x' + y' = 0 \to \boxed{y' = -2x'}$

$\hookrightarrow$ 1 grado de libertad

3) Fijo $x' = 1 \Rightarrow w = (1,-2)$

4) $\|(1,0)\| = \sqrt{2}$, $\|(1,-2)\| = \sqrt{\langle (1,-2), (1,-2) \rangle}$

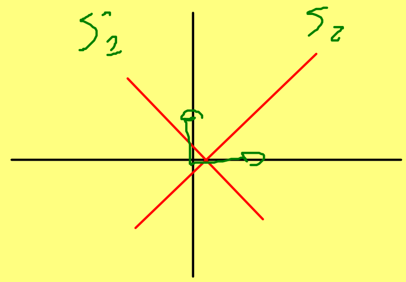
$= \sqrt{2 - 4 + 4}$

$= \sqrt{2}$

Entonces $\mathcal{B} = \left\{ \frac{(1,0)}{\sqrt{2}}, \frac{(1,-2)}{\sqrt{2}} \right\}$ es una

base ortonormal.

$V = \bigoplus_{i=1}^{n} S_i$ $\qquad$ $V = S_1 \oplus S_2$



$\mathcal{B} = \{(1,0),(0,1)\}$

$a_1 v_1 + \dots + a_n v_n = 0 \Rightarrow a_1 = \dots = a_n = 0$

$v_i \in S_i$

---

1) $\mathcal{B} = \{(2,3),(1,2)\}$.

Hallar $\langle\ ,\ \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ t.q.

$\mathcal{B}$ sea una base ortonormal

- Sea $(x,y) \in \mathbb{R}^2$. $\Rightarrow (x,y) = \lambda_1 (2,3) + \lambda_2 (1,2)$

$$\begin{cases} 2\lambda_1 + 1\cdot\lambda_2 = x & \text{i} \\ 3\lambda_1 + 2\lambda_2 = y & \text{ii} \end{cases}$$

ii - i: $\lambda_1 = y - x \rightarrow \boxed{\lambda_1 = y - x}$

$2\lambda_2 = x - 2\lambda_1 = x - 2y + 2x = 3x - 2y$

$\boxed{\lambda_2 = \frac{3x - 2y}{2}}$

Todo mal

$\text{Coord}_{\mathcal{B}}(x,y) = \left(y - x, \frac{3x-2y}{2}\right)$

$$\langle (x,y),(x',y')\rangle = (y-x)(y'-x') + \left(\frac{3x-2y}{2}\right)\left(\frac{3x'-2y'}{2}\right)$$

$$= yy' - yx' - xy' + xx' \qquad -\frac{6}{4}-\frac{4}{4} = -\frac{10}{4}$$

$$+ \frac{1}{4}\left(9xx' + 4yy' - 6xy' - 6x'y\right)$$

$$= \frac{13}{4}xx' - \frac{10}{4}xy' - \frac{10}{4}yx' + 2yy'$$

$$\langle (x,y),(x',y')\rangle = \frac{13}{4}xx' - \frac{5}{2}xy' - \frac{5}{2}yx' + 2yy'$$

$$\langle (2,3),(2,3)\rangle = \frac{13}{4}\cdot 4 - \frac{5}{\cancel{2}}\cdot \cancel{6}^{3} - \frac{5}{\cancel{2}}\cdot\cancel{6}^{3} + 2\cdot 9$$

$$= 13 - 30 + 18 = 1 = \sqrt{\|(2,3)\|}$$

$$\langle (2,3),(1,2)\rangle = \frac{13}{\cancel{4}_{2}}\cdot\cancel{2} - \frac{5}{\cancel{2}}\cdot\cancel{4}^{2} - \frac{5}{2}\cdot 3 + \overset{12}{\cancel{2}}\cdot 6$$

$$= \frac{13}{2} - 10 - \frac{15}{2} + 12$$

$$= -1 + 2 \to$$

Esto tiene que dar cero.

$$\beta = \{(2,3),(1,2)\}$$

$$\langle (x,y),(x',y')\rangle := \langle \mathrm{Coord}_\beta(x,y), \mathrm{Coord}_\beta(x',y')\rangle_{\text{usual}}$$

$$\mathrm{Coord}_\beta(2,3) = (1,0), \quad \|(2,3)\| = \sqrt{\langle (2,3),(2,3)\rangle} = \sqrt{\langle (1,0),(1,0)\rangle_{\text{usual}}} = \sqrt{1} = 1$$

$$\mathrm{Coord}_\beta(1,2) = (0,1) \to \text{Idem}$$

$\langle (2,3),(2,2)\rangle = \langle (2,0),(0,2)\rangle_{usual} = 0$

Repasamos la idea.

1) Ver si el conjunto S es una base

(si es seguimos, si no, no funciona)

2) $S = \{v_1, v_2, \dots, v_n\}$.

Definir $\langle w, u\rangle := \langle coord_S(w), coord_S(u)\rangle_{usual}$

$w, u \in [S] = V$

---

3) $\|v_i\| = \sqrt{\langle v_i, v_i\rangle} = \sqrt{\langle e_i, e_i\rangle_{usual}} = \sqrt{1} = 1$

$coord_S(v_i) = e_i = (0, 0, \dots, \underset{\substack{\uparrow \\ i\text{-esimo lugar}}}{1}, \dots, 0)$

$\langle v_i, v_j\rangle = \langle e_i, e_j\rangle_{usual} = \delta_{ij} = \begin{cases} 1 & si\ i = j \\ 0 & si\ i \neq j \end{cases}$

$coord_S(v_i) = e_i$

$coord_S(v_j) = e_j$

Obs: Si pide fórmula explicita, hay que hallar $\langle coord_S(w), coord_S(u)\rangle_{usual}$