

OPERADORES VECTORIALES EN DIFERENTES SISTEMAS DE COORDENADAS

Coordenadas cartesianas

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}, \quad d\tau = dx dy dz$$

$$\text{Gradiente: } \nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}. \quad \text{Divergencia: } \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\text{Rotor: } \nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}} = \det \begin{pmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{pmatrix}$$

$$\text{Laplaciano: } \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Coordenadas esféricas

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin\theta d\varphi \hat{\boldsymbol{\phi}}, \quad d\tau = r^2 \sin\theta dr d\theta d\varphi$$

$$\text{Gradiente: } \nabla t = \frac{\partial t}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin\theta} \frac{\partial t}{\partial \varphi} \hat{\boldsymbol{\phi}}$$

$$\text{Divergencia: } \nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta v_\theta) + \frac{1}{r \sin\theta} \frac{\partial v_\varphi}{\partial \varphi}$$

Rotor:

$$\nabla \times \mathbf{v} = \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta v_\varphi) - \frac{\partial v_\theta}{\partial \varphi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r} (r v_\varphi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}}$$

$$\text{Laplaciano: } \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \varphi^2}$$

Coordenadas cilíndricas

$$d\mathbf{l} = d\rho \hat{\boldsymbol{\rho}} + \rho d\varphi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}}, \quad d\tau = \rho d\rho d\varphi dz$$

$$\text{Gradiente: } \nabla t = \frac{\partial t}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial t}{\partial \varphi} \hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$$

$$\text{Divergencia: } \nabla \cdot \mathbf{v} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho v_\rho) + \frac{1}{\rho} \frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z}$$

$$\text{Rotor: } \nabla \times \mathbf{v} = \left[\frac{1}{\rho} \frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z} \right] \hat{\boldsymbol{\rho}} + \left[\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho} \right] \hat{\boldsymbol{\phi}} + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho v_\varphi) - \frac{\partial v_\rho}{\partial \varphi} \right] \hat{\mathbf{z}}$$

$$\text{Laplaciano: } \nabla^2 t = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial t}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2}$$

IDENTIDADES VECTORIALES

f, g son campos escalares, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ son campos vectoriales

Productos triples

$$(1) \quad \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$(2) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Derivación de productos

$$(3) \quad \nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$(4) \quad \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$(5) \quad \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$(6) \quad \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$(7) \quad \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(8) \quad \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

Segundas derivadas

$$(9) \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(10) \quad \nabla \times (\nabla f) = 0$$

$$(11) \quad \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

TEOREMAS FUNDAMENTALES

Teorema del gradiente:
$$\int_a^b (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$$

Teorema de la divergencia o de Gauss:
$$\int (\nabla \cdot \mathbf{A}) d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$$

Teorema del rotor o de Stokes:
$$\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$