$$\frac{v_1}{v_2}\tilde{E}_{OT2}e^{ik_2d} - \frac{v_1}{v_2}\tilde{E}_{OR2}e^{-ik_2d} = \tilde{E}_{OT3}e^{ik_1d} \qquad (4')$$

Sumamos (2) + (4'):

$$\tilde{E}_{OT2}\left(e^{ik_2d} + \frac{v_1}{v_2}e^{ik_2d}\right) + \tilde{E}_{OR2}e^{-ik_2d}\left(1 + \frac{v_1}{v_2}\right) = 2\tilde{E}_{OT3}e^{ik_1d}$$

$$\Rightarrow \tilde{E}_{OT2}e^{ik_2d}\left(1 + \frac{v_1}{v_2}\right) + \tilde{E}_{OR2}e^{-ik_2d}\left(1 - \frac{v_1}{v_2}\right) = 2\tilde{E}_{OT3}e^{ik_1d}$$

$\Rightarrow$ Como $v_1 = c$, $v_2 = c/n$ $\Rightarrow v_1/v_2 = n$

$$\Rightarrow \tilde{E}_{OT2}e^{ik_2d}(1+n) + \tilde{E}_{OR2}e^{-ik_2d}(1-n) = 2\tilde{E}_{OT3}e^{ik_1d}$$

Restamos (2) − (4'):

$$\tilde{E}_{OT2}e^{ik_2d} - \frac{v_1}{v_2}\tilde{E}_{OT2}e^{ik_2d} + \tilde{E}_{OR2}e^{-ik_2d} + \frac{v_1}{v_2}\tilde{E}_{OR2}e^{-ik_2d} = 0$$

$$\tilde{E}_{OT2}e^{ik_2d}\left(1 - v_1/v_2\right) + \tilde{E}_{OR2}e^{-ik_2d}\left(1 + v_1/v_2\right) = 0$$

$$\Rightarrow \tilde{E}_{OT2}e^{ik_2d}(1-n) + \tilde{E}_{OR2}e^{-ik_2d}(1+n) = 0$$

Así tenemos dos ecs:

$$\begin{cases} \tilde{E}_{OT2}e^{ik_2d}(1+n) + \tilde{E}_{OR2}e^{-ik_2d}(1-n) = 2\tilde{E}_{OT3}e^{ik_1d} & (I) \\ \tilde{E}_{OT2}e^{ik_2d}(1-n) + \tilde{E}_{OR2}e^{-ik_2d}(1+n) = 0 & (II) \end{cases}$$

De (II) $\tilde{E}_{OT2} = -\tilde{E}_{OR2}e^{-2ik_2d}\dfrac{(1+n)}{(1-n)}$

Sustit. en (I) $\quad -\tilde{E}_{OR2}e^{-ik_2d}\dfrac{(1+n)^2}{(1-n)} + \tilde{E}_{OR2}e^{-ik_2d}(1-n) = 2\tilde{E}_{OT3}e^{ik_1d}$

$$\Rightarrow \tilde{E}_{OR2}e^{-ik_2d}\left(-\frac{(1+n)^2}{(1-n)} + (1-n)\right) = 2\tilde{E}_{OT3}e^{ik_1d}$$

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$$\Rightarrow \tilde{E}_{OR2}\left(\frac{-(1+n)^2 + (1-n)^2}{(1-n)}\right) = 2\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}$$

$$-(1+n)(1+n) = -\left[1+2n+n^2\right] = -n^2-2n-1$$

$$(1-n)^2 = (1-n)(1-n) = 1-2n+n^2$$

$$\Rightarrow \tilde{E}_{OR2}\left(\frac{-n^2-2n-1+1-2n+n^2}{(1-n)}\right) = 2\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}$$

$$\tilde{E}_{OR2}\,\frac{-4n}{(1-n)} = 2\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}$$

$$\Rightarrow \tilde{E}_{OR2} = -\frac{(1-n)}{2n}\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}$$

Volviendo a (I):

$$\tilde{E}_{OT2}\, e^{ik_2d}(1+n) - \frac{(1-n)}{2n}\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}\, e^{-ik_2d}(1-n) = 2\tilde{E}_{OT3}\, e^{ik_1d}$$

$$\tilde{E}_{OT2}\, e^{ik_2d}(1+n) = 2\tilde{E}_{OT3}\, e^{ik_1d} + \frac{(1-n)^2}{2n}\tilde{E}_{OT3}\, e^{ik_1d}$$

$$\tilde{E}_{OT2} = \frac{2\tilde{E}_{OT3}\, e^{ik_1d}}{(1+n)\, e^{ik_2d}} + \frac{(1-n)^2}{2n(1+n)} \cdot \frac{\tilde{E}_{OT3}\, e^{ik_1d}}{e^{ik_2d}}$$

$$\tilde{E}_{OT2} = \left[\frac{2}{(1+n)} + \frac{(1-n)^2}{2n(1+n)}\right]\tilde{E}_{OT3}\, e^{i(k_1-k_2)d}$$

$$\left[\frac{2}{(1+n)} \cdot \frac{2n}{2n} + \frac{(1-n)(1-n)}{2n(1+n)}\right] = \frac{4n+1-2n+n^2}{2n(1+n)}$$

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$$\Rightarrow \tilde{E}_{OT2} = \left[\frac{n^2+2n+1}{2n(1+n)}\right] \tilde{E}_{OT3}\, e^{i(k_1-k_2)d}$$

Ahora como $(n+1)^2 = n^2+2n+1$ $\Rightarrow$ Sustituimos y queda:

$$\tilde{E}_{OT2} = \frac{(n+1)^2}{2n(1+n)} \tilde{E}_{OT3}\, e^{i(k_1-k_2)d}$$

$$\Rightarrow \tilde{E}_{OT2} = \frac{(1+n)}{2n} \tilde{E}_{OT3}\, e^{i(k_1-k_2)d}$$

Ahora hacemos (3) $\cdot\, n_1/n_2$:

$$\tilde{E}_{O1} - \tilde{E}_{OR} = \frac{n_1}{n_2}\left(\tilde{E}_{OT2} - \tilde{E}_{OR2}\right) \qquad (3')$$

Ahora sumamos (1) + (3')

$$2\tilde{E}_{O1} = \tilde{E}_{OT2} + \tilde{E}_{OR2} + \frac{n_1}{n_2}\tilde{E}_{OT2} - \frac{n_1}{n_2}\tilde{E}_{OR2}$$

$$2\tilde{E}_{O1} = \tilde{E}_{OT2}\left(1+\frac{n_1}{n_2}\right) + \tilde{E}_{OR2}\left(1-\frac{n_1}{n_2}\right) = \tilde{E}_{OT2}(1+n) + \tilde{E}_{OR2}(1-n)$$

Ahora sustituimos los $\tilde{E}_{OT2}$ y $\tilde{E}_{OR2}$:

$$\Rightarrow 2\tilde{E}_{O1} = \frac{(1+n)^2}{2n}\tilde{E}_{OT3}\, e^{i(k_1-k_2)d} + \frac{(1-n)^2}{2n}\tilde{E}_{OT3}\, e^{i(k_1+k_2)d}$$

$$\Rightarrow \tilde{E}_{O1} = \frac{\tilde{E}_{OT3}}{4n}\left[(1+n)^2 e^{i(k_1-k_2)d} - (1-n)^2 e^{i(k_1+k_2)d}\right]$$

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$$\Rightarrow \frac{\tilde{E}_{OT3}}{\tilde{E}_{OI}} = \frac{4n}{(1+n)^2 e^{i(\gamma_1-\gamma_2)\delta} - (1-n)^2 e^{i(\gamma_1+\gamma_2)\delta}}$$

$$\left|\frac{\tilde{E}_{OT3}}{\tilde{E}_{OI}}\right|^2 = \frac{16n^2}{\left((n+1)^2\operatorname{sen}((\gamma_1-\gamma_2)\delta) - (1-n)^2\operatorname{sen}((\gamma_1+\gamma_2)\delta)\right)^2 + \left((n+1)^2\cos((\gamma_1-\gamma_2)\delta) - (1-n)^2\cos((\gamma_1+\gamma_2)\delta)\right)^2}$$

$$= \frac{16n^2}{(1+n)^4 + (1-n)^4 - 2(1+n)^2(1-n)^2\operatorname{sen}(a)\operatorname{sen}(b) - 2(1+n)^2(1-n)^2\cos(a)\cos(b)}$$

$$= \frac{16n^2}{(1+n)^4 + (1-n)^4 - 2(1+n)^2(1-n)^2\left[\operatorname{sen}(a)\operatorname{sen}(b) + \cos(a)\cos(b)\right]}$$

$$= \frac{16n^2}{(1+n)^4 + (1-n)^4 - 2(1+n)^2(1-n)^2\cos(a-b)}$$

$$\cos(a-b) = \cos(\gamma_1 - \gamma_2 - \gamma_1 + \gamma_2) = \cos(-2\gamma_2) = \cos(2\gamma_2)$$

$$\Rightarrow \left|\frac{\tilde{E}_{OT3}}{\tilde{E}_{OI}}\right|^2 = \frac{16n^2}{(1+n)^4 + (1-n)^4 - 2(1+n)^2(1-n)^2\cos(2\gamma_2)}$$

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