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The Variation of Scattered X-Rays with Density in an Irradiated Body

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§ 1. INTRODUCTION

THE dose delivered by primary x-radiation to a point in an inhomogeneous body can be calculated by applying appropriate exponential factors to the percentage depth dose values of the primary beam in water. The general problem of calculating the dose delivered by scattered radiation to a point in an inhomogeneous body cannot be solved by the use of simple exponential factors. Even in water equivalent materials the pattern of scattered radiation is not known in detail so that it is difficult to calculate the effects which will be produced by the presence of other materials. The non-water-equivalent materials which are of greatest practical importance in radiotherapy are bone and lung tissue. Bone, which differs from water in both density and effective atomic number, presents a more difficult problem than lung tissue, which differs from water only in its density. The present work is concerned with this simpler problem, that is with the calculation of the dosage contribution made by scattered radiation in bodies which differ from water only in density.

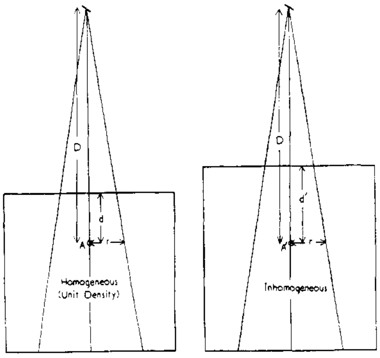


Fig. 1

The method of calculation which is described below was developed in connection with a transit dose technique which has been described elsewhere (O'Connor 1956). In this context the problem presents itself in the following form. Compare a point A (fig. 1) which lies D cm from the anode and d cm below the surface of a water equivalent body, on the central axis of a field whose radius at the level of A is r cm, with a point A′ which lies at the same distance from the anode and which is irradiated by a field of the same dimensions but which lies d' cm below the surface of an inhomogeneous body. If $\int_0^{d'} \rho_x \, dx = d$ where ρ_x is the density of the material x cm above A′, the primary radiation at A and A′ will be the same. It is required to find the value of the scattered radiation at A′.

§ 2. Homogeneous Bodies

In the case of a homogeneous body which differs from water only in its density the scattered radiation at a given point can be found in the following way. Consider two systems, system I (fig. 2) in which the body material is water equivalent and system II in which the body material has a density ρ and the linear dimensions are $1/\rho$ times those of system I. Let P and P′ be corresponding points in the systems. For every volume element, V, of unit density material in system I there is a corresponding volume element, V', in system II which has the same angular relationship to the source of radiation and the reference point. Scattering angles and wavelength changes will be the same in both systems. Furthermore absorption between the volume element and the reference point will be the same in both systems since in system II the density has been changed to ρ and the linear dimensions have been multiplied by $1/\rho$. The ratio of the primary radiation reaching the volume element to the primary radiation reaching the reference point will be the same in both systems. The radiation scattered per unit incident primary per electron in the volume element V', and reaching the reference point P′ will differ by a factor ρ^2 from the radiation scattered per unit incident primary per electron in the volume element V and reaching the reference point P, since the linear dimensions of system II are $1/\rho \times$ those of system I. But the number of electrons in the volume element V' is $\rho/\rho^3 = 1/\rho^2$ times the number of electrons in the volume of element V. These factors cancel out with the result that the ratio of single scatter to primary is the same at P and P′. A similar constancy of angular relationships and cancelling of the inverse square law factor with the number of electrons in corresponding volume elements occurs at every scattering so that the constancy of the ratio of scatter to primary is maintained for multiple scatter. This general relationship between density of the material and geometry of the system is not of direct practical value since it applies only to homogeneous bodies. It does, however, indicate the order of magnitude and general character of the relationship between density and scatter dose.

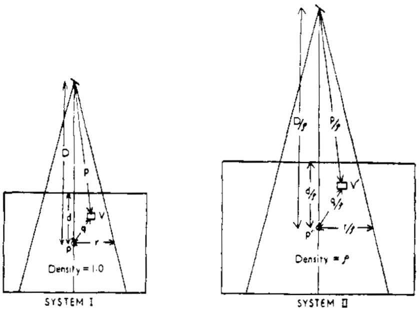


Fig. 2

§ 3. DEFINITION OF 'RELATIVE SCATTER ABSORPTION FACTORS'

The scattered radiation, ΔS, which originates in the thin upper layer of thickness Δx and density ρ of a homogeneous body whose total thickness is $x+\Delta x$ and reaches a reference point Q (fig. 3) on the lower surface of the body can be denoted by

$$\Delta S=\rho P_0 G'(x)F(x)\Delta x$$

where P_0 is the intensity of the primary which reaches the layer, $G'(x)$ is a term which depends on the radiation quality and on the geometry of the system and $F(x)$ is a term which allows for the effective absorption in the rest of the body of the scatter which originates in the layer, i.e. allows for the absorption of this single scatter, and also takes account of the multiple scatter which arises from it. If ΔS is expressed in terms of the primary radiation, P_Q, at the reference point we have

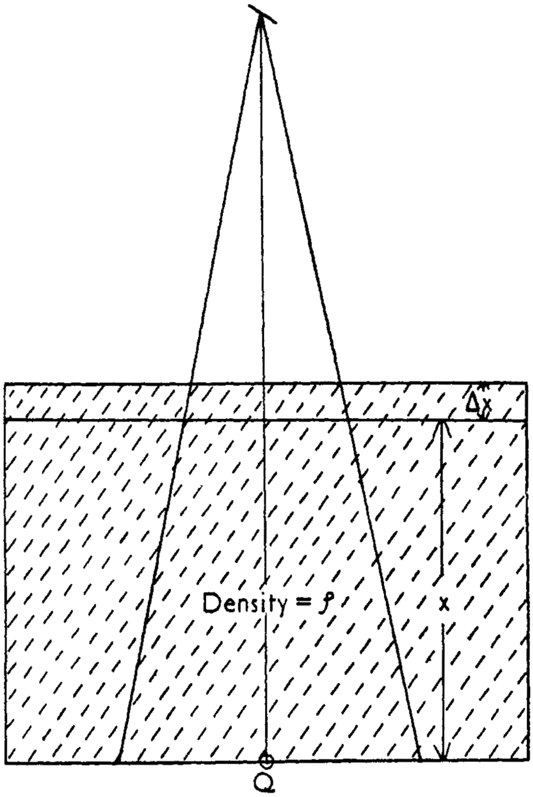


Fig. 3

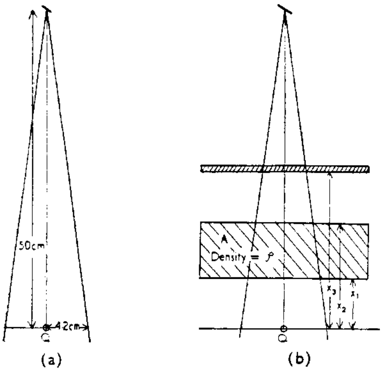
$$\Delta S=\rho P_Q \exp(\mu x)G(x)F(x)\Delta x$$

where μ is the linear absorption coefficient of the primary beam, and $G(x)$ is again a term which depends on the radiation quality and geometry.

At the reference point the ratio of scatter which originates in the layer to primary radiation is given by

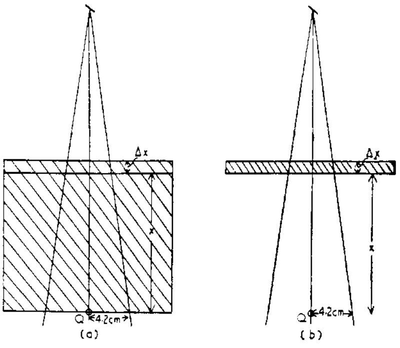
$$\Delta S/P_Q = \rho G(x) \exp(\mu x) F(x) \Delta x.$$

The term $\exp(\mu x)F(x)$, which represents the ratio of the effective absorption of the scatter which originates in the layer to the absorption of the primary beam, will be described as the relative scatter absorption factor and denoted in general by the symbol ϕ. The value of ϕ will depend on the radiation quality, the geometry of the system and the density of the absorbing material. In what follows only a primary beam whose half-value layer is 1·5 mm Cu will be considered and the geometrical conditions will be limited to those shown in fig. 4 (*a*), i.e. we will be concerned only with the scattered radiation reaching a point Q which lies 50 cm from the anode and on the central axis of a circular field whose radius at that distance is 4·2 cm. Under these conditions the relative scatter absorption factor for the scatter which originates in a layer x_3 cm above the reference point Q (fig. 4 (*b*)) in the block of material A, whose density is ρ and whose lower and upper boundaries lie respectively x_1 and x_2 cm above Q, will be denoted by $\phi(x_1, x_2, \rho, 4{\cdot}2, 50, x_3)$ and the geometrical term appropriate to the layer will be denoted by $G(x_3, 4{\cdot}2, 50)$.



(a) (b)

Fig. 4



(a) (b)

Fig. 5

Using this notation the ratio to primary radiation, at the reference point Q in fig. 5 (*a*), of the scatter originating in a thin layer of water equivalent material of thickness Δx which forms the upper layer of a water equivalent body whose total thickness is $x + \Delta x$, is given by

$$R_W = G(x, 4{\cdot}2, 50)\phi(0, x, 1{\cdot}0, 4{\cdot}2, 50, x)\Delta x. \quad . \quad . \quad . \quad . \quad (1)$$

If the position of this layer is unaltered but all the material lying between it and the reference point is removed (fig. 5 (*b*)) the relative scatter absorption factor becomes unity and the ratio at the reference point becomes

$$R_{\text{air}} = G(x, 4{\cdot}2, 50)\Delta x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From eqns. (1) and (2) we have

$$\phi(0,\, x,\, 1{\cdot}0,\, 4{\cdot}2,\, 50,\, x)=R_{\mathrm{w}}/R_{\mathrm{air}}.$$

R_{air} may be determined directly by measuring the total radiation at the reference point when a block of water equivalent material is held in air x cm above it, the primary contribution being found from it's known linear absorption coefficient. R_{w} was evaluated by determining the difference between the ratio of scattered to primary radiation at the lower surfaces of water equivalent bodies $x+\Delta x$ and x cm thick respectively. This procedure may not give an accurate value of the quantity sought. The observed increase in the ratio of scatter to primary as the thickness of the body is increased may be in part attributable to the reflection in the added layer of some of the scattered radiation which had previously escaped from the surface. While this type of scatter comes from the layer it does not originate in the layer, i.e. the initial scattering event does not take place in the layer. Despite this disadvantage the procedure described above was used to determine the value of R_{w}. It is believed that no great errors will arise from this source since any errors will cancel out when the contributions from all layers of a homogeneous body are summed and will tend to cancel out even in inhomogeneous bodies.

Values of R_{w} and R_{air} were found in the manner described above for values of x ranging from 1·5 to 17·5 cm. From them the corresponding values of $\phi(0,\, x,\, 1{\cdot}0,\, 4{\cdot}2,\, 50,\, x)$ were calculated and are given in table 1. A Siemens ionization chamber, with a volume of 4·5 cm^3 and a negligible quality dependence was used in these experiments. The absorbing material consisted of sheets of Mix D wax. The experimental readings were reproducible to within $1\frac{1}{2}\%$. Initially a block 2·5 cm thick was used as the fixed scattering layer, i.e. $\Delta x=2{\cdot}5$ cm. Since the scatter originating in a block of this thickness forms only a small proportion of the total radiation at the reference point, errors of $1\frac{1}{2}\%$ in the measurement of the total radiation can lead to large errors in the scatter contribution and hence to large errors in the values of the relative scatter absorption factors. These errors can be reduced by increasing the thickness of the fixed scattering block but if this is done the values of the relative scatter absorption factors which are obtained will be those appropriate to the single scatter and multiple scatter produced throughout the thick layer rather than those appropriate to the single scatter which originates in a clearly defined thin layer. Since the combined multiple and single scatter produced throughout the thick layer will differ in wavelength, direction of incidence and distribution of intensity across the layer, from the single scatter originating in a thin layer, it is to be expected that the relative scatter absorption factors will also differ. To examine whether this difference would lead to serious errors the experiments described above were repeated using blocks 5·0, 10·0 and 15 cm thick as the fixed scattering layer. It was found that the differences in the values of the relative scatter absorption factors were within the limits set by the experimental error of $1\frac{1}{2}\%$ to which the measurement of the total radiation was subject. This result does not necessarily indicate that the diffuse origin of the single scatter and the presence of multiple scatter from the thick scattering blocks does not alter the value of the relative scatter absorption factors. It does, however, imply that any such effect can be neglected without introducing major errors into the computation of total doses.

Table 1. Relative Scatter Absorption Factors $\phi(0, x, 1{\cdot}0, 4{\cdot}2, 50, x)$

x (cm)	ϕ	x (cm)	ϕ	x (cm)	ϕ
1·5	1·29	7·5	1·91	13·5	2·42
2·5	1·33	8·5	1·99	14·5	2·46
3·5	1·40	9·5	2·10	15·5	2·51
4·5	1·52	10·5	2·20	16·5	2·57
5·5	1·64	11·5	2·28	17·5	2·58
6·5	1·78	12·5	2·36		

This result greatly simplifies the further analysis of the relative scatter absorption factors. It means that the layer of origin of the scatter need not be specified so that the factor appropriate to the block A in fig. 4 (*b*) can be written as $\phi(x_1, x_2, \rho, 4{\cdot}2, 50)$ instead of $\phi(x_1, x_2, \rho, 4{\cdot}2, 50, x_3)$ as previously given. This in turn, combined with the knowledge that the value of the factor is unaffected by the presence of multiple scatter incident on the block, leads to the conclusion that the factor appropriate to any given block of material is equal to the product of the factors of its component layers. By the use of this relationship the relative scatter absorption factor for any given block of water equivalent material can be found from the values given in table 1. For example if the block of material in fig. 4 (*b*) were of unit density its relative scatter absorption factor would be $\phi(x_1, x_2, 1{\cdot}0, 4{\cdot}2, 50)$, the value of which can be found as follows. We have

$$\phi(0, x_2, 1{\cdot}0, 4{\cdot}2, 50)=\phi(0, x_1, 1{\cdot}0, 4{\cdot}2, 50)\times\phi(x_1, x_2, 1{\cdot}0, 4{\cdot}2, 50)$$

whence

$$\phi(x_1, x_2, 1{\cdot}0, 4{\cdot}2, 50)=\frac{\phi(0, x_2, 1{\cdot}0, 4{\cdot}2, 50)}{\phi(0, x_1, 1{\cdot}0, 4{\cdot}2, 50)}.$$

The values of both the terms on the right-hand side of this equation can be found from table 1 and hence $\phi(x_1, x_2, 1{\cdot}0, 4{\cdot}2, 50)$ can be evaluated.

§ 4. DEPENDENCE OF VALUES OF ϕ ON DENSITY

The ratio, at the reference point, of the scatter originating in a thin layer of homogeneous material of density ρ and thickness Δx which forms the upper layer of a homogeneous body of density ρ and thickness $x+\Delta x$, to the primary radiation is given by

$$R_\rho = \rho G(x,\ 4{\cdot}2,\ 50)\phi(0,\ x,\ \rho,\ 4{\cdot}2,\ 50)\Delta x. \quad . \quad . \quad . \quad . \quad (3)$$

The corresponding expression for a water equivalent body is,

$$R_{\rm w} = G(x,\ 4{\cdot}2,\ 50)\phi(0,\ x,\ 1{\cdot}0,\ 4{\cdot}2,\ 50)\Delta x. \quad . \quad . \quad . \quad . \quad (4)$$

Hence from eqns. (3) and (4) we have

$$\phi(0,\ x,\ \rho,\ 4{\cdot}2,\ 50) = \frac{1}{\rho}\frac{R_\rho}{R_{\rm w}}\phi(0,\ x,\ 1{\cdot}0,\ 4{\cdot}2,\ 50). \quad . \quad . \quad . \quad (5)$$

Since values of $\phi(0, x, 1{\cdot}0, 4{\cdot}2, 50)$ have already been obtained, $\phi(0, x, \rho, 4{\cdot}2, 50)$ can be evaluated if R_ρ is known. As described above the procedure employed for determining the value of $R_{\rm w}$ involved the measurement of the exit dose from a water equivalent block and the required information could be obtained from the standard depth dose tables if allowance could be made for the dosage reduction caused by the removal of back scattering material from behind the point of measurement. According to Clarkson and Herbert (1948) this effect can be allowed for by assuming that the removal of the back scattering material reduces the scatter contribution to half its original value. It was, however, found that for the particular radiation quality now under consideration and for a range of field areas from 20 cm² to 300 cm² the effect of removing the back scattering material was to reduce the total radiation by a factor whose value depended only on the cross section of the field at the point of measurement and was independent of the depth below the surface except in so far as this affected the area of the cross section at the level of the point of measurement. This relationship was valid for depths from 0 to 20 cm below the surface and the value of the factor was accordingly given by the reciprocal of the surface back scatter factor corresponding to the field area at the point of measurement. This relationship gives higher values of the exit dose than those given by Clarkson and Herberts' relationship.

Exit doses for water equivalent materials were calculated in this way from the standard depth dose tables for a series of field areas at the exit point. From these in turn the ratio at the exit point of scattered radiation to primary radiation was calculated and evaluated as a function of the thickness of the body for each value of cross section at the exit point. It is of interest to note that the results obtained were independent of the value of the full scale deflection. From this information the ratio at the exit point of scattered radiation originating in successive layers to primary radiation can be found.

The corresponding information for a homogeneous body of density ρ can be obtained by applying the general relationship between geometry of the system and density of the body, which is described in § 2, to the results obtained above. In theory account should be taken in the application of the general relationship of the effect of changing the f.s.d., but as has been noted, the ratio of scattered radiation to primary radiation shows a negligible dependence on the f.s.d. provided the cross section at the level of interest is constant and hence this effect may be neglected. It is thus possible to obtain from the information contained in the standard depth dose tables values of R_ρ. From eqn. (5), $\phi(0, x, \rho, 4\cdot2, 50)$ can then be evaluated. In table 2 values of $\phi(0, x, \rho, 4\cdot2, 50)$ thus obtained are given for values of x ranging from 0·5 cm to 19·5 cm and densities ranging from 0 to 2·4. In table 3 values of the ratio of scatter to primary, S_F/P, at the exit surfaces of bodies of various thicknesses and densities are given, together with the rates of change $d/dx(S_F/P)$, of this ratio with body thickness.

A similar analysis was applied to back scattered radiation. This analysis was carried out in the same way as was described above for the forward scatter except that the dependence of the back scatter contribution on the thickness of the body was found by experiment, this information not being obtainable from the standard depth dose tables. Values of the relative scatter absorption factor for back scattered radiation, $\phi_B(0, x, \rho, 4\cdot2, 50)$, are given in table 4 and values of the ratio of back scattered to forward moving radiation and the rate of change of this quantity with body thickness are given in table 5.

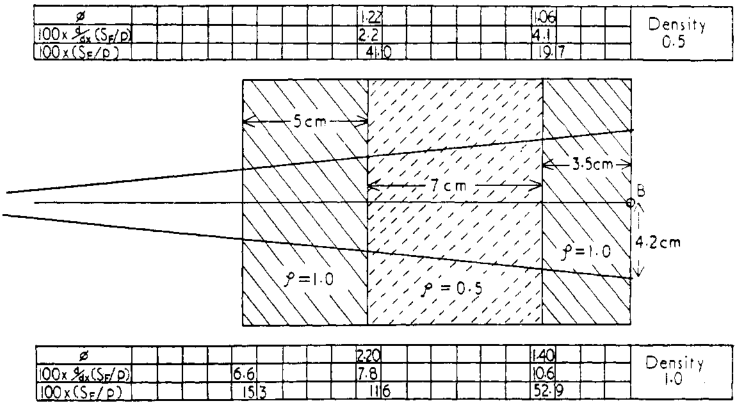


Fig. 6

§ 5. METHOD OF CALCULATION

5.1. *Standard Geometrical Conditions*

Suppose it is required to calculate the dose at the point B in fig. 6 which lies in a body whose construction is shown in the figure. It is convenient to construct Perspex rulers of the type shown in fig. 6 for each value of the density given in tables 2 and 3. These rulers are divided into centimetre intervals and in each interval the corresponding values of ϕ, of $d/dx(S_F/P)$ and of S_F/P are inscribed. For the sake of clarity only the values relevant to the present example are shown on the rulers in fig. 6. The scatter contribution is calculated as follows. The ruler

Table 2. Relative Scatter Absorption Factors for Forward Scatter. 4·2 cm radius reference field : Half-value Layer 1·5 mm Cu

x \ ρ	0·0 to 0·3	0·4	0·5	0·6	0·7	0·8	1·0	1·2	1·4	1·7	2·0	2·4
0·5 (cm)	1·00	0·94	0·94	1·00	1·08	1·15	1·25	1·30	1·34	1·36	1·37	1·39
1·5	1·00	0·96	0·97	1·03	1·12	1·18	1·29	1·36	1·42	1·51	1·60	1·68
2·5	1·00	1·00	1·01	1·08	1·15	1·22	1·33	1·42	1·52	1·70	1·89	2·10
3·5	1·00	1·02	1·06	1·13	1·22	1·29	1·40	1·52	1·66	1·96	2·30	2·95
4·5	1·00	1·04	1·11	1·20	1·29	1·36	1·52	1·70	1·93	2·30	2·70	3·65
5·5	1·00	1·06	1·14	1·24	1·35	1·43	1·64	1·88	2·14	2·59	3·11	4·15
6·5	1·00	1·09	1·17	1·30	1·42	1·53	1·78	2·07	2·40	2·92	3·51	4·60
7·5	1·00	1·10	1·22	1·35	1·47	1·61	1·91	2·25	2·62	3·19	3·88	5·00
8·5	1·00	1·10	1·22	1·38	1·53	1·67	1·99	2·40	2·84	4·48	4·23	5·60
9·5	1·00	1·10	1·22	1·40	1·56	1·72	2.10	2·54	3·03	3·75	4·53	
10·5	1·00	1·10	1·22	1·40	1·61	1·78	2·20	2·67	3·25	4·10	4·80	
11·5	1·00	1·10	1·22	1·40	1·65	1·84	2·28	2·78	3·43	4·30		
12·5	1·00	1·10	1·22	1·40	1·65	1·89	2·36	2·92	3·66	4·60		
13·5	1·00	1·10	1·22	1·40	1·65	1·89	2·42	3·04	3·86			
14·5	1·00	1·10	1·22	1·40	1·65	1·89	2·46	3·13	4·07			
15·5	1·00	1·10	1·22	1·40	1·65	1·89	2·51	3·20				
16·5	1·00	1·10	1·22	1·40	1·65	1·89	2·57	3·30				
17·5	1·00	1·10	1·22	1·40	1·65	1·89	2·58					
18·5	1·00	1·10	1·22	1·40	1·65	1·89	2·59					
19·5	1·00	1·10	1·22	1·40	1·65	1·89	2·60					

Table 3. Ratios of Forward Scatter to Primary. 4·2 cm radius reference field : Half-value Layer 1·5 mm Cu

ρ / x	0·0 to 0·3		0·4		0·5		0·6		0·7		0·8		1·0		1·2		1·4		1·7		2·0		2·4	
(cm)	A′	B′	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1	17·5	17·5	6·6	6·6	8·0	8·0	10·5	10·5	13·2	13·2	16·1	16·1	21·9	21·9	27·3	27·3	32·8	32·8	40·5	40·5	48·0	48·0	58·2	58·2
2	13·5	31·0	5·2	11·8	6·6	14·6	8·3	18·8	10·6	23·8	12·7	28·8	17·4	39·3	22·0	49·3	26·8	59·6	34·5	75·0	43·2	91·2	55·0	113
3	10·2	41·2	4·1	15·9	5·1	19·7	6·9	25·7	8·2	32·0	9·9	38·7	13·6	52·9	17·4	66·7	21·7	81·3	29·5	105	38·6	130	55·0	168
4	7·6	48·8	3·1	19·0	4·1	23·8	5·2	30·9	6·5	38·5	7·9	46·6	10·6	63·5	13·9	80·6	17·7	99·0	25·4	130	35·0	165	55·0	223
5	6·3	55·1	2·6	21·6	3·5	27·3	4·6	35·5	5·7	44·2	6·9	53·5	9·6	73·1	12·9	93·5	17·2	116	24·9	155	35·0	200	55·0	278
6	5·6	60·7	2·4	24·0	3·2	30·5	4·1	39·6	5·3	49·5	6·1	59·9	9·2	82·3	12·6	106	16·8	133	24·6	179	35·0	235	55·0	333
7	5·0	65·7	2·2	26·2	2·9	33·4	3·9	43·5	5·0	54·5	6·1	66·0	8·9	91·2	12·4	118	16·8	150	24·6	204	35·0	270	55·0	388
8	4·6	70·3	1·8	28·0	2·8	36·2	3·7	47·2	4·7	59·2	5·8	71·8	8·7	99·9	12·3	131	16·7	167	24·6	229	35·0	305	55·0	443
9	4·2	74·5	1·8	29·8	2·5	38·7	3·5	50·7	4·4	63·6	5·6	77·4	8·3	108	12·0	143	16·6	183	24·6	254	35·0	340		
10	3·8	78·3	1·7	31·5	2·3	41·0	3·2	53·9	4·2	67·8	5·2	82·6	8·0	116	11·6	155	16·2	199	24·2	278	34·5	375		
11	3·5	81·5	1·5	33·0	2·2	43·2	2·9	56·8	4·0	71·8	5·1	87·7	7·8	124	11·3	166	16·1	215	24·2	302				
12	3·3	85·1	1·5	34·5	2·0	45·2	2·8	59·6	3·8	75·6	4·9	92·6	7·5	132	11·0	177	15·9	231	24·2	326				
13	3·1	88·2	1·4	35·9	1·9	47·1	2·6	62·2	3·6	79·2	4·7	97·3	7·3	139	10·8	188	15·9	247	24·2	350				
14	2·9	90·1	1·3	37·2	1·8	48·9	2·4	64·6	3·3	82·5	4·4	102	7·0	146	10·6	198	15·7	263						
15	2·7	92·8	1·2	38·4	1·6	50·5	2·3	66·9	3·1	85·6	4·1	106	6·7	153	10·2	208	15·7	279						
16	2·6	95·4	1·1	39·5	1·6	52·1	2·2	69·1	3·1	88·7	4·0	110	6·6	159	10·2	218								
17	2·6	98·0	1·1	40·6	1·6	53·7	2·1	71·2	2·9	91·6	3·8	114	6·5	166	10·2	228								
18	2·4	100	1·1	41·7	1·5	55·2	2·1	73·3	2·8	94·4	3·7	117	6·3	172										
19	2·4	103	1·0	42·7	1·5	56·7	2·0	75·3	2·7	97·1	3·6	121	6·1	178										
20	2·3	105	1·0	43·7	1·5	58·2	2·0	77·3	2·7	99·8	3·5	124	6·0	184										

Columns A $=100\times d/dx\,(S_F/P)$ Columns B $=100\times S_F/P$ Column A′ $=1/\rho\times 100\,d/dx\,(S_F/P)$ Column B′ $=1/\rho\times 100\,(S_F/P)$

Table 4. ‘ Relative Scatter Absorption Factors ’ for Back Scatter. 4·2 cm radius reference field : Half-value Layer 1·5 mm Cu

ρ x	0·0 to 0·3	0·4	0·5	0·6	0·7	0·8	1·0	1·2	1·4	1·7	2·0	2·4
0·5 (cm)	1·02	1·03	1·04	1·04	1·04	1·04	1·02	1·00	0·96	0·90	0·82	0·70
1·5	1·00	1·00	0·99	0·97	0·96	0·94	0·90	0·85	0·81	0·74	0·68	0·59
2·5	0·91	0·84	0·81	0·78	0·76	0·73	0·68	0·65	0·61	0·56	0·50	0·44
3·5	0·80	0·70	0·66	0·62	0·59	0·57	0·52	0·48	0·45	0·40	0·36	0·32
4·5	0·72	0·60	0·56	0·53	0·48	0·46	0·41	0·37	0·34	0·30	0·27	0·24
5·5	0·63	0·51	0·46	0·42	0·39	0·36	0·32	0·28	0·26	0·23	0·21	0·19
6·5	0·55	0·43	0·38	0·34	0·31	0·28	0·24	0·21	0·19	0·17	0·15	0·13
7·5	0·45	0·34	0·30	0·27	0·24	0·22	0·18	0·16	0·14	0·12	0·11	0·09

Table 5. Ratios of Back Scatter to Forward Moving Radiation. 4·2 cm radius reference field : Half-value Layer 1·5 mm Cu

x \ ρ	0·0 to 0·3		0·4		0·5		0·6		0·7		0·8		1·0		1·2		1·4		1·7		2·0		2·4	
(cm)	A′	B′	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1	11·0	11·0	4·5	4·5	5·7	5·7	6·9	6·9	8·0	8·0	9·1	9·1	11·2	11·2	13·2	13·2	14·8	14·8	16·8	16·8	18·1	18·1	18·5	18·5
2	7·0	18·0	2·9	7·4	3·6	9·3	4·2	11·1	4·8	12·8	5·5	14·6	6·5	17·7	7·3	20·5	8·1	22·9	9·1	25·8	9·8	27·9	10·2	28.7
3	5·0	23·0	1·8	9·2	2·2	11·5	2·5	13·6	2·9	15·7	3·1	17·7	3·7	21·4	4·2	24·7	4·7	27·6	5·1	31·0	5·4	33·3	5·7	34·4
4	3·0	26·0	1·2	10·4	1·3	12·8	1·5	15·1	1·7	17·4	1·9	19·6	2·1	23·5	2·4	27·1	2·6	30·2	2·8	33·8	3·0	36·3	3·2	37·6
5	2·0	28·0	0·7	11·1	0·8	13·6	1·0	16·1	1·0	18·4	1·1	20·7	1·3	24·8	1·3	28·4	1·4	31·6	1·5	35·3	1·6	37·9	1·7	39·3
6	2·0	30·0	0·4	11·5	0·5	14·1	0·5	16·6	0·6	19·0	0·6	21·3	0·7	25·5	0·8	29·2	0·8	32·4	0·9	36·2	0·9	38·8	1·0	40·3
7	1·0	31·0	0·3	11·8	0·4	14·5	0·4	17·0	0·4	19·4	0·4	21·7	0·4	25·9	0·4	29·6	0·4	32·8	0·5	36·7	0·5	39·3	0·5	40·8
8	0·5	32·0	0·2	12·0	0·2	14·7	0·2	17·2	0·3	19·7	0·3	22·0	0·2	26·1	0·2	29·8	0·2	33·0	0·3	37·0	0·3	39·6	0·3	41·2

Columns $A = 100 \times d/dx\ (S_B/F)$ Columns $B = 100 \times (S_B/F)$ Column $A' = 1/\rho \times 100\ d/dx\ (S_B/F)$ Column $B' = 1/\rho \times 100\ (S_B/F)$

corresponding to density 1·0 is placed over a drawing of the body, with its zero point level with the point under consideration. The scatter contribution from the first 3·5 cm of material is read off from this ruler as

$$(0{\cdot}529+0{\cdot}5\times 0{\cdot}106)P_B=0{\cdot}582P_B.$$

From the ruler corresponding to a density of 0·5, the contribution from the next 7 cm is read off as

$$[0{\cdot}410+0{\cdot}5\times 0{\cdot}022-(0{\cdot}197+0{\cdot}5\times 0{\cdot}041)]P_B=0{\cdot}204P_B.$$

This is the contribution which the block of material would make if it formed part of a homogeneous body of density 0·5. If the block were suspended in air 3·5 cm above the point B its contribution would be reduced by a factor $1/\phi(0,\ 3{\cdot}5,\ 0{\cdot}5,\ 4{\cdot}2,\ 50)$. In fact the space between the block of material of density 0·5 and the point B is occupied by unit density material and hence the contribution is given by

$$0{\cdot}204P_B\times\frac{\phi(0,\ 3{\cdot}5,\ 1{\cdot}0,\ 4{\cdot}2,\ 50)}{\phi(0,\ 3{\cdot}5,\ 0{\cdot}5,\ 4{\cdot}2,\ 50)}.$$

$\phi(0,\ 3{\cdot}5,\ 1{\cdot}0,\ 4{\cdot}2,\ 50)$ is read off as 1·4 and $\phi(0,\ 3{\cdot}5,\ 0{\cdot}5,\ 4{\cdot}2,\ 50)$ as 1·06. Hence the contribution from the material of density 0·5 is given by

$$0{\cdot}204P_B\times\frac{1{\cdot}4}{1{\cdot}06}=0{\cdot}27P_B.$$

In a similar way the contribution from the next 5 cm is given by

$$(1{\cdot}53+0{\cdot}066\times 0{\cdot}5-(1{\cdot}16+0{\cdot}5\times 0{\cdot}078))\times\frac{\phi(0,\ 10{\cdot}5,\ 0{\cdot}5,\ 4{\cdot}2,\ 50)}{\phi(0,\ 10{\cdot}5,\ 1{\cdot}0,\ 4{\cdot}2,\ 50)}$$
$$\times\frac{\phi(0,\ 3{\cdot}5,\ 1{\cdot}0,\ 4{\cdot}2,\ 50)}{\phi(0,\ 3{\cdot}5,\ 0{\cdot}5,\ 4{\cdot}2,\ 50)}P_B=0{\cdot}36\times\frac{1{\cdot}22}{2{\cdot}20}\times\frac{1{\cdot}4}{1{\cdot}06}P_B=0{\cdot}264P_B.$$

The total scatter contribution is given by the sum of the contributions from the three blocks, i.e. by $(0{\cdot}58+0{\cdot}27+0{\cdot}264)P_B=1{\cdot}11P_B$. P_B, the primary at the point B, may be determined from the known linear absorption coefficients of the primary beam in the materials and hence the total dose from forward moving radiation at B can be found. It has so far been assumed that no back scattering material lies behind the point B. If there is back scattering material the total dose at the point B is given by multiplying the value found above for the forward moving radiation by the appropriate back scatter factor. If the back scattering material is water equivalent the back scatter factor is simply the surface back scatter factor for a field of radius 4·2 cm. If the back scattering material is inhomogeneous the appropriate value of the back scatter factor can be found from tables 4 and 5 in a manner similar to that described above.

5.2. *Extensions to other Geometrical Conditions*

It has been noted that in water equivalent bodies the ratio of scattered to primary radiation at a point is independent of the f.s.d. provided the field area at the level of the point is constant and hence the restriction that the reference point be 50 cm from the anode can be removed. This result will also hold true in the case of a homogeneous body of any given density. It appeared likely that it would also hold true in the case of a inhomogeneous body, although this cannot be formally demonstrated. Experimental measurements of the doses in inhomogeneous bodies support this suggestion.

The method of calculation can be extended to include circular fields of any given radius at the reference point by applying the general relationship between geometry and density. For example if the radius of the field at the reference point were r cm the correct value of the ratio of scatter to primary would be obtained by applying the procedure described in § 5.1 above to a body whose dimensions were $4{\cdot}2/r$ times those of the real body and whose densities were $r/4{\cdot}2$ times the densities of the real body.

The extension to fields of other than circular cross section cannot be so easily accomplished but the following procedure will not lead to great errors. We define an ‘ equivalent ’ body as the water equivalent body constructed about the reference point so that the value of the primary at the reference point is unchanged. Suppose a case in which the dimensions of the field at the reference point are 10×16 cm. We select the circular field which has the same area, i.e. a field of radius 7·1 cm and calculate in the manner described above the dose received by the reference point in both the real and the ‘ equivalent ’ bodies. Let these doses be respectively D_a and D_b. The ratio D_a/D_b will be described as the scatter correction factor. The dose, D_c, delivered to the reference point in the ‘ equivalent ’ body by a field of dimensions 10×16 cm is found by one of the methods of calculation which have been devised for use in rotation therapy, i.e. methods which do not require a fixed value of the f.s.d. The dose received by the reference point in the real body is then given by $D_c\times D_a/D_b$. The method thus consists of determining the dose in the ‘ equivalent ’ body and multiplying this by a correction factor which is determined by applying the data given in the tables. The correction factor varies much more slowly with the shape of the field cross section than does the ratio of scatter to primary and hence errors arising from the non circular shape of the cross section are minimized.

So far only cases in which the density of the body material was constant in any plane perpendicular to the central axis have been considered. The removal of this restriction greatly complicates the problem and no attempt has been made to devize a formal analysis which would be applicable to these conditions. In practice these conditions are approached when for example an air passage lies in the beam but does not extend across the full width of the beam. In these circumstances we have divided the beam into two parts, one of which just covers the non-water equivalent material the other being the remainder of the beam. The methods of calculation described above are then applied separately to each part of the beam, the total scatter contribution being taken as the sum of the contributions from the two parts of the beam. This admittedly approximate method appears to give results which are accurate to within about 5%.

§ 6. Experimental Tests and Practical Results

The accuracy of the above methods of calculation was tested by comparison with the results of measurements made in inhomogeneous bodies. The experiments were carried out using sheets of Mix D wax, plywood sheets with a density of 0·69 and light wallboard sheets with a density of 0·22. The mass absorption coefficients of the three materials were measured at two different radiation qualities and were found to be equal to each other at each quality so that the effective atomic numbers could be taken as being equal. The materials were prepared in sheets of constant mass per unit area so that the absorption of the primary beam was the same in every sheet. In each set of experiments the ionization chamber was placed at a fixed distance, whose value varied from 40 cm to 60 cm, from the anode and a fixed number of sheets was placed between the chamber and the anode. The primary dose at the chamber therefore depended only on the number of sheets and was independent of the density or arrangement of the individual sheets. The dose for a series of different arrangements of the sheets and for a variety of field sizes was calculated in the manner described. In order to avoid introducing additional errors the dose in the 'equivalent' body was measured directly, i.e. by using sheets of Mix D wax only.

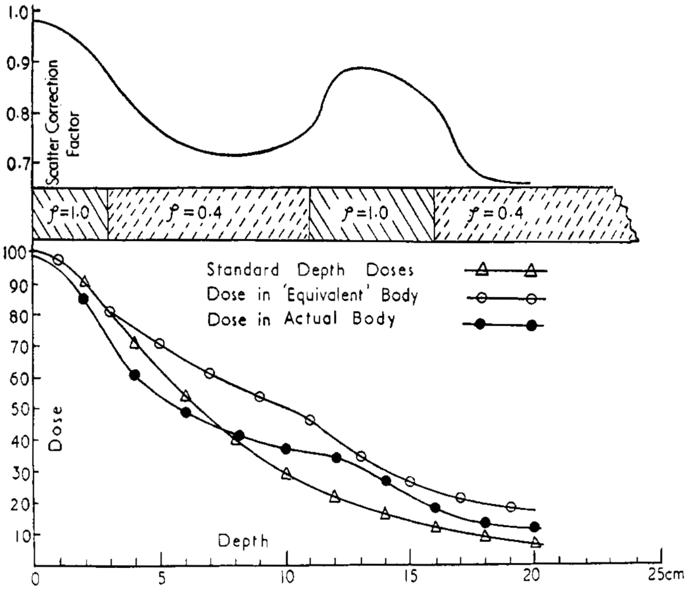


Fig. 7

The calculated dose was compared with the value found experimentally for the corresponding arrangement of the material. The individual measurements were reproducible to within $1\frac{1}{2}\%$ so that the experimental error in the measurement of the ratio of dose in the actual body to dose in the equivalent body was $\pm 3\%$. In the course of a large number of experiments performed in bodies of widely differing construction and using a series of field sizes ranging from 20 cm^2 to 150 cm^2 the maximum discrepancy between the calculated and measured ratios of the dose in the actual body to the dose in the equivalent body was 5%, most of the results agreeing more closely than this. This can be regarded as satisfactory confirmation of the validity of the methods of calculation.

These methods of calculation find their greatest practical application in the estimation of doses delivered to points in the thorax. In the thorax the scatter correction factors may range from 0·7 for points in the centre of the lungs to 0·95 for points in the centre of the body, e.g. at the Oesophagus. As is shown in fig. 7, which illustrates the conditions when a beam is directed laterally through the thorax, these correction factors result in the reduction of the estimate of the dose which is obtained by the use of simple transit dose techniques. The effect is of particular importance in the calculation of doses delivered to the lungs since in this case the reduction of the scatter contribution may more than compensate for the increased transmission of the primary beam.

§ 7. NOTE ON THE EFFECT OF BONE

Bone differs from water in both electron density and effective atomic number. For radiation with a half-value layer of 1·5 mm Cu the principal effect of an increase in the effective atomic number will be to increase the photo electric absorption and hence to reduce the ratio of scattered to primary radiation. The degree of this reduction will depend on the proportion of multiple scatter since a reduction will occur at each stage of scatter production. On the other hand the increased electron density will tend to increase the ratio of scatter to primary. It is of interest to see what the net effect of these opposing tendencies is.

Bony material was simulated by a mixture of equal parts by weight of plaster of Paris and water. The electron density of this material was $3{\cdot}18 \times 10^{23}$ electrons per g and the effective atomic number was 12·4. The material was prepared in a sheet $30 \times 30 \times 5{\cdot}6$ cm and the dose received by a chamber which lay under this sheet was compared with the dose received under the 'equivalent' thickness of Mix D wax, back-scattering material being provided in each case by a thick block of Mix D wax. For a field area of 25 cm^2 the dose under the ' bone ' was 5% greater than the dose under the equivalent wax. For a field area of 56 cm^2 the dose under the ' bone ' was 1% less than under the equivalent wax and for field area of 150 cm^2 it was 12% less. A similar experiment was performed in which the back scatter from the ' bone ' was compared with the back scatter from wax. Under these circumstances the ratios of the total doses were 1·0 for 25 cm^2 field, 1·0 for a 56 cm^2 field and 0·95 for a 150 cm^2 field. These results indicate that the opposing effects of the increased effective atomic number and increased electron density do depend on the field size in the general manner which is to be expected from simple theoretical considerations. They suggest that for bone the two effects cancel out for a field area of about 50 cm^2.

Summary

It is shown that, provided the effective atomic number remains constant, there is a general relationship between the variation of scatter with density and the variation with geometry of the system. A simplified analysis of the scatter contribution in water equivalent bodies is presented. This analysis leads to the definition of relative scatter absorption factors and the values of these factors in water equivalent materials are found by experiment for radiation with a half-value layer of 1·5 mm Cu. By the application of the general relationship between density of the body and geometry of the system values of the relative scatter absorption factors in bodies of other than unit density are deduced. These values may be employed in the calculation of the scatter dose delivered to points which lie in an inhomogeneous body. The practical importance of the results obtained is discussed with particular reference to the use of Transit Dose techniques.

Résumé

On prouve que, pourvu que le nombre atomique effectif demeure constant, il existe une relation générale entre la variation de la diffusion avec la densité et la variation avec la géométrie du système. On présente une analyse simplifiée de la contribution de la diffusion dans les corps équivalents à l'eau. Cette analyse mène à la définition des facteurs relatifs de l'absorption de diffusion, dont les valeurs dans des matériaux équivalents à l'eau sont trouvés par la voie expérimentale pour une radiation possédant une épaisseur-moitié de 1,5 mm Cu. Les valeurs des facteurs relatifs de l'absorption de diffusion dans les corps dont la densité diffère de l'unité sont déduites en employant la relation générale entre la densité du corps et la géométrie du système. Ces valeurs peuvent être employées pour calculer la dose de diffusion, délivrée aux endroits dans un corps non-homogène. On discute l'importance pratique des résultats obtenus, en particulier en ce qui concerne l'emploi des techniques dites de ' transit dose '.

Zusammenfassung

Es wird gezeigt, dass, solange die effektive Atomzahl konstant bleibt, eine allgemeine Beziehung zwischen der Veränderung der Streuung mit der Dichte und der Veränderung der Streuung mit der Geometrie des

Systems besteht. Es wird eine vereinfachte Analyse des Streuungsbeitrages in wassergleichwertigen Körpern dargestellt. Diese Analyse führt zur Definition der relativen Streuungsabsorptionszahlen, deren Werte in wassergleichwertigen Stoffen für die Strahlung mit einer Halbwertsschicht von 1,5 mm Cu experimentell gefunden worden sind. Die Werte der relativen Streuungsabsorptionszahlen in Körpern mit einer von Eins verschiedenen Dichte werden durch Anwendung der allgemeinen Beziehung zwischen der Körperdichte und der Geometrie des Systems abgeleitet. Diese Werte können für die Berechnung der Streuungsdosis, die an die in einem ungleichförmigen Körper liegenden Punkte geliefert wird, verwendet werden. Die praktische Bedeutung der erhaltenen Ergebnisse wird erörtert, insbesondere in Bezug auf die Durchgangsdosis-Verfahren.

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