Acoustic Resonators for Far-Field Control of Sound on a Subwavelength Scale

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(Received 8 April 2011; revised manuscript received 19 May 2011; published 3 August 2011)

We prove experimentally that broadband sounds can be controlled and focused at will on a subwavelength scale by using acoustic resonators. We demonstrate our approach in the audible range with soda cans, that is, Helmholtz resonators, and commercial computer speakers. We show that diffraction-limited sound fields convert efficiently into subdiffraction modes in the collection of cans that can be controlled coherently in order to obtain focal spots as thin as $1/25$ of a wavelength in air. We establish that subwavelength acoustic pressure spots are responsible for a strong enhancement of the acoustic displacement at focus, which permits us to conclude with a visual experiment exemplifying the interest of our concept for subwavelength sensors and actuators.

DOI: 10.1103/PhysRevLett.107.064301 PACS numbers: 43.20.+g, 43.35.+d, 46.40.Ff

Sound, just as light, is subject to diffraction and interference effects [1]. This explains why a marching band can be easily heard from distant streets in a dense urban environment. The latter are also responsible for the puzzle that represents designing a room for optimal acoustic performances [2]. From a more fundamental point of view, those phenomena impose severe restrictions regarding the control and observation of acoustic waves. Namely, sonic and ultrasonic waves are subject to the so-called diffraction limit, which sets a critical bound to any imaging or focusing technique [3]. For example, sound focusing in the audible range is limited by diffraction effects, and the typical focal spot size in room acoustics is metric [4].

This limit arises from the fact that spatial details thinner than half a wavelength are carried by evanescent waves. Those waves stick to their source, their amplitude decreasing exponentially with distance, and are negligible about a wavelength away from it [5]. There have been a few proposals for subwavelength imaging in acoustics, based on superlens or hyperlens designs [6,7] or on phononic crystals [8]. Regarding focusing, there was one proposal based on an acoustic analog of the ''bull's eye'' structure in optics [9,10] as well as demonstrations of ultrasound focusing using a negative index material [11], but none of those methods demonstrated subdiffraction focal spots. The only experimental evidence of subwavelength focusing of acoustic waves either required the use of an acoustic sink [12] or were obtained from the near field [8].

Here we show that broadband audible range sound can be manipulated and focused on a subwavelength scale, that is, on a scale much smaller than the wavelength in air, and from the far field by using subwavelength acoustic resonators. We use for this purpose a collection of simple everyday life objects: an array of soda cans. The concept is based on our theoretical proposal of the resonant metalens, which was demonstrated with electromagnetic waves at radio frequencies by using metallic wires [13,14]. We show that the strong coupling between neighboring soda cans induces a splitting of the resonant frequency of a single Helmholtz resonator. We demonstrate that monochromatic diffraction-limited waves emitted by commercial computer loudspeakers excite efficiently in the array of cans resonant periodic modes of spatial periods much smaller than the wavelength in air. Those subwavelength Bloch modes have different resonant frequencies and radiation patterns depending on their wave vectors. Harnessing those subdiffraction wave fields by using broadband sounds, we are able to experimentally obtain from the far field subwavelength focusing of sound onto spots as small as $1/25$ of a wavelength in air, with a position resolution of $1/15$ of the wavelength, i.e., the center to center distance between soda cans. Finally, we establish that subwavelength focusing of sound results in strong exaltations of the acoustic displacements, and we prove it experimentally through a visual experiment.

Because evanescent waves are bound to an object or a source, measuring them in the far field requires one to convert them into propagating ones by lessening their momentum. Such a conversion can be obtained by using tailored media that offer hyperbolic dispersion relations [15,16] and more generally anisotropic media [17,18]. However, the most common conversion from evanescent waves to propagating ones or vice versa is due to scattering off sharp corners and wedges and, more generally, off any finite-size object. The efficiency of these processes remains usually quite poor, yet here we show that it can be exploited and drastically enhanced by using strongly coupled subwavelength acoustic resonators. Our approach is based on a sonic analogue of the resonant metalens we recently introduced with microwaves [13,14]. The resonant metalens, a cluster of coupled subwavelength resonators, constitutes an ideal evanescent to propagating wave converter; its finite size and resonant nature ensure a very efficient conversion of diffraction-limited waves to evanescent waves, while its dispersive behavior literally allows us to code a subwavelength wave field into the complex spectrum of the far field [14].

In electromagnetism, numerous works have been devoted to the design of resonant elements of typical size much smaller than the equivalent wavelength at resonance which serve as building bricks for metamaterials [19,20]. In acoustics, some resonant and subwavelength unit cells have been reported [21,22]. Yet a simple subwavelength resonator was introduced more than a century ago and reported recently to be a very good candidate for negative modulus and negative index ultrasonic materials [11,23]: the Helmholtz resonator [24].

A Helmholtz resonator, the acoustic equivalent of an electric LC resonator, is a subwavelength open cavity. Everyday life bottles and cans exhibit a Helmholtz resonance in the audible range of the spectrum. We work with soda cans for two reasons; first, they possess a truly subwavelength diameter of 6.6 cm at the resonance frequency of 420 Hz, corresponding to $\lambda_R = 0.8$ m [25]. But, moreover, its dimensions make it relatively low loss [25].

First, we demonstrate that an array of cans constitutes a good evanescent to propagating wave converter. In other words, we show that harmonic sounds emitted from the far field excite efficiently in the array subwavelength resonant modes with spatial variations as small as the diameter of a can. We present the experimental setup in Fig. 1(a). We study a periodic ensemble of 7×7 soda cans (2) arranged on a square matrix of period 6.6 cm, that is, the width of a single can, and placed in the xy plane. Around it we disposed 8 computer speakers (1), 1 m (more than one wavelength) away so that only propagating waves reach the cans [26]. The speakers are connected to a multichannel sound card (4) controllable via MATLAB. We measure the pressure at any location above the soda cans with a 1 cm wide microphone (3) mounted on a 3D moving stage (5).

As an initial step, we acquire the time domain Green's functions between the 8 speakers and the top of the 49 cans, by using a 10 ms pulse centered around 400 Hz. A typical recorded sound is plotted in Figs. 1(b) and 1(c). It presents

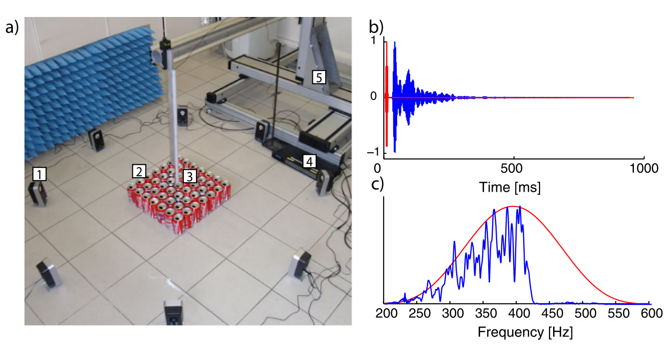

FIG. 1 (color online). (a) Experimental setup detailed in the text. (b) Typical emitted pulse (red) and measured pressure (blue) on top of one can. (c) Spectra of (b).

resonance peaks spanning roughly from 340 Hz up to the resonance frequency of a single soda can, 420 Hz. Those peaks, which are much thinner than the resonance of a single can [25], are associated with periodic modes which possess spatial variations ranging from the dimension of the can array down to the period of the structure. Because of the finite size of the can array, only 49 modes with discrete frequencies and wave vectors exist whose dispersion relation is presented in Ref. [25]. One can understand their origin as follows: Like an ensemble of coupled mass spring oscillators, the system of N coupled resonators supports N modes that present N different resonant frequencies. Here, because the resonators are much smaller than the wavelength, those modes are subwavelength. Stated differently, the resonance of the cans hybridizes with the air continuum, which creates a surface wave polariton [27]. Contrary to intuition, the cans couple to each other via air rather than mechanically albeit being compactly packed [25].

In order to verify the subwavelength nature of the resonant excited modes, we measure the pressure field on top of the collection of Helmholtz resonators at various frequencies. To that end, we evaluate by using our set of temporal Green's functions the resonant frequencies of the modes [25]. Because of its square symmetry and subwavelength period, a mode can be scattered in the far field into four different patterns depending on its number of nodes, that is, its parity. Monopolar radiation patterns are obtained for even-even modes, X- and Y-oriented dipolar fields are generated by even-odd modes, and odd-odd modes scatter into quadrupolar far-field radiations.

With our setup, we generate monochromatic wave fields approximating the radiation patterns of the modes at their resonant frequencies. Then, using the microphone and the moving stage, we measure the pressure for the desired mode. In Fig. 2(a), we first show the array of cans surrounded by the speakers in close view. Then we exemplify the far-field generation of resonant modes in the collection of Helmholtz resonators, sketching the created wave field with schematized speakers and wave fronts. We represent the fields generated in the array of cans at four different frequencies and with wave fronts approximating the four possible radiation patterns of the subwavelength modes. The field of Fig. 2(b), measured at 398 Hz and corresponding to a monopolar radiation pattern, already has a subwavelength spatial period since it presents two nodes within about 0.6 wavelengths at this frequency. Conversely, Figs. 2(c)–2(e) show pressure fields that all present subwavelength spatial periods from 0.5λ down to 0.3λ and whose symmetries are consistent with the generated wave fronts.

We point out here that measuring experimentally the most subwavelength modes, those at the edge of the 1st Brillouin zone, is very challenging because of dissipation. Indeed, since the most subwavelength resonant modes scatter less efficiently into propagating waves, they resonate longer inside the array of cans. Without viscous and thermic losses, all the modes should be resolved because their linewidth decreases as the dispersion relation flattens [14]. Here, however, even though soda cans present acceptable amounts of dissipation, the higher-frequency, or most subwavelength, modes are unresolved and cannot be excited independently. Nevertheless, we will see that this does not hamper our ability to control sound on a scale even smaller than the period of the medium.

We have proved that incident diffraction-limited and monochromatic sound fields generate subdiffraction resonant modes that extend over the entire collection of cans. Now we want to harness those modes in order to create subwavelength focal spots. This becomes possible if, instead of utilizing the usual monochromatic approach of metamaterials, a broadband one is preferred. Indeed, one very simple solution for focusing waves in a medium presenting modes consists in adding them coherently at a specific time and a given location, while they add up incoherently at other positions and other times. This can be realized by using time reversal (TR), which has been widely studied in acoustics and electromagnetism within the past decades [28,29]. The principles of TR are quite straightforward. The temporal responses between an array of sensors and a location in space are measured and stored. Then these sounds are flipped in time and sent back into the medium by using the same sensors. Obviously, reversing a signal in time results in conjugating its phase at each frequency of the spectrum. Therefore, achieving TR from an array of sensors toward one point literally amounts to summing all the modes with zero relative phase at this specific position, hence creating a spatiotemporally focused wave.

We have performed this experiment on several locations on top of the ensemble of Helmholtz resonators using the 8 computer speakers. We present two maps of the acoustic intensities obtained after time reversal on two positions of the array [respectively, (2, 2) and (3, 5), Figs. 3(c) and 3(d)]. Clearly, the focal spots obtained, which display similar widths at half maximum, prove that the diffraction limit was overcome by a factor of 4. Although we show only two maps because of space limitation, we stress that those foci

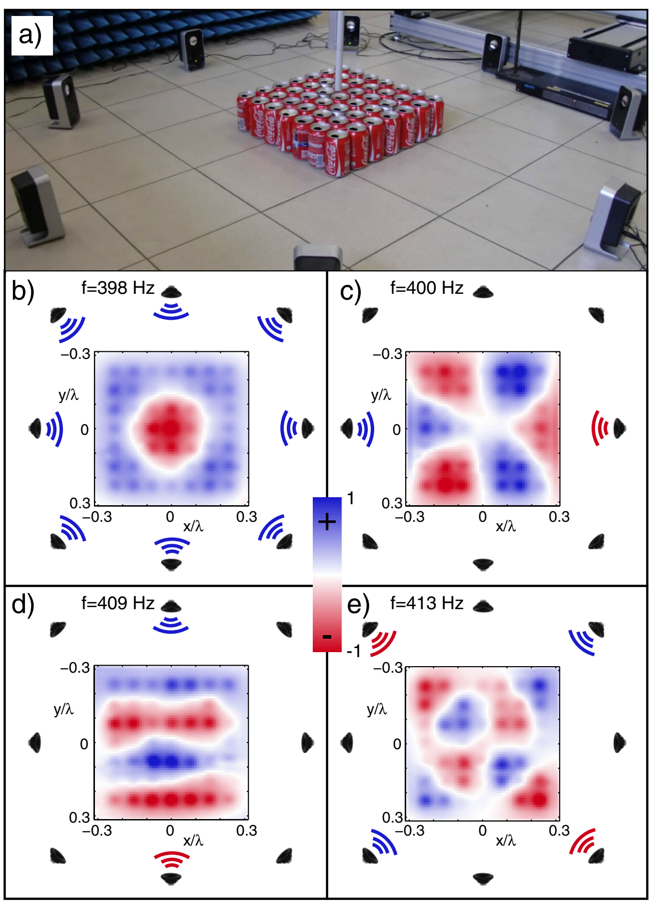

FIG. 2 (color online). (a) A close view of the setup. (b)–(e) Monochromatic subwavelength modes of various radiation patterns and frequencies.

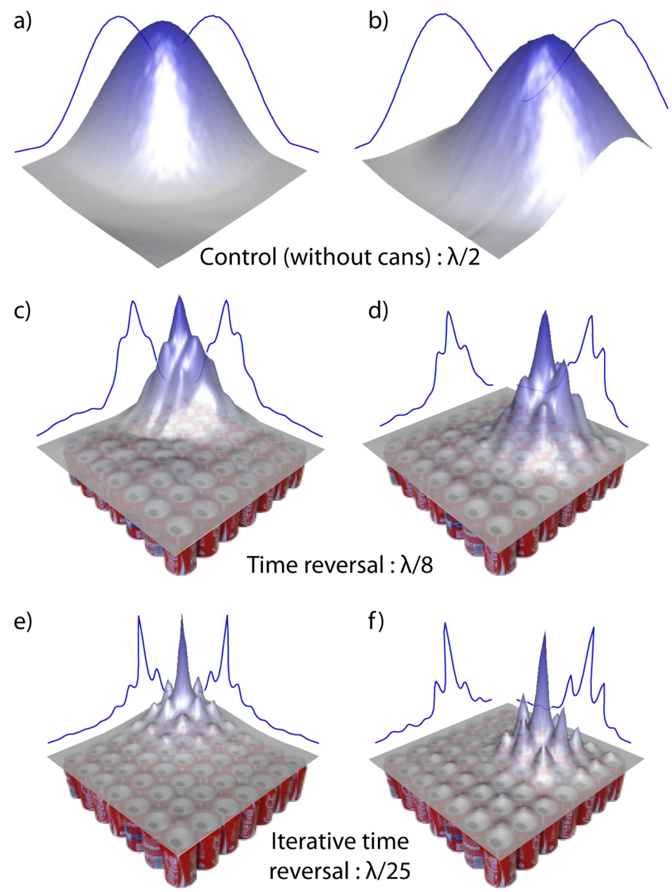

FIG. 3 (color online). Normalized acoustic intensities of (a), (b) diffraction-limited spots obtained without the cans, (c),(d) $\lambda/8$ spots obtained in the array of cans using TR, and (e),(f) $\lambda/25$ foci measured with iterative TR. The peak acoustic intensities for equal emitted energies are around 6.3 AU for (a),(b), 3.5 for (c),(d), and 1.9 for (e),(f). Movies are available [25].

can be placed on any can of the array at will by changing the sounds emitted with the speakers. As a control experiment, we also plot in Figs. 3(a) and 3(b) the maps of the acoustic intensities measured after TR in our laboratory room without the soda cans and on the same locations. This map unambiguously proves that the subwavelength focusing arises from the ensemble of resonators, since here we obtain typical $\lambda/2$ wide diffraction-limited foci. We underline that focusing sound in the array of cans results in much thinner foci than with a single can [25].

Even though TR constitutes a very simple and powerful method for subwavelength control of sound, its performances can be surpassed by using more elaborate methods. Indeed, TR is optimal in nondissipative media, because it does not modify the relative amplitudes of the spectrum of a signal and simply acts on its phases. However, we have shown that the more subwavelength a mode, the higher the impact of dissipation. As a consequence, waves can be controlled onto sharper foci provided that highly attenuated modes are compensated at emission. We use an iterative version of TR [30] to generate a bank of sounds which can focus onto any point of the collection of cans [25]. Again, the 8 speakers emit the calculated sounds, and we measure the intensity on top of the array of resonators. We have repeated the operation for several locations, and we map here the intensities obtained for two chosen ones, respectively, (2, 2) and (3, 5), in Figs. 3(e) and 3(f). Now, measuring the width at half maximum of the acoustic intensity, we obtain focal spots as thin as $1/25$ of the wavelength, corresponding roughly to the size of the opening of a can. Of course, because we cannot focus waves in between cans, the focusing resolution is limited by the period of the medium. Overall, we prove that we can beat the diffraction limit by a factor of 12 with a positioning accuracy of $\lambda/15$. Moreover, the acoustic intensities obtained with and without cans are of the same order of magnitude, hence demonstrating the efficiency of the approach (see the caption of Fig. 3).

Apart from its obvious interest in terms of energy deposition, the obtained subwavelength focusing of pressure fields has a direct consequence. Analogous to the exaltation of the electric field in subwavelength-varying potentials, it leads to a strong enhancement of the acoustic displacement, a key property for efficient actuators and sensors. We prove it in the last part of the Letter.

To do so, we perform a visual experiment that recalls the works of Chladni, by using a high-speed camera (see details in Ref. [25]). We suspend a 20 μm thin sheet of metallized Mylar on top of the array of cans. A white light projector illuminates the sheet of Mylar, oriented a few degrees off its normal, while the camera is placed exactly at normal incidence. Because of the small angle between the projector and the camera, the Mylar sheet appears dark except for the direct image of the projector's bulb. Now, we deposit one layer of glass beads (diameter around 120 μm) on top of the Mylar sheet and take another picture with the camera: The Mylar sheet appears very shiny due to the glass beads [25]. This effect occurs thanks to the retroreflection of the light on the glass beads placed on top of the metallized surface.

It is widely known that Mylar is transparent to sound especially at low frequencies: A sheet of Mylar follows the displacement of a sound field. We use a trick to film this displacement by using the high-speed camera and the retroreflective effect of the glass beads. When the upward displacement of the sound field is large enough, the glass beads have too much kinetic energy to follow the downward displacement of the Mylar sheet, and they dissociate from it. This in turns cancels the retroreflective effect, which darkens the related area of the Mylar sheet. This way we can image the acoustic displacement generated by the array of Helmholtz resonators with a good optical contrast [25].

We have utilized this experimental procedure to make movies of the field created when we focus onto various locations on top of the collection of cans. Figure 4 shows pictures taken from films obtained by using time reversal (2nd row) and our modified time reversal scheme (3rd row), both films being available [25]. We verify that the beads were lifted from the Mylar sheet onto areas of subwavelength sizes. Utilizing the same experimental procedure without the soda cans, we did not observe any

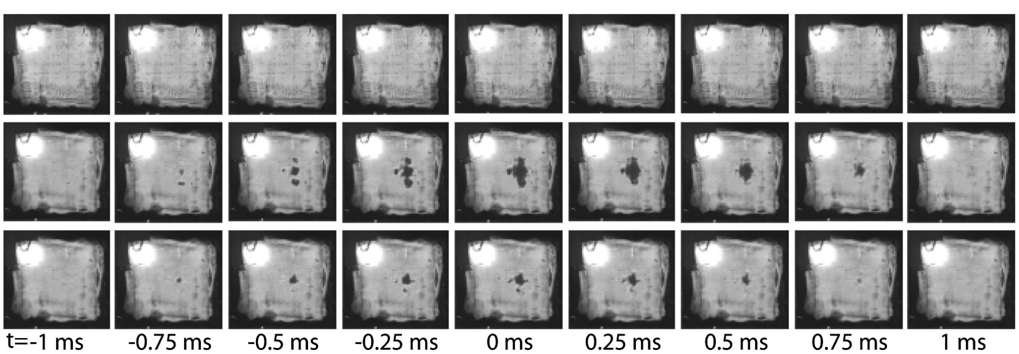

FIG. 4. 2nd and 3rd rows: Subwavelength enhanced acoustic displacements obtained with TR and iterative TR in the array of cans and filmed with a high-speed camera. Without cans, no movement is observed (1st row). Movies are available [25].

movement of the glass beads despite some reverberation of sound in the room (1st row of Fig. 4). This means that, even though the deposited pressures are of the same order of magnitude with and without the cans, our approach results in much higher displacements of the acoustic field. Indeed, since the displacement depends on the gradient of the pressure, subwavelength pressure fields induce strong exaltations of the latter. In our case, focusing on the soda cans, for the same emitted energy and the aforementioned acoustic intensities at foci, we get 7.4 (TR) and 5.4 (iterative TR) times the acoustic displacement obtained without the cans. This suggests that our approach can be utilized for efficient subwavelength size actuators and microelectromechanical systems.

Our experiment is of fundamental interest, and its main results can be reproduced easily. Furthermore, using subwavelength coupled resonators offers three tremendous advantages. First, it introduces the possibility to engineer matrix of actuators or sensors that are arranged on a subwavelength scale. Second, because our approach takes advantage of dispersion, it allows us to address independently many sensors by using their temporal signature and a few sources. Third, subwavelength pressure fields create enhanced acoustic displacements with numerous potential applications. This approach, which is valid for a broad range of resonators throughout the spectrum, can be easily generalized to elastic waves in solids. We believe that it will open up many fascinating avenues in the audible domain for sound control, generation, and engineering but also at any frequency for the design of arrays of actuators, micromechanical actuators in general, and, by reciprocity, sensors.

We thank J. Biscaras for the soda cans, J. de Rosny and F. Ramaz for the sound card and the high-speed camera, and A. C. Boccara for useful discussions. F. L. acknowledges funding from French "Direction Générale de l'Armement." G. L. acknowledges funding from Orange Labs.

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