

3. (a) Sea $A \in P(X)$.

$A^2 = A \cap A = A$. Entonces es anillo de Boole.

(b)

Afirmación: $(-x)^2 = x^2$, $\forall x \in A$.

dem Se tiene $(-1)x = x(-1) = -x$, $\forall x$

$$\text{dem } x + (-1)x = (1 + (-1))x = 0x = 0$$

$$x + x(-1) = x(1 + (-1)) = x0 = 0$$

También sabemos $(-1)^2 = 1$

$$\text{dem } (-1)^2 + (-1) = (-1)(-1 + 1) = (-1) \cdot 0 = 0$$

Entonces

$$\begin{aligned} (-x)^2 &= (-x)(-x) \\ &= x(-1)(-1)x \\ &= x(-1)^2x \\ &= x \cdot 1 \cdot x \\ &= x^2 \end{aligned}$$

De donde: $-x = (-x)^2 = x^2 = x$, por lo tanto

$$x + x = 0, \forall x \in A.$$

Otra forma: $x + x = (x + x)^2$

$$\begin{aligned} &= x + x^2 + x^2 + x \\ &= x + x + x + x \end{aligned}$$

$$\rightarrow 0 = x + x$$

c) $(x + y) \stackrel{\text{Boole}}{=} (x + y)^2 = x^2 + xy + yx + y^2 \stackrel{\text{Boole}}{=} x + xy + yx + y$

$$\Leftrightarrow 0 = xy + yx \Leftrightarrow xy = -yx \stackrel{\text{parte (b)}}{=} yx$$