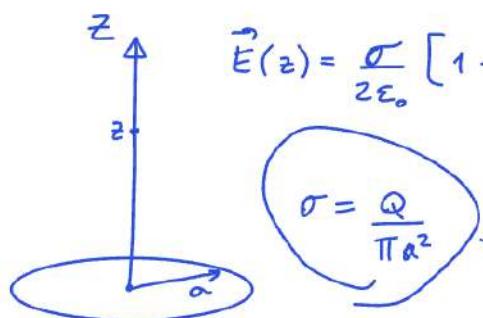


Ej 2.2.1b)

$$\vec{E}(z) = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}} \right] \hat{k} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{1 + \frac{a^2}{z^2}}} \right] \hat{k}$$

$\approx \frac{\sigma}{2\epsilon_0} \left[1 - \left(1 - \frac{1}{2} \frac{1}{2} \frac{a^2}{z^2} \right) \right] = \frac{\left(\frac{\sigma}{2\epsilon_0} \dots \frac{a^2}{z^2} \right)}{4\epsilon_0} = \frac{\frac{\sigma}{2\epsilon_0} \frac{a^2}{z^2}}{4\pi\epsilon_0 a^2}$



$$f(x) \underset{\substack{\uparrow \\ \text{Taylor}}}{\approx} f(a) + f'(a)(x-a) + \dots$$

Paréntesis

$$\frac{1}{\sqrt{1+x^2}} = 1 - \frac{x}{(1+x^2)^{3/2}}$$

$\left. \begin{array}{l} f(0) \\ f'(0) \\ f''(0) \end{array} \right|_{x=0}$

$$= 1 - \frac{x}{(1+x^2)^{3/2}} \Big|_{x=0} + o(x^3)$$

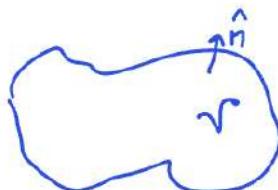
$\left. \begin{array}{l} \frac{1}{(1+x^2)^{3/2}} \\ \frac{3x}{(1+x^2)^{5/2}} \end{array} \right|_{x=0}$

$$g(x^2) = f(x) \quad g(y) \quad y = x^2$$
$$\frac{1}{\sqrt{1+y^2}} = 1 - \frac{y}{2(1+y^2)^{3/2}} \Big|_{y=0} + o(y^2)$$

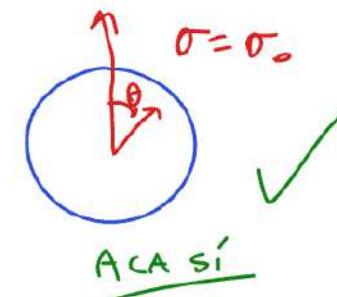
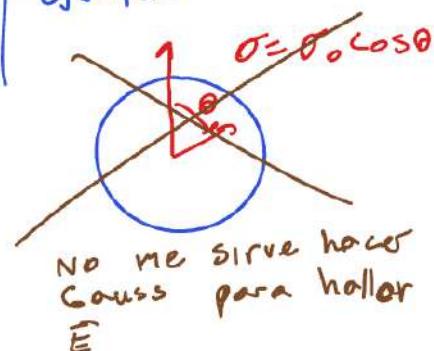
$\left. \begin{array}{l} f(0) \\ f'(0) \end{array} \right|_{y=0}$

¿Cuándo es útil la Ley de Gauss?

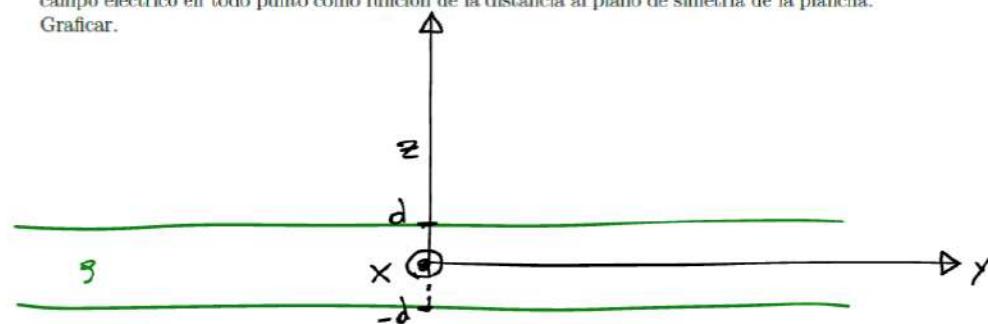
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$



si tengo simetría
EN LAS CARGAS
⇒ puedo buscar la forma
de sacarle provecho a la ley
de Gauss
Ejemplo:



7. Una plancha plana infinita de espesor $2d$ tiene una densidad de carga uniforme ρ . Hallar el campo eléctrico en todo punto como función de la distancia al plano de simetría de la plancha.
Graficar.



Quiero $\vec{E}(x, y, z)$.

es $\vec{E}(x, y, z)$ depende de x ? No
depende de y ? No

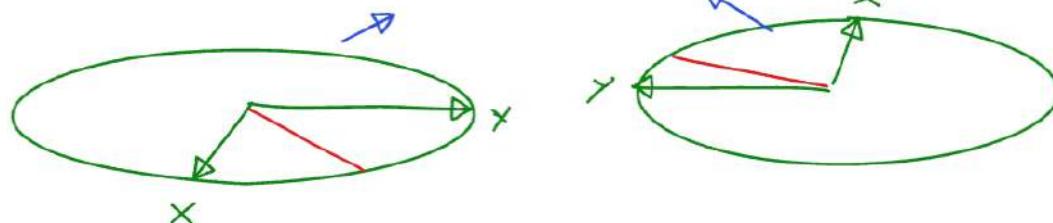
$$\Rightarrow \vec{E}(x, y, z) = \vec{E}(z)$$

¿Puede $\vec{E} \cdot \hat{i} \neq 0$? No

¿Puede $\vec{E} \cdot \hat{j} \neq 0$? No

$$\Rightarrow \boxed{\vec{E}(z) = E(z) \hat{k}}$$

Roto el sistema.



$\oint \vec{E} \cdot \hat{n} da = \sum_{S_1} \vec{E} \cdot \hat{n} da + \sum_{S_2} \vec{E} \cdot \hat{n} da = \int_{S_1} E(z) dx dy - \int_{S_2} E(-z) dx dy$

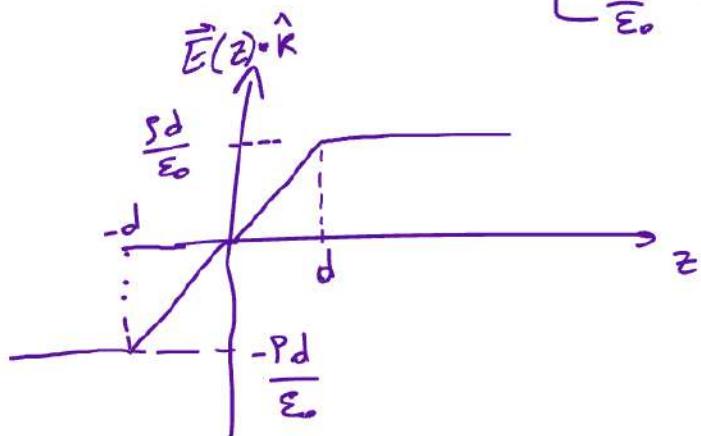
tapas: $\hat{n} = -\hat{k}$
 $\vec{E} = E(-z)\hat{k}$

costados: $\hat{n} \perp \hat{k}$
 $\vec{E} \cdot \hat{n} = 0$

$\Rightarrow \oint \vec{E} \cdot \hat{n} da = E(z) \overbrace{\int_{S_1} dx dy}^S - E(-z) \overbrace{\int_{S_2} dx dy}^S = S(E(z) - E(-z))$

Por simetría del problema es $E(z) = -E(-z)$

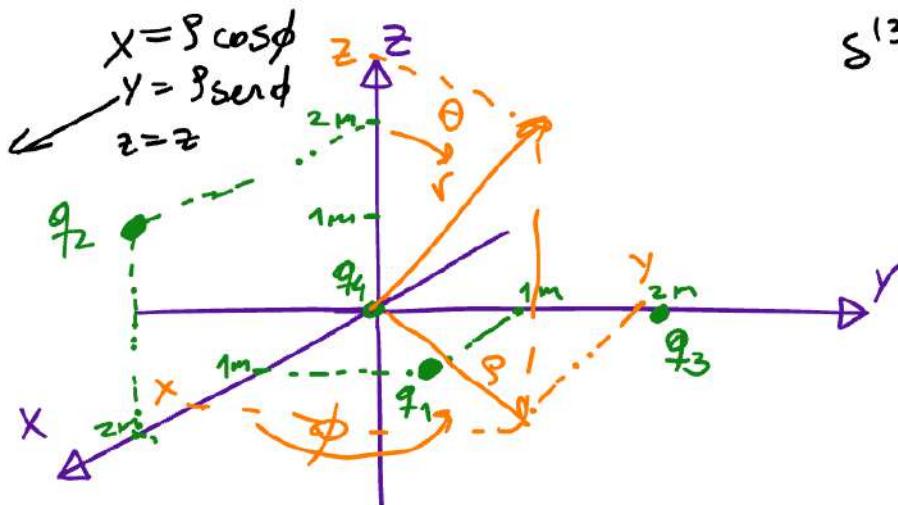
$$\Rightarrow \oint \vec{E} \cdot d\vec{a} = 2S E(z) = \frac{Q_{enc}}{\epsilon_0} \xrightarrow[\text{Gauss finalmente}]{S: S(2d)} \frac{q \cdot S(2d)}{\epsilon_0}$$
$$\Rightarrow |E(z)| = \begin{cases} \frac{pd}{\epsilon_0} & \text{si } |z| > d \\ \frac{qz}{\epsilon_0} & \text{si } |z| \leq d \end{cases} \Rightarrow z \geq 0$$



10. Considere los siguientes datos: $q_1 = 1C$, $q_2 = -2C$, $q_3 = 3C$, $q_4 = 4C$, $\vec{r}_1 = 1\hat{m} + 1\hat{m}$, $\vec{r}_2 = 2\hat{m} + 2\hat{k}$, $\vec{r}_3 = 2\hat{m}$ y $\vec{r}_4 = 0$. ¿Cuál será la densidad volumétrica de carga ρ en coordenadas:

- a) Cartesianas
- b) Cilíndricas
- c) Esféricas

$$\begin{aligned}x &= r \sin \theta \cos \phi \\y &= r \sin \theta \sin \phi \\z &= r \cos \theta\end{aligned}$$



$$\delta^{(3)}(\vec{r} - \vec{r}_i) = \delta(x - x_i) \delta(y - y_i) \delta(z - z_i)$$

$$[\delta] = \frac{1}{m}$$

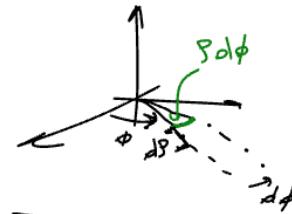
$$\alpha) \quad \delta(\vec{r}) = \sum_{i=1}^4 q_i \delta^{(3)}(\vec{r} - \vec{r}_i) = \left\{ \begin{array}{l} \delta(x-1m) \delta(y-1m) \delta(z) \\ -2 \delta(x-2m) \delta(y) \delta(z-2m) \\ +3 \delta(x) \delta(y-2m) \delta(z) \\ +4 \delta(x) \delta(y) \delta(z) \end{array} \right\} c_{lin1}$$

b) $\delta^{(3)}(\vec{r} - \vec{r}_0) = \underbrace{\delta(r - r_0)}_{\text{cilind.}} \underbrace{\delta(\phi - \phi_0)}_{\text{cylind.}} \underbrace{\delta(z - z_0)}_{\text{cylind.}} + 4 \delta(x) \delta(y) \delta(z) \in C$

$$f(\vec{r}_0) = \int \delta^{(3)}(\vec{r} - \vec{r}_0) f(\vec{r}) d\vec{r}$$

$$= \int \underbrace{\delta(r - r_0)}_{\rho} \underbrace{\delta(\phi - \phi_0)}_{\rho d\phi} \underbrace{\delta(z - z_0)}_{dz} f(\vec{r}_0) d\rho d\phi dz$$

$$\begin{aligned} &= f(\underbrace{r_0 \cos \phi_0 \hat{i} + r_0 \sin \phi_0 \hat{j} + z_0 \hat{k}}_{\vec{r}_0}) \\ &\quad \rho \rightarrow r_0 \\ &\quad \phi \rightarrow \phi_0 \\ &\quad z \rightarrow z_0 \end{aligned}$$

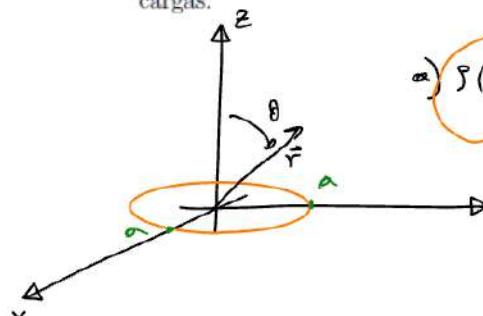


$$\begin{aligned} &\int d\rho \delta(r - r_0) \int \delta(\phi - \phi_0) d\phi \left[\int \delta(z - z_0) f(p \cos \phi \hat{i} + p \sin \phi \hat{j} + z \hat{k}) dz \right] \\ &\quad \downarrow \\ &\int \delta(\phi - \phi_0) \underbrace{f(p \cos \phi \hat{i} + p \sin \phi \hat{j} + z_0 \hat{k})}_{h(\phi)} d\phi \\ &\quad \downarrow \\ &\int d\rho \delta(r - r_0) \underbrace{f(p \cos \phi_0 \hat{i} + p \sin \phi_0 \hat{j} + z_0 \hat{k})}_{s(r)} \\ &\quad \downarrow \\ &f(p_0 \cos \phi_0 \hat{i} + p_0 \sin \phi_0 \hat{j} + z_0 \hat{k}) = f(\vec{r}_0) \end{aligned}$$

11. Considere la distribución de cargas continua y unidimensional dada por un anillo infinitesimalmente delgado, de radio a , centrado en el origen y ubicado en el plano xy .

a) Determine la densidad de carga volumétrica en coordenadas esféricas.

b) Suponga ahora que el anillo se encuentra en el plano yz y determine la distribución de cargas.



$$\text{a) } \rho(\vec{r}) = \lambda \delta(r-a) \frac{\delta(\theta - \frac{\pi}{2})}{r}$$

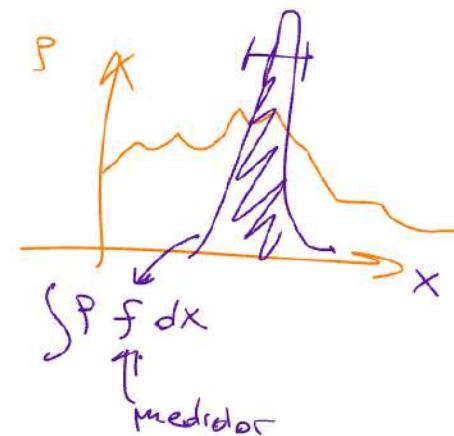
$$\int \rho(\vec{r}) d^3\vec{r} = Q$$

$$\lambda \int_a^\infty r dr \delta(r-a) \int_0^{2\pi} \delta(\theta - \frac{\pi}{2}) \sin \theta d\theta \int_0^{2\pi} d\phi$$

$$\Rightarrow Q = \lambda 2\pi a \Rightarrow \lambda = \frac{Q}{2\pi a}$$

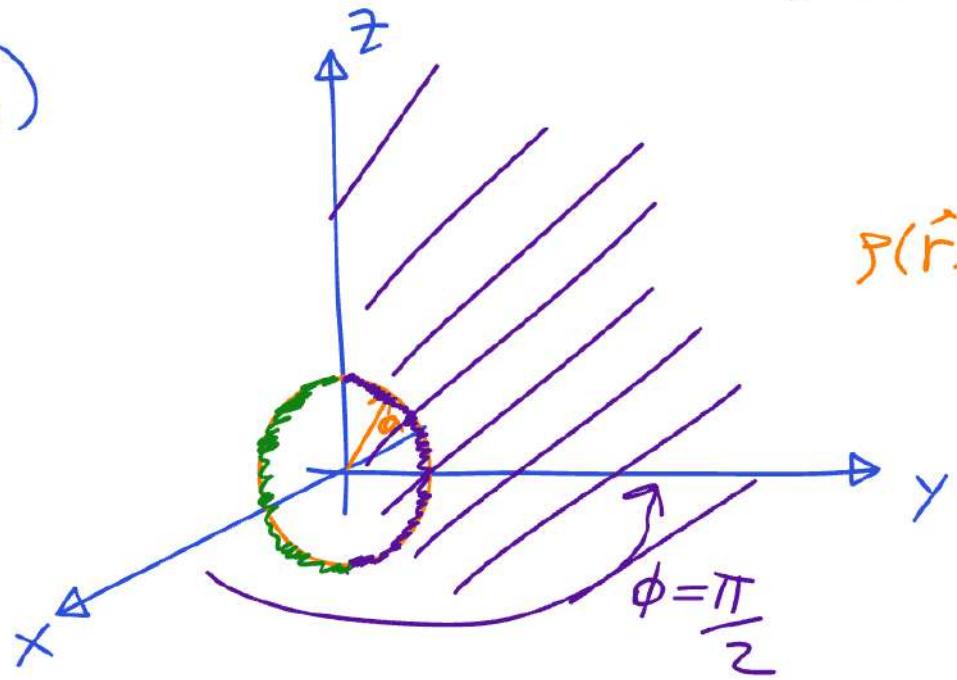
$$\Rightarrow \rho(\vec{r}) = \frac{Q}{2\pi a} \delta(r-a) \frac{\delta(\theta - \frac{\pi}{2})}{r}$$

b)



$$\Rightarrow \rho(\vec{r}) = \frac{\rho_0}{2\pi a} \delta(r-a) \frac{\delta(\theta - \frac{\pi}{2})}{r}$$

b)



$$\rho(\vec{r}) = \lambda \delta(r-a) \left\{ \frac{\delta(\phi - \frac{\pi}{2})}{r \sin \theta} + \frac{\delta(\phi - \frac{3\pi}{2})}{r \sin \theta} \right\}$$