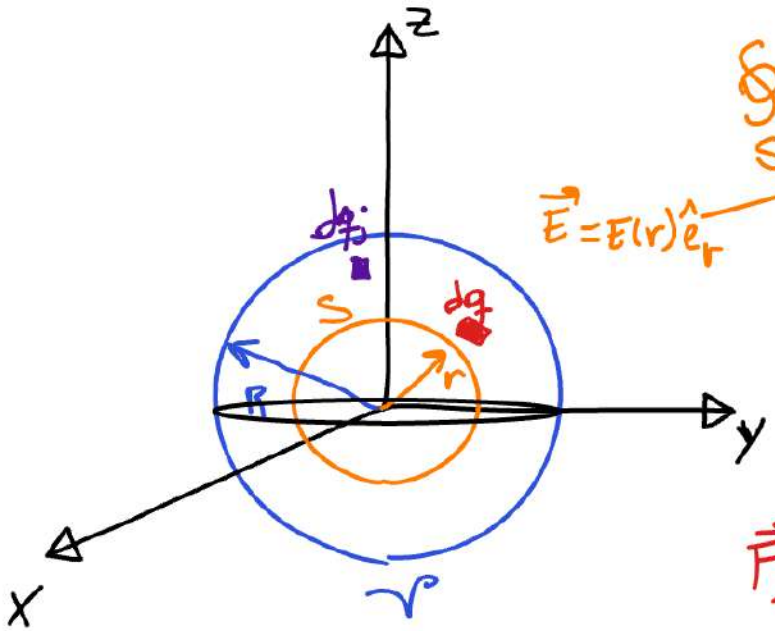
Problem 2.43 Find the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere. Express your answer in terms of the radius R and the total charge Q. [*Answer:* $(1/4\pi\epsilon_0)(3Q^2/16R^2)$]

Griffiths (3ª ed)

Encuentre la fuerza neta que el hemisferio sur de una esfera uniformemente cargada realiza sobre el hemisferio norte.



$$\oint_S \vec{E}\cdot\hat{n}\,da = \frac{Q_{enc}}{\varepsilon_0} = \frac{\int \rho\, d^3\vec{r}}{\varepsilon_0} \overset{\rho \text{ cte}}{=} \frac{\rho\,\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\vec{E} = E(r)\hat{e}_r \quad \Rightarrow \quad E(r)\int_S da = 4\pi r^2 E(r)$$

$$\Rightarrow \vec{E}(r) = \frac{\rho r}{3\varepsilon_0}\hat{e}_r$$

$$\vec{F}_{/dq} = \vec{E}(r)\,dq = \vec{E}(r)\,\rho\, d^3\vec{r} = \frac{\rho^2 r}{3\varepsilon_0}\hat{e}_r$$

$$\hat{k} \cdot \vec{F}_{\substack{Neta \\ Norte}} = \hat{k} \cdot \sum_i \vec{F}_{dq_i} = \int_{V_{Norte}} \frac{\rho^2 r}{3\varepsilon_0} \hat{e}_r \, d^3\vec{r} \cdot \hat{k}$$

$$\hat{e}_r \cdot \hat{k} = \cos\theta$$

$$= \int_0^{\pi/2} d\theta \int_0^{2\pi} d\phi \int_0^{R} dr \; \frac{\rho^2 r}{3\varepsilon_0} \cos\theta \; r^2 \operatorname{sen}\theta$$

$$\Rightarrow \vec{F}_{\substack{Neta \\ Norte}} = \underbrace{2\pi}_{\int_\phi} \; \underbrace{\frac{1}{2}}_{\int_\theta} \; \underbrace{\frac{R^4}{4}}_{\int_r} \; \frac{\rho^2}{3\varepsilon_0} \hat{k} = \frac{\pi R^4 \rho^2}{12\varepsilon_0} \hat{k} = \frac{16\pi^2 R^6 \rho^2}{9} \cdot \frac{1}{\varepsilon_0} \cdot \frac{1}{16} \cdot \frac{3}{4} \cdot \frac{1}{\pi} \cdot \frac{1}{R^2} \hat{k}$$

(Q^2)

$$\Rightarrow \boxed{\vec{F}_{\substack{Neta \\ Norte}} = \frac{Q^2}{4\pi\varepsilon_0 R^2} \cdot \frac{3}{16} \hat{k}}$$