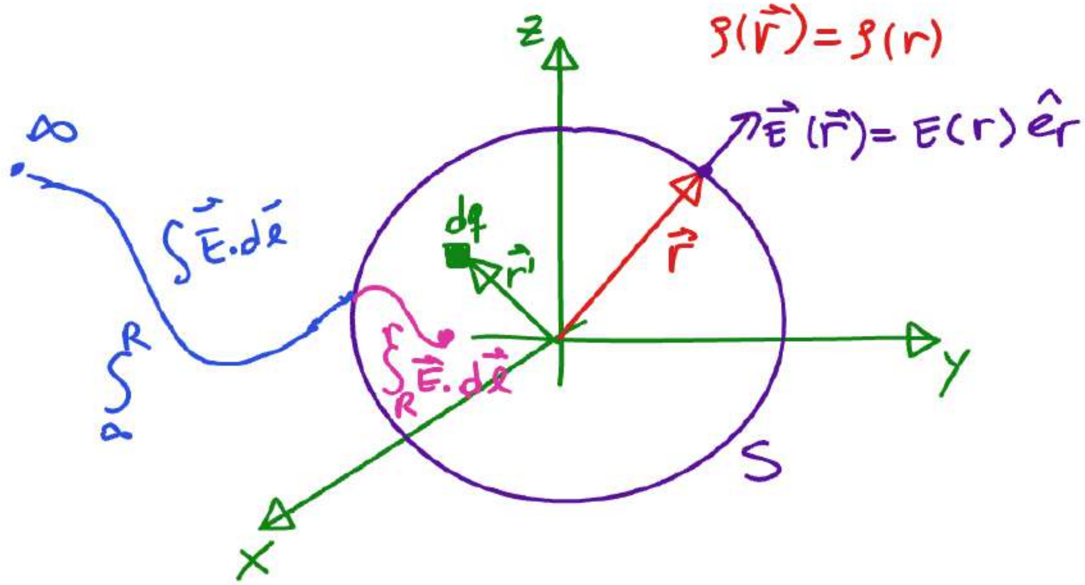
Reitz
(4ta ed)

2.15 Una distribución de carga esférica tiene una densidad de carga volumétrica que es función únicamente de r, la distancia al centro de la distribución. En otras palabras, $\rho = \rho(r)$. Si $\rho(r)$ tiene los valores dados a continuación, determine el campo eléctrico en función de r. Integre el resultado para obtener una expresión para el potencial electrostático $\varphi(r)$, sujeto a la restricción de que $\varphi(\infty) = 0$. (a) $\rho = A/r$, siendo A constante para $0 \le r \le R$; $\rho = 0$ para $r > R$. (b) $\rho = \rho_0$ (es decir, constante) para $0 \le r \le R$; $\rho = 0$ para $r > R$.



$$d\vec E(\vec r) = \frac{\overbrace{dq}^{\rho(r')\,d^3\vec r'}\,(\vec r - \vec r')}{4\pi\varepsilon_0|\vec r - \vec r'|^3} \to \int$$

$$\underbrace{\oint_S \vec E\cdot \overset{\hat e_r}{\hat n}\,da}_{E(r)\,4\pi r^2} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\int_{enc.S}\rho(r')\,d^3\vec r'}{\varepsilon_0}$$

$$\Rightarrow \vec E(\vec r) = \hat e_r \frac{\int_{enc\,S}\rho(r')\,d^3\vec r'}{4\pi\varepsilon_0 r^2}$$

a)
$$\rho(r) = \begin{cases} \frac{A}{r} & \text{si } r \le R \\ 0 & \text{si } r > R \end{cases}$$

b
$$\rho = \begin{cases} \rho_0 & \text{si } r \le R \\ 0 & \text{si } r > R \end{cases}$$

$$\vec{E}(\vec{r}) = \frac{\hat{e}_r}{4\pi\varepsilon_0 r^2} \underbrace{\int \operatorname{sen}\theta \, d\theta \, d\phi}_{4\pi} \int_0^r \rho(r') r'^2 dr'$$

$$\vec{E}(\vec{r}) = \frac{\hat{e}_r}{\varepsilon_0 r^2} \begin{cases} \int_0^R A r' dr & \text{si } r > R \\ \int_0^r A r' dr & \text{si } r \leq R \end{cases}$$

$$\vec{E}(\vec{r}) = \begin{cases} \frac{A R^2}{2\varepsilon_0 r^2} \hat{e}_r & \text{si } r > R \\ \frac{A}{2\varepsilon_0} \hat{e}_r & \text{si } r \leq R \end{cases}$$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{\hat{e}_r}{4\pi\varepsilon_0 r^2} \underbrace{\int \operatorname{sen}\theta \, d\theta \, d\phi}_{4\pi} \int_0^r \rho(r') r'^2 dr'$$

$$\vec{E}(\vec{r}) = \frac{\hat{e}_r}{\varepsilon_0 r^2} \begin{cases} \rho_0 \int_0^R r'^2 dr' & \text{si } r > R \\ \rho_0 \int_0^r r'^2 dr' & \text{si } r \leq R \end{cases}$$

$$\Rightarrow \vec{E}(\vec{r}) = \begin{cases} \frac{\rho_0 R^3}{3\varepsilon_0 r^2} \hat{e}_r & \text{si } r > R \\ \frac{\rho_0 r}{3\varepsilon_0} \hat{e}_r & \text{si } r \leq R \end{cases}$$

$$\vec{E} = -\nabla\varphi$$

$$\underbrace{\varphi(\vec{r})}_{\varphi(r)} - \cancelto{0}{\varphi(\vec{r}_{ref})} = -\int_{\vec{r}_{ref}}^{\vec{r}} \vec{E}\cdot d\vec{l}$$

$$\varphi(r) = \int_{\vec{r}_{ref}}^{\vec{r}} \vec{E}\cdot d\vec{l} = -\int_{r_{ref}}^{r} E(r')\,dr'$$

$$d\vec{l} = dr'\,\hat{e}_{r'} + ds\,\hat{u} \qquad (\hat{u} \perp \hat{e}_r)$$

a) $\varphi(r) = -\int_{\infty}^{r} \frac{AR^2}{2\varepsilon_0 r'^2}dr'$ si $r > R$

$\varphi(r) = -\left[\int_{\infty}^{R} \frac{AR^2}{2\varepsilon_0 r'^2}dr' + \int_{R}^{r} \frac{A}{2\varepsilon_0}dr'\right]$ si $r \leq R$

$$\varphi(r) = \begin{cases} \frac{AR^2}{2\varepsilon_0 r} & \text{si } r > R \\ \frac{AR}{2\varepsilon_0} - \frac{A(r-R)}{2\varepsilon_0} & \text{si } r \leq R \end{cases}$$

b) $\varphi(r) = -\int_{\infty}^{r} \frac{\rho_0 R^2}{3\varepsilon_0 r'^2}dr'$ si $r > R$

$\varphi(r) = -\left[\int_{\infty}^{R} \frac{\rho_0 R^3}{3\varepsilon_0 r'^2}dr' + \int_{R}^{r} \frac{\rho_0 r'}{3\varepsilon_0}dr'\right]$ si $r \leq R$

$$\varphi(r) = \begin{cases} \frac{\rho_0 R^3}{3\varepsilon_0 r} & \text{si } r > R \\ \frac{\rho_0 R^2}{3\varepsilon_0} - \frac{\rho_0 (r^2 - R^2)}{6\varepsilon_0} & \text{si } r \leq R \end{cases}$$