

## Práctico 5

6. Un dipolo puntual se localiza en el centro de un cascarón esférico conductor conectado a tierra. Calcular el potencial en el interior de la esfera usando el método de separación de variables.

$$\nabla^2 \varphi = 0$$

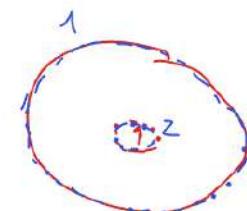
C.B. 1  $\varphi |_{r=R} = 0$

$$\varphi = \varphi(r, \theta, \phi)$$

C.B. 2  $\lim_{r \rightarrow 0} \varphi = \varphi_{\text{dip}} = \frac{q \cos \theta}{4\pi \epsilon_0 r^2} + o\left(\frac{1}{r}\right)$

coord esf.  $(r, \theta, \phi)$

$$\nabla^2 \varphi = 0 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \varphi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \varphi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \varphi}{\partial \phi^2} = 0$$



Supongo  
 $\varphi = R(r) \Theta(\theta)$

Sol más  
 gral a  
 la ec.  $\nabla^2 \varphi = 0$

$$\boxed{\varphi(r, \theta) = \sum_{n=0}^{+\infty} \left( A_n r^n + \frac{B_n}{r^{n+1}} \right) P_n(\cos \theta)}$$

que puedo  
 escribir de esta forma

TODO SE JUSTIFICA POR LA UNICIDAD

$\sim C_1$

$$C.B.1) \quad \varphi(r, \theta) = 0 \rightarrow$$

$$\langle P_m(\cos\theta), P_m(\cos\theta) \rangle$$

$$\boxed{\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta) \sin\theta d\theta = S_{nm}}$$

$$\sum_{n=0}^{\infty} \left( A_n R^n + \frac{B_n}{R^{n+1}} \right) P_n(\cos\theta) = 0$$

Espacio vectorial de funciones en  $\theta$

$$\sum_n c_n \vec{v}_n(\theta) = 0$$

dónde  $\vec{v}_n \cdot \vec{v}_m = S_{nm}$

$$\boxed{\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta) \sin\theta d\theta = S_{nm}}$$

$$\begin{aligned} & \int_0^{\pi} d\theta \sin\theta P_n(\cos\theta) \sum_{n=0}^{\infty} c_n P_n(\cos\theta) = \int_0^{\pi} P_m(\cos\theta) \sin\theta d\theta = 0 \\ & \sum_{n=0}^{\infty} c_n \int_0^{\pi} d\theta \sin\theta P_n(\cos\theta) P_m(\cos\theta) \\ & \quad \stackrel{n=m}{=} 1 \quad \stackrel{n \neq m}{=} 0 \\ & \sum_{n=0}^{\infty} c_n \frac{2}{2n+1} S_{nm} = c_m \frac{2}{2n+1} \end{aligned}$$

$$\Rightarrow c_m = 0 \Rightarrow A_m = -\frac{B_m}{R^{m+1}}$$

$$\varphi(r, \theta) = \sum_{n=0}^{\infty} B_n \left( \frac{r^n}{R^{n+1}} + \frac{1}{R^n} \right) P_n(\cos\theta)$$

$$C.B.2) \quad \lim_{r \rightarrow 0} \varphi = \frac{P \cos\theta}{4\pi\epsilon_0 r^2} = \int_0^{\pi} \frac{P}{4\pi\epsilon_0 r^2} P_m(\cos\theta) \sin\theta d\theta = \frac{P}{4\pi\epsilon_0 r^2} S_{1,m} \frac{2}{2m+1}$$

$$P_1(\cos\theta) = \cos\theta \quad \int_0^{\pi} \cos\theta \sum_{n=0}^{\infty} \frac{B_n}{R^{n+1}} P_n(\cos\theta) \sin\theta d\theta$$

$$\sum_{n=0}^{\infty} \frac{B_n}{R^{n+1}} \int_0^{\pi} P_m P_n \sin\theta d\theta = \frac{B_m}{R^{m+1}} \cancel{\frac{2}{2m+1}} \Rightarrow \text{si } m \neq 1 \Rightarrow B_m = 0$$

$$\Rightarrow B_m = 0 \quad \forall m > 1$$

$$(B_0 = 0)$$

$$\Rightarrow A_m = 0 \quad \text{si } m \neq 1$$

$$\boxed{\varphi(r, \theta) = \left( \frac{P}{4\pi\epsilon_0} \left[ -\frac{r}{R^2} + \frac{1}{R^2} \right] \right) \overset{\cos\theta}{P}_1(\cos\theta) = \frac{P \cos\theta}{4\pi\epsilon_0} \left[ \frac{1}{R^2} - \frac{r}{R^2} \right]}$$