REP. 4 ejercicio 2a

Thursday, September 16, 2021 8:51 AM

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$ ~~es invertible~~

con a_0 invertible.

Vas a querer $b_0 + b_1 x + b_2 x^2 + \dots$ t.q.

$$(a_0 + a_1 x + a_2 x^2 + \dots)(b_0 + b_1 x + b_2 x^2 + \dots) = 1$$

$$\begin{cases} a_0 b_0 = 1 \\ a_0 b_1 + a_1 b_0 = 0 \\ a_0 b_2 + a_1 b_1 + a_2 b_0 = 0 \\ \vdots \end{cases}$$

⇝ entonces considero

$$\begin{cases} b_0 = a_0^{-1} \\ b_1 = (-a_1 b_0) a_0^{-1} \\ \vdots \end{cases}$$

entonces $a_0 + a_1 x + a_2 x^2 + \dots$ invertible.

Ejercicio 4

$$\begin{tikzcd} D_1 \arrow[r, "\varphi"] \arrow[d, "\eta_1"'] \arrow[dr, phantom, "\circlearrowleft"] & D_2 \arrow[d, "\eta_2"] \\ \mathrm{Frac}(D_1) \arrow[r, dashed] & \mathrm{Frac}(D_2) \end{tikzcd}$$

⚠ iny

$$\begin{tikzcd} D_1 \arrow[r, "\eta_2 \circ \varphi"] \arrow[d, "\eta_1"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{Frac}(D_2) \\ \mathrm{Frac}(D_1) \arrow[ur, dashed, "\exists! f"'] & \end{tikzcd}$$

$$\begin{tikzcd} D_2 \arrow[r, "\eta_1 \circ \varphi^{-1}"] \arrow[d, "\eta_2"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{Frac}(D_1) \\ \mathrm{Frac}(D_2) \arrow[ur, dashed, "\exists! g"'] & \end{tikzcd}$$

Ahora queremos $fg = id$ y $g \circ f = id$

$$\begin{tikzcd} D_1 \arrow[r, "\varphi"] \arrow[d, "\eta_1"'] & D_2 \arrow[r, "\varphi^{-1}"] \arrow[d, "\eta_2"] & D_1 \arrow[d, "\eta_1"] \\ \mathrm{Frac}(D_1) \arrow[r, "f"'] & \mathrm{Frac}(D_2) \arrow[r, "g"'] & \mathrm{Frac}(D_1) \end{tikzcd}$$

$$g f \eta_1 = \eta_1 \varphi^{-1} \varphi = \eta_1 \quad (*)$$

$$\begin{tikzcd} D_1 \arrow[r, "g f \eta_1"] \arrow[d, "\eta_1"'] & \mathrm{Frac}(D_1) \\ \mathrm{Frac}(D_1) \arrow[ur, dashed, "\exists!"'] & \end{tikzcd}$$

- obviamente gf hace conmutar ese diagrama.
- La id también.

Por unicidad, terminamos.

⚠ Este es un tipo de argumento útil y "limpio" con el que probablemente se encuentren en otros contextos también.

felicidad.