4. Considere dos cascarones conductores, uno a potencial V_a y otra a potencial V_b, determinar el potencial electroestático en la región comprendida entre los cascarones en las siguientes circunstancias:

   a) Los cascarones son esféricos con radios a y b ($b > a$).

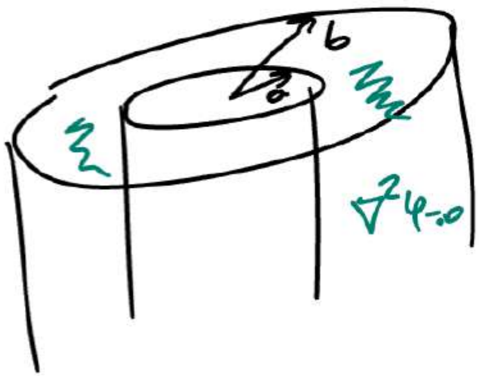
   b) Los cascarones son cilíndricos (infinitos) con radios a y b ($b > a$).

b)

1) Identifico las coord: cilíndricas

2) Planteo la sol. gral:

$$\varphi(\rho, \phi) = \sum_{n=1}^{+\infty} \left(A_n \rho^n + \frac{B_n}{\rho^n}\right)\left(C_n \operatorname{sen}(n\phi) + D_n \cos(n\phi)\right) + C_0 + D_0 \ln(\rho)$$



3) Escribo las cond. de borde

$$\varphi(a, \phi) = V_a$$
$$\varphi(b, \phi) = V_b$$

4) Impongo 3) en 2)

con $A_n' =$
$B_n' =$

$\nabla^2\varphi=0$

$\varphi(a,\phi) = V_a$

$\varphi(b,\phi) = V_b$

con $A_n' = A_n C_n$

$B_n' = B_n C_n$

$\forall\, n \neq 0$

4) Impongo 3) en 2)

$$\varphi(a,\phi) = V_a = \sum_{n=1}^{+\infty}\left(A_n a^n + \frac{B_n}{a^n}\right)\left(C_n \operatorname{sen}(n\phi) + D_n \cos(n\phi)\right) + C_0 + D_0 \operatorname{Ln}(a) \Rightarrow A_n' a^n + \frac{B_n'}{a^n} = 0$$

xq' no hay dep ang. a la izq.

$\int_0^{2\pi} [\quad = \quad] \operatorname{sen}(n_0\phi)\, d\phi$

$\int_0^{2\pi} [\quad = \quad] \cos(n_0\phi)\, d\phi$

Lado Izquierdo

$$\Rightarrow \int_0^{2\pi} V_a \operatorname{sen}(n_0\phi)\, d\phi = 0 \quad \text{con } n_0 \in \mathbb{N}$$

$\mathbb{N}-\{0\}$

$$\int_0^{2\pi} V_a \cos(n_0\phi)\, d\phi = \begin{cases} 0 & \text{con } n_0 \in \mathbb{N}^* \\ V_a 2\pi & \text{si } n_0 = 0 \end{cases}$$

Lado derecho

$$\sum_{n=1}^{+\infty}\left(A_n a^n + \frac{B_n}{a^n}\right)\left\{\underbrace{\int_0^{2\pi} C_n \operatorname{sen}(n\phi)\operatorname{sen}(n_0\phi)\, d\phi}_{C_n \delta_{n n_0} \pi} + \underbrace{\int_0^{2\pi} D_n \cos(n\phi)\operatorname{sen}(n_0\phi)\, d\phi}_{0}\right\} + \{C_0 + D_0 \operatorname{Ln}(a)\}\left\{\cancel{\int_0^{2\pi} \operatorname{sen}(n_0\phi)\, d\phi}^{\,0}\right\} = 0$$

se resuelven si: $n \neq n_0$

usando $\operatorname{sen}(a \pm b) = \operatorname{sen}(a)\cos(b) \pm \cos(a)\operatorname{sen}(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \operatorname{sen}(a)\operatorname{sen}(b)$

lado IZQUIERDO

Lado derecho

$$\sum_{n=1}^{+\infty}\left(A_n a^n + \frac{B_n}{a^n}\right)\left\{\underbrace{\int_0^{2\pi} C_n \operatorname{sen}(n\phi)\operatorname{sen}(n_o\phi)\,d\phi}_{C_n\delta_{nn_o}\pi} + \underbrace{\int_0^{2\pi} D_n \cos(n\phi)\operatorname{sen}(n_o\phi)\,d\phi}_{0}\right\} + \{C_o + D_o \ln(a)\}\left\{\cancel{\int_0^{2\pi}\operatorname{sen}(n_o\phi)\,d\phi}^{\,0}\right\} = 0$$

lado IZQUIERDO

se resuelven si: $n \neq n_o$

usando $\operatorname{sen}(a \pm b) = \operatorname{sen}(a)\cos(b) \pm \cos(a)\operatorname{sen}(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \operatorname{sen}(a)\operatorname{sen}(b)$

$$\Rightarrow \boxed{\pi\left(A_{n_o} a^{n_o} + \frac{B_{n_o}}{a^{n_o}}\right) C_{n_o} = 0 \quad \forall\ n_o \in \mathbb{N}} \quad C1$$

$$\sum_{n=1}^{+\infty}\left(A_n a^n + \frac{B_n}{a^n}\right)\left\{\underbrace{\int_0^{2\pi} C_n \operatorname{sen}(n\phi)\cos(n_o\phi)\,d\phi}_{0} + \underbrace{\int_0^{2\pi} D_n \cos(n\phi)\cos(n_o\phi)\,d\phi}_{\delta_{nn_o}\pi D_n}\right\} + \{C_o + D_o \ln(a)\}\left\{\underbrace{\int_0^{2\pi}\cos(n_o\phi)\,d\phi}_{\delta_{on_o}2\pi}\right\} = \begin{cases} 0 & \text{si } n_o \neq 0 \\ V_a 2\pi & \text{si } n_o = 0 \end{cases}$$

Si $n_o \neq 0 \Rightarrow \boxed{\left(A_{n_o} a^{n_o} + \frac{B_{n_o}}{a^{n_o}}\right)\pi D_{n_o} = 0}$ C2

Si: $n_o = 0$ $\boxed{(C_o + D_o \ln(a))\, 2\pi = 2\pi V_a}$ C3

Impongo la c.b en $\rho = b$

$$(**) \quad V_b = \sum_{n=1}^{+\infty} \left(A_n b^n + \frac{B_n}{b^n}\right)\left(C_n \operatorname{sen}(n\phi) + D_n \cos(n\phi)\right) + C_0 + D_0 \ln(b)$$

$\Rightarrow$ C4 $$\boxed{\pi\left(A_{n_0} b^{n_0} + \frac{B_{n_0}}{b^{n_0}}\right) C_{n_0} = 0 \qquad \forall\, n_0}$$ (Análoga a C1)

C5 $$\boxed{\left(A_{n_0} b^{n_0} + \frac{B_{n_0}}{b^{n_0}}\right) \pi D_{n_0} = 0}$$ (Análoga a C2)

C6 $$\boxed{\left(C_0 + D_0 \ln(b)\right) 2\pi = 2\pi V_b}$$ (Análogo a C3)

C6 $\boxed{(C_0 + D_0 \ln(b))\, 2\pi = 2\pi\, V_b}$ (Análogo a C3)

combinemos C1 y C4

$\cancel{\pi}\left(A_{n_0} a^{n_0} + \frac{B_{n_0}}{a^{n_0}}\right) C_{n_0} = 0$ | $\cancel{\pi}\left(A_{n_0} b^{n_0} + \frac{B_{n_0}}{b^{n_0}}\right) C_{n_0} = 0$

si $C_{n_0} \neq 0$

$$\Rightarrow \left.\begin{aligned} A_{n_0} a^{2n_0} &= -B_{n_0} \\ A_{n_0} b^{2n_0} &= -B_{n_0} \end{aligned}\right\} \Rightarrow A_{n_0} = B_{n_0} = 0 \quad \text{o} \quad a = b \quad \text{absurdo}$$

combinemos C2 y C5 $n_0 \neq 0$

$\pi\left(A_{n_0} a^{n_0} + \frac{B_{n_0}}{a^{n_0}}\right) D_{n_0} = 0$ $\pi\left(A_{n_0} b^{n_0} + \frac{B_{n_0}}{b^{n_0}}\right) D_{n_0} = 0$

$\Rightarrow$ si $D_{n_0} \neq 0$

$\Rightarrow$ $A_{n_0} = B_{n_0} = 0$

o $a = b$ absurdo

VÁLIDO $\forall\, n \neq 0$

$$\Rightarrow \varphi(\rho, \phi) = \cancel{\sum_{n=1}^{\infty} \left(A_n \rho^n + \frac{B_n}{\rho^n}\right)\left(C_n \operatorname{sen}(n\phi) + D_n \cos(n\phi)\right)} + C_0 + D_0 \ln(\rho)$$

0

$$\Downarrow$$

$$\varphi(\rho, \phi) = C_0 + D_0 \ln(\rho)$$

los fijo con C3 y C6

$$C3 \longrightarrow \quad C_0 + D_0 \ln(a) = V_a$$

$$C6 \longrightarrow \quad C_0 + D_0 \ln(b) = V_b$$

$$\Rightarrow \quad V_a - D_0 \ln(a) + D_0 \ln(b) = V_b$$

$$\Rightarrow \quad D_0 \ln\left(\frac{b}{a}\right) = V_b - V_a$$

$$y \quad D_0 = \frac{V_b - V_a}{\ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_0 = V_a - \frac{V_b - V_a}{\ln\left(\frac{b}{a}\right)} \ln(a)$$

$$\Rightarrow \quad \varphi(\rho, \phi) = V_a - \frac{V_b - V_a}{\ln\left(\frac{b}{a}\right)} \ln(a) + \frac{V_b - V_a}{\ln\left(\frac{b}{a}\right)} \ln(\rho)$$

$$\hookrightarrow \boxed{\varphi(\rho, \phi) = V_a + \frac{V_b - V_a}{\ln\left(\frac{b}{a}\right)} \ln\left(\frac{\rho}{a}\right)}$$

a) 1) esf.

2) $\varphi(r,\theta) = \sum_{n=0}^{+\infty} \left(A_n r^n + \frac{B_n}{r^{n+1}} \right) P_n(\cos\theta)$

3) $\varphi(a,\theta) = V_a$ $*1$

$\varphi(b,\theta) = V_b$ $*2$

4) $V_a = \sum_{n=0}^{+\infty} \left(A_n a^n + \frac{B_n}{a^{n+1}} \right) P_n(\cos\theta)$ $\Rightarrow$ Por ortog. de los P_n

$V_a = V_a \underbrace{P_0(\cos\theta)}_{=1}$

c1) $A_n a^n + \frac{B_n}{a^{n+1}} = 0 \quad \forall\, n \neq 0$

c2) $V_a = A_0 + \frac{B_0}{a}$

$V_b P_0(\cos\theta) = V_b = \sum_{n=0}^{+\infty} \left(A_n b^n + \frac{B_n}{b^{n+1}} \right) P_n(\cos\theta) \Rightarrow$ Por ortog de los P_n

c3) $A_n b^n + \frac{B_n}{b^{n+1}} = 0 \quad \forall\, n \neq 0$

c4) $V_b = A_0 + \frac{B_0}{b}$

de C1 y C3 es $A_n a^{2n+1} = -B_n = A_n b^{2n+1} \Rightarrow A_n = B_n = 0 \quad \forall\, n \neq 0$

de C2 y C4 es $V_a = A_0 + \frac{B_0}{a} = \left(V_b - \frac{B_0}{b}\right) + \frac{B_0}{a}$

$$\Downarrow$$

$$B_0 = (V_a - V_b) \cdot \frac{1}{\frac{1}{a} - \frac{1}{b}} = (V_a - V_b)\frac{ab}{b-a}$$

$$\Rightarrow \quad A_0 = V_a - \frac{(V_a - V_b)}{\frac{1}{a} - \frac{1}{b}} \cdot \frac{1}{a} = V_a - \frac{(V_a - V_b)\, b}{b-a}$$

$$\Rightarrow \quad \varphi(r,\theta) = V_a - \frac{(V_a - V_b)\, b}{b-a} + \frac{(V_a - V_b)}{b-a}\frac{ab}{r}$$

$$\varphi(r,\theta) = \frac{V_b b - V_a a + (V_a - V_b)\frac{ab}{r}}{b-a} \begin{cases} \nearrow V_a & \text{si } r = a \\ \searrow V_b & \text{si } r = b \end{cases}$$