Soluciones práctico 2

1.

$f = x + y$
$f_x = 1 = f_y$

$f = 2x - 3y$
$f_x = 2, f_y = -3$

$f = xy$
$f_x = y, f_y = x$

$f = x^2 + y^2$
$f_x = 2x, f_y = 2y$

$f = (x + y)^2$
$f_x = 2(x + y) = f_y$

$f = 1/(x + y)$
$f_x = -1/(x + y)^2 = f_y$

$f = x/y$
$f_x = 1/y, f_y = -x/y^2$

$f = (x + y)/(x - y)$
$f_x = -2y/(x - y)^2$
$f_y = 2x/(x - y)^2$

$f = x/(x^2 + y^2)$
$f_x = (y^2 - x^2)/(x^2 + y^2)^2$
$f_y = -2xy/(x^2 + y^2)^2$

$f = (x - y)/(x^2 - y^2)$
$f_x = f_y = \frac{-(x-y)^2}{(x^2-y^2)^2}$

$f = e^{x+y}$
$f_x = e^{x+y} = f_y$

$f = e^{x^2 y}$
$f_x = 2xy \cdot e^{x^2 y}$
$f_y = x^2 \cdot e^{x^2 y}$

$f = \log(x + y)$
$f_x = 1/(x + y) = f_y$

$f = e^x + \log(y)$
$f_x = e^x$
$f_y = 1/y$

$f = \log(x^2 + 3y)$
$f_x = 2x/(x^2 + 3y)$
$f_y = 3/(x^2 + 3y)$

$f = \operatorname{sen}(x + y)$
$f_x = \cos(x + y) = f_y$

$f = \cos(x - 2y)$
$f_x = -\operatorname{sen}(x - 2y)$
$f_y = 2\operatorname{sen}(x - 2y)$

$f = \operatorname{sen}(x^2 + y)$
$f_x = 2x \cdot \cos(x^2 + y)$
$f_y = \cos(x^2 + y)$

$f = \operatorname{sen}(x) + \cos(y)$
$f_x = \cos(x)$
$f_y = -\operatorname{sen}(y)$

$f = \operatorname{sen}(x)$
$f_x = \cos(x)$
$f_y = 0$

$f = \operatorname{sen}(e^x + y^2)$
$f_x = e^x \cdot \cos(e^x + y^2)$
$f_y = 2y \cdot \cos(e^x + y^2)$

$f = \operatorname{sen}\Big(\cos(x + 1/y)\Big)$
$f_x = -\operatorname{sen}(x + 1/y) \cdot \cos\Big(\cos(x + 1/y)\Big)$
$f_y = \frac{1}{y^2}\operatorname{sen}(x + 1/y) \cdot \cos\Big(\cos(x + 1/y)\Big)$

$f = e^{\operatorname{sen}(x)}\cos(x)$
$f_x = \cos^2(x)e^{\operatorname{sen}(x)} - \operatorname{sen}(x)\cos(x)e^{\operatorname{sen}(x)}$
$f_y = 0$

$f = \log(e^{x^2} + y)$
$f_x = (2xe^{x^2})/(e^{x^2} + y)$
$f_y = 1/(e^{x^2} + y)$

$f = \log(\operatorname{sen}(x) + y^2)$
$f_x = \cos(x)/(\operatorname{sen}(x) + y^2)$
$f_y = 2y/(\operatorname{sen}(x) + y^2)$

2.

$$f(d,t) = \frac{\alpha d T_0}{1+\alpha dt}$$

$$f_d = \alpha T_0 \frac{1}{(1+\alpha dt)^2} > 0$$

$$f_t = \alpha^2 T_0 \frac{-d^2}{(1+\alpha dt)^2} < 0$$

3.

$$\frac{\partial P}{\partial V} = \frac{-kT}{V^2}$$

$$\frac{\partial V}{\partial T} = \frac{k}{P}$$

$$\frac{\partial T}{\partial P} = \frac{V}{k}$$

4.

$v_0 = (\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$

$D_{v_0}(x+y) = \dfrac{3}{\sqrt{5}}$

$D_{v_0}(2x-3y) = -\dfrac{4}{\sqrt{5}}$

$D_{v_0}(xy) = \dfrac{1}{\sqrt{5}}(y+2x)$

$D_{v_0}(x^2+y^2) = \dfrac{2}{\sqrt{5}}(x+2y)$

$D_{v_0}(x+y)^2 = \dfrac{6}{\sqrt{5}}(x+y)$

$v_0 = (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$

$D_{v_0}\left(\dfrac{1}{x+y}\right) = \dfrac{-4}{\sqrt{10}} \dfrac{1}{(x+y)^2}$

$D_{v_0}\left(\dfrac{x}{y}\right) = \dfrac{1}{\sqrt{10}} \left(\dfrac{3}{y} - \dfrac{x}{y^2}\right)$

$D_{v_0}\left(\dfrac{x+y}{x-y}\right) = \dfrac{2}{\sqrt{10}} \dfrac{-3y+x}{(x-y)^2}$

$D_{v_0}\left(\dfrac{x}{x^2+y^2}\right) = \dfrac{1}{\sqrt{10}} \dfrac{3(y^2-x^2)-2xy}{(x^2+y^2)^2}$

$D_{v_0}\left(\dfrac{x-y}{x^2-y^2}\right) = \dfrac{-4}{\sqrt{10}} \dfrac{(x-y)^2}{(x^2-y^2)^2}$

5.

$f = x/y,\ v = (3/5, 4/5)$

$f_x(3,3) = 1/3$

$f_y(1,2) = -1/4$

$D_v f(-1,1) = 7/5$

6.

a) $(0,0)$

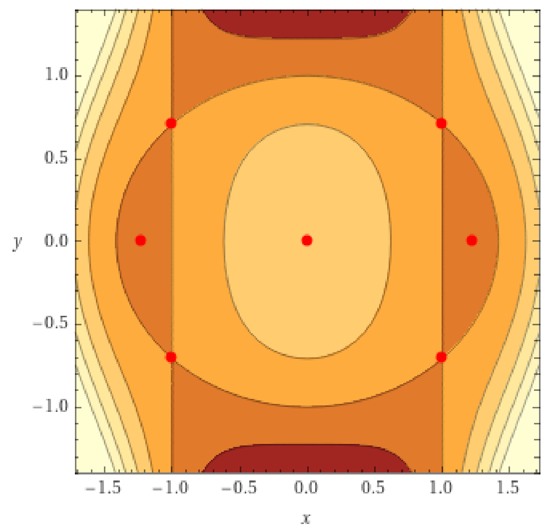
b) No hay puntos estacionarios

c) $(0,0)$

d) No hay puntos estacionarios

e) $(0,0)$, $\left(\pm\sqrt{\frac{3}{2}},0\right)$, $\left(\pm 1,\pm\frac{1}{\sqrt{2}}\right)$ (7 puntos en total)

Puntos estacionarios de la parte (*e*)



7.

a) $H=\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix}$

b) $H=\begin{pmatrix}e^x & 0\\ 0 & 0\end{pmatrix}$

c) $H=\begin{pmatrix}(4x^2-2)e^{-x^2-y^2} & 4xye^{-x^2-y^2}\\ 4xye^{-x^2-y^2} & (4y^2-2)e^{-x^2-y^2}\end{pmatrix}$

d) $H=\begin{pmatrix}\frac{-8x^2+4y^2}{(2x^2+y^2)^2} & \frac{8xy}{(2x^2+y^2)^2}\\ \frac{8xy}{(2x^2+y^2)^2} & \frac{4x^2-2y^2}{(2x^2+y^2)^2}\end{pmatrix}$

e) $H=\begin{pmatrix}12x^2+4y^2-6 & 8xy\\ 8xy & 4x^2-4\end{pmatrix}$

8.

a) Punto silla en $(0,0)$

b) No hay puntos estacionarios

c) Máximo relativo en $(0,0)$

d) No hay puntos estacionarios

e)
- par de mínimos relativos en $\left(\pm\sqrt{\frac{3}{2}},0\right)$
- cuatro puntos silla en $\left(\pm 1,\pm\frac{1}{\sqrt{2}}\right)$
- máximo local en $(0,0)$

Puntos estacionarios de la parte (e)