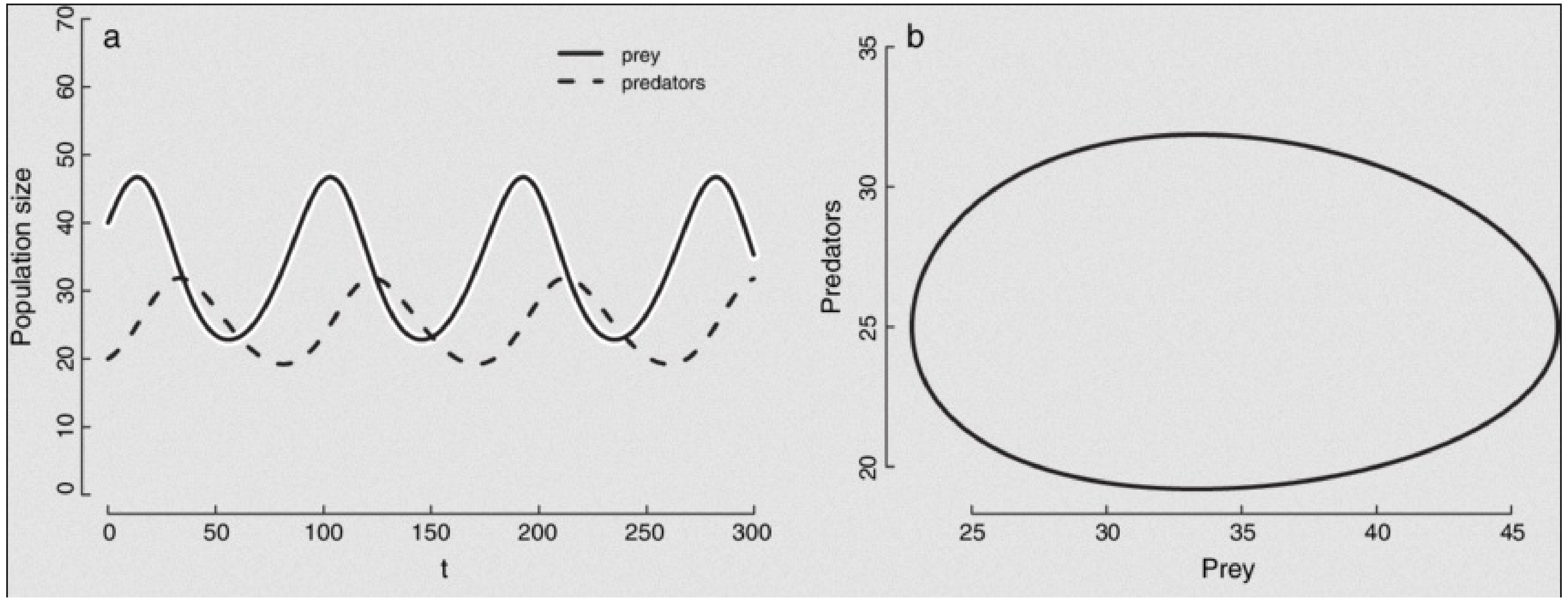
Interacción entre dos poblaciones: predador-presa

Modelo de Lotka-Volterra



Taller de Modelización Matemática y Computacional

2021

Oscilaciones en poblaciones de depredadores y presas

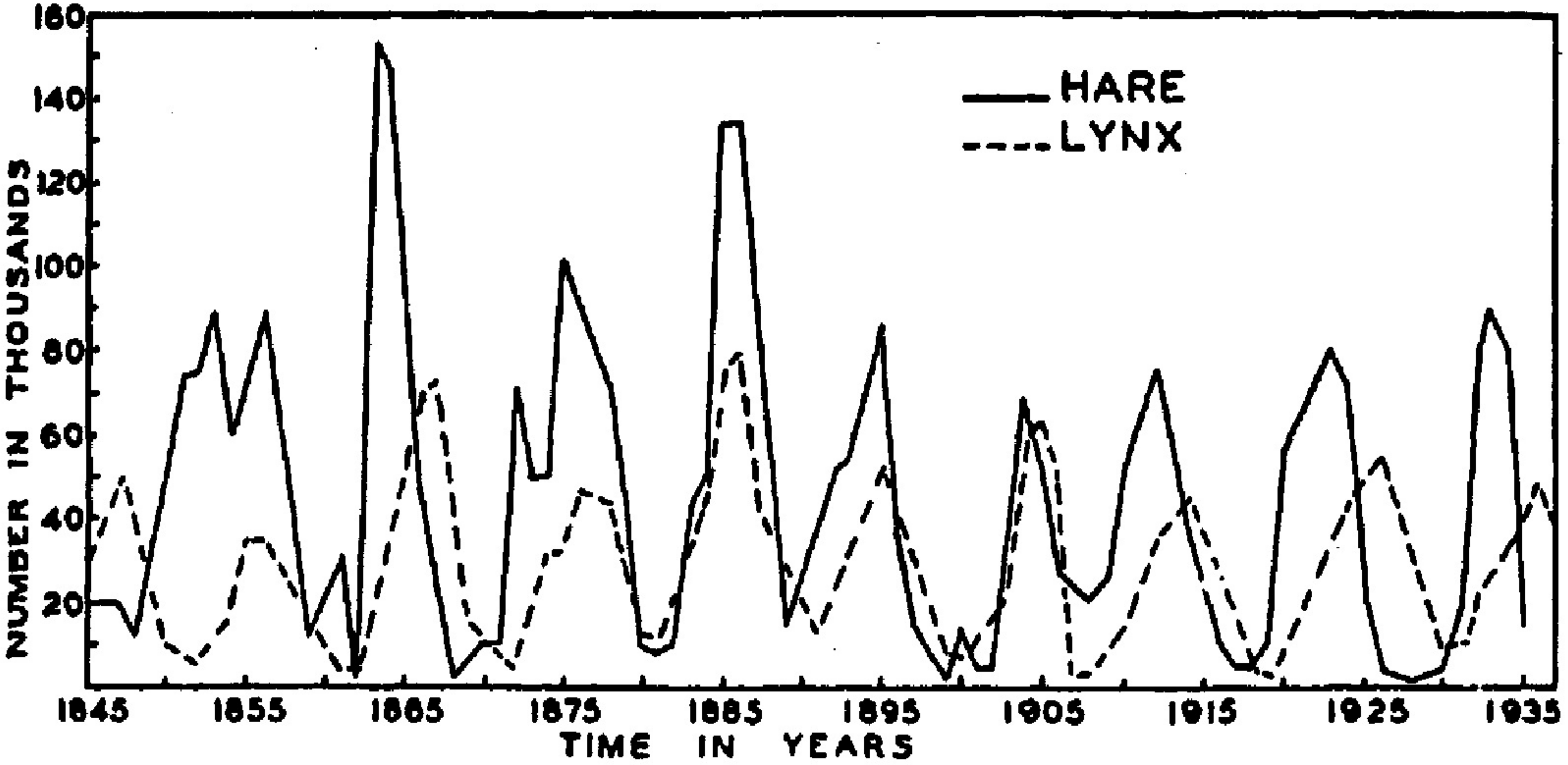
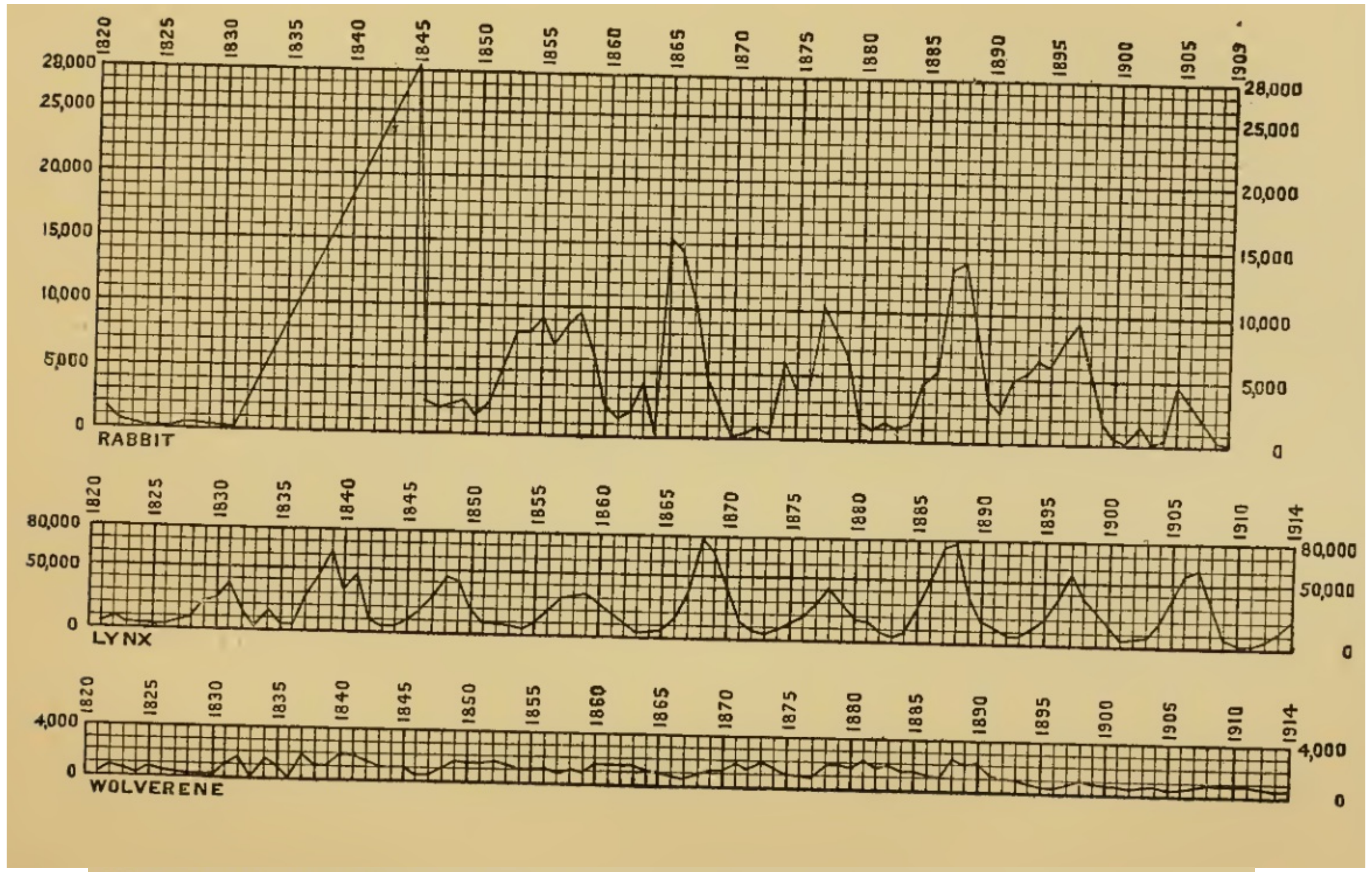
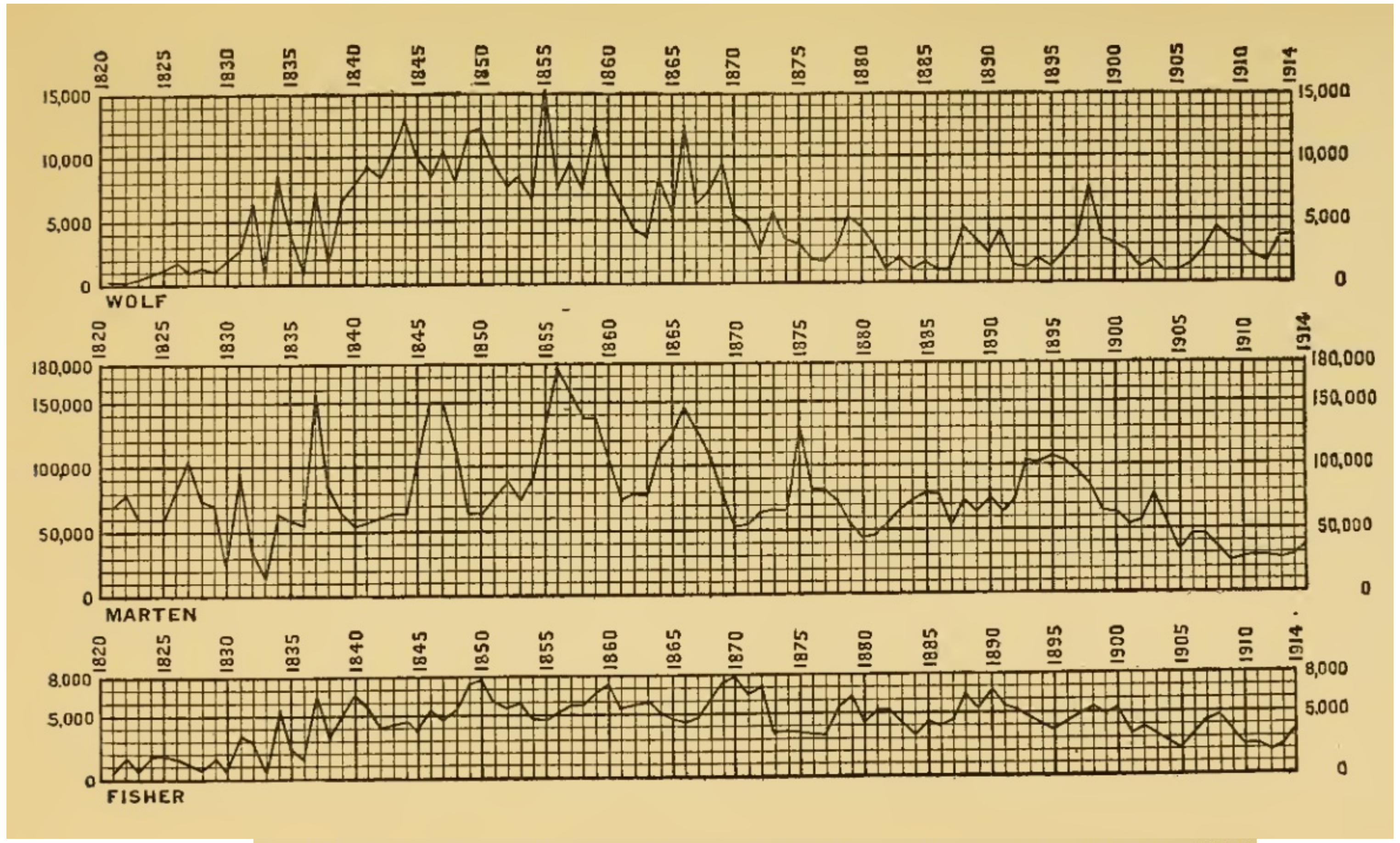


Figure 6.3 Records dating back to the 1840s kept by the Hudson Bay Company. Their trade in pelts of the snowshoe hare and its predator the lynx reveals that the relative abundance of the two species undergoes dramatic cycles. The period of these cycles is roughly 10 years. [From E. P. Odum (1953), fig. 39.]

Oscilaciones en poblaciones de depredadores y presas



PERIODIC FLUCTUATIONS OF RABBIT, LYNX, AND WOLVERENE IN CANADA



PERIODIC FLUCTUATIONS OF WOLF, MARTEN, AND FISHER IN CANADA

Oscilaciones en poblaciones de depredadores y presas

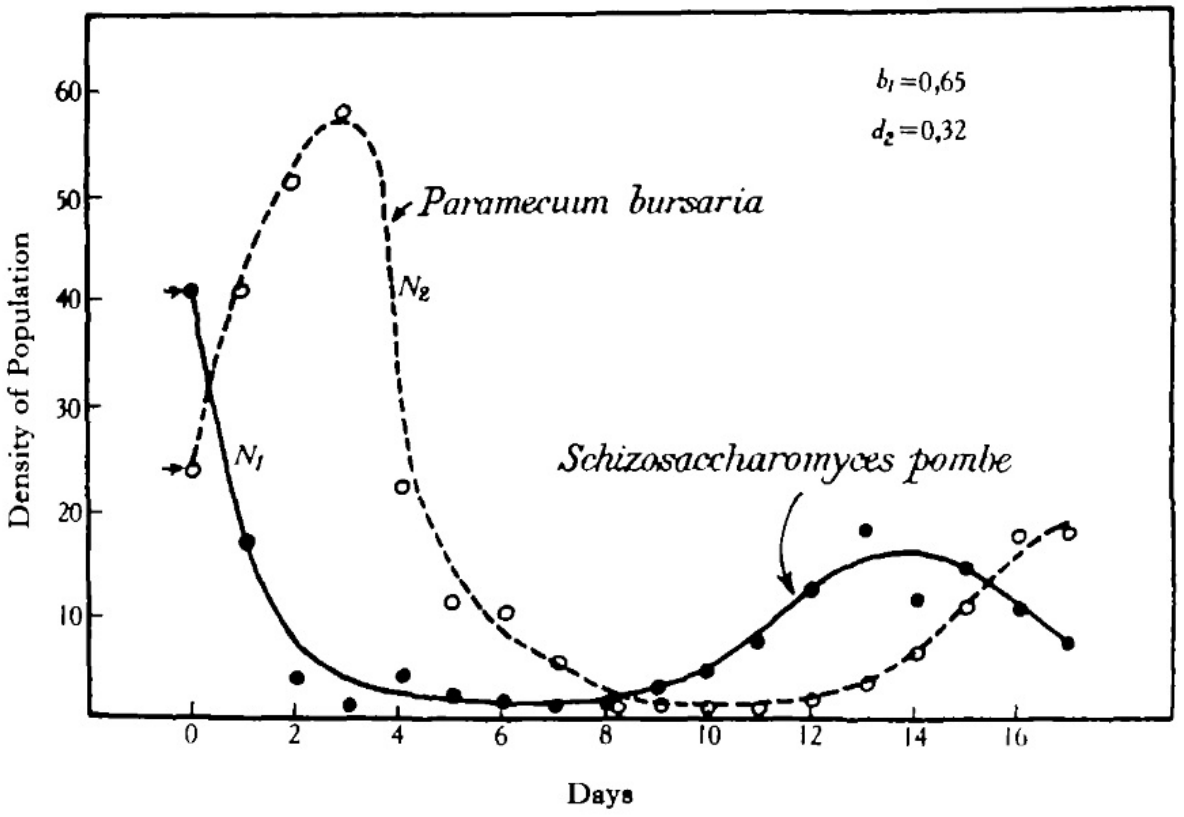


Fig. 1. Fluctuations in the density of population of *Paramecium bursaria* (number of organisms per 0·5 c.c.) and *Schizosaccharomyces pombe* (per 0·1 c.mm.).

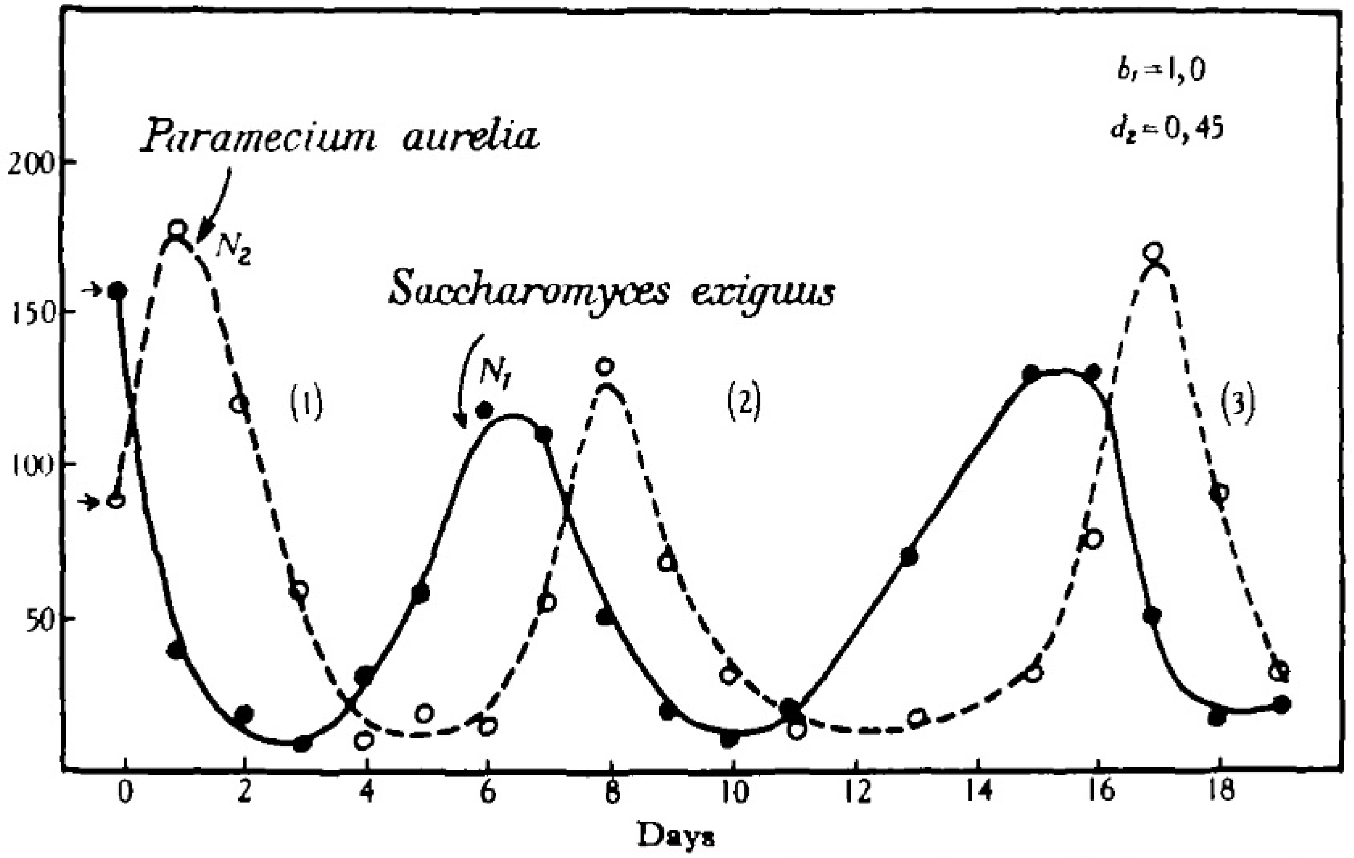


Fig. 3. Fluctuations in the density of population of *Paramecium aurelia* (per 15 c.c.) and *Saccharomyces exiguus* (per 0·1 c.mm.).

N : presa

P : depredador

Ecuaciones de Lotka-Volterra

1880-1949

$$\begin{cases} \dfrac{dN}{dt} = \alpha N - \beta P N \\[2em] \dfrac{dP}{dt} = -\delta P + \gamma P N \end{cases}$$

Vito Volterra
1860-1940

Puntos de equilibrio y Nulclinas

$\dot{N} = 0$

$$0 = \alpha N - \beta P N$$
$$\Rightarrow N(\alpha - \beta P) = 0$$
$$\Rightarrow N = 0 \quad o \quad \alpha - \beta P = 0 \iff P = \alpha/\beta$$

$\dot{P} = 0$

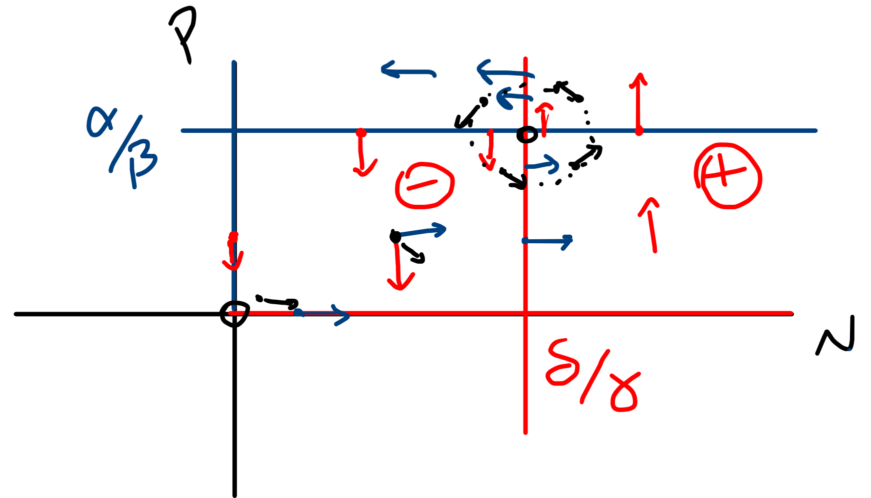
$$0 = -\delta P + \gamma P N = P(\gamma N - \delta) \rightarrow P = 0$$
$$\hookrightarrow N\gamma = \delta$$
$$N = \delta/\gamma$$

Ptos. eq: (N, P)

$(0,0)$

$(\delta/\gamma, \alpha/\beta)$

Nulclinas: Los valores (N,P) que hacen que $\frac{dN}{dt}$ o $\frac{dP}{dt}$ se anulen.



$\frac{dN}{dt} = 0$

$\frac{dP}{dt} = 0$

Estabilidad de los puntos de equilibrio

Def: $\begin{pmatrix} f \\ g \end{pmatrix}(x,y)$

$$J(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}$$

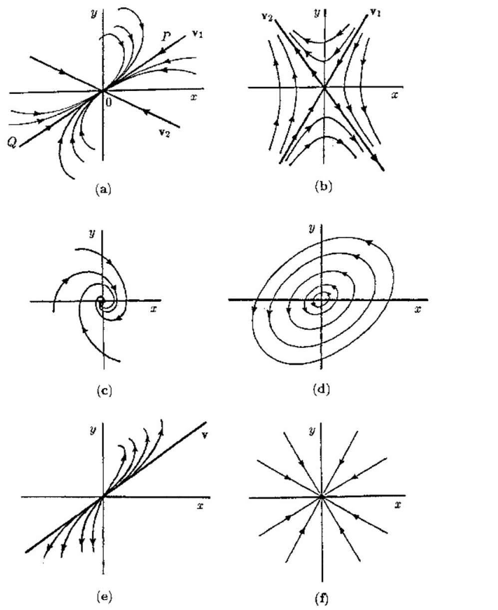


Figure A.1. Typical examples of the basic linear singularities of the phase plane solutions of (A.4). **(a)** Node (Type I): these can be stable (as shown) or unstable. **(b)** Saddle point: these are always unstable. **(c)** Spiral: these can be stable or unstable. **(d)** Centre: this is neutrally stable. **(e)** Node (Type II): these can be stable or unstable. **(f)** Star: these can be stable or unstable.

$$\dot{N} = \alpha N - \beta P N$$
$$\dot{P} = -\delta P + \gamma P N$$

$$J(N,P) = \begin{bmatrix} \alpha - \beta P & -\beta N \\ \gamma P & -\delta + \gamma N \end{bmatrix}$$

$$J(0,0) = \begin{bmatrix} \alpha & 0 \\ 0 & -\delta \end{bmatrix}$$

$$\chi_J(\lambda) := \det(J - \lambda I)$$

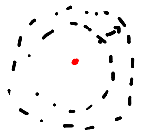
$$\boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc}$$

$$= \begin{vmatrix} \alpha - \lambda & 0 \\ 0 & -\delta - \lambda \end{vmatrix}$$

$$\chi_J(\lambda) = (\alpha - \lambda)\cdot(-\delta - \lambda) = -\underbrace{(\alpha - \lambda)}_{\lambda = \alpha > 0}\underbrace{(\delta + \lambda)}_{-\delta < 0} = 0$$

$$J(\delta/\gamma, \alpha/\beta) = \begin{bmatrix} 0 & -\beta\delta/\gamma \\ \gamma\alpha/\beta & 0 \end{bmatrix} \qquad \chi_J(\lambda) = \begin{vmatrix} -\lambda & -\beta\delta/\gamma \\ \gamma\alpha/\beta & -\lambda \end{vmatrix}$$

$$= \lambda^2 + \frac{\cancel{\beta}\delta}{\cancel{\gamma}}\frac{\cancel{\gamma}\alpha}{\cancel{\beta}} = \lambda^2 + \delta\alpha$$



$$\chi_J(\lambda) = \lambda^2 + \delta\alpha = 0$$

$$\hookrightarrow \lambda^2 = -\delta\alpha$$

$$AX^2 + BX + C = 0$$

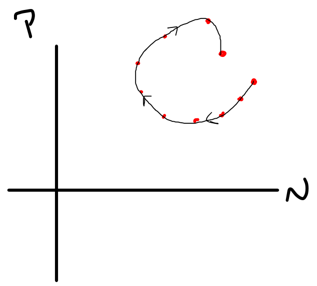
$$\Delta = B^2 - 4AC$$

$$\lambda = \pm\sqrt{-\delta\alpha} = 0 \pm i\sqrt{\delta\alpha} \Rightarrow (\delta/\gamma, \alpha/\beta) \text{ es un "centro"}$$

$$\sqrt{-\delta\alpha} = \underbrace{\sqrt{-1}}_{i} \cdot \sqrt{\delta\alpha}$$

cerca del mismo hay oscilaciones estables

Cantidades conservadas en el plano de fase



$$¿\frac{dP}{dN}?$$

$$\frac{dP}{dN} = \frac{dP}{dt}\underbrace{\frac{dt}{dN}}_{=\left(1/\frac{dN}{dt}\right)} = \frac{dP}{dt}\Big/\frac{dN}{dt} = \boxed{\frac{-\delta P + \gamma P N}{\alpha N - \beta P N} = \frac{dP}{dN}}$$

$$\frac{P(-\delta + \gamma N)}{N(\alpha - \beta P)} = \left(\frac{P}{\alpha - \beta P}\right)\left(\frac{\gamma N - \delta}{N}\right) = \frac{dP}{dN}$$

$$\int_{N_0}^{N} \underbrace{\frac{\gamma N - \delta}{N}}_{\gamma - \frac{\delta}{N}}\, dN = \int_{P_0}^{P} \underbrace{\frac{\alpha - \beta P}{P}}_{\frac{\alpha}{P} - \beta}\, dP$$

$$\boxed{\gamma N_0 + \beta P_0 - \ln\left(N_0^{\delta} P_0^{\alpha}\right) = cte. = H}$$