An Introduction to Evolutionary Games 8

In an evolutionary game, players are interpreted as populations—of animals or individuals. The probabilities in a mixed strategy of a player in a bimatrix game are interpreted as shares of the population. Individuals within the same part of the population play the same pure strategy. The main 'solution' concept is the concept of an evolutionary stable strategy.

Evolutionary game theory originated in biology. The developed evolutionary biological concepts were later applied to boundedly rational human behavior, and a connection was established with dynamic systems and with game-theoretic concepts such as Nash equilibrium.

This chapter presents a short introduction to evolutionary game theory. For a more advanced continuation see Chap. 15.

In Sect. 8.1 we consider symmetric two-player games and evolutionary stable strategies. Evolutionary stability is meant to capture the idea of *mutation* from the theory of evolution. We also establish that an evolutionary stable strategy is part of a symmetric Nash equilibrium. In Sect. 8.2 the connection with the so-called replicator dynamics is studied. Replicator dynamics intends to capture the evolutionary idea of *selection* based on fitness. In Sect. 8.3 asymmetric games are considered. Specifically, a connection between replicator dynamics and strict Nash equilibrium is discussed.

8.1 Symmetric Two-Player Games and Evolutionary Stable Strategies

A famous example from evolutionary game theory is the *Hawk-Dove game*:

$$\begin{array}{c} \\ \text{Hawk} \\ \text{Dove} \end{array} \begin{array}{c} \begin{array}{cc} \text{Hawk} & \text{Dove} \end{array} \\ \begin{pmatrix} 0,0 & 3,1 \\ 1,3 & 2,2 \end{pmatrix} \end{array}.$$

H. Peters, *Game Theory*, Springer Texts in Business and Economics,
DOI 10.1007/978-3-662-46950-7_8

This game models the following situation. Individuals of the same large population meet randomly, in pairs, and behave either aggressively (Hawk) or passively (Dove)—the fight is about nest sites or territories, for instance. This behavior is genetically determined, so an individual does not really choose between the two modes of behavior. The payoffs reflect (Darwinian) fitness, e.g., the number of offspring. In this context, players 1 and 2 are just two different members of the same population who meet: indeed, the game is symmetric—see below for the formal definition. A mixed strategy $\mathbf{p} = (p_1, p_2)$ (of player 1 or player 2) is naturally interpreted as expressing the population shares of individuals characterized by the same type of behavior. In other words, $p_1 \times 100\,\%$ of the population are Hawks and $p_2 \times 100\,\%$ are Doves. In view of this interpretation, in what follows we are particularly interested in symmetric Nash equilibria, i.e., Nash equilibria in which the players use the same strategy. The Hawk-Dove game has three Nash equilibria, only one of which is symmetric namely $((\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}))$.

Remark 8.1 The Hawk-Dove game can also be interpreted as a *Game of Chicken*. Two car drivers approach each other on a road, each one driving in the middle. The driver who is the first to return to his own lane (Dove) 'loses' the game, the one who stays in the middle 'wins' (Hawk). With this interpretation also the asymmetric equilibria are of interest. The asymmetric equilibria can also be of interest within the evolutionary approach: the Hawk-Dove game can be interpreted as modelling competition between two species, represented by the row and the column player. Within each species, there are again two types of behavior. See Sect. 8.3. □

The definitions of a symmetric game and a symmetric Nash equilibrium are as follows.

Definition 8.2 Let $G = (A, B)$ be an $m \times n$ bimatrix game. Then G is *symmetric* if $m = n$ and $b_{ij} = a_{ji}$ for all $i, j = 1, \ldots, m$. A Nash equilibrium $(\mathbf{p}^*, \mathbf{q}^*)$ of G is *symmetric* if $\mathbf{p}^* = \mathbf{q}^*$. □

In other words, a bimatrix game (A, B) is symmetric if both players have the same number of pure strategies and the payoff matrix of player 2 is the transpose of the payoff matrix of player 1, i.e., we obtain B by interchanging the rows and columns of A. This will also be denoted by $B = A^T$, where 'T' stands for 'transpose'. A Nash equilibrium is symmetric if both players play the same strategy.

We state the following fact without a proof (see Chap. 15).

Proposition 8.3 *Every symmetric bimatrix game G has a symmetric Nash equilibrium.*

With the interpretation above—different types of behavior within one and the same population—it is only meaningful to consider symmetric Nash equilibria. But in fact, we will require more.

Let $G = (A, B)$ be a symmetric game. Knowing that the game is symmetric, it is sufficient to know the payoff matrix A, since $B = A^T$. In what follows, when we consider a symmetric game A we mean the game $G = (A, A^T)$. Let A be an $m \times m$ matrix. Recall (Chaps. 2 and 3) that Δ^m denotes the set of mixed strategies (for player 1 or player 2).

The main concept in evolutionary game theory is that of an *evolutionary stable strategy*. The original concept will be formally introduced in Chap. 15. Here, we give an equivalent but easier to handle definition. With some abuse of language we give it the same name.

Definition 8.4 Let A be an $m \times m$ matrix. A strategy $\mathbf{x} \in \Delta^m$ is an *evolutionary stable strategy* (*ESS*) if the following two conditions hold.

(a) $(\mathbf{x}, \mathbf{x})$ is a Nash equilibrium in (A, A^T).
(b) For every $\mathbf{y} \in \Delta^m$ with $\mathbf{y} \neq \mathbf{x}$ we have:

$$\mathbf{x}A\mathbf{x} = \mathbf{y}A\mathbf{x} \Rightarrow \mathbf{x}A\mathbf{y} > \mathbf{y}A\mathbf{y} \ . \tag{8.1}$$

□

To interpret this definition, think again of $\mathbf{x}$ as shares of one and the same large population. The first condition says that this population is in equilibrium: $\mathbf{x}$ is one of the possible distributions of shares that maximize average fitness against $\mathbf{x}$. The second condition concerns *mutations*. Suppose there is another distribution of shares $\mathbf{y}$ (a mutation) that fares equally well against $\mathbf{x}$ as $\mathbf{x}$ itself does: $\mathbf{y}$ is an alternative 'best reply' to $\mathbf{x}$. Then (8.1) says that $\mathbf{x}$ fares better against $\mathbf{y}$ than $\mathbf{y}$ does against itself. Hence, $\mathbf{y}$ does not take over: the 'mutation' $\mathbf{y}$ is not successful. The original definition of *ESS* is phrased in terms of *small* mutations, but this turns out to be equivalent to the definition above (Chap. 15).

The evolutionary stable strategies for an $m \times m$ matrix A can be found as follows. First, compute the symmetric Nash equilibria of the game $G = (A, B)$ with $B = A^T$. This can be done using the methods developed in Chap. 3. Second, for each such equilibrium $(\mathbf{x}, \mathbf{x})$, check whether (8.1) holds. If it does, then $\mathbf{x}$ is an evolutionary stable strategy.

We apply this method to the Hawk-Dove game. For this game,

$$A = \begin{matrix} & \text{Hawk} & \text{Dove} \\ \text{Hawk} & 0 & 3 \\ \text{Dove} & 1 & 2 \end{matrix} \ .$$

The unique symmetric equilibrium strategy was $\mathbf{x} = (\frac{1}{2}, \frac{1}{2})$. The condition $\mathbf{x}A\mathbf{x} = \mathbf{y}A\mathbf{x}$ in (8.1) is satisfied for every $\mathbf{y} = (y, 1 - y)$. This can be seen by direct computation but it also follows from the fact that $(\mathbf{x}, \mathbf{x})$ is a Nash equilibrium and $\mathbf{x}$

has all coordinates positive (how?). Hence, we have to check if

$$\mathbf{x}A\mathbf{y} > \mathbf{y}A\mathbf{y}$$

for all $\mathbf{y} = (y, 1 - y) \neq \mathbf{x}$. This inequality reduces (check!) to:

$$(2y - 1)^2 > 0 ,$$

which is true for all $y \neq \frac{1}{2}$. Thus, $\mathbf{x} = (\frac{1}{2}, \frac{1}{2})$ is the unique ESS in A.

8.2 Replicator Dynamics and Evolutionary Stability

Central in the theory of evolution are the concepts of *mutation* and *selection*. While the idea of mutation is meant to be captured by the concept of evolutionary stability, the idea of selection is captured by the so-called replicator dynamics. We illustrate the concept of replicator dynamics by considering again the Hawk-Dove game

$$\begin{array}{c} \\ \text{Hawk} \\ \text{Dove} \end{array} \begin{array}{c} \begin{array}{cc} \text{Hawk} & \text{Dove} \end{array} \\ \begin{pmatrix} 0,0 & 3,1 \\ 1,3 & 2,2 \end{pmatrix} \end{array} .$$

Consider a mixed strategy or, in the present context, vector of population shares $\mathbf{x} = (x, 1 - x)$. Consider an arbitrary individual of the population. Playing 'Hawk' against the population $\mathbf{x}$ yields an expected payoff or 'fitness' of

$$0 \cdot x + 3 \cdot (1 - x) = 3(1 - x)$$

and playing 'Dove' yields

$$1 \cdot x + 2 \cdot (1 - x) = 2 - x .$$

Hence, the *average fitness* of the population is

$$x \cdot 3(1 - x) + (1 - x) \cdot (2 - x) = 2 - 2x^2 .$$

We now assume that the population shares develop over time, i.e., that x is a function of time t, and that the change in x, described by the time derivative $\dot{x} = \dot{x}(t) = dx(t)/dt$, is proportional to the difference between Hawk's fitness and average fitness. In particular, if Hawk's fitness is larger than average fitness, then the percentage of Hawks increases, and if Hawk's fitness is smaller than average fitness, then the percentage of Hawks decreases. In case of equality the population is at rest.

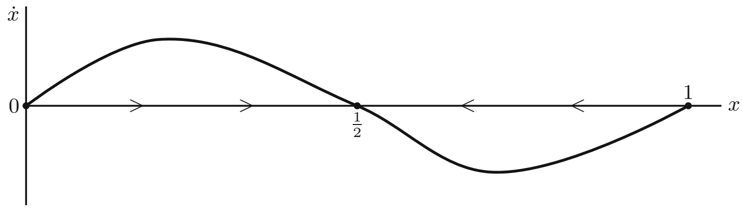

Fig. 8.1 Replicator dynamics for the Hawk-Dove game

Formally, we assume that $\dot{x}$ is given by the following equation.

$$\dot{x}(t) = dx(t)/dt = x(t)\left[3(1 - x(t)) - (2 - 2x(t)^2)\right] . \tag{8.2}$$

Equation (8.2) is the *replicator dynamics* for the Hawk-Dove game. The equation says that the population of Hawks changes continuously (described by $dx(t)/dt$), and that this change is proportional to the difference between the fitness at time t—which is equal to $3(1 - x(t))$—and the average fitness of the population—which is equal to $2 - 2x(t)^2$. Simplifying (8.2) and writing x instead of $x(t)$ yields

$$\dot{x} = dx/dt = x(x - 1)(2x - 1).$$

This makes it possible to draw a diagram of dx/dt as a function of x (a so-called *phase diagram*). See Fig. 8.1. We see that this replicator dynamics has three different roots, the so-called *rest points*[1] $x = 0$, $x = \frac{1}{2}$, and $x = 1$. For these values of x, the derivative dx/dt is equal to zero, so the population shares do not change: the system is at rest. In case $x = 0$ all members of the species are Doves, their fitness is equal to the average fitness, and so nothing changes. This rest point, however, is not stable. A slight disturbance, e.g., a genetic mutation resulting in a Hawk, makes the number of Hawks increase because dx/dt becomes positive. This increase will go on until the rest point $x = \frac{1}{2}$ is reached. A similar story holds for the rest point $x = 1$, where the population consists of only Hawks. Now suppose the system is at the rest point $x = \frac{1}{2}$. Note that, after a disturbance in either direction, the system will move back again to the state where half the population consists of Doves. Thus, of the three rest points, only $x = \frac{1}{2}$ is *stable*. (A formal definition of stability of a rest point is provided in Chap. 15.)

Recall from the previous section that $\mathbf{x} = (\frac{1}{2}, \frac{1}{2})$ is also the unique evolutionary stable strategy of the Hawk-Dove game. That this is no coincidence follows from the next proposition, which we state here without a proof (see Chap. 15 for a proof). (The definition of replicator dynamics is analogous to the one in the Hawk-Dove game.)

[1] In the literature also called equilibrium points, critical points, stationary points.

Proposition 8.5 *Let* $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ *be a* 2×2 *matrix with* $a_{11} \neq a_{21}$ *and* $a_{12} \neq a_{22}$. *Then:*

(*a*) *A has at least one evolutionary stable strategy.*
(*b*) $\mathbf{x} = (x, 1-x)$ *is an evolutionary stable strategy of A if and only if* $\mathbf{x}$ *is a stable rest point of the replicator dynamics.*

Remark 8.6 For general $m \times m$ matrices the set of completely mixed (i.e., with all coordinates positive) rest points of the replicator dynamics coincides with the set of completely mixed strategies in symmetric Nash equilibria. There are also connections between stability of rest points and further properties of Nash equilibria. See Chap. 15 for details. □

Example 8.7 As another example, consider the matrix

$$A = \begin{array}{c} \\ V \\ W \end{array} \begin{array}{c} \begin{array}{cc} V & W \end{array} \\ \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \end{array} .$$

The bimatrix game (A, A^T) has three Nash equilibria all of which are symmetric, namely: (V, V), (W, W), and $((1/3, 2/3), (1/3, 2/3))$. Against V the unique best reply is V, so that V is an *ESS*: (8.1) is satisfied trivially. By a similar argument, W is an *ESS*.

Against $(1/3, 2/3)$, any $\mathbf{y} = (y, 1-y)$ is a best reply. For $(1/3, 2/3)$ to be an *ESS* we therefore need

$$\begin{pmatrix} 1/3 & 2/3 \end{pmatrix} A \begin{pmatrix} y \\ 1-y \end{pmatrix} > \begin{pmatrix} y & 1-y \end{pmatrix} A \begin{pmatrix} y \\ 1-y \end{pmatrix}$$

for all $0 \leq y \leq 1$ with $y \neq 1/3$. The inequality simplifies (check!) to the inequality $(3y-1)^2 < 0$, which never holds. Hence, $(1/3, 2/3)$ is not an ESS.

We now investigate the replicator dynamics. The expected payoff of V against $(x, 1-x)$ is equal to $3x + 1 \cdot (1-x) = 2x + 1$. The expected payoff of W against $(x, 1-x)$ is equal to $x + 2 \cdot (1-x) = 2 - x$. The average payoff is $x(2x+1) + (1-x)(2-x) = 3x^2 - 2x + 2$. Hence, the replicator dynamics is

$$dx/dt = x(2x + 1 - (3x^2 - 2x + 2)) = -x(x-1)(3x-1) .$$

Figure 8.2 presents the phase diagram, which shows that $x = 0$ and $x = 1$ are stable rest points, and that the rest point $x = 1/3$ is not stable, in accordance with Proposition 8.5. □

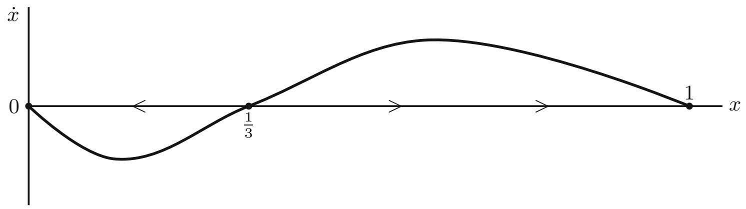

Fig. 8.2 Replicator dynamics for Example 8.7

8.3 Asymmetric Games

The evolutionary approach to game theory is not necessarily restricted to symmetric situations, i.e., bimatrix games of the form (A, A^T) in which the row and column players play identical strategies. In biology as well as economics one can find many examples of asymmetric situations. Think, for instance, of two different species competing about territory in biology; and see Problem 8.6 for an example from economics.

Consider the 2×2-bimatrix game

$$(A, B) = \begin{array}{c} \\ U \\ D \end{array} \begin{array}{c} \begin{array}{cc} L & R \end{array} \\ \begin{pmatrix} 0,0 & 2,2 \\ 1,5 & 1,5 \end{pmatrix} \end{array} .$$

Think of two populations, the row population and the column population. In each population there are two different types: U and D in the row population and L and R in the column population. Individuals of one population are continuously and randomly matched with individuals of the other population, and we are interested again in the development of the population shares. To start with, assume the shares of U and D types in the row population are x and $1-x$, respectively, and the shares of L and R types in the column population are y and $1-y$. The expected payoff of a U type individual is given by:

$$0 \cdot y + 2 \cdot (1-y) = 2 - 2y .$$

For a D type individual it is

$$1 \cdot y + 1 \cdot (1-y) = 1 .$$

For an L type individual it is

$$0 \cdot x + 5 \cdot (1-x) = 5 - 5x .$$

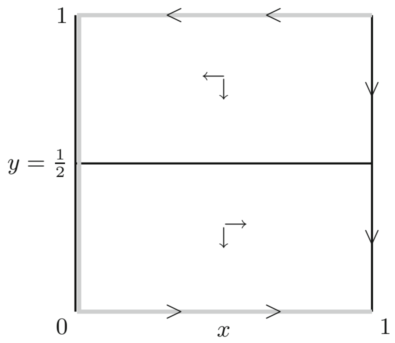

Fig. 8.3 Phase diagram of the asymmetric evolutionary game

And for an R type individual:

$$2 \cdot x + 5 \cdot (1 - x) = 5 - 3x \ .$$

The average of the row types is therefore:

$$x[2(1 - y)] + (1 - x) \cdot 1$$

and the replicator dynamics for the population share $x(t)$ of U individuals is given by

$$dx/dt = x[2(1 - y) - x[2(1 - y)] - (1 - x)] = x(1 - x)(1 - 2y) \ . \tag{8.3}$$

Here, we write x and y instead of $x(t)$ and $y(t)$. Similarly one can calculate the replicator dynamics for the column population (check this result!):

$$dy/dt = y(1 - y)(-2x) \ . \tag{8.4}$$

We are interested in the rest points of the dynamic system described by Eqs. (8.3) and (8.4), and, in particular, by the stable rest points. Figure 8.3 presents a diagram of the possible values of x and y. In this diagram, the black lines are the values of x and y for which the derivative in (8.3) is equal to 0, i.e., for which the row population is at rest. The gray lines are the values of x and y for which the derivative in (8.4) is equal to 0: there, the column population is at rest. The points of intersection are the points where the whole system is at rest; this is the set

$$\{(0, y) \mid 0 \leq y \leq 1\} \cup \{(1, 0)\} \cup \{(1, 1)\} \ .$$

In Fig. 8.3, arrows indicate the direction in which x and y move. For instance, if $1 > y \geq \frac{1}{2}$ and $0 < x < 1$ we have $dx/dt < 0$ and $dy/dt < 0$, so that in that region x as well as y decrease. A *stable rest point* is a rest point such that, if the system is slightly disturbed and moves to some point close to the rest point in question, then it should move back again to this rest point. In terms of the arrows in Fig. 8.3

this means that a stable rest point is one where all arrows in the neighborhood point towards that point. It is obvious that in our example the point $(1, 0)$ is the only such point. So the situation where the row population consists only of U type individuals $(x = 1)$ and the column population consists only of R type individuals $(y = 0)$ is the only stable situation with respect to the replicator dynamics.

Is there a relation with Nash equilibrium? One can check (!) that the set of Nash equilibria in this example is the set:

$$\{(U, R), (D, L)\} \cup \{(D, (q, 1-q)) \mid \frac{1}{2} \leq q \leq 1\} .$$

So the stable rest point (U, R) is a Nash equilibrium. Furthermore, it has a special characteristic, namely, it is the only strict Nash equilibrium of the game. A *strict* Nash equilibrium in a game is a Nash equilibrium where each player not only does not gain but in fact strictly looses by deviating. For instance, if the row player deviates from U in the Nash equilibrium (U, R) then he obtains strictly less than 2. All the other equilibria in this game do not have this property. For instance, if the column player deviates from L to R in the Nash equilibrium (D, L), then he still obtains 5.

The observation that the stable rest point of the replicator dynamics coincides with a strict Nash equilibrium is not a coincidence. The following proposition is stated here without a proof.

Proposition 8.8 *In a* 2×2 *bimatrix game a pair of strategies is a stable rest point of the replicator dynamics if and only if it is a strict Nash equilibrium. For larger games, any stable rest point of the replicator dynamics is a strict Nash equilibrium, but the converse does not necessarily hold.*

Remark 8.9 A strict Nash equilibrium in a bimatrix game must be a pure Nash equilibrium, for the following reason. If a player plays two or more pure strategies with positive probability in a Nash equilibrium, then he must be indifferent between these pure strategies and, thus, can deviate to any of them while keeping the same payoff. This holds true in any arbitrary game, not only in bimatrix games. □

8.4 Problems

8.1. *Symmetric Games*
Compute the evolutionary stable strategies for the following payoff matrices A.

(a) $A = \begin{pmatrix} 4 & 0 \\ 5 & 3 \end{pmatrix}$ (Prisoners' Dilemma)

(b) $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ (Coordination game)

8.2. *More Symmetric Games*
For each of the following two matrices, determine the replicator dynamics, rest points and stable rest points, and evolutionary stable strategies. Include phase diagrams for the replicator dynamics. For the evolutionary stable strategies, provide independent arguments to show evolutionary stability by using Definition 8.4.

(a) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $A = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$

8.3. *Asymmetric Games*
For each of the following two asymmetric situations (i.e., row and column populations are assumed to be different and we do not only consider symmetric population shares), determine the replicator dynamics, rest points and stable rest points, including phase diagrams. Also determine all Nash and strict Nash equilibria.

(a) $(A, A^T) = \begin{pmatrix} 0,0 & 1,1 \\ 1,1 & 0,0 \end{pmatrix}$

(b) $(A, A^T) = \begin{pmatrix} 2,2 & 0,1 \\ 1,0 & 0,0 \end{pmatrix}$

8.4. *More Asymmetric Games*
For each of the following two bimatrix games, determine the replicator dynamics and all rest points and stable rest points. Also compute all Nash equilibria, and discuss the relation using Proposition 8.8.

(a) $(A, B) = \begin{pmatrix} 3,2 & 8,0 \\ 4,0 & 6,2 \end{pmatrix}$

(b) $(A, B) = \begin{pmatrix} 4,3 & 3,4 \\ 5,5 & 2,4 \end{pmatrix}$

8.5. *Frogs Call For Mates*
Consider the following game played by male frogs who Call or Don't Call their mates.

$$\begin{array}{c} \\ \text{Call} \\ \text{Don't Call} \end{array} \begin{array}{c} \begin{array}{cc} \text{Call} & \text{Don't Call} \end{array} \\ \begin{pmatrix} P-z, P-z & m-z, 1-m \\ 1-m, m-z & 0,0 \end{pmatrix} \end{array}$$

The payoffs are in units of 'fitness', measured by the frog's offspring. Here z denotes the cost of Calling (danger of becoming prey, danger of running out of energy); and m is the probability that the male who calls in a pair of males, the other of whom is not calling, gets a mate. Typically, $m \geq \frac{1}{2}$. Next, if no male calls then no female is attracted, and if both call returns diminish and they each attract P females with $0 < P < 1$.

(a) Show that there are several possible evolutionary stable strategies for this game, depending on the parameters (m, z, P).
(b) Set $m = 0.6$ and $P = 0.8$. Find values for z for each of the following situations: (i) Don't Call is an evolutionary stable strategy (ESS); (ii) Call is an ESS; (iii) A mixture of Call and Don't Call is an ESS.
(c) Suppose there are two kinds of frogs in *Frogs Call For Mates*. Large frogs have a larger cost of calling (z_1) than do small frogs (z_2). Determine the corresponding asymmetric bimatrix game. Determine the possible stable rest points of the replicator dynamics.

8.6. *Video Market Game*
Two boundedly rational video companies are playing the following asymmetric game:

$$\begin{array}{l} \\ \text{Open system} \\ \text{Lockout system} \end{array} \begin{array}{c} \begin{array}{cc} \text{Open system} & \text{Lockout system} \end{array} \\ \begin{pmatrix} 6,4 & \quad 5,5 \\ 9,1 & \quad 10,0 \end{pmatrix} \end{array}$$

Company I (the row company) has to decide whether to have an open system or a lockout system. Company II (the column company) has to decide whether to create its own system or copy that of company I. What is a rest point of the replicator dynamics for this system?

8.5 Notes

Evolutionary game theory originated in the work of the biologists Maynard Smith and Price (1973). Taylor and Jonker (1978) and Selten (1983), among others, played an important role in applying the developed evolutionary biological concepts to boundedly rational human behavior, and in establishing the connection with dynamic systems and with game-theoretic concepts such as Nash equilibrium. A comprehensive treatment is Weibull (1995).

The original definition of evolutionary stable strategy (Chap. 15) is due to Maynard Smith and Price (1973). Taylor and Jonker (1978) introduced the replicator dynamics.

For more economic applications of asymmetric evolutionary games see for instance Gardner (1995). In the literature the concept of evolutionary stable strategy is extended to asymmetric games. See Selten (1980) or Hofbauer and Sigmund (1988) for details, also for the relation between stable rest points and strict Nash equilibrium.

Problems 8.5 and 8.6 are taken from Gardner (1995).

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