(3)

$E^2 - p^2 = m^2$ En referencia Lorentziano en espacio tiempo plano y por lo tanto en referencia Lorentziano local en espacio tiempo curvo.

4-momento: Necesitamos base de referencia Lorentziano de reposo del observador.

$e_{\hat{0}}$ = 4-velocidad del observador $= u$

$\propto \partial_t$ porque r, θ, ϕ del observador son constantes

$u = a\,\partial_t \qquad -1 = u\cdot u = a^2 g(\partial_t, \partial_t) = a^2 g_{tt} = -a^2\left(1 - \frac{2M}{r}\right)$

$\Rightarrow a = \left(1 - \frac{2M}{r}\right)^{-1/2}$

$\rightarrow e_{\hat{0}} = u = \left(1 - \frac{2M}{r}\right)^{-1/2} \partial_t$

$e_{\hat{r}} = \frac{1}{\sqrt{g_{rr}}}\partial_r \qquad \Rightarrow g(e_{\hat{r}}, e_{\hat{r}}) = \frac{1}{g_{rr}} g(\partial_r, \partial_r) = 1$

$= \left(1 - \frac{2M}{r}\right)^{1/2} \partial_r$

$e_{\hat{\theta}} = \frac{1}{\sqrt{g_{\theta\theta}}}\partial_\theta \qquad e_{\hat{\phi}} = \frac{1}{\sqrt{g_{\phi\phi}}}\partial_\phi$

$g(e_{\hat{\theta}}, e_{\hat{r}})$

$= \frac{1}{\sqrt{g_{\theta\theta}}}\frac{1}{\sqrt{g_{rr}}} g(\partial_\theta, \partial_r)$

$= \frac{1}{\sqrt{g_{\theta\theta}}}\frac{1}{\sqrt{g_{rr}}} g_{\theta r} = 0$

$$E = p^{\hat{0}} = -p\cdot e_{\hat{0}} \qquad p\cdot e_{\hat{0}} = p^{\hat{\alpha}} e_{\hat{\alpha}}\cdot e_{\hat{0}} = p^{\hat{0}} e_{\hat{0}}\cdot e_{\hat{0}} = -p^{\hat{0}}$$

$$= -g(p, e_{\hat{0}}) = -g(p, \partial_t)\left(1-\frac{2M}{r}\right)^{-1/2}$$

$$= -g_{\mu t}\, p^{\mu}\left(1-\frac{2M}{r}\right)^{-1/2}$$

$$= \left(1-\frac{2M}{r}\right)p^{t}\left(1-\frac{2M}{r}\right)^{-1/2}$$

$$\left(1-\frac{2M}{r}\right)^{1/2} p^{t}$$

$$\|\vec{p}\| = \|p^{\hat{r}} e_{\hat{r}}\| \qquad p\cdot e_{\hat{r}} = p^{\hat{\alpha}} e_{\hat{\alpha}}\cdot e_{\hat{r}} = p^{\hat{r}} e_{\hat{r}}\cdot e_{\hat{r}} = p^{\hat{r}}$$

$$= |p^{\hat{r}}|$$

$$= |p\cdot e_{\hat{r}}|$$

$$= |p\cdot \partial_r|\left(1-\frac{2M}{r}\right)^{\frac{1}{2}} \qquad p\cdot\partial_r = g_{\mu r}p^{\mu} = g_{rr}p^{r} = \left(1-\frac{2M}{r}\right)^{-1}p^{r}$$

$$= |p^{r}|\left(1-\frac{2M}{r}\right)^{-1/2}$$

$$p = [p^{t}, p^{r}, 0, 0] = \left[E\left(1-\frac{2M}{r}\right)^{-1/2}, \|\vec{p}\|\left(1-\frac{2M}{r}\right)^{1/2}, 0, 0\right]$$

4 Altura de la persona $l = 2m$

$$l\,\partial_r g^{r} < 200\ m/s^2$$

$$\Rightarrow \partial_r g^{r} < 100\ s^{-2}$$

En teoría Newtoniana

$$g^{r} = -\frac{GM}{r^2} \Rightarrow \partial_r g^{r} = \frac{2GM}{r^3} = \frac{r_s c^2}{r^3}$$

$\partial_r g^{r}$ es maximizada cuando r es minimizado, En el trayecto

de la persona en $r = R$ = radio del cascaron.
Pero $R \simeq r_s$.

$$\frac{R}{R^3} c^2 < 100\, s^{-2} \Rightarrow R^2 > c^2/100\, s^{-2}$$
$$\Rightarrow R > c/10\, s^{-1} = 3 \times 10^7 m = 39\,000\, km$$

$$\partial_r g^r = R^r{}_{\nu\mu r} = -g^{rr} R_{rtrt} (u^t)^2$$
$$= -\left(1 - \frac{2M}{r}\right)(u^t)^2 R_{rtrt}$$

(6) $e = 1$, $\mathcal{E} = \frac{e^2 - 1}{2} = 0$ corresponde a velocidad 0 en ∞.

$$\mathcal{E} = 0 = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{eff}$$

Caida es radial, entonces $\frac{d\phi}{d\tau} = 0 \Rightarrow \ell = 0$

$$\Rightarrow V_{eff}(r) = -\frac{M}{r}$$

$$\Rightarrow \left(\frac{dr}{d\tau}\right)^2 = \frac{2M}{r} \qquad \frac{dr}{d\tau} = -\sqrt{\frac{2M}{r}}$$

$$\frac{d\tau}{dr} = -\sqrt{\frac{r}{2M}}$$

$$\Delta\tau = \int_{6M}^{2M} \frac{d\tau}{dr} dr = \int_{6M}^{2M} -\sqrt{\frac{r}{r_s}}\, dr \quad \leftarrow \text{definir } \rho = \frac{r}{r_s}$$

$$= \int_{2M}^{6M} \sqrt{\frac{r}{r_s}}\, dr = r_s \int_1^3 \sqrt{\rho}\, d\rho = r_s \left[\frac{2}{3} \rho^{3/2}\right]_1^3$$

$$= 2M \frac{2}{3}\left(3\sqrt{3} - 1\right)$$

(8) $\ell = r^2 \frac{d\phi}{d\tau} \Rightarrow \frac{d\tau}{d\phi} = \frac{r^2}{\ell}$ $\quad \Delta\tau_{orbita} = 2\pi \frac{r^2}{\ell} = \frac{2\pi (7M)^2}{\ell}$

$$r_{est} = \frac{l^2}{2M}\left(1 + \sqrt{1 - 12\left(\frac{M}{l}\right)^2}\right)$$

$$r_{inst} = \frac{l^2}{2M}\left(1 - \sqrt{1 - 12\left(\frac{M}{l}\right)^2}\right)$$

$$\approx \frac{l^2}{2M}\left(\cancel{1} - \left(\cancel{1} - 12\left(\frac{M}{l}\right)^2\right)\right) = \frac{12}{2}\frac{l^2}{M}\left(\frac{M}{l}\right)^2 = 6M$$

$\Rightarrow$ si $r = 7M$ debe ser orbita estable