El mapa unidimensional de Robert May

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Taller MMC-Bio 2021

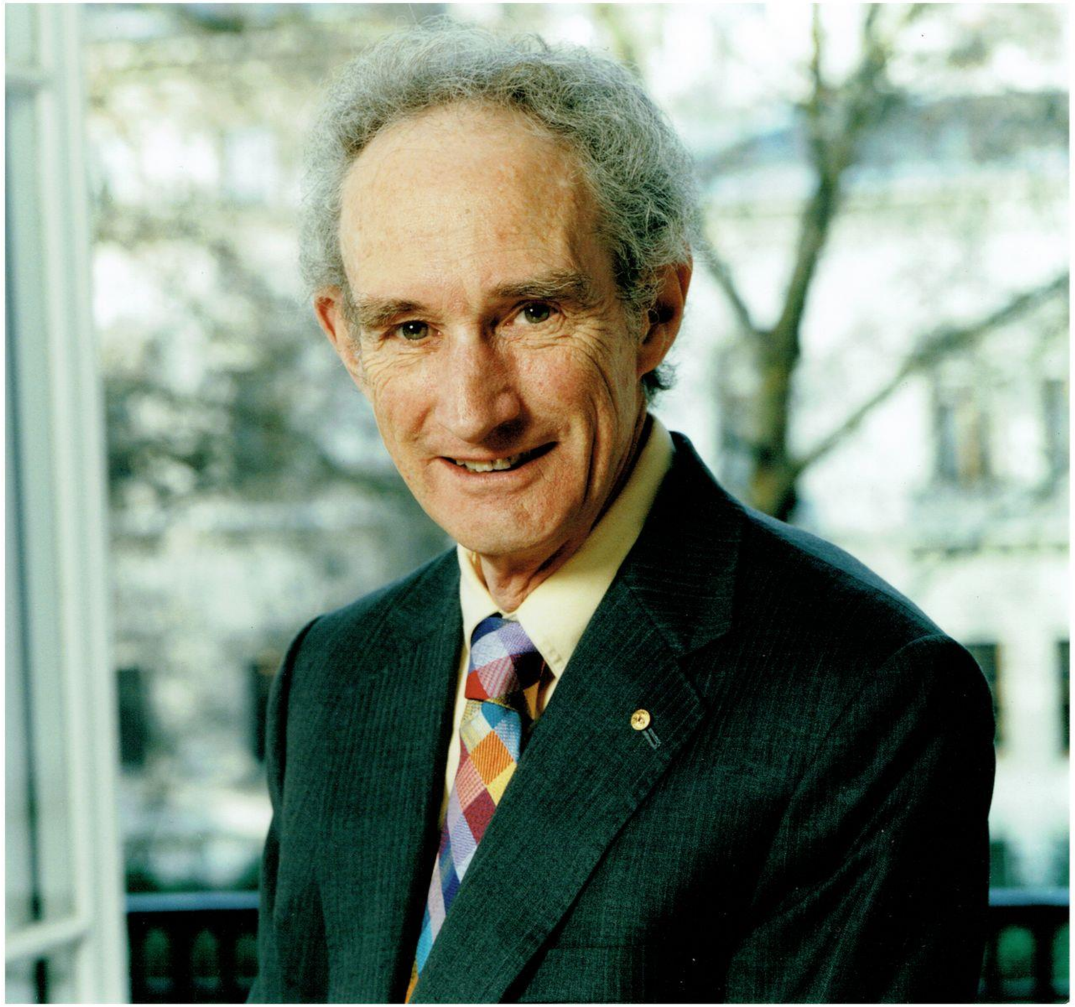
Robert May (1936-2020)

Robert May, Baron May of Oxford

From Wikipedia, the free encyclopedia

Robert McCredie May, Baron May of Oxford, OM, AC, FRS, FAA, FTSE, FRSN, HonFAIB (8 January 1936 – 28 April 2020) was an Australian scientist who was Chief Scientific Adviser to the UK Government, President of the Royal Society,[8] and a professor at the University of Sydney and Princeton University. He held joint professorships at the University of Oxford and Imperial College London. He was also a crossbench member of the House of Lords from 2001 until his retirement in 2017.

May was a Fellow of Merton College, Oxford, and an appointed member of the council of the British Science Association. He was also a member of the advisory council for the Campaign for Science and Engineering.[9]



review article

Simple mathematical models with very complicated dynamics

Robert M. May*

First-order difference equations arise in many contexts in the biological, economic and social sciences. Such equations, even though simple and deterministic, can exhibit a surprising array of dynamical behaviour, from stable points, to a bifurcating hierarchy of stable cycles, to apparently random fluctuations. There are consequently many fascinating problems, some concerned with delicate mathematical aspects of the fine structure of the trajectories, and some concerned with the practical implications and applications. This is an interpretive review of them.

THERE are many situations, in many disciplines, which can be described, at least to a crude first approximation, by a simple first-order difference equation. Studies of the dynamical properties of such models usually consist of finding constant equilibrium solutions, and then conducting a linearised analysis

Fourth, there is a very brief review of the literature pertaining to the way this spectrum of behaviour—stable points, stable cycles, chaos—can arise in second or higher order difference equations (that is, two or more dimensions; two or more interacting species), where the onset of chaos usually requires

Ecuaciones a diferencias finitas

- Tiempos discretos

$$N_{t+1} = f(N_t).$$

- Tasa de reproducción R

$$N_{t+1} = RN_t.$$

- Iterando:

$$N_1 = RN_0,$$
$$N_2 = RN_1 = R^2N_0,$$
$$N_3 = RN_2 = R^2N_1 = R^3N_0,$$
$$\vdots$$
$$N_t = R^tN_0.$$

Comportamientos

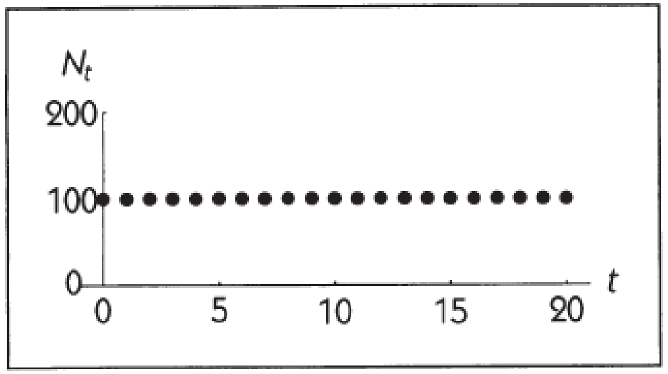


Figure 1.4
The solution to $N_{t+1} = 1.00N_t$.

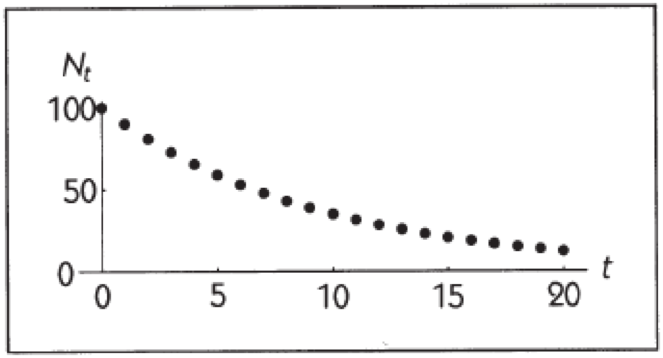


Figure 1.2
The solution to $N_{t+1} = 0.90N_t$.

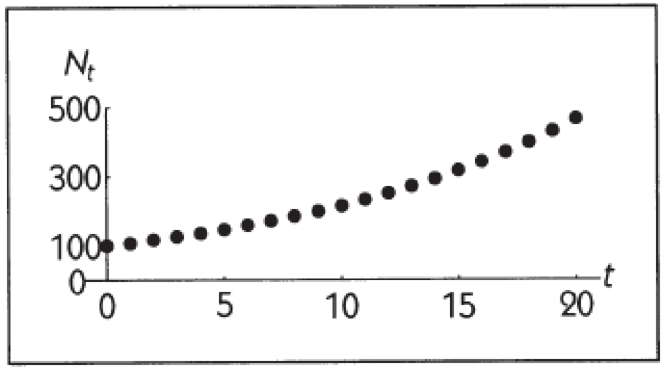


Figure 1.3
The solution to $N_{t+1} = 1.08N_t$.

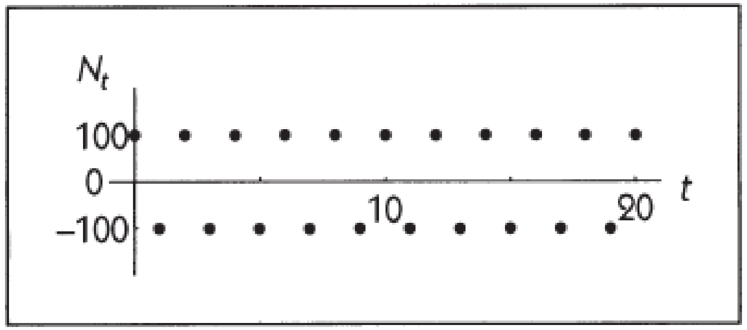


Figure 1.7
The solution to $N_{t+1} = -1.00N_t$.

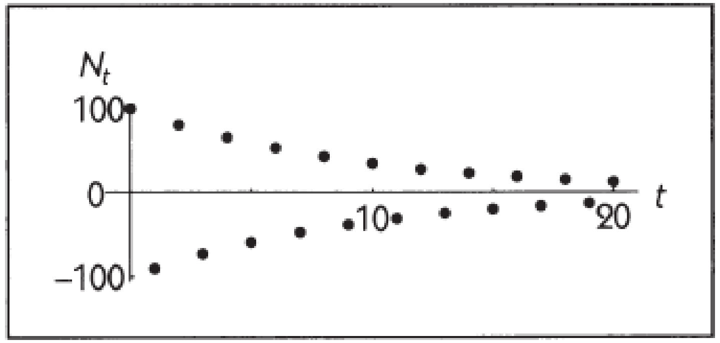


Figure 1.5
The solution to $N_{t+1} = -0.90N_t$.

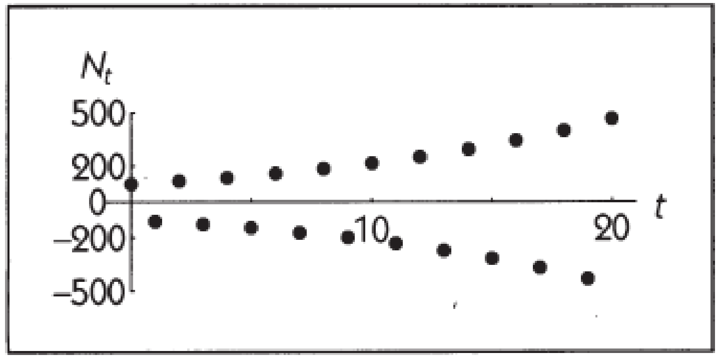
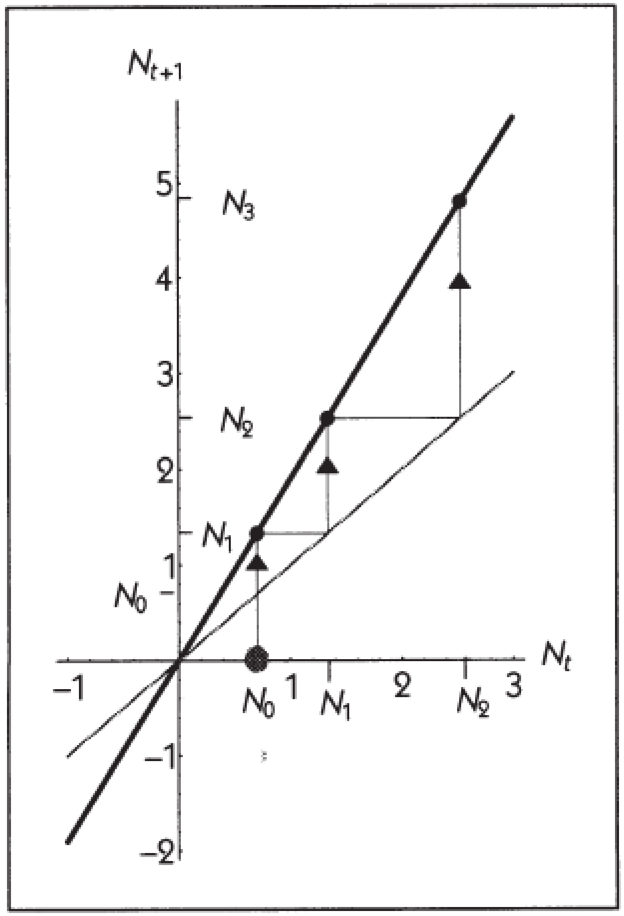


Figure 1.6
The solution to $N_{t+1} = -1.08N_t$.

Iteración gráfica: método Cobweb



Logística discreta (o mapa cuadrático)

$$x_{t+1} = Rx_t(1 - x_t).$$

- Variable x
- Carácter ‘logístico’
- Valores de R

La parábola

$$y = R\,x\,(1 - x)$$

- Raíces
- Derivadas
- ¿Tiene extremo? ¿De qué tipo?

Puntos fijos

- Cuando x vale x*, x(t+1) también vale x*

$$x^* = R\, x^* \,(1 - x^*)$$

$x_1^*=$

$x_2^*=$

Iteración en la logística discreta

- ver script orbitas_mod.m

- ejemplo R=1.5
- ¿A qué valor tiende?

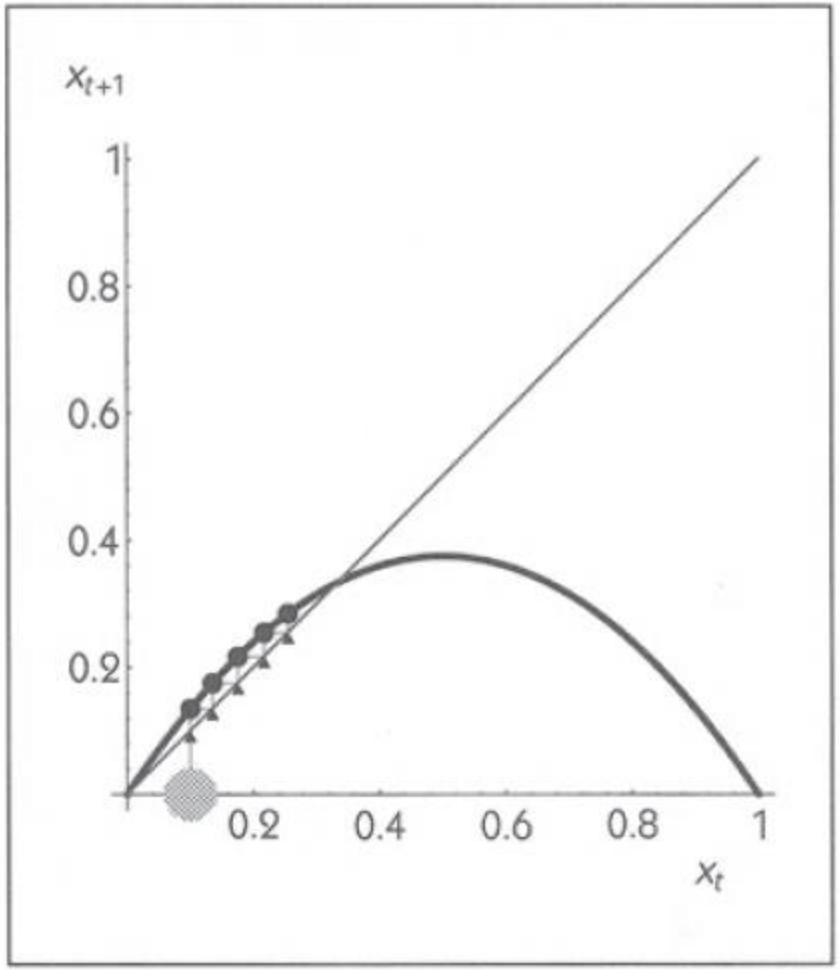


Figure 1.9
Cobweb iteration of $x_{t+1} = 1.5(1 - x_t)x_t$.

Exploración de la dinámica con orbitas_mod.m

- Cubrir valores progresivamente desde <1 hasta 4
- ¿Qué comportamientos encuentra?

Exploración detallada con cobweb_logi.m

- Ver valores estacionarios en el vector orbit
- Anotar los valores de R en que aparecen duplicaciones de período

Comportamiento aperiódico (¿caos?)

- ¿Qué pasa más allá de R=…?
- Ver programa bifurcaciones.m
- Universalidad (carácter fractálico)

Ruta al caos por cascada de duplicaciones de período. Constante de Feigenbaum

- For $3.0000 < R < 3.4495$, there is a stable cycle of period 2.
- For $3.4495 < R < 3.5441$, there is a stable cycle of period 4.
- For $3.5441 < R < 3.5644$, there is a stable cycle of period 8.
- For $3.5644 < R < 3.5688$, there is a stable cycle of period 16.
- As R is increased closer to 3.570, there are stable cycles of period 2^n, where the period of the cycles increases as 3.570 is approached.
- For values of $R > 3.570$, there are narrow ranges of periodic solutions as well as aperiodic behavior.

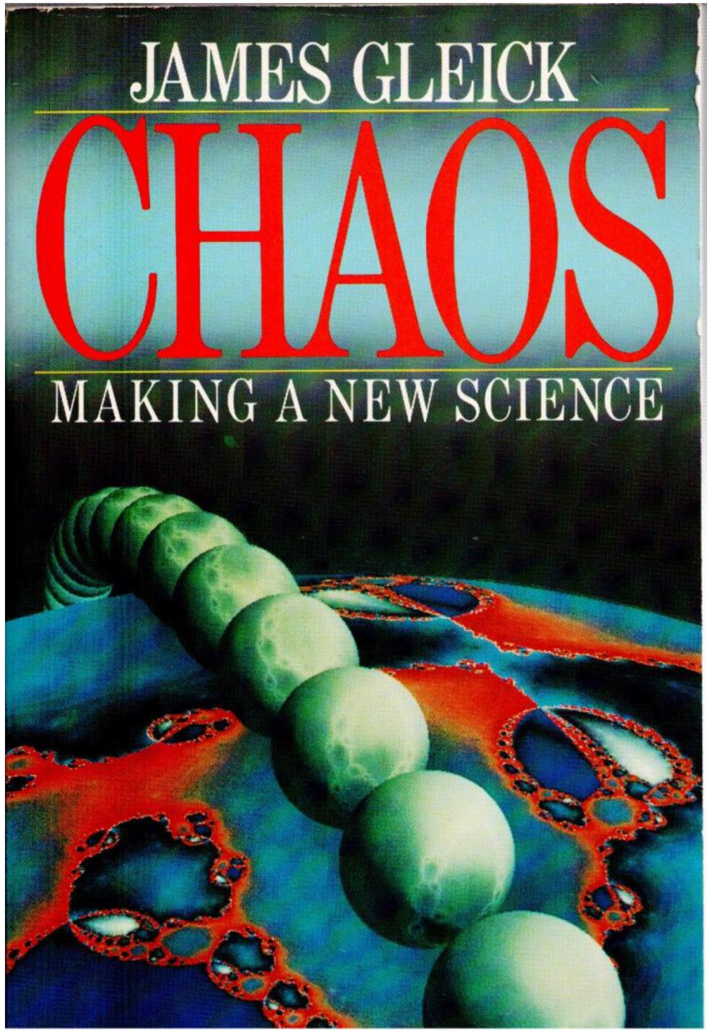
- Número de Feigenbaum

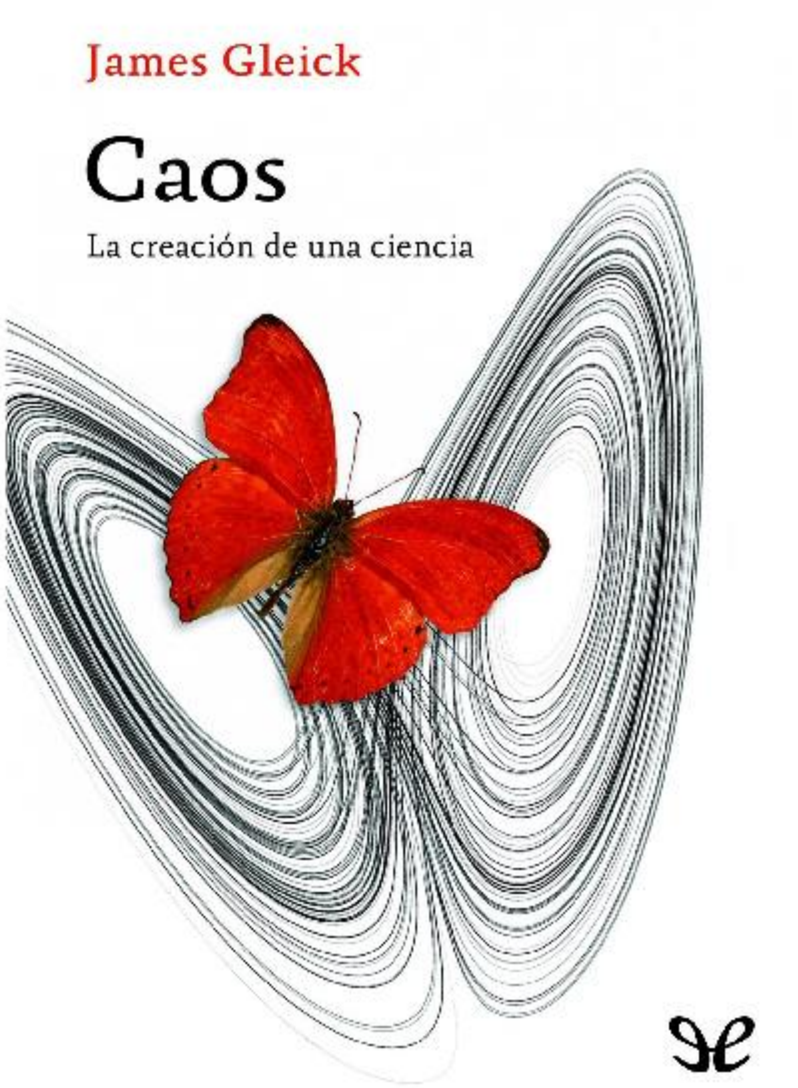
$$\lim_{n\to\infty} \frac{\Delta_n}{\Delta_{2n}} = 4.6692\ldots.$$

Sensibilidad extrema a las condiciones iniciales

- El ‘efecto mariposa’
- Ver programa orbitas2.m

Para continuar leyendo…





El atractor de Edward Lorenz (c. 1963)

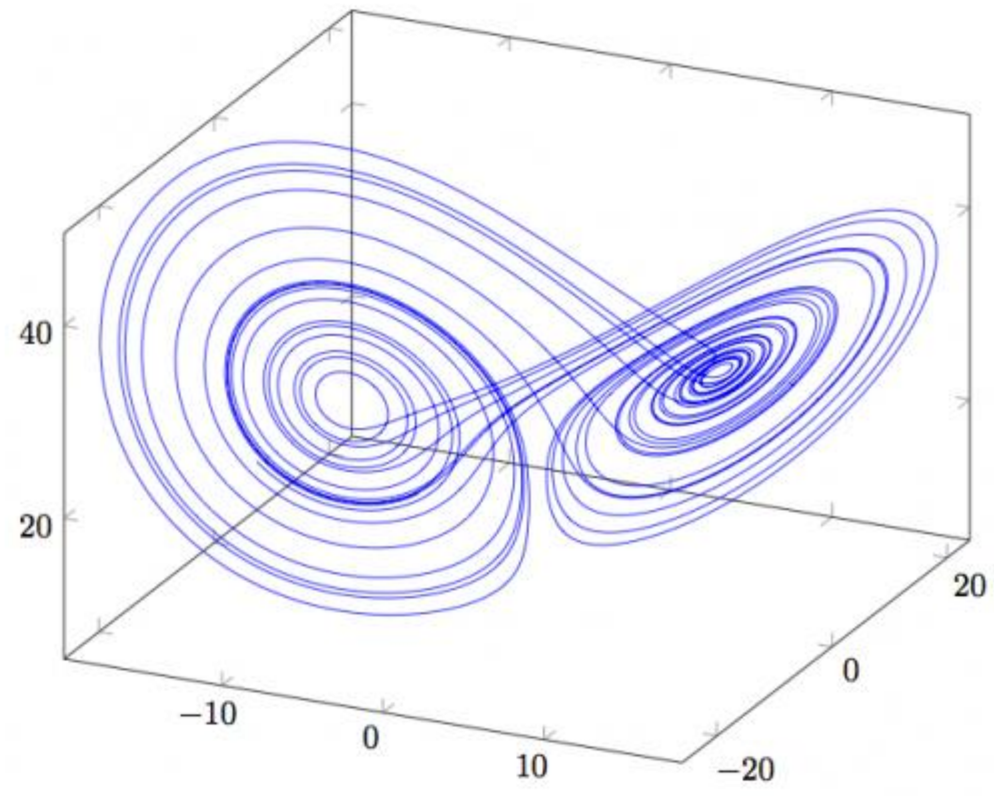
- La impredictibilidad de los fenómenos atmosféricos

$$\frac{dx}{dt} = \sigma (y - x)$$

$$\frac{dy}{dt} = \rho x - y - xz$$

$$\frac{dz}{dt} = xy - \beta z$$

Atractores extraños



- “Lo impredecible y lo predeterminado evolucionan juntos, haciendo que cada cosa sea como es. Éste es el modo en que la naturaleza se crea a sí misma, a todas las escalas, desde el copo a la tormenta de nieve”

Tom Stoppard (Arcadia, Acto I, Escena 4)