$d\tau^2 = -g_{\mu\nu}\, dx^\mu dx^\nu$

$\Longleftrightarrow \left(\frac{d\tau}{d\lambda}\right)^2 = -g_{\mu\nu} \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$. Si $\lambda = \tau$ $\quad 1 = -g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -g(u,u)$

$\Rightarrow \quad g(u,u) = -1$

Hartle ejemplo 9.1

Base ON de referencia Lorentziano local de reposo de observador

$= u_{obs},\, e_1,\, e_2,\, e_3$ $\qquad u_{obs}$ = 4-velocidad observador

4-momento $\quad p = E\, u_{obs} + p^i e_i$

con E la energía y $\bar{p}$ el 3-momento en este referencia

$\Rightarrow \quad p \cdot u_{obs} = E \underbrace{u_{obs}\cdot u_{obs}}_{=-1} + p^i \overbrace{e_i \cdot u_{obs}}^{=0 \text{ pq base ON}}$

$= -E$

$\Rightarrow \quad E = -p \cdot u_{obs}$

Para movimiento radial:

$0 = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 - \frac{M}{r}$ $\quad \ell = 0$ $\quad \Rightarrow \quad \frac{dr}{d\tau} = \pm\sqrt{\frac{2M}{r}}$ $\qquad \frac{d\theta}{d\tau} = \frac{d\phi}{d\tau} = 0$

$1 = e = \left(1 - \frac{2M}{r}\right)\frac{dt}{d\tau} \quad \Rightarrow \quad \frac{dt}{d\tau} = \left(1 - \frac{2M}{r}\right)^{-1}$

Relación u_{obs} con ∂_t:

$u_{obs} = u^t_{obs}\,\partial_t$ porque observador está en reposo en coordenadas Schwarzschild

$$-1 = u_{obs}\cdot u_{obs} = {u^t_{obs}}^2\,\partial_t\cdot\partial_t = {u^t_{obs}}^2\, g(\partial_t,\partial_t)$$

$$= {u^t_{obs}}^2\, g_{tt} = -\left(1-\frac{2M}{r}\right){u^t_{obs}}^2$$

$$\Rightarrow u^t_{obs} = \left(1-\frac{2M}{r}\right)^{-1/2}$$

$$E = -p\cdot u_{obs} = -g_{tt}\, m u^t\, u^t_{obs} = \left(1-\frac{2M}{r}\right)^{1/2} m u^t$$

$$= m\left(1-\frac{2M}{r}\right)^{-1/2}$$

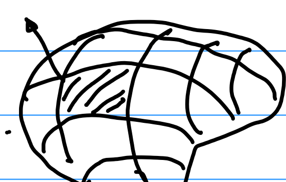
$$E = m\gamma = \frac{m}{\sqrt{1-v^2}} \Rightarrow v^2 = \frac{2M}{r} \qquad v = \sqrt{\frac{2M}{r}}$$

Ejercicio 3

$\int_{\partial\Sigma} T^{ij} n_j \, d\text{area}$ | T^{ij} = densidad de corriente de p^i

= la tasa con que p^i escapa a Σ

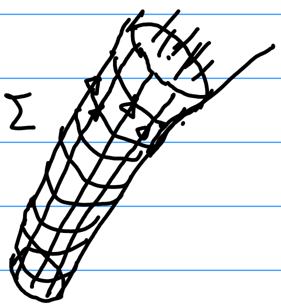
$\vec{n}$



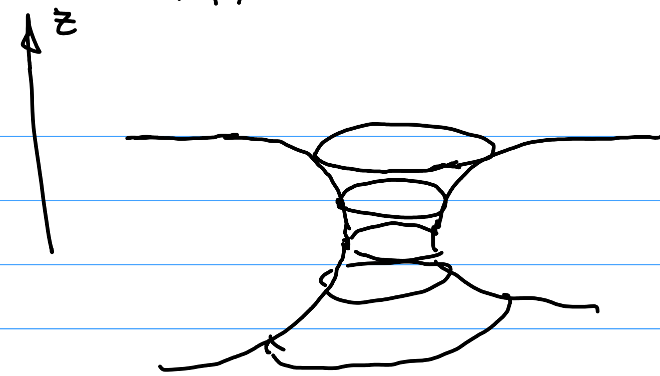
= F^i ← fuerza sobre exterior

b)

Σ



R, ϕ, z



$$dr^2 + (b^2 + r^2)\, d\phi^2$$

$$= dz^2 + dR^2 + R^2 d\phi^2$$

$$\left(d\phi = d\phi \qquad dz = \frac{\partial z}{\partial r} dr \quad dR = \frac{\partial R}{\partial r} dr \right)$$

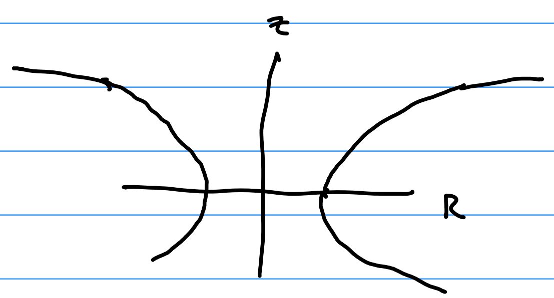
$$= \left(\frac{\partial z}{\partial r}\right)^2 dr^2 + \left(\frac{\partial R}{\partial r}\right)^2 dr^2 + R^2 d\phi^2 \qquad R = \sqrt{b^2 + r^2}$$

$$\frac{\partial R}{\partial r} = \frac{1}{2} \frac{1}{\sqrt{b^2 + r^2}} 2r = \frac{r}{\sqrt{b^2 + r^2}} \qquad \left(\frac{\partial R}{\partial r}\right)^2 = \frac{r^2}{r^2 + b^2}$$

$$\left(\frac{\partial z}{\partial r}\right)^2 + \left(\frac{\partial R}{\partial r}\right)^2 = 1 \Rightarrow \left(\frac{\partial z}{\partial r}\right)^2 = 1 - \frac{r^2}{r^2 + b^2} = \frac{b^2}{r^2 + b^2}$$

$$\frac{\partial z}{\partial r} = \frac{b}{\sqrt{r^2 + b^2}} \qquad z(r) = b \int_0^{\frac{r}{b}} \frac{1}{\sqrt{1 + \left(\frac{r}{b}\right)^2}}\, d\frac{r}{b} = b \int_0^{\operatorname{senh}^{-1}\frac{r}{b}} d\psi$$

$$= b \operatorname{senh}^{-1} \frac{r}{b}$$



$$R = \sqrt{b^2 + r^2} = b\sqrt{1 + \left(\frac{r}{b}\right)^2}$$

$$= b \cosh \frac{z}{b}$$