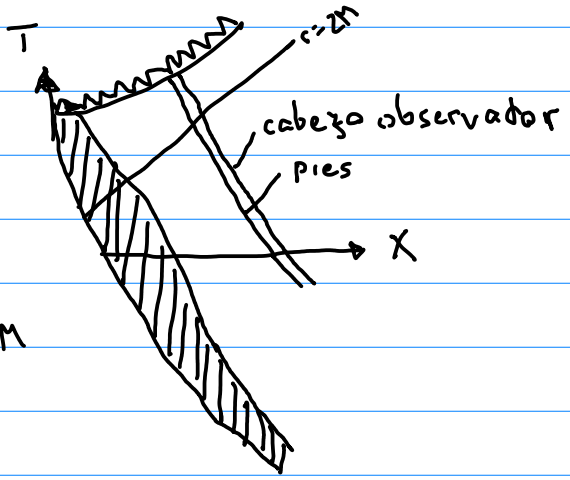
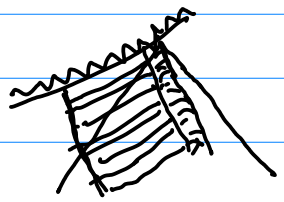
(13) a)



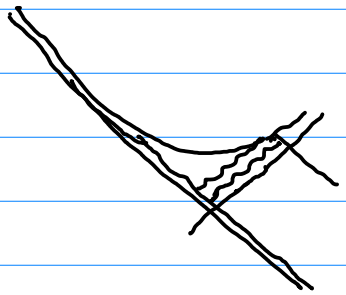
Siempre ve pies.

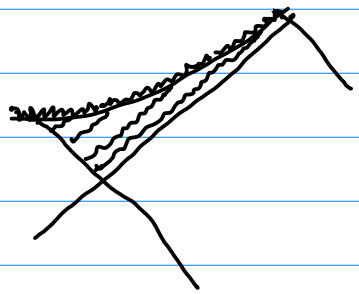
Cuando cabeza esta en $r = 2M$ ve pies en $r = 2M$



No ve los llegar a la singularidad

b)

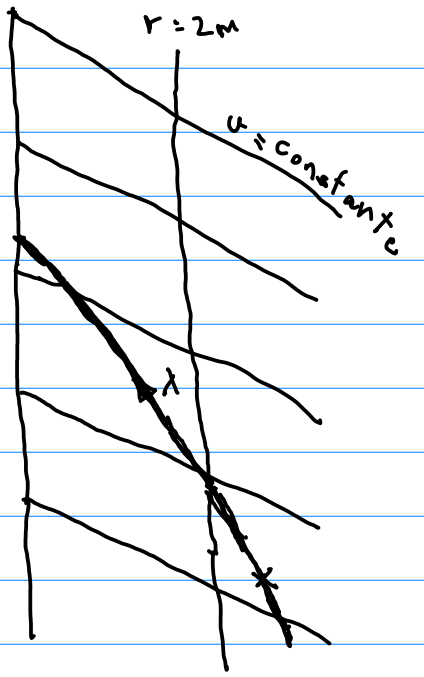




Dentro de agujero negro no es negro pero si el agujero fue formado mucho antes que el observador cae adentro sería poca la luz de la estrella que le llega.

18 a) Exterior

$$ds^2 = -\left(1-\frac{2M}{r}\right)dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$



Interior

$$ds^2 = -dv'^2 + 2\,dv'\,dr + r^2 d\Omega^2$$

$$-\frac{dv'^2}{d\lambda^2} + 2\frac{dv'}{d\lambda}\frac{dr}{d\lambda}$$

$$= -\left(1-\frac{2M}{r}\right)\left(\frac{dv}{d\lambda}\right)^2 + 2\frac{dv}{d\lambda}\frac{dr}{d\lambda}$$

$$-\left(\frac{dv'}{dv}\right)^2 + 2\frac{dv'}{dv}\frac{dr}{dv} = -\left(1-\frac{2M}{r}\right) + 2\frac{dr}{dv} \quad \text{en cascaron}$$

Metrica interior con coordenadas v, r $\quad v(v')$ que concuerda con v en el cascaron

$$ds^2 = -\left(\frac{dv'}{dv}\right)_c^2 dv^2 + 2\left(\frac{dv'}{dv}\right)_c dv\,dr + r^2 d\Omega^2$$

$$= \left[-2\left(\frac{dv'}{dv}\right)_c\left(\frac{dr}{dv}\right)_c - \left(1-\frac{2M}{r_c}\right) + 2\left(\frac{dr}{dv}\right)_c\right]dv^2 + 2\left(\frac{dv'}{dv}\right)_c dv\,dr + r^2 d\Omega^2$$

b) Horizonte en el interior

$$0 = -dv'^2 + 2\,dv'\,dr = dv'\left(-dv' + 2dr\right)$$

$\Rightarrow dv' = 0$ ← rayos de luz entrantes

$dv' = 2dr$ ← rayos de luz salientes

$\Rightarrow dv'/dv\; dv = 2dr \longrightarrow \Rightarrow \dfrac{dr_{luz}}{dv} = \dfrac{1}{2}\left(\dfrac{dv'}{dv}\right)_c$

(26) # Diagrama de Penrose

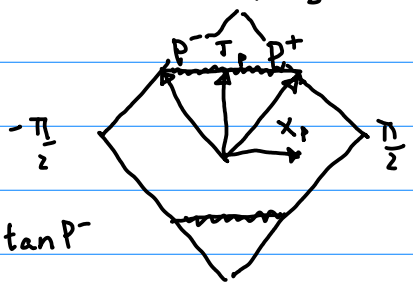
$$X^+ = e^{v/4M} \qquad T = \frac{x^+ - x^-}{2}$$

$$X^- = e^{-u/4M} \qquad X = \frac{x^+ + x^-}{2}$$

$$p^+ = \tan^{-1} x^+ = \tan^{-1} e^{v/4M} \qquad -\frac{\pi}{2} \leq p^+ \leq \frac{\pi}{2}$$

$$p^- = \tan^{-1}(-x^-) = \tan^{-1}\left(-e^{-u/4M}\right) \qquad -\frac{\pi}{2} \leq p^- \leq \frac{\pi}{2}$$

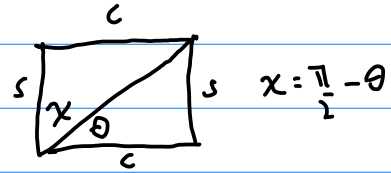
$$T_p = \frac{p^+ + p^-}{2} \qquad X_p = \frac{p^+ - p^-}{2}$$



Singularidades $1 = T^2 - X^2 = -x^+ x^- = \tan p^+ \tan p^-$

$\tan p^+ = 1 / \tan p^- \iff p^+ = \frac{\pi}{2} - p^-$

$\Rightarrow p^+ + p^- = \frac{\pi}{2}$ en singularidad $\Rightarrow T^p = \frac{\pi}{4}$



$\Theta \quad p^+ + p^- = -\frac{\pi}{2}$ $\qquad T^p = -\frac{\pi}{4}$